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Settlement calculation for long-short composite piled raft foundation

ZHAO Ming-hua(赵明华), ZHANG Ling(张 玲), YANG Ming-hui(杨明辉)
(College of Civil Engineering, Hunan University, Changsha 410082, China)

Abstract: The mechanism of long-short composite piled raft foundation was discussed. Assuming the relationship between shear stress and shear strain of the surrounding soil was elasto-plastic, shear displacement method was employed to establish the different explicit relational equations between the load and the displacement at the top of pile in either elastic or elasto-plastic period. Then Mylonakis & Gazetas model was introduced to simulate the interaction between two piles or between piles and soil. Considering the effect of cushion, the flexible coefficients of interaction were provided. With the addition of a relevant program, the settlement calculation for long-short composite piled raft foundation was developed which could be used to account for the interaction of piles, soil and cushion. Finally, the calculation method was used to analyze an engineering example. The calculated value of settlement is 10.2 mm, which is close to the observed value 8.8 mm.

Key words: long-short composite piled raft foundation; shear displacement method; settlement; Mylonakis & Gazetas model
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1 INTRODUCTION

In recent years, along with the theoretical study and practical experience[1−4], a new type of foundation named long-short composite piled raft foundation has been developed[5−6]. In the system of this composite foundation, various types of piles, such as long-rigid piles, short-rigid piles, semi-rigid short piles and flexible piles were utilized synthetically. Furthermore, this type of foundation has been successfully applied in most of economically developed cities, which are located in coastal areas, especially in Shanghai Economic Circle of China.

However, the research on working mechanism of long-short composite piled raft foundation is still at an initial stage. Especially, there is no explicit settlement calculation in engineering practices, although many theories concerning the analysis of piled raft foundation have been proposed by numbers of researchers[6−7]. Usually, the composite modulus method is resorted to calculate the settlement of long-short composite piled raft foundations. In contrast to settlement calculation for conventional composite foundations, different modulus should be used to calculate the settlement of long-short composite piled raft foundation. In the meantime, surrounding soil is divided into two reinforced sections while treating the plane of short piles end as an interface. The shortcomings of the modulus method are listed as follows: the stress ratio should be given before the calculation; if there are no field data, the ratio has to be determined by experiment. Furthermore, this method ignores the pile-soil-cushion interaction. From the above mentioned, to develop a new settlement calculation is necessary.

In this paper, the working mechanism of composite piled raft foundation was discussed. Then based on the shear displacement method, the Mylonakis & Gazetas model for interaction between two piles or between piles and soil was introduced, and a settlement calculation for long-short composite piled raft foundation was proposed, which could be used to account for the interaction among piles, soil and cushion integrally.

2 WORKING MECHANISM

Compared with other composite piled raft foundations, long-short composite piled raft foundation has its own working mechanism[6], owing to the different pile lengths. In consideration of the arrangement of long pile with the interval of short pile, three different working sections are developed, as shown in Fig.1.

1) Section Ⅰ, where long piles and short piles are working together, is mainly used to enhance bearing capacity of the foundation.

2) Section Ⅱ, where long piles are working solely,

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Corresponding author: ZHAO Ming-hua, Professor; Tel: +86-731-8821590; E-mail: mhzhaohd@21cn.com

is mainly used to reduce settlement.

3) Section Ⅲ, where there are soil layers without piles, is mainly used to bear pile body load.

It should be noted that while these three sections work simultaneously, bearing capacity of the foundation will be enhanced, and displacement will be reduced as well. Moreover, long piles and short piles play different roles in the foundation.

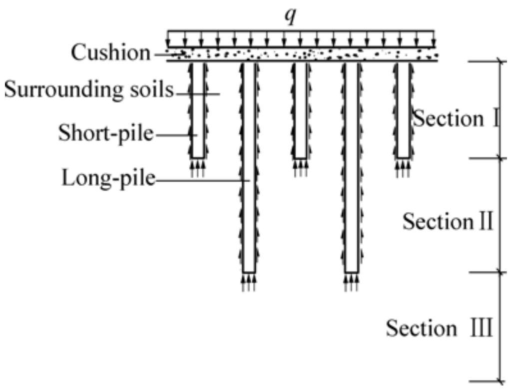

Fig.1 Working mechanism of long-short-pile composite foundations

2.1 Mechanisms of long-piles

In long-short composite piled raft foundations, long piles are mainly used to transfer the load from the piles to the deep ground, to reduce the deformation of the compressive soil layers, to protect the short-flexible piles, and to prevent the soil from protuberating while working together with short piles. In section Ⅰ, "holding effect" and "shielding effect" appeared obviously among the piles. What's more, soil and piles deformed simultaneously in this section. While in section Ⅱ, different displacement occurred in the pile-soil interface near the long pile tip, then long piles stabbed into the subjacent bed.

2.2 Mechanisms of short-piles

According to the different physics mechanical properties of soil, short piles have two functions.

1) When the soft soil layer under the foundation is thick, short piles are used to enhance the bearing capacity of soft soil layer.

2) If there are two ideal bearing stratums under the basis, long piles and short piles can stand on these two stratums respectively, so that bearing capacity of the stratums can be brought into full play. Moreover, in the later case, short piles are mainly used to reinforce bearing capacity of the foundation, while long piles are mainly used to reduce settlement. Meanwhile, the working load of the foundation treatments can be reduced on the premise that the design requirement is satisfied.

Generally, a cushion is laid to coordinate the displacement of the pile and the soil, so the pile and the soil can stand the structural load together.

3 BASIC ASSUMPTIONS AND DIFFERENTIAL EQUATIONS

The behavior of long-short composite piled raft foundation is complex, so in order to simplify the calculation, four presumptions are made as follows.

1) The foundation structure is absolutely rigid. In other words, every point under the raft has the same vertical displacement when the calculated domain is scattered into limited grids.

2) Every long-pile has the same material characteristic and the same geometry size, and so does the short-pile.

3) An elasto-plastic model is suggested for the surrounding soil（Fig. 2）, the relationship between shear stress and shear strain of the soil is

$$\tau = G_s\gamma,\ \gamma \leqslant \gamma_u$$
$$\tau = \tau_u,\ \gamma > \gamma_u$$

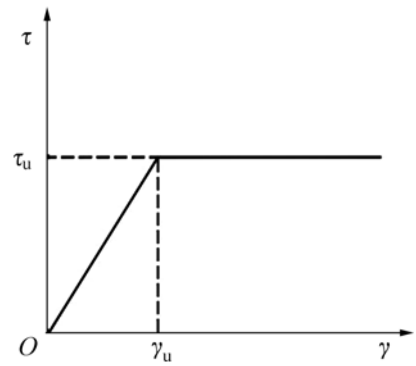

Fig.2 Shear stress-strain relationship of surrounding soil

4) Piles, soil and cushion only have the vertical movement respectively, and only their interaction under the vertical load is regarded.

When the foundation is subjected to the vertical loads, the settlement and the load of piles decrease as the depth increases, and the side friction force escalates from top to bottom. Taking a small segment of the pile in direction z for analysis, the following equilibrium equation can be obtained:

$$\frac{\mathrm{d}P(z)}{\mathrm{d}z} = -U\tau_0(z) \qquad (1)$$

By analyzing compression of the pile segment, it can be gotten that

$$P(z) = -A_p E_p \frac{\partial w_p}{\partial z} \qquad (2)$$

Finally, governing differential equation of a single pile is expressed as follows:

$$\frac{d^2 w_p}{dz^2} = \frac{U\tau_0}{A_p E_p} \tag{3}$$

where w_p is vertical displacement of the pile; U, A_p and E_p are perimeter, cross-sectional area and elastic modulus of the pile; τ_0 is shear stress of the soil near pile shaft.

4 STRESS ANALYSIS OF PILES

4.1 Elastic analysis of piles

Based on the shear displacement method and with $\frac{\partial \sigma_z}{\partial z}$ skipped, the transfer equation of the shear stress of surrounding soil in radial direction is [8]

$$\tau = \frac{\tau_0 r_0}{r} \tag{4}$$

Adding the elastic relationship between shear stress and the shear stress $\gamma = \tau/G_s$ into Eqn.(4), the vertical displacement of surrounding soil is obtained as follows:

$$w_s(z,r) = \begin{cases} \frac{\tau_0 r_0}{G_s} \ln \frac{r_m}{r}, r_0 \le r \le r_m \\ 0, \quad r > r_m \end{cases} \tag{5}$$

where G_s is shear modulus of the soil, $G_s = E_s/(1+\upsilon_s)$; E_s and υ_s are elastic modulus and Poisson's ratio of the soil; r_0 is radius of the pile; r is distance between axes of two nearby piles; r_m is the maximum radius within which shear strain in soil is negligible, and it can be taken as $6d$[9] (d is diameter of the pile).

When w_p equals to w_s, the settlement equilibrium equation of a single pile is

$$\frac{\partial^2 w_p}{\partial z^2} - \frac{k_1}{A_p E_p} w_p = 0 \tag{6}$$

where $k_1 = 2\pi G_s / \ln(r_m / r_0)$.

The equation calculating displacement of the pile tip is obtained by using the Randolph formula[10] :

$$W_b = \eta [P_b(1-\upsilon_b)]/(4 r_0 G_b) = P_b/k_2 \tag{7}$$

where $k_2 = 4 r_0 G_b / [\eta (1-\upsilon_b)]$; P_b is pile tip resistance value; υ_b and G_b are Poisson's ratio and shear modulus of the soil under the pile. Coefficient η ranges from 0.85 to 1.00.

By using Eqns.(6) and (7), the following equations can be obtained:

$$w_p(z) = P_b \left\{ \frac{1}{A_p E_p \mu} \mathrm{sh}[\mu(l-z)] + \frac{1}{k_2} \mathrm{ch}[\mu(l-z)] \right\}$$

$$P(z) = P_b \left\{ \frac{A_p E_p \mu}{k_2} \mathrm{sh}[\mu(l-z)] + \mathrm{ch}[\mu(l-z)] \right\} \tag{8}$$

where $\mu = \sqrt{k_1/(A_p E_p)}$, and l is length of the pile. Then, the load-settlement relation on the pile top is expressed as follows:

$$w_p(0) = P(0) \left[\frac{1}{A_p E_p \mu} \mathrm{sh}(\mu l) + \frac{1}{k_2} \mathrm{ch}(\mu l) \right] / \left[\frac{A_p E_p \mu}{k_2} \mathrm{sh}(\mu l) + \mathrm{ch}(\mu l) \right] \tag{9}$$

4.2 Elasto-plastic analysis of piles

The soil around piles enters into plastic state or slipping state gradually from top to bottom as the load increases. It's assumed that $z = l_{cr}$, shear stress τ of the soil achieves its ultimate value τ_u, then the soil slips. When $z > l_{cr}$, the soil near pile shafts is in elastic state, Eqn.(9) can be used to calculate displacement of piles. When $z \leqslant l_{cr}$, the soil near pile shafts is in elasto-plastic state. Substituting τ_u and $l - l_{cr}$ for τ_{0a} and l separately into Eqns.(1), (2), (5) and (9), the following equations can be obtained:

$$w_p(0) = \frac{\tau_\mu r_0}{G_s} \ln(\frac{r_m}{r_0}) + \frac{1}{A_p E_p} P(0) l_{cr} - \frac{A_p E_p}{2U\tau_u} l_{cr}^2 \tag{10}$$

$$P(0) = U\tau_u l_{cr} + \frac{\tau_u r_0}{G_s} \ln(\frac{r_m}{r_0}) \cdot \left\{ \frac{A_p E_p \mu}{k_2} \mathrm{sh}[\mu(l-l_{cr})] + \mathrm{ch}[\mu(l-l_{cr})] \right\} / \left\{ \frac{1}{A_p E_p \mu} \mathrm{sh}[\mu(l-l_{cr})] + \frac{1}{k_2} \mathrm{ch}[\mu(l-l_{cr})] \right\} \tag{11}$$

From the above equations, it can be seen that the relationship between $P(0)$ and $w(0)$ is nonlinear.

4.3 Pile-pile interaction

From Ref.[11], it can be found that when the pile is subject to the vertical load, the soil nearby the pile is in high-strain area and has plastic displacement. It is assumed that this plastic zone is only in a certain limited range, and won't affect the neighboring piles and most of the surrounding soil. So the interaction of pile-pile, pile-soil can be calculated by the elastic method. Take a note that soil here isn't in high-strain area.

According to the Mylonakis & Gazetas model (Fig.3) and the same principles in single pile (Eqn.(6)), the settlement equilibrium equation of unloaded receiver pile i, with the distance of s from loaded source pile j, is obtained as follows:

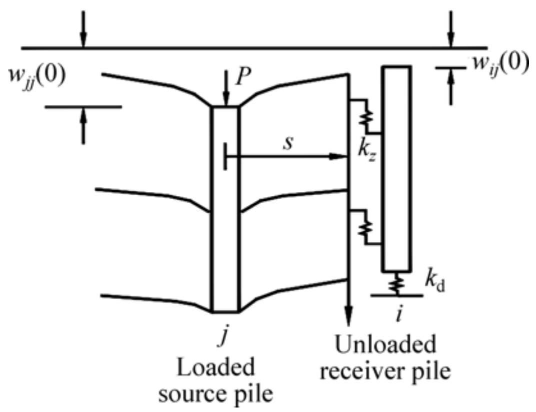

Fig.3 Interaction model between two piles

$$A_{Pi}E_{pi}\frac{\partial^2 w_{ij}(z)}{\partial z^2}-k_{1i}[w_{ij}(z)-w_s(z)]=0 \quad (12)$$

where $w_s(z)=\psi(s)w_{jj}(z)$[12], in which $\psi(s)$ is the decreasing function in displacement field, and the expressions for $\psi(s)$ and $w_{jj}(z)$ are as follows:

$$\psi(s)=\begin{cases}\dfrac{\ln r_m-\ln s}{\ln r_m-\ln r_j}, r_j<s<r_m\\ 0, \qquad s\geq r_m\end{cases} \quad (13)$$

$$w_{jj}(z)=c_1e^{\mu_j z}+c_2e^{-\mu_j z} \quad (14)$$

Adding $\mu_i=\sqrt{k_{1i}/(A_{pi}E_{pi})}$ into Eqn.(12), the following equation can be obtained:

$$\frac{\partial^2 w_{ij}}{\partial z^2}-\mu_i^2 w_{ij}+\mu_i^2\psi(s)(c_1e^{\mu_j z}+c_2e^{-\mu_j z})=0 \quad (15)$$

When $\mu_i=\mu_j=\mu$, according to the boundary conditions $\begin{cases}P_i(0)=0\\ P_{bi}=k_{2i}W_{bi}\end{cases}$, Eqn.(15) can be solved for $w_{ij}(0)$:

$$w_{ij}(0)=\frac{(h-g)I_1+2I_2}{g+h} \quad (16)$$

where $g=(k_{2i}+A_{pi}E_{pi}\mu)e^{\mu l_i}$, $h=(k_{2i}-A_{pi}E_{pi}\mu)e^{\mu l_i}$,

$$I_1=-\frac{\psi(s)P_{jj}(0)}{A_{pi}E_{pi}\mu},$$

$$I_2=\frac{\psi(s)}{2}\left[-P_{jj}(l_i)+k_{1i}l_iw_{jj}(l_i)\right]-\frac{k_{2i}l_i}{2A_{pi}E_{pi}}P_{jj}(l_i).$$

The following equation can be obtained in a similar way in the case of $\mu_i\neq\mu_j$:

$$w_{ij}(0)=\frac{(h'-g')I_1'+2I_2'}{g'+h'}+\beta w_{jj}(0) \quad (17)$$

where $g'=(k_{2i}+A_{pi}E_{pi}\mu_i)e^{\mu_i l_i}$,

$h'=(k_{2i}-A_{pi}E_{pi}\mu_i)e^{-\mu_i l_i}$, $I_1'=\beta\dfrac{P_{jj}(0)}{A_{pi}E_{pi}\mu_i}$,

$I_2'=\beta P_{jj}(l_i)-\beta k_{2i}w_{jj}(l_i)$, $\beta=\dfrac{\mu_i^2\psi(s)}{\mu_i^2-\mu_j^2}$.

5 SETTLEMENT CALCULATION

5.1 Basic equation for settlement calculation

The basic equation for calculating pile-soil-cushion interaction is

$$[\boldsymbol{K}]\{\boldsymbol{w}\}=\{\boldsymbol{P}\} \quad (18)$$

When the calculation domain is discretized into limited grids (Fig.4), $[\boldsymbol{K}]$, $\{\boldsymbol{w}\}$, $\{\boldsymbol{P}\}$ are correlated rigid matrix, displacements vector and loads vector respectively.

$$[K]=\begin{bmatrix}k_{11} & \cdots & k_{1i} & \cdots & k_{1N}\\ \vdots & \cdots & \vdots & \vdots & \vdots\\ k_{i1} & \cdots & k_{il} & \cdots & k_{iN}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ k_{N1} & \cdots & k_{Ni} & \cdots & k_{NN}\end{bmatrix}=\begin{bmatrix}K_{pp} & K_{pq} & K_{ps}\\ K_{qp} & K_{qq} & K_{qs}\\ K_{sp} & K_{sq} & K_{ss}\end{bmatrix} \quad (19)$$

where subscripting letters p, q, s stand for shortpile, long-pile, and soil, respectively.

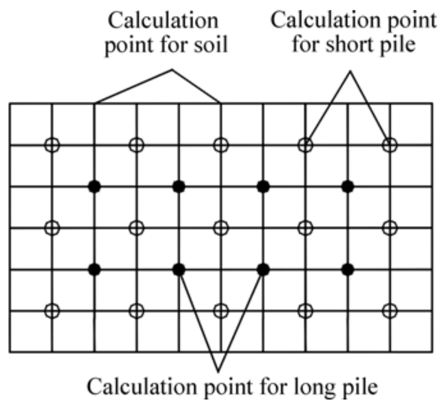

Fig.4 Calculation grids of foundation

When $w_1=w_2=\cdots=w_N=w$, Eqn.(18) can also be written as follows:

$$w\sum_{i=1}^{N}\sum_{j=1}^{N}k_{ij}=\sum_{i=1}^{N}P_i \quad (20)$$

Namely,

$$w=\sum_{i=1}^{N}P_i\Big/\left(\sum_{i=1}^{N}\sum_{j=1}^{N}k_{ij}\right)=P\Big/\left(\sum_{i=1}^{N}\sum_{j=1}^{N}k_{ij}\right) \tag{21}$$

The above equation is the final equation for settlement calculation, in which P is the total load of the foundation.

5.2 Calculation for flexible matrix

When the load P is given, calculation of the rigid matrix **[*K*]** is core of the whole problem. Generally, it can be obtained from its corresponding flexible matrix **[*δ*]**. When $\delta_{ij}=\delta_{ji}$, only 6 independent flexible factors should be gotten. The flexible factors $\delta_{\mathrm{pp},jj}$ and $\delta_{\mathrm{qq},jj}$ can be obtained by using Eqns.(9) and (10), while interaction flexible factors $\delta_{\mathrm{pp},ij}(i\neq j)$,$\delta_{\mathrm{qq},ij}(i\neq j)$ and $\delta_{\mathrm{pq},ij}$ can be obtained by using Eqns.(16) and (17). The flexible factors $\delta_{\mathrm{sp},ij}$ and $\delta_{\mathrm{sq},ij}$ can be obtained by using the following equations $\delta_{\mathrm{sp},ij}=\psi(s)\delta_{\mathrm{pp},jj}$ and $\delta_{\mathrm{sq},ij}=\psi(s)\delta_{\mathrm{qq},ij}$.

Flexibility coefficients of the surrounding soil are based on the choice of foundation models. Considering that the soil under cushion is restricted by piles, the lateral deformation is small, and the neighboring influence can be neglected. It can be supposed that the foundation coincides with the Winkler foundation model, so the flexible coefficients of the soil-soil interaction are:

$$\begin{cases}\delta_{\mathrm{ss}.ij}=1/c, i=j\\ \delta_{\mathrm{ss},ij}=0, i\neq j\end{cases} \tag{22}$$

where c is the Winkler bedding value.

According to Ref.[13], taking the cushion as the distributed spring, its influence can be considered by adding the decrement of the cushion into the diagonal elements of the matrix. So the diagonal elements can be modified as follows:

$$\overline{\delta}_{ii}=\delta_{ii}+h_{\mathrm{c}}/(E\overline{A}_i) \tag{23}$$

where h_{c} is thickness of the cushion; $\overline{A}$ is relevant area of the pile cross-section or area of soil node points; E is modulus, it is lateral compressive modulus of the soil node point, and also is deformation modulus of the pile point.

5.3 Modification of flexible factors

The radial displacement of the vertically loaded discrete-material piles (such as stone piles, etc.) can not be neglected (Fig.5), and the modifications are made as follows.

As the stress-strain relation of the vertically loaded discrete-material piles is extremely complex, the following assumes are made to simplify the calculation.

1) The radial strain along the pile shaft is a constant[14].

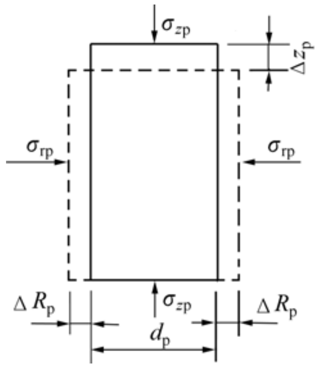

Fig.5 Stress-strain relationship of discrete-material pile

2) The rigid displacement of the pile is ignored.

3) The radial strain $\varepsilon_{\mathrm{rp}}$ equals the radial strain $\varepsilon_{\theta\mathrm{p}}$.

4) Define $K_{\mathrm{p}}=-\varepsilon_{\mathrm{rp}}/\varepsilon_{\mathrm{zp}}$, where K_{p} is radio of the radial strain and the vertical strain. Then vertical stress of the pile is obtained as follows:

$$\sigma_{\mathrm{zp}}=\frac{E_{\mathrm{p}}}{(1+\upsilon_{\mathrm{p}})(1-2\upsilon_{\mathrm{p}})}\left[2\upsilon_{\mathrm{p}}-\frac{(1-\upsilon_{\mathrm{p}})}{K_{\mathrm{p}}}\right]\varepsilon_{\mathrm{zp}}=\alpha\varepsilon_{\mathrm{rp}} \tag{24}$$

Substituting $\sigma_{zp}=P/A$, $\varepsilon_{rp}=\Delta r/r_i$, and $\varepsilon_{zp}=\Delta z/l_i$ into the above equation yields:

$$\Delta z=\alpha\frac{l_i}{r_iK_{\mathrm{p}}}\frac{P}{A_{\mathrm{p}i}} \tag{25}$$

in which $\alpha=\dfrac{E_{\mathrm{p}}}{(1+\upsilon_{\mathrm{p}})(1-2\upsilon_{\mathrm{p}})}\left[2\upsilon_{\mathrm{p}}+\dfrac{(1-\upsilon_{\mathrm{p}})}{K_{\mathrm{p}}}\right]$.

Corrected flexible factor of the short-pile is:

$$\delta'_{\mathrm{pp}}=\delta_{\mathrm{pp}}+\Delta z\big|_{P=1} \tag{26}$$

6 ENGINEERING APPLICATION

To validate the settlement calculation method proposed in this paper, a case mentioned in Ref.[15] was illustrated. The 12-story building is located on the soft clay ground in Hangzhou. Some instruments were installed to observe the behavior of the composite piled foundation in situ.

6.1 Soil

To simplify the problem, surrounding soils are assumed to be homogenous. Shear modulus and Poisson's ratio of soils are summarized in Table 1. Bedding value c of the soil is 1.5×10^4 kN/m^3, which is based on the recommendation of Ref.[16] (The main geological information of the foundation was not quoted in this paper, detailed information can be gotten from Ref.[15]).

Table 1 Properties of soils of case history

Soils	Shear modulus/MPa	Poisson's ratio
Surrounding soils	2.0	0.3
Bearing stratum	3.75×10^3	0.2

6.2 Piles

Soil-cement piles, which have the number of 44, diameter of 0.5 m, cement mixing depth of about 40 m, elastic modulus of 0.36 GPa, are used to improve the shallow subgrade. Concrete piles, which have the number of 60, diameter of 0.6 m, length of about 9 m, elastic modulus of 30 GPa, are used to reduce settlement.

6.3 Cushion

The thickness of broken stone cushion is 0.15 m. The compressive modulus of cushion E_0 is 45 MPa, and the corresponding elastic modulus E_s is 50 MPa.

6.4 Loading

The plane area of the raft is 14.7×30.84 m^2 and the average distributed load acted on the raft is about 163.6 kPa, so the total load of foundation is about 74.168 MN.

Layout of the piles and calculation grids are shown in Fig.6.

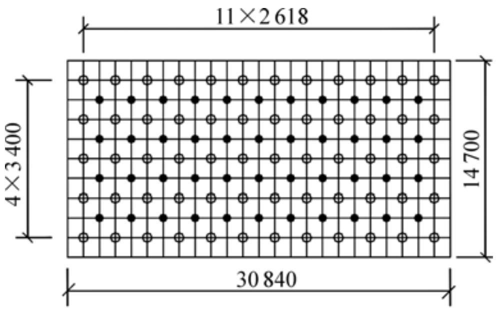

Fig.6 Layout of piles and calculation grids

The calculated value of settlement is 10.2 mm, while the observed value is 8.8 mm, which indicates the feasibility of the method.

7 CONCLUSIONS

1) Based on the shear deformation method, the Mylonakis & Gazetas model was introduced. Considering the effects of cushion, the flexible factors of interactions were provided. Then the settlement calculation for long-short composite piled raft foundations was developed.

2) The calculation result of a case shows that the calculated value is very closed to the observed value based on the method in this paper. Besides that, the choices of certain parameters still need to be calibrated.

3) The influence of the superstructure is ignored, and the farther studies still need to be carried out.

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