Article ID: 1005 - 9784(2005)S1 - 0222 - 05

Improved PSO algorithm and its application①

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Abstract: The mechanism of particle swarm optimization algorithm is studied, and one can draw the conclusion that the best particle found by the swarm falling into local minima is one of the main reasons for premature convergence. Therefore, an improved particle swarm optimization algorithm is proposed. This algorithm selects the best particle with roulette wheel selection method, so premature converging to local optima is avoided. At last, the improved particle swarm optimization algorithm is applied to optimization of time-sharing power supply for zinc electrolytic process. Simulation and practical results show that the global search ability of IPSO is improved greatly and optimization of time-sharing power supply for zinc electrolytic process can bring about outstanding economic benefit for plant.

Key words: particle swarm optimization; premature convergence; roulette wheel; zinc electrolytic process; time-sharing power supply
CLC number: TP273 **Document code**: A

1 INTRODUCTION

Particle swarm optimization (PSO) is a global optimization evolutionary algorithm first proposed by Kennedy et al in 1995 [1,2]. This algorithm simulates social behavior such as a school of flying birds in searching of food. The basic idea of PSO is to search the optimum through cooperation and information sharing among individuals. PSO possesses several highly desirable attributes, including the fact that the basic algorithm is very easy to understand and implement, fast speed and good robustness. Until now, PSO has been widely used in many fields such as power system optimization[3], neural network training [4], system identification [5] and optimization of PID parameters[6] with excellent results.

Like other global optimization evolutionary algorithm such as genetic algorithm[7], premature convergence exists in PSO algorithm, especially for complex multimodal search problems. In order to avoid premature convergence, many improved algorithms were proposed[8-11]. In Refs. [8] and [9], mutation operator is adopted to maintain the population diversity and escape local optima. In Ref. [10], a new hybrid PSO algorithm is introduced which makes use of gradient information to achieve faster convergence without falling into local optima. In Ref. [11], a new technique, named function stretching, is used in PSO algorithm to transform the objective function and alleviate the local optima.

To PSO algorithm, an important reason for premature convergence is that the best particle found by the swarm often fall into local minima. In this paper, an improved PSO (IPSO) algorithm is proposed. During the iteration of IPSO algorithm, we do not simply select the particle with the highest fitness as the best one, but select a particle from several particles with the highest fitness usingroulette wheel method. Thus, it is avoided that all particles approach a possible local optimum very quickly, and the probability of premature converging to local optima is decreased. Optimization of time-sharing power supply for zinc electrolytic process (TPSZEP) is a typical global optimization problem with nonlinear constraint. In this paper, IPSO is applied to optimization of TPSZEP. Industrial practical result shows that the optimal power supply scheme has brought out notable economic benefit for plant.

2 PSO ALGORITHM AND ITS PREMATURE CONVERGENCE

PSO is population and fitness based algorithm like genetic algorithm. It is initialized with a population of random solutions of the objective function. The individuals in the population are called as particles. Each particle searches the optimum position like the behavior of a bird searching food that it "flown" through the problem space by tracking two extrema. One is the unit extremum which is

① **Foundation item**: Project (2002CB312200) supported by the National Key Research 973 Program of China
Received date: 2005 - 05 - 25; **Accepted date**: 2005 - 07 - 10
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the optimal solution achieved by the particle so far. The other is the global extremum which is the optimal achieved by the whole population at present. PSO algorithm can be depicted as follows:

Suppose that there are m particles in D-dimensional search space. The position of the ith particle is represented as $\boldsymbol{X}_i=[x_{i1}, x_{i2}, \cdots, x_{iD}]^T$, ant its best position (the position giving the highest fitness value) found so far is recorded and represented as $\boldsymbol{P}_i=[p_{i1}, p_{i2}, \cdots, p_{iD}]^T$. The best position found by the swarm so far is also recorded and represented as $\boldsymbol{P}_g=[p_{g1}, p_{g2}, \cdots, p_{gD}]^T$. During the iteration, the ith particle flies in the search space with the velocity $\boldsymbol{V}_i=[v_{i1}, v_{i2}, \cdots, v_{iD}]^T$. Then, the velocity and position of the ith particle update with the following equations:

$$v_{id}(t+1)=w^* v_{id}(t)+c_1^* r_1^* (p_{id}(t)- x_{id}(t))+c_2^* r_2^* (p_{gd}(t)-x_{id}(t))$$
$$x_{id}(t+1)=x_{id}(t)+v_{id}(t+1)$$
$$1\leqslant i\leqslant m,\ 1\leqslant d\leqslant D) \quad (1)$$

where c_1 and c_2 called acceleration factors are two positive constants, r_1 and r_2 are two random values in the range [0, 1], w is the inertia weight. During the iteration, the velocity vector $\boldsymbol{V}$ is clamped to the range $[-\boldsymbol{V}_{max}, \boldsymbol{V}_{max}]$ to reduce the likelihood of the particle leaving the search space, and the position vector $\boldsymbol{X}$ is clamped to the range $[\boldsymbol{X}_{min}, \boldsymbol{X}_{max}]$, where $\boldsymbol{X}_{min}$ and $\boldsymbol{X}_{max}$ are decided according to the practical problem and $\boldsymbol{V}_{max}$ is usually chosen to be $k\times \boldsymbol{X}_{max}$, with $0.1\leqslant k\leqslant 1.0$[12].

Like other evolutionary algorithm, to PSO algorithm, if the particles converge too quickly, it will result in premature convergence and losing further search ability. Through studying the mechanism of PSO algorithm, one can conclude that PSO algorithm search optimal solution by tracking two extrema $\boldsymbol{P}_i$ and $\boldsymbol{P}_g$. so, during the search process, if $\boldsymbol{P}_g$ lies in a local optimum position, all particles will approach $\boldsymbol{P}_g$ very quickly. This will result in premature convergence. In order to solve this question, during the iteration, it must be avoided that $\boldsymbol{P}_g$ falls into local optimum position.

3 IMPROVED PSO ALGORITHM AND ITS IMPLEMENTATION

From previous analysis, we know that in order to solve the question of premature convergence for PSO algorithm, it must be avoided that $\boldsymbol{P}_g$ falls into local optimum position. In this paper, the scheme of selecting $\boldsymbol{P}_g$ is improved. During the iteration of IPSO algorithm, we do not simply select the particle with the highest fitness as $\boldsymbol{P}_g$, but select a particle from several particles with the highest fitness using roulette wheel method.

Concretely, let the positions of the first L particles with highest fitness in particle swarm be $\boldsymbol{P}_g^1$, $\boldsymbol{P}_g^2$, $\cdots$, $\boldsymbol{P}_g^L$ and their fitness be g^1, g^2, $\cdots$, g^L. According to roulette wheel selection method, the probability of $\boldsymbol{P}_g^i (i=1, 2, \cdots, L)$ being selected as p_g is:

$$O^i = g^i / \sum_{j=1}^{L} g^j \quad (i=1,2,\cdots,L) \quad (2)$$

Thus, the IPSO algorithm maintains the character of basic PSO algorithm. That is to say, IPSO algorithm also searches optimal solution by tracking two extrema. However, when the best particle found by the swarm falls into local minima, other different particles maybe selected as $\boldsymbol{P}_g$, and the particles will fly to other direction. Then, it is avoidedthat all particles approach the local optimum very quickly, and the probability of premature converging to local optima is decreased. Although the selected particle is not the optimal particle, it is a suboptimal particle. This ensures that the particle swarm flies to a good direction.

According to the previous idea, the improved PSO algorithm is implemented as follows:

Step 1 Determine the position boundary $[\boldsymbol{X}_{min}, \boldsymbol{X}_{max}]$ and flying velocity boundary $[-\boldsymbol{V}_{max}, \boldsymbol{V}_{max}]$ according to practical problem and set the maximum iterative times. Initialize the position $\boldsymbol{X}_i (i=1, 2, \cdots, m)$ and flying velocity $\boldsymbol{V}_i$ of m particles randomly, and compute their fitness F_i.

Step 2 Let the best position achieved by the ith $(i=1, 2, \cdots, m)$ particle so far be $\boldsymbol{P}_i=\boldsymbol{X}_i$ and its fitness be $Fbest_i=F_i$. Suppose that the position and fitness of the L particles with highest fitness in the swarm are $\boldsymbol{P}_g^1$, $\boldsymbol{P}_g^2$, $\cdots$, $\boldsymbol{P}_g^L$ and g^1, g^2, $\cdots$, g^L separately and satisfy $g^1\geqslant g^2\geqslant\cdots\geqslant g^L$.

Step 3 Calculate the probability that $P_g^i (i=1, 2, \cdots, L)$ is selected as $\boldsymbol{P}_g$ according to Eqn. (2).

Step 4 Generate a random value ζ in the range [0, 1]. If ζ satisfies $0\leqslant\zeta<O^1$, then let $\boldsymbol{P}_g=\boldsymbol{P}_g^1$. Else if ζ satisfies $\sum_{j=1}^{k-1} O^j \leqslant \zeta < \sum_{j=1}^{k} O^j$, then let $\boldsymbol{P}_g=P_g^k (k=2, 3, \cdots, L)$.

Step 5 Update the flying velocity and position of particles according to Eqn. (1), and clamp velocity vectors and position vectors to the range $[-\boldsymbol{V}_{max}, \boldsymbol{V}_{max}]$ and $[\boldsymbol{X}_{min}, \boldsymbol{X}_{max}]$ respectively.

Step 6 To the ith $(i=1, 2, \cdots, m)$ particle, if $F_i>Fbest_i$, the let $\boldsymbol{P}_i=\boldsymbol{X}_i$ and $Fbest_i=F_i$.

Step 7 To the ith $(i=1, 2, \cdots, m)$ particle, if $F_i>g_1$, then let $\boldsymbol{P}_g^1=\boldsymbol{X}_i$ and $g^1=F_i$; else if $g^{j-1}>F_i>g^j (j=2, 3, \cdots, L)$, then let $\boldsymbol{P}_g^j=\boldsymbol{X}_i$ and $g^j=F_i$.

Step 8 If the least error criterion or the maximum iterative times are met then stop. Otherwise go to step 3.

In IPSO algorithm, L is a very important value. If is too small, the roulette wheel selection method will have very little effect on this algorithm. $L=1$ is a very special case. In this case, the

IPSO degrades to basic particle swarm optimization. On the other hand, if L is too large, the particle selected as $\boldsymbol{P}_g$ is perhaps a bad particle. This will affect the search ability of IPSO. In application, L is set as $[m/10]$, where $[x]$ means the largest integer do not exceed x.

4 OPTIMIZATION OF TIME-SHARING POWER SUPPLY FOR ZINC ELECTROLYTIC PROCESS

4.1 Description of optimization of TPSZEP

According to time-sharing price policy drawn by the power department, one day is classified into four load-duration periods, and the power price changes in different periods. The power cost in zinc electrolytic process contains two parts: the first part called power-consumption-cost is defined as product of power consumption and power price; the second part called basic-cost is defined as product of maximal load in different periods and a load coefficient. At the same time, in order to further encourage end-users to consume more power in valley-load period, if the load in valley-load period is the maximum, the load is halved and compared with the load in other periods, then the maximal load after compared is used to calculate the basic-cost. Optimization of TPSZEP will adopt rational power supply scheme to minimize the power cost under the condition of ensuring output and quality of zinc. So, the optimization model of TPSZEP can be depicted as follows:

$$\min J = \min(J_1 + J_2) = \min\left[\sum_{i=1}^{4} p_i U_i D_i b S_0 n t_i + k\max(L_1, L_2, L_3, \frac{1}{2}L_4)\right]$$

$$\text{subject to} \begin{cases} L_i = U_i D_i b S_0 n \\ \eta_i = b_i^0 + b_i^1 D_i + b_i^2 D_i^2 + b_i^3 D_i^3 + b_i^4 D_i^4 \\ U_i = a_i^0 + a_i^1 D_i \\ \sum_{i=1}^{4} G_i = \sum_{i=1}^{4} q D_i b S_0 n t_i \eta_i = G \\ I_{\min} \leqslant D_i b S_0 \leqslant I_{\max} \end{cases} \tag{3}$$

where J_1 and J_2 denote power-consumption-cost and basic-cost respectively, p_i and t_i denote the power price (yuan/(kW · h)) and time (h) of the ith period respectively, U_i and D_i denote the cell voltage (V) and current density (A/m^2) of the ith period, b and n denote the number of plates in a cell and number of cells, S_0 is the area of a negative plate (m^2), k is the load coefficient, L_i is the load (kW · h) of the ith period, $q = 1.2202$ g/(A · h) is electrochemical equivalent of zinc, η_i is current-efficiency of the ith period. G is the expected daily output of zinc (t), $I_{\max}$ and $I_{\min}$ are the allowed maximum and minimum current, they are determined by empirical knowledge of the process. According to the process data of zinc electrolytic process, the relationships between cell voltage, current-efficiency and current density can be obtained through linear regression and four order polynomial regression. $a_i^k (k = 0, 1)$ and $b_i^k (k=0, 1, 2, 3, 4)$ are the regression coefficient.

Optimization of TPSZEP is a typical nonlinear global optimization problem including equality and inequality constraints. In order to apply IPSO effectively, the optimization model must be processed. In this paper, penalty function method is used to reconstruct the optimization objective function. The inequality constraints have been considered in PSO algorithm, so only the equality constraint must be considered. Then, optimization objective function is reconstructed with exterior point method[13] as follows:

$$\min J = \min\Big\{\sum_{i=1}^{4} P_i U_i D_i b S_0 n t_i + k\max(L_1, L_2, L_3, \frac{1}{2}L_4) + M(\sum_{i=1}^{4}\sum_{j=1}^{N} q D_i b S_0 n t_i \eta_i - G)^2\Big\} \tag{4}$$

where M is penalty factor. In IPSO algorithm, M is a large value such as $M = 1\,000$.

4.2 Simulation research

In this paper, optimization problem (3) is solved by basic PSO and IPSO respectively. In the algorithm, the basic parameters are set as: $m = 100$, $c_1 = c_2 = 1.49$, $w = 0.729$, $L = 10$, maximum iterative time is 2000, expected daily output of zinc is $G = 600$. In order to compare basic PSO algorithm with IPSO algorithm more clearly, the stop condition is that the maximum iterative times is met. The simulation results are shown in Fig. 1. Figs. 1(a) — (d) are the search results of current density of different periods of the best particle, and (e) is the search result of objective value of the best particle. In Fig. 1, the solid line is the result of basic PSO algorithm and the dashed line is the result of IPSO algorithm. From Fig. 1, we know that for basic PSO algorithm, after some iterations, because that a local optima (its objective value is 758 548) is searched, the optimized values and the position of the best particle will not update any more, it results in the premature convergence. To IPSO algorithm, although the best particle falls into the local optima (its objective value is 759 787), it is can enlarge the search space and find the better solution (its last objective value is 727 154).

4.3 Industrial application

According to the process data obtained from a certain zinc smelter, regression method is used to establish the relationship between current efficiency, cell voltage and current density at different ratio

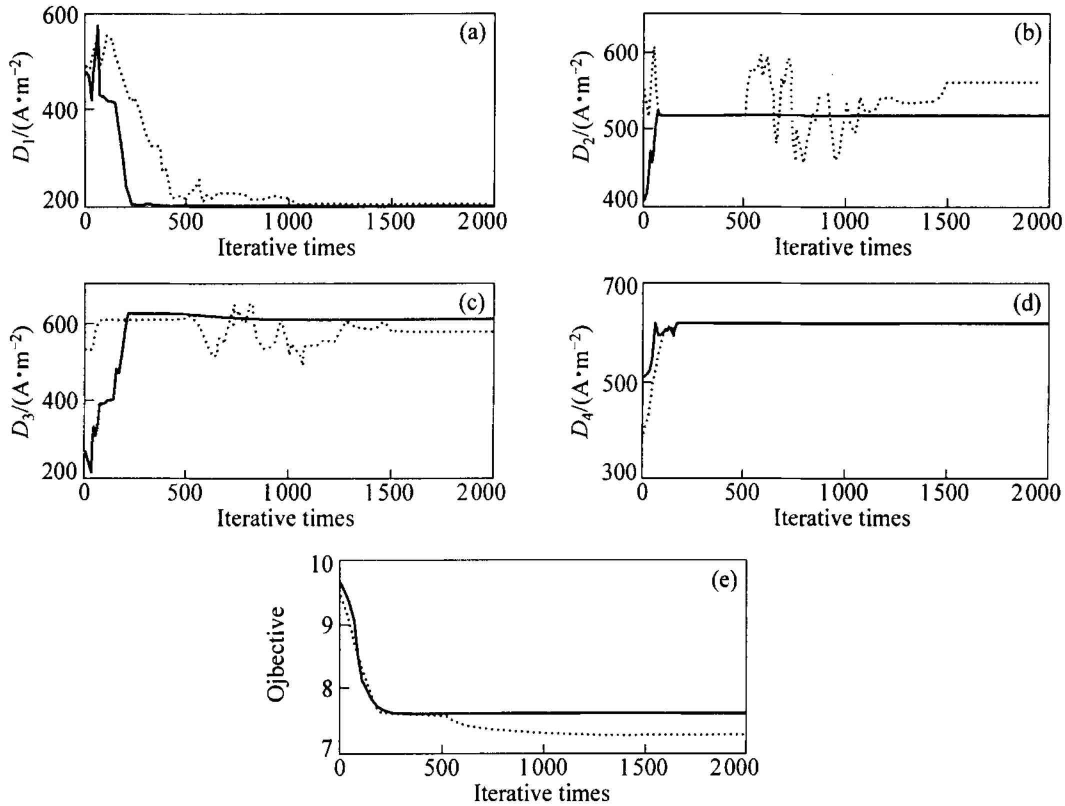

Fig. 1 Comparison of results of basic PSO algorithm and IPSO algorithm
(a)—D_1; (b)—D_2; (c)—D_3; (d)—D_4; (e)—Objective
······—IPSO; ——Basic PSO

of sulfuric acid and zinc in electrolyte and the optimization model of TPSZEP. Then the IPSO algorithm is applied to optimize the scheme of power supply. Table 1 shows the practical data in a consecutive week. At the same time, the time-sharing benefit is calculated. From Table 1 we know that the optimization scheme of TPSZEP not only meet the expected output of zinc but also bring out huge benefit for plant.

Table 1 Practical sesult of optimization of TPSZEP

Date	Expected output/ t	Practical output/ t	Current density/ (A·m^{-2})				Time-sharing benefit/ yuan
			D_1	D_2	D_3	D_4	
2005-04-01	565	564.4	200	500	512	620	72 531
2005-04-02	610	612.3	200	575	591	620	60 588
2005-04-03	585	586.4	200	534	550	620	67 170
2005-04-04	575	575.9	200	518	530	620	69 864
2005-04-05	585	586.2	200	534	550	620	67 168
2005-04-06	580	581.8	200	530	541	620	68 399
2005-04-07	570	572.5	200	511	521	620	70 592

5 CONCLUSIONS

1) To the question that premature convergence of PSO algorithm because of the best particle found by the swarm falling into local minima, an improved PSO algorithm is proposed. In the improved PSO algorithm, the best particle is selected from several particles with the highest fitness usingroulette wheel method. Thus, it is avoided that all particles approach a possible local optimum very quickly, and the probability of premature converging to local optima is decreased.

2) According to time-sharing price counting policy of electric power, the relationship current efficiency, cell voltage and current density at different ratio of sulfuric acid and zinc electrolyte are obtained though the process data of zinc electrolytic process. Then the optimization model of time-sharing power supply of zinc electrolytic process is established. Improved PSO algorithm is used to optimize the scheme of time-sharing power supply and good practical result is obtained.

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(Edited by CHEN Ai-hua)