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# A bi-objective model for job-shop scheduling problem to minimize both energy consumption and makespan①

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**Abstract:** The issue of reducing energy consumption for the job-shop scheduling problem in machining systems is addressed, whose dual objectives are to minimize both the energy consumption and the makespan. First, the bi-objective model for the job-shop scheduling problem is proposed. The objective function value of the model represents synthesized optimization of energy consumption and makespan. Then, a heuristic algorithm is developed to locate the optimal or near optimal solutions of the model based on the Tabu search mechanism. Finally, the experimental case is presented to demonstrate the effectiveness of the proposed model and the algorithm.

**Key words:** green manufacturing; job-shop scheduling; tabu search; energy-saving

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## 1 INTRODUCTION

In 21st century, the green manufacturing is a developing trend in the manufacturing industry, and reducing energy consumption of machining systems is important for the implementation of "Green Manufacturing". The job-shop scheduling problem is a wide range and large scale problem in the production processes, which is defined by scheduling $n$ jobs on $m$ machines to optimize some factors[1]. Most of the existing research on the job-shop scheduling problem is only aimed at the time factor in the production processes[2-9], but seldom considers environmental factors, such as energy consumption. It has been discovered that energy consumption of machining systems can be reduced by scheduling jobs on different machines without any additional cost in our former research[10,11]. However, the scheduling model proposed in the Ref. [10] only considered single factor of minimizing energy consumption, but did not take into account time factor.

This paper addresses the bi-objective job-shop scheduling model to optimize both energy consumption and makespan aiming at the problem mentioned above and a heuristic algorithm is introduced to locate the optimal or near optimal solutions of the model based on the tabu search mechanism.

## 2 BI-OBJECTIVE MODEL FOR JOB-SHOP SCHEDULING PROBLEM AND PARAMETER DESCRIPTION

### 2.1 Bi-objective model for job-shop scheduling problem

Optimizing "Energy Consumption and Makespan for the Job-Shop Scheduling Problem" (E&MJSSP) can be described as follows: The machining system is composed of $m$ identical machines, and $n$ jobs need to be machined. Only one operation, which can be machined by any machine, needs to be done for each job. The process of machining one job can not be interrupted, and one machine can do one job at a time. The bi-objective model for job-shop scheduling problem to simultaneously optimize energy consumption and makespan (E&MJSSP model) is proposed by synthesizing two factors of energy and time as follows:

$$Z(E(x),\ T(x)) = \min\left\{ \sum_{i=1}^{m}\sum_{j=1}^{n} x(i,\ j) \cdot \int_0^{t'(i,\ j)} P_{(i,\ j)} \cdot \mathrm{d}t,\ \max_{i=1}^{m}\left\{ \sum_{j=1}^{n} x(i,\ j) \cdot t(i,\ j) \right\} \right\} \tag{1}$$

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$$\begin{cases} n \geq m, \ \sum_{j=1}^{n} x(i,j) \geq 1 \text{ and } \sum_{i=1}^{m} x(i,j) = 1 \\ n < m, \ \sum_{j=1}^{n} x(i,j) \leq 1 \text{ and } \sum_{i=1}^{m} x(i,j) = 1 \\ x(i,j) = \begin{cases} 1, & j \text{ on } i \\ 0, & \text{otherwise} \end{cases} \end{cases} \quad (2)$$

where $i$ is the serial number of machines, $i=1, 2, \cdots, m$; $j$ is the serial number of jobs, $j=1, 2, \cdots, n$; $t(i, j)$ is the total time for the job $j$ on the machine $i$; $t'(i, j)$ is the machining time for machining the job $j$ on the machine $i$; $P_{(i,j)}$ is the power of the machine $i$ when machining the job $j$.

The objective function consists of two objectives according the formula (1), the first function $E(x)$ is to optimize the energy consumption of the machining system, and the second function $T(x)$ is to optimize the makespan of it. The constrain equation (the formula (2)) states that not only one job is done on one machine when the number of jobs is more than the number of machines, otherwise, no job is done on some machines when the number of jobs is less than the number of machines.

## 2.2 Parameter description of E&MJSSP model

### 2.2.1 Decision variable

The decision variable of the E&MJSSP model is denoted by matrix $\boldsymbol{X}$ that is described by the 0 - 1 integral variable of $x(i, j)$. Therefore, the decision variable matrix $\boldsymbol{X}$, which denotes the solution of the E&MJSSP model with scheduling $n$ jobs on $m$ machines, is shown as follows:

$$\boldsymbol{X}_{m\times n} = \begin{bmatrix} x(1,1) & x(1,2) & \cdots & x(1,n) \\ x(2,1) & x(2,2) & \cdots & x(2,n) \\ \vdots & & x(i,j) & \vdots \\ x(m,1) & x(m,2) & \cdots & x(m,n) \end{bmatrix}$$

In the decision variable matrix $\boldsymbol{X}$, the number of rows denotes the number of machines, and the number of columns denotes the number of jobs.

### 2.2.2 Total time of jobs

The total time for each job includes two parts: one is the machining time of the job, the other is the auxiliary time (preparation time) of the job. The energy consumption is only related with the machining time, and the total time is the sum of the machining time and the auxiliary time of the job. The total time matrix $\boldsymbol{T}$ is written as follows:

$$\boldsymbol{T}_{m\times n} = \begin{bmatrix} t(1,1) & t(1,2) & \cdots & t(1,n) \\ t(2,1) & t(2,2) & \cdots & t(2,n) \\ \vdots & & t(i,j) & \vdots \\ t(m,1) & t(m,2) & \cdots & t(m,n) \end{bmatrix}$$

### 2.2.3 Energy consumption in E&MJSSP model

LIU et al[10,11] proposed that if the process parameters of the job were kept the same, the part of energy consumption for machining the job was approximately equal, and the other part of energy consumption for running the machines was different when scheduling the same job on different machines. Therefore, reducing the total energy consumption of the machining system can be approximately transformed into optimizing the unloaded energy consumption of the machining system. In process parameters, unload power mostly depends on the spindle speed of machines, namely, the unload power is the same when keeping the same spindle speed for a machine. Unload power of the machine $i$ performing the job $j$ is denoted by $P_{u(i,j)}$, then:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x(i,j)\int_{0}^{t(i,j)} P_{(i,j)} \cdot \mathrm{d}t \approx \sum_{i=1}^{m}\sum_{j=1}^{n} x(i,j)\{P_{u(i,j)} \cdot t(i,j)\} \quad (3)$$

Accordingly, the energy consumption matrix $\boldsymbol{E}_u$ in the formula (3) is given as follows:

$$\boldsymbol{E}_{u(m\times n)} = \begin{bmatrix} P_{u(1,1)} \cdot t(1,1) & P_{u(1,2)} \cdot t(1,2) & \cdots & P_{u(1,n)} \cdot t(1,n) \\ P_{u(2,1)} \cdot t(2,1) & P_{u(2,2)} \cdot t(2,2) & \cdots & P_{u(2,n)} \cdot t(2,n) \\ \vdots & & P_{u(i,j)} \cdot t(i,j) & \vdots \\ P_{u(m,1)} \cdot t(m,1) & P_{u(m,2)} \cdot t(m,2) & \cdots & P_{u(m,n)} \cdot t(m,n) \end{bmatrix}$$

# 3 HEURISTIC ALGORITHM BASED ON TABU SEARCH FOR E&MJSSP MODEL

The job-shop scheduling problem is a NP-hard combinatorial optimization problem. Researchers are enlightened from the mechanism of biological evolution, and have developed a series of new approaches to solve the complicated combinatorial optimization problems including tabu search algorithm, genetic algorithm, ant colony optimization, and so on, which has been proved to be effective through a large amount of experimentation[12]. In this paper, tabu search algorithm is adopted to solve the E&MJSSP model.

Tabu search is a meta-heuristic that guides a local heuristic search procedure to explore the solution space beyond the local optimality[13-15]. The totally deterministic tabu search algorithm tailored to the E&MJSSP model are discussed including initial solution, neighborhood generation, tabu classification, tabu tenure, aspiration criterion and stopping criterion.

1) Initial solution. The search procedure is started with an initial solution, and the initial solution can be any reasonable solution. However, an appropriate initial solution makes the search procedure more effective. The approach to obtain the initial solution of the E&MJSSP model is presented according to the priority rule of minimizing energy consumption of machining each job and balancing the number of jobs performed by each machine. The procedure for setting the initial solution of the E&MJSSP model is presented as follows:

**Step 1** Set the decision variable matrix $\boldsymbol{X}_{m\times n}$ to 0, and denote the unperformed jobs by $\boldsymbol{M}$.

**Step 2** Calculate matrix $\boldsymbol{x}^{\mathrm{sum}}_{m\times 1}$ which states the rule of balancing the number of jobs performed by each machine. The approach is: add all the numbers of each row of decision variable matrix $\boldsymbol{X}_{m\times n}$ and select the row, of which the sum is maximal; set this row of matrix $\boldsymbol{x}^{\mathrm{sum}}_{m\times 1}$ to 1, and the rest of rows $\boldsymbol{x}^{\mathrm{sum}}_{m\times 1}$ to 0 (set all the rows of matrix to 0 if the sums of the rows are equal).

**Step 3** Determinate the energy consumption matrix $\boldsymbol{E}^{M}_{u}$ according to $\boldsymbol{M}$. Select the minimal $e(i, j)$ of matrix $\boldsymbol{E}^{M}_{u}$ from the rows of $\boldsymbol{x}^{\mathrm{sum}}_{m\times 1}$, where the number is 0, and set $\boldsymbol{x}_{m\times n}(i, j)$ to 1.

**Step 4** Delete the job $j$ from the $\boldsymbol{M}$.

**Step 5** If the $\boldsymbol{M}$ is null, end this; otherwise, go to Step 2.

2) Neighborhood generation. Insert moves and swaps moves are two of the frequently used move types in permutation problems[16]. In the E&MJSSP model, the sequence of jobs on a machine doesn't affect the objective of optimization. Therefore, insert moves is to remove the job from one machine to another machine, and the new solution can be obtained by changing the 0 - 1 state of the job, which represents whether the job is performed by the machines. Swaps moves can be regarded as twice the insert moves for two jobs; namely, obtaining the new solution that is needed to change the 0 - 1 states of the two jobs at the same time.

3) Tabu classification. In the E&MJSSP model, there is one tabu classification which is related to the movement path of the search. Define "Insert $(i, k)$" as the path of the insert move which represents "inserting job $i$ into machine $k$". The path of the swap move is defined with "Insert $(i, r)$ &Insert$(j, k)$" which denotes "inserting job i on machine $k$ into machine $r$ and inserting job $j$ on machine $r$ into machine $k$". Tabu list consists of the two kinds of movement paths. In tabu tenure, the movement path stored in tabu list is a tabu.

4) Tabu tenure. Tabu tenure is the number of iterations where moves remain tabu active, whose objective is to avoid the reappearance of these recent moves in a finite number of iterations determined by the tabu list size. Two approaches to tenure selection: one is using a single tenure value throughout the search; the other is systematically varying the tenure among a number of values. In this study, the former approach is adopted, and the single tenure value is set in terms of the number of jobs and machines in the E&MJSSP model.

5) Aspiration criterion and stopping criterion. The objective function value in the E&MJSSP model, $Z(E(x), T(x))$, is used to measure the aspiration criterion of the tabu search solution $\boldsymbol{X}$. If a certain move is tabu while the corresponding $Z(E(x), T(x))$ passes the aspiration criterion, then the move will be performed. Stopping criterion can be determined through more than one approach. In the E&MJSSP model, the search procedure stops after a given number of iterations which is set according to the number of jobs and machines.

With the above introduced key concepts of tabu search, the short term tabu search procedure is presented below:

**Step 1** Determine the initial solution of tabu search for the E&MJSSP model according to 2).

**Step 2** Set the tabu list size according to 4), and the initial value of the tabu list is "null"; initialize the optimal objective value $T^{*}_{c}$ and the responding optimal solution $x^{*}$ with the initial objective value and the initial solution.

**Step 3** Set the number of iterations according to 5).

**Step 4** Generate the neighborhood of solution x according to 2).

**Step 5** Select the optimal objective value from the neighborhood.

**Step 6** Judge the move by the tabu list, and obtain the new solution according to 4).

**Step 7** Update correlative factors of tabu search, such as the Tabu list, the optimal objective value $T_c^*$, the optimal solution $x^*$ and the solution $x$.

**Step 8** If the number of iterations reaches stopping criterion, end the search; otherwise, return to Step 4.

## 4 CASE STUDY

In the gear machining workshop of a certain factory, there is a batch of gear need to hobbing, which includes 5 species of gear. The process parameters of the 5 species of gear are given in Table 1.

There are 4 hobbing machines in the gear machining workshop: YKX3132, YB2130, Y3180H and Y3150E. The "unload power-spindle speed" data of the machines are measured to calculate the total energy consumption in the E&MJSSP model. According to the "unload power-spindle speed" data and the spindle speed given in Table 1, the unload power data of the machines can be obtained as shown in Table 2.

The tabu list size is set to 5, and the number of iterations to 50 because the number of jobs and machines, both are not large. The parameters of this case are the inputs for the E&MJSSP model, and the result is obtained as shown in Table 3.

**Table 1** Machining process parameters hobbing 5 species of gear

| No. | Spindle speed/ $(r \cdot min^{-1})$ | Machining time/ min | Auxiliary time/ min | Total time/ min |
|---|---|---|---|---|
| Job 1 | 200 | 22.39 | 5.40 | 27.79 |
| Job 2 | 150 | 8.39 | 4.90 | 13.29 |
| Job 3 | 250 | 17.12 | 6.70 | 23.82 |
| Job 4 | 300 | 10.82 | 6.30 | 17.12 |
| Job 5 | 125 | 28.10 | 5.80 | 33.90 |

**Table 2** Unload power data of hobbing gear on machines kW

| Machine | Job1 | Job 2 | Job 3 | Job 4 | Job 5 |
|---|---|---|---|---|---|
| YKX3132 | 3.25 | 2.80 | 3.90 | 4.40 | 2.50 |
| YB2130 | 3.26 | 3.04 | 3.68 | 4.48 | 2.88 |
| Y3180H | 3.84 | 3.36 | 4.10 | 4.58 | 3.04 |
| Y3150E | 3.28 | 3.01 | 3.64 | 4.25 | 2.38 |

**Table 3** Solution data of two models

| Model | Solution matrix | Energy consumption/ (kW · h) | Makespan time/min |
|---|---|---|---|
| E&MJSSP model | $X_{min}=\begin{bmatrix}0&0&0&0&1\\1&0&0&0&0\\0&0&1&0&0\\0&1&0&1&0\end{bmatrix}$ | 4.692 | 33.90 |
| Model proposed in Ref. [10] | $X_{min}=\begin{bmatrix}1&1&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&1&1&1\end{bmatrix}$ | 4.526 | 74.84 |

Meanwhile, the scheduling model proposed in Ref. [10] is been used for this case. Both of the results are shown in Table 3.

From Table 3, comparing with the scheduling model proposed in Ref. [10], the time of makespan of the E&MJSSP model proposed in this paper is much shorter while the discrepancy of the energy consumption value is not large. Therefore, we can conclude that the E&MJSSP model proposed in this paper can optimize energy consumption as well as makespan time, which makes the E&MJSSP model closer to the attainment of green manufacturing goals.

## 5 CONCLUSIONS

A bi-objective job-shop scheduling model is developed to optimize both the energy consumption and the makespan in machining systems based on the method of "Energy-Saving Job-Shop Scheduling". The objective function value of the model represents synthesized optimization of energy consumption and makespan. A heuristic algorithm based on tabu search, which is tailored to the E&MJSSP model proposed and is developed to search for optimal or near optimal solutions of this problem. Some key concepts of the heuristic algorithm are discussed including initial solution, neighborhood generation, tabu classification, tabu tenure, aspiration criterion and stopping criterion. By solving the case problem in the gear machining workshop, it shows that the E&MJSSP model proposed in this paper satisfies the goals of green manufacturing to greater extent than the model presented in authors' former research.

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