Vol. 10 No. 3 Sep. 2003
Article ID: 1005-9784(2003)03-0226-05

J. CENT. SOUTH UNIV. TECHNOL.

Simulation of spatially coupling dynamic response of train-track time-variant system

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Abstract: There exist three problems in the calculation of lateral vibration of the train-track time-variant system at home and abroad and the method to solve them is presented. Spatially coupling vibration analysis model of train-track time-variant system is put forward. Each vehicle is modeled as a multi-body system with 26 degrees of freedom and the action of coupler is also considered. The track structure is modeled as an assembly of track elements with 30 degrees of freedom, then the spatially coupling vibration matrix equation of the train-track time-variant system is established on the basis of the principle of total potential energy with stationary value and the "set-in-right-position" rule. The track vertical geometric irregularity is considered as the excitation source of the vertical vibration of the system, and the hunting wave of car bogie frame is taken as the excitation source of lateral vibration of the system. The spatially coupling vibration matrix equation of the system is solved by Wilson-θ direct integration method. The approximation of the calculated results to the spot test results demonstrates the feasibility and effectiveness of the presented analysis method. Finally, some other vibration responses of the system are also obtained.

Key words: vibration; train; track; time-variant system; hunting wave; car bogie frame; excitation source
CLC number: U211.3 **Document code:** A

1 INTRODUCTION

Train-track spatially coupling system (hereinafter referred to as the System) is a complex typical dynamic system. Following are the main methods used in and out of China in analyzing the vibration of the System[1-5]. 1) The vehicle vibration equations and track vibration equations are set up respectively. Then these two groups of vibration equations are connected by the wheel-rail interaction forces. Finally, the solution of these differential equations is obtained by the iterative method. Because of the clearance between rail and wheel flange, the lateral wheel-rail contact condition could not be formulated. So the sole nature of the solution to the lateral vibration equations of the System would not be ensured. Consequently, no satisfactory calculation results of the System have been gained. 2) The lateral track irregularity is mostly treated as the excitation source of lateral vibration of the System. In fact, the lateral vibration of the System may be caused by many factors, such as lateral track irregularity, conicity of wheel tread, defects of wheels and rails, errors in production of vehicles, eccentricity of mass and loading of cars. Sole consideration of the lateral track irregularity results in negligence of many other factors. 3) The lateral track irregularity function is simulated at random according to the power spectrum density. Then, the random lateral vibration of the System is analyzed. This method only considers the random function of lateral track irregularity but neglects the influence of other random factors on the lateral vibration of the System. In fact, it is difficult to carry out the random vibration analysis of the System, as the theory of random vibration analysis of the System has not been established. In this paper the method to solve the three problems is presented and the model of vibration analysis of train-track time-variant system is put forward.

2 METHODS TO SOLVE PROBLEMS

Following are the methods to solve the three problems mentioned above.

1) Train and track should be treated as a

Foundation item: Project (50078006) supported by the National Natural Science Foundation of China; Project (2001G029) supported by the Foundation of the Science and Technology Section of the Railway Bureau
Received date: 2002-08-30; **Accepted date:** 2002-11-18
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unitary system in order to set up the spatial vibration equations of the System on the basis of the principle of total potential energy with stationary value in elastic dynamics and the rule of "set-in-right-position" [6]. Consequently, the sole solution to the lateral vibration equations of the System can be ensured. Since in these cases the boundary condition of the System is that of the track, not that of the wheel-rail contact surface, the lateral wheel-rail contact condition is not required.

2) The hunting wave of car bogie frame is considered as the excitation source of lateral vibration of the System according to the homogeneous nature of the lateral vibration equation group of the System. Such excitation source includes all the factors leading to the lateral vibration of the System.

3) The method of random energy analysis of lateral vibration of the System is established to solve the problem of random analysis of the lateral vibration of the System[7].

3 MATHEMATICAL MODEL

3.1 Track element model

Track structure is modeled as an assembly of track elements. Each track element has 30 degrees of freedom(DOFs). Fig. 1 shows a track element consisting of rails and sleepers. In Fig. 1, sleepers are supported by elastic ballast, and the rails and sleepers are both considered as elastic bodies. The notations in Fig. 1 are listed as follows:

K_0, K_1—Vertical stiffness of ballast;

K_2—Lateral stiffness of ballast;

K_3—Longitudinal stiffness of ballast;

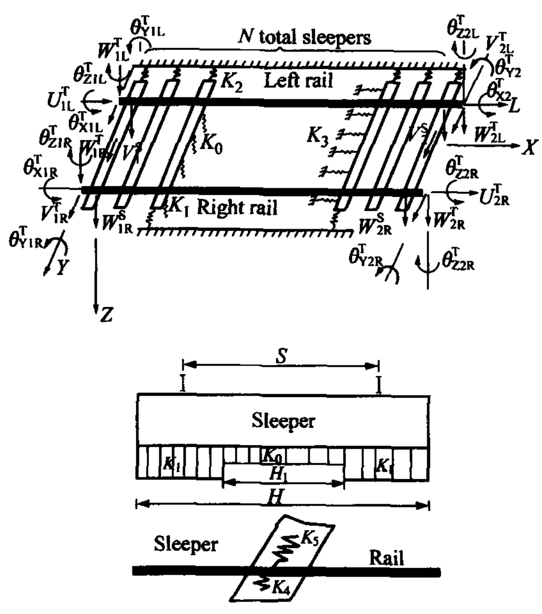

Fig. 1 Track element model

K_4—Vertical stiffness of pad;

K_5—Lateral stiffness of pad;

L—Length of track element;

N—Number of sleepers of track element;

a—Span of two sleepers;

S—Distance between two wheel-rail contact points;

H_0—Ineffective supporting length of sleeper;

H—Length of sleeper

The vector of the node displacements of the track element in Fig. 1 is shown as follows:

$$\{u\}_e=\begin{bmatrix}\{u_1\}\\ \{u_2\}\end{bmatrix}_{30\times 1}$$

where

$$\{u_1\}=[U_{1R}^T, V_{1R}^T, W_{1R}^T, \theta_{X1R}^T, \theta_{Y1R}^T, \theta_{Z1R}^T, U_{1L}^T, V_{1L}^T, W_{1L}^T, \theta_{X1L}^T, \theta_{Y1L}^T, \theta_{Z1L}^T, V_1^S, W_{1R}^S, W_{1L}^S]^T \quad (1)$$

$$\{u_2\}=[U_{2R}^T, V_{2R}^T, W_{2R}^T, \theta_{X2R}^T, \theta_{Y2R}^T, \theta_{Z2R}^T, U_{2L}^T, V_{2L}^T, W_{2L}^T, \theta_{X2L}^T, \theta_{Y2L}^T, \theta_{Z2L}^T, V_2^S, W_{2R}^S, W_{2L}^S]^T \quad (2)$$

where U_{1R}^T, V_{1R}^T, W_{1R}^T are longitudinal, lateral and vertical displacements of first node of right rail of track element, respectively; U_{1L}^T, V_{1L}^T, W_{1L}^T are longitudinal, lateral and vertical displacements of first node of left rail of track element, respectively; U_{2R}^T, V_{2R}^T, W_{2R}^T are longitudinal, lateral and vertical displacements of second node of right rail of track element, respectively; U_{2L}^T, V_{2L}^T, W_{2L}^T are longitudinal, lateral and vertical displacements of second node of left rail of track element, respectively; θ_{X1R}^T, θ_{Y1R}^T, θ_{Z1R}^T are rotation displacements of first node of right rail of track element along longitudinal, lateral and vertical direction, respectively; θ_{X1L}^T, θ_{Y1L}^T, θ_{Z1L}^T are rotation displacements of first node of left rail of track element along longitudinal, lateral and vertical direction, respectively; θ_{X2R}^T, θ_{Y2R}^T, θ_{Z2R}^T are rotation displacements of second node of right rail of track element along longitudinal, lateral and vertical direction, respectively; θ_{X2L}^T, θ_{Y2L}^T, θ_{Z2L}^T are rotation displacements of second node of left rail of track element along longitudinal, lateral and vertical direction, respectively; V_1^S, W_{1R}^S, W_{1L}^S are lateral and vertical displacements of first sleeper of track element, respectively; V_2^S, W_{2R}^S, W_{2L}^S are lateral and vertical displacements of second sleeper of track element, respectively.

3.2 Vehicle model

Each vehicle is reproduced with 7 rigid bodies (car body, two bogies and four wheel sets), and connected with each other by means of elastic and damping linear elements (see Fig. 2). The car body and bogies of the general vehicle have 6 DOFs each, being imposed the longitudinal motion with constant velocity v. For each wheel set longitudinal and lateral coordinates are considered. This results in a total of 26 DOFs for each vehicle[7].

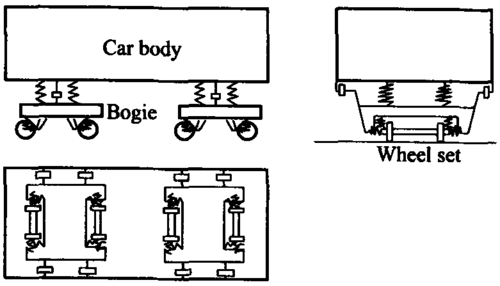

Fig. 2 Vehicle model

3.3 Vehicle-track coupling model

It can be seen from Fig. 3 that each wheel set of the k-th vehicle is in touch with track element e, f, g, h, respectively. Therefore, the vertical displacements of each wheel of the k-th vehicle are shown as follows:

$$Z_{wi}^{R}=W_{ij}^{R}+Z_{ioR} \quad (3)$$

$$Z_{wi}^{L}=W_{ij}^{L}+Z_{ioL} \quad (4)$$

$$(i=1,2,3,4;\ j=e,f,g,h)$$

where Z_{wi}^{R} is vertical displacement of right wheel of i-th wheel set; Z_{wi}^{L} is vertical displacement of left wheel of i-th wheel set; W_{ij}^{R} is vertical displacement of right rail of j-th track element being in touch with right wheel of i-th wheel set; W_{ij}^{L} is vertical displacement of left rail of j-th track element being in touch with left wheel of i-th wheel set; Z_{ioR}, Z_{ioL} are track vertical irregularity.

Fig. 3 Vehicle-track coupling model

4 EXCITATION SOURCE

It is reasonable for track vertical irregularity to be treated as the excitation source of vertical vibration of the System. But the lateral vibration of the System may be caused by many factors, such as lateral track irregularity, conicity of wheel tread, defects of wheels and rails, errors in production of vehicles, eccentricity of mass and loading of vehicles. Sole consideration of the lateral track irregularity results in negligence of many other factors. In this paper, lateral vibration wave of car bogie frame (usually referred to as hunting wave) is considered as the excitation source of lateral vibration of the System.

The following is the matrix equation of the lateral vibration of the System, which is established by means of the principle of the total potential energy with the stationary value in elastic system dynamics and the rule of "set-in-right-position" for formulating system matrixes [6] and under the condition of no effect of wind load.

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{\delta}}\}+[\boldsymbol{C}]\{\dot{\boldsymbol{\delta}}\}+[\boldsymbol{K}]\{\boldsymbol{\delta}\}=0 \quad (5)$$

It is a homogeneous equation, by which only relative values of responses can be found. In the case that a part of the responses of the System are given, vibration responses of other parts of the System can be obtained from the matrix equation. So the lateral vibration displacement parameter of the System $\boldsymbol{\delta}$ can be divided into k number of known parameters δk and n number of unknown parameters δ_n, then

$$\{\boldsymbol{\delta}\}=\{\boldsymbol{\delta}_k \quad \boldsymbol{\delta}_n\}^{\mathrm{T}}.$$

Owing to matrix partitioning, the matrix equation is changed into

$$\begin{bmatrix}\boldsymbol{M}_{kk} & \boldsymbol{M}_{kn}\\ \boldsymbol{M}_{nk} & \boldsymbol{M}_{nn}\end{bmatrix}\begin{Bmatrix}\ddot{\boldsymbol{\delta}}_k\\ \ddot{\boldsymbol{\delta}}_n\end{Bmatrix}+\begin{bmatrix}\boldsymbol{C}_{kk} & \boldsymbol{C}_{kn}\\ \boldsymbol{C}_{nk} & \boldsymbol{C}_{nn}\end{bmatrix}\begin{Bmatrix}\dot{\boldsymbol{\delta}}_k\\ \dot{\boldsymbol{\delta}}_n\end{Bmatrix}+\begin{bmatrix}\boldsymbol{K}_{kk} & \boldsymbol{K}_{kn}\\ \boldsymbol{K}_{nk} & \boldsymbol{K}_{nn}\end{bmatrix}\begin{Bmatrix}\boldsymbol{\delta}_k\\ \boldsymbol{\delta}_n\end{Bmatrix}=0 \quad (6)$$

Therefore, we obtain

$$[\boldsymbol{M}_{nn}]\{\ddot{\boldsymbol{\delta}}_n\}+[\boldsymbol{C}_{nn}]\{\dot{\boldsymbol{\delta}}_n\}+[\boldsymbol{K}_{nn}]\{\boldsymbol{\delta}_n\}=-[\boldsymbol{M}_{nk}]\{\ddot{\boldsymbol{\delta}}_k\}-[\boldsymbol{C}_{nk}]\{\dot{\boldsymbol{\delta}}_k\}-[\boldsymbol{K}_{nk}]\{\boldsymbol{\delta}_k\} \quad (7)$$

$$[\boldsymbol{M}_{kk}]\{\ddot{\boldsymbol{\delta}}_k\}+[\boldsymbol{C}_{kk}]\{\dot{\boldsymbol{\delta}}_k\}+[\boldsymbol{K}_{kk}]\{\boldsymbol{\delta}_k\}+[\boldsymbol{M}_{kn}]\{\ddot{\boldsymbol{\delta}}_n\}+[\boldsymbol{C}_{kn}]\{\dot{\boldsymbol{\delta}}_n\}+[\boldsymbol{K}_{kn}]\{\boldsymbol{\delta}_n\}=0 \quad (8)$$

Eq. (8), a non-independent equation, should be taken off. A solution of n number of unknown lateral vibration responses of the System is found from Eq. (7) where all the items on the right side of the equation are known. Thus, $\{\ddot{\boldsymbol{\delta}}\}$, $\{\dot{\boldsymbol{\delta}}\}$ and $\{\boldsymbol{\delta}\}$ are derived as the excitation sources of lateral vibration of the System.

The lateral displacement parameter of car bogie frame should be set as a known number. The most direct approach is to measure the lateral vibration displacement of a wheel set with no distribution of sensors on it. From the view of vibration test, it is convenient for the hunting wave of car bogie frame to be measured. So we come to realize that the hunting wave of car bogie frame can be treated as the excitation source of lateral vibration of the System.

The following points should also be mentioned. 1) Our study is made from response to response. That is true. For example, the measured earthquake acceleration waves have been taken as the excitation source in calculation of responses of the tectonic earthquake. Therefore, the study from response to response has good grounds. 2) The measured hunting wave of car bogie frame accurately reflects the influence of all factors, which give rise to the lateral vibration of the System. This is impracticable when the lateral track irregularity is taken as the excitation source of lateral vibration of the System. 3) Being deterministic itself, the measured hunting wave of car bogie frame can be only used in deterministic analysis of lateral vibration of the System. On the other hand, the artificial hunting wave of car bogie frame can be used in random analysis of lateral vibration of the System[7].

5 VIBRATION EQUATION OF SYSTEM AND ITS SOLUTION

When the potential energy of the track element and vehicle is respectively formulated at time t, the total potential energy of the System can also be obtained at time t. The matrix equation of spatial vibration of the System is well established at time t on the basis of the principle of total potential energy with stationary value in elastic system dynamics and the rule of "set-in-right-position" [6].

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{u}}\}+[\boldsymbol{C}]\{\dot{\boldsymbol{u}}\}+[\boldsymbol{K}]\{\boldsymbol{u}\}=\{\boldsymbol{P}\} \tag{9}$$

where $\{\ddot{\boldsymbol{u}}\}$, $\{\dot{\boldsymbol{u}}\}$, $\{\boldsymbol{u}\}$ and $\{\boldsymbol{P}\}$ are the acceleration, velocity, displacement and load vector of the System, respectively. $[\boldsymbol{M}]$, $[\boldsymbol{C}]$ and $[\boldsymbol{K}]$ are the mass, damping and stiffness matrix of the System, respectively.

In order to solve Eq. (9), the Wilson-θ direct integration method is used. Because it is not necessary to use iterative method in vibration calculation of the System in each step, the calculation efficiency is greatly enhanced.

6 RESULTS

The comparison between calculated results and measured results is made to verify the feasibility and effectiveness of the calculation method mentioned above. The dynamic responses of the passenger train formed with 4 passenger cars hauled by a locomotive DF11 running at 150 km/h on the 100 m track are calculated. Fig. 4 and Fig. 5 show the comparison between the calculated results and measured results of the lateral and vertical vibration waves of a sleeper.

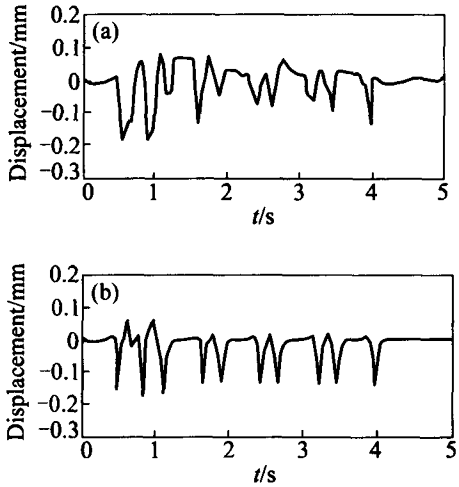

Fig. 4 Measured and calculated lateral displacement wave figures of sleeper (v=150 km/h)
(a)—Measured; (b)—Calculated

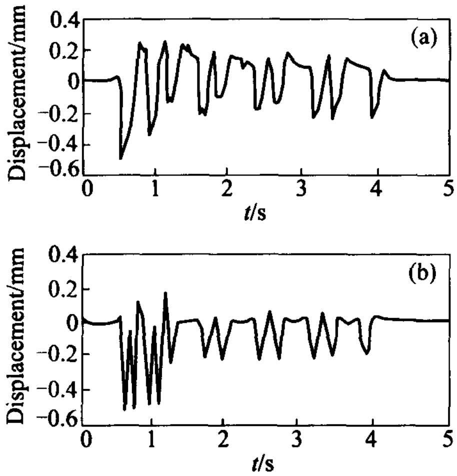

Fig. 5 Measured and calculated vertical displacement wave figures of sleeper (v=150 km/h)
(a)—Measured; (b)—Calculated

The comparison between the maximum calculated and measured values of lateral wheel-rail forces, the lateral and vertical accelerations of car body are listed in Table 1.

Table 1 Comparison between maximum calculated and measured values (v=150 km/h)

Maximum value	Lateral wheel-rail force/kN	Lateral car body acceleration /($m \cdot s^{-2}$)	Vertical car body acceleration /($m \cdot s^{-2}$)
Calculated	5.73	0.71	0.80
Measured	5.21	0.70	0.90

The comparisons listed above have shown good correspondence between the calculated and measured results.

Some other calculated results, such as lateral rail acceleration, lateral and vertical wheel-rail force, derailment coefficient, Sperling index are also obtained. These results are shown in Fig. 6 to Fig. 10.

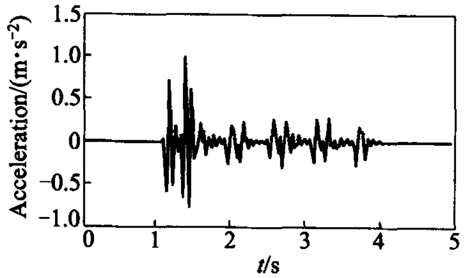

Fig. 6 Lateral wheel-rail acceleration wave figure (v=150 km/h)

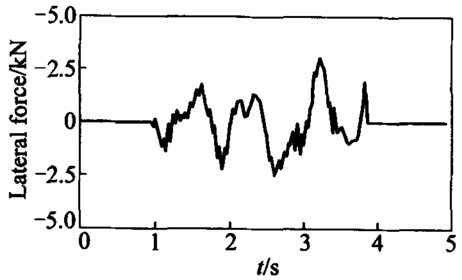

Fig. 7 Lateral wheel-rail force wave figure (v=150 km/h)

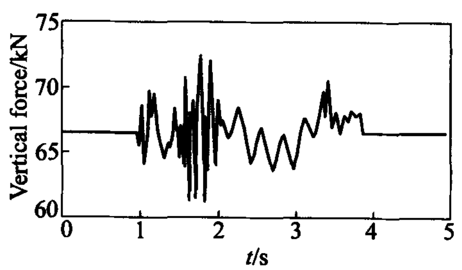

Fig. 8 Vertical wheel-rail force wave figure (v=150 km/h)

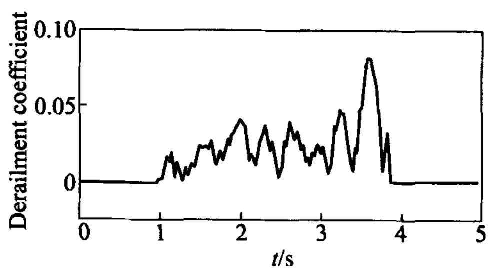

Fig. 9 Maximum derailment coefficient wave figure (v=150 km/h)

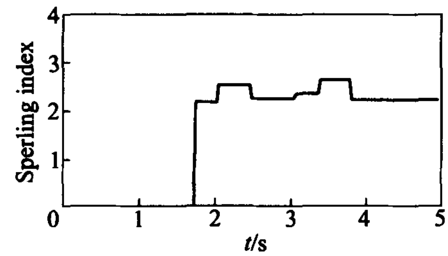

Fig. 10 Lateral Sperling index wave figure (v=150 km/h)

7 CONCLUSIONS

The calculated lateral and vertical displacement wave figures of sleeper have good correspondence with the measured results. The maximum calculated values of the lateral wheel-rail force, lateral and vertical car body acceleration also have good correspondence with the maximum measured results. In the end of the paper, some other calculated results, such as rail lateral acceleration, lateral and vertical wheel-rail force, derailment coefficient and Sperling index are also obtained. The approximation of the numerical results to the spot test results demonstrates the feasibility and effectiveness of the analysis method.

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(Edited by PENG Chao-qun)