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N-th root slant stack for enhancing weak seismic signals

Li Fei[1,2], Xie Jun-fa[2*], Yao Zong-hui[1], Li Mei[1], Zhao Yu-lian[2], Liu Wei-ming[2], and Chen Juan[1]

Abstract: Seismic imaging of complicated underground structures with severe surface undulation (i.e., double complex areas) is challenging owing to the difficulty of collecting the very weak reflected signal. Enhancing the weak signal is difficult even with state-of-the-art multi-domain and multidimensional prestack denoising techniques. This paper presents a time–space dip analysis of offset vector tile (OVT) domain data based on the *τ-p* transform. The proposed N-th root slant stack method enhances the signal in a three-dimensional *τ-p* domain by establishing a zero-offset time-dip seismic attribute trace and calculating the coherence values of a given data sub-volume (i.e., inline, crossline, time), which are then used to recalculate the data. After sorting, the new data provide a solid foundation for obtaining the optimal N value of the N-th root slant stack, which is used to enhance a weak signal. The proposed method was applied to denoising low signal-to-noise ratio (SNR) data from Western China. The optimal N value was determined for improving the SNR in deep strata, and the weak seismic signal was enhanced. The results showed that the proposed method effectively suppressed noise in low-SNR data.
Keywords: N-th root, Weak seismic signal, *τ-p*, OVT

Introduction

In China, seismic imaging for oil and gas exploration in locations such as the Gobi Desert and Loess Plateau has reached double complex areas with a complex surface and complex subsurface. The complicated underground geological conditions of double complex areas make the signal-to-noise ratio (SNR) of the seismic data very low, which greatly affects the imaging accuracy. Difficulties with calibrating and tracking seismic horizons increase the uncertainty of the structural model and limit exploration and development (Guo et al., 2013). To increase the accuracy of seismic imaging in double complex areas, some experts have focused on developing methods to enhance a weak signal. Different approaches can be used to eliminate noise depending on the characteristics of the seismic signal and noise distribution (Mu, 2012; Liu and Zhang, 1996; Hu and White, 1993). For example, the denoising method in the one-dimensional frequency domain and filtering method in the *f-k* domain exploit differences in the frequency and apparent velocity of the signal and noise. However, denoising in the *f-k* domain also generates false frequencies, which damages the signal fidelity. The *f-x* predictive filtering method (Zhao et al., 1998) uses inversion to predict noise and false events. However, the assumption of linear events is rather harsh, and the method has difficulty dealing with nonlinear events. The hyperbolic filtering method in the time domain uses the theory of least-squares filtering and the chaotic matrix algorithm to suppress random noise in the shot record and effectively improve the SNR of seismic data. However, the amplitude spectra of the processed signal

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1.Exploration and Development Research Institute of Changqing Oil Company, Petrochina, Xi'an 710021, China.
2.Northwest Branch of Research Institute of Petroleum Exploration and Development,PetroChina , Lanzhou 730020, China.
*Corresponding author: Xie Jun-fa (Email: xiejunfa@petrochina.com.cn) .

easily produce the phenomenon of a “notch frequency.” In addition, the method has low computational efficiency, and it is poor at reducing noise from irregular events. Time–frequency peak filtering (TFPF) is independent of the signal waveform and involves modulating the collected signal to an analytic signal with an instantaneous frequency and then estimating the signal from the time–frequency distributions. It offers the advantages of fewer instantaneous constraints, and it effectively suppresses strong background noise. However, TFPF does not consider the actual direction of events during the denoising process. Empirical mode decomposition (EMD) is an adaptive frequency noise suppression method based on the Hilbert transform, which decomposes the signal to obtain several inherent modal functions that contain details of the signal in different frequency segments, which often change according to the signal. EMD is relatively effective at noise suppression, but obtaining the components of the inherent modal functions results in the mode aliasing phenomenon. This increases the difficulty of processing late signals, which are inevitable during noise suppression.

Various methods have been applied to eliminate surface waves, multiples, random noise, and coherent noise in seismic data, including the curvelet transform (Donoho et al., 1993, 2002; Li et al., 2007), wavelet transform (Liu and Zhang, 1996), regional filtering (Hu et al., 2016), and cross-sectional filtering (Xue, 2019). The curvelet transform involves determining the denoising threshold by combining the corresponding statistical values of the curvelet coefficients because the signal and noise coefficients are located in different decomposition layers and have different characteristics. It is effective at denoising but has strict requirements to achieve a suitable threshold value. The wavelet transform is a multiscale analysis method that transforms the data into the frequency domain while retaining the time characteristics so that the frequency characteristics of the signal can be presented at different scales. The angular resolution is not high (Mallat, 1989), and this method cannot express the directional features of image edges well (Mu, 2012; Liu and Zhang, 1996). Fan et al. (2008) proposed combining the wavelet transform and multiple autocorrelation to extract weak signals from a background of strong white or colored noise. However, while the wavelet transform is effective at local time–frequency analysis of one-dimensional data, it has limited applicability to processing data in two dimensions or higher. The two-dimensional wavelet is simply the tensor product of one-dimensional wavelets, so it is able to describe point-like singularities in two-dimensional signals but cannot accurately describe the characteristics of edge noise, such as straight lines. Thus, it is not effective for processing weak seismic signals. Regional filtering exploits differences in the velocity and offset range of the surface wave and signal to obtain the maximum frequency of the signal, which is used to suppress the surface wave. However, this method loses precision and accuracy when the high-frequency part of the surface wave overlaps with the low-frequency part of the signal.

Other methods have focused on both suppressing noise and extracting weak signals with varying degrees of success, such as the S-transform (Liu, 2018), fast Fourier transform *K-L* transform (Peng et al., 2007), singular value decomposition (Lu, 2006), polynomial fitting (Zhong et al., 2006), nonlocal mean filtering (Hu, 2014), and chaotic system detection (Gao et al., 2006). The rise of artificial intelligence led Yang et al. (2020) to apply a residual convolutional neural network to suppress random noise. Yang et al. (2021) later improved upon the method by using an adaptive convolutional neural network instead. Zheng et al. (2021) applied residual learning to suppress random noise in microseismic data (Zheng et al., 2021). However, these artificial intelligence methods require large amounts of training data and do not have strong generalization ability, so their applicability to processing actual data is currently limited.

The above denoising methods have generally used the time–frequency, frequency, and other transformation domains to suppress noise and extract weak signals, or they have combined traditional denoising methods with signal processing and applied mathematics to improve the denoising performance. Some of these methods have been applied to actual seismic data, and they have demonstrated their effectiveness in suppressing noise. However, when the seismic data contains strong random noise, the weak signal is often lost, and existing denoising methods are ineffective. In this paper, the N-th root slant stack method is combined with analytical methods for array data (Kanasewich et al., 2012; Muirhead and Datt, 1976; McFadden et al., 1986) to suppress noise and enhance the weak signal in seismic data to effectively solve the problems with seismic imaging in double complex areas.

Method

The slant stack is commonly used to suppress random noise in seismic data. The *τ-p* transformation is used to perform a local stack, which strengthens the signal and weakens random noise to suppress noise. For weak seismic signals, the N-th root slant stack can be used to improve the suppression of strong noise, which greatly improves the SNR and enhances the signal clarity of the processed data. The conventional slant stack is usually applied to denoising in the shot gather or common midpoint (CMP) gather rather than in a three-dimensional data volume. In this paper, the N-th root slant stack is applied to denoising seismic data in the offset vector tile (OVT) domain.

N-th root slant stack

The N-th root slant stack addresses the shortcomings of the linear slant stack, which has difficulty with extracting very weak seismic signals. The N-th root slant stack is a nonlinear filter applied to a single sample of prestack seismic data that can handle the nonlinear characteristics of seismic noise. The N-th root slant stack can be considered a *τ-p* transformation that is defined as follows (Muirhead and Datt, 1976; Zang and Zhou, 2002):

$$G^N(t) = \frac{g(t)}{\left|g(t)\right|}\left|g(t)\right|^{1/N}, \tag{1}$$

where $g(t)$ is the input data and $G^N(t)$ is equivalent to the N-th root of g(t). $G^N(t)$ and $g(t)$ can both be positive or negative. When N > 1, the dynamic range of the data is compressed, which means that the N-th root of each sampling point is stacked. When N < 1, the transformation expands the dynamic range of the data. This means that N is set as the reciprocal of the corresponding value in the positive transformation. Then, the N-th root slant stack can be defined as (Muirhead and Datt, 1976; Zang and Zhou, 2002)

$$S(t) = R(t)\left|R(t)\right|^{N-1}, \tag{2}$$

where $R(t) = \frac{1}{k}\sum_{j=1}^{k} G_j^N(t+\tau_j)$. Thus, the N-th root slant stack is a nonlinear transformation function depending on the N value.

Although the signal in seismic data has a very weak signal, it has a certain coherence like signals with strong energy. In contrast, random noise in seismic data is usually not coherent. Thus, the slant stack can enhance the energy of a weak signal relative to random noise to improve the SNR of seismic data. For seismic data with a very low SNR, a large N value is selected to compress the dynamic range of the data, as given in equation (1), which increases the energy of the signal relative to the noise. The energy of the signal can be further increased by using the slant stack as given in equation (2). For seismic data with a high SNR, selecting a small N value can increase the energy of the signal and weaken the energy of the noise.

Offset vector tile domain

The offset vector tile (OVT) was conceived to sort data before migration and generate a new domain. The OVT domain is an extension of the cross-arranged gather and can be considered as a subset of this gather. The OVT domain can be combined with the N-th root slant stack to enhance weak seismic imaging signals for oil and gas exploration. Two specific methods can be applied to noise suppression.

First, based on the similarity between the OVT domain and post-stack data (Li et al., 2015), the N-th root slant stack and its *τ-p* transform (Zang and Zhou, 2002) can be applied to the data volume in the OVT domain to suppress noise and enhance the signal. In the data analysis stage, the coherence values of the *x-t* domain are calculated using either the conventional coherence method or a similar coherence method. The conventional coherence method is given by (Li et al., 2015)

$$\mathrm{N}_L = \sum_{j=1}^{t} \frac{\left[\sum_{i=1}^{n} A_{ij}\right]^2}{K_j \sum_{i=1}^{n}\left(A_{ij}\right)^2}, \tag{3}$$

where *τ-p* is the conventional coherence at the position L, A_{ij} is the amplitude of the j-th sampling point and i-th trace in the data volume, t is the number of sampling points in the time window, n is the total trace number of the child data volume, and K_j is the valid trace number of the j-th sampling point.

A similar coherence method is given by (Li et al., 2015)

$$\mathrm{S}_L = \sum_{j=1}^{t} \frac{\left[\sum_{i=1}^{n} A_{ij}\right]^2 - \left(\sum_{i=1}^{n} A_{ij}A_{ij}\right)}{\left(K_j - 1\right)\left(\sum_{i=1}^{n} A_{ij}A_{ij}\right)}. \tag{4}$$

The coherence values are usually calculated by using the conventional coherence method. As shown in Figure 1, after the coherence value of a given child data volume is calculated, the coherence value of the next child data volume is calculated according to a given increment in a given spatial direction. To obtain a smoothing effect, there needs to be some overlap between two adjacent child data volumes.

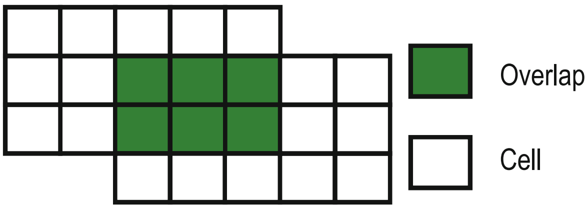

Figure 1. Movement between two child data volumes.

Second, the calculated coherence values can be sorted to optimize the sequencing of the corresponding OVT gathers. Alternatively, the grouping of the coherence values of the OVT gathers can be optimized. The mathematical heap sort method (Fan, 2008; Li et al., 2017; Yu et al., 2016) is used to sort the calculated coherence values in an orderly manner to make the dataset more coherent, as shown in Figure 2. This makes the sorted results more conducive to the selection of N. If is the data to be sorted, then the detailed sorting steps are as follows:

1.A large root heap is built on the raw data to serve as the initial disordered region.

2. Exchange the top elements of the heap (i.e., OVT[1] and OVT[n]) to obtain a new disordered region OVT[1...$n-1$] and ordered region OVT[1...n] while satisfying OVT[1...$n-1$] $\leq$ OVT[n].

3. Adjust OVT[1...n] to make a new heap.

4. Exchange OVT[1] with the last data of the disordered region again to obtain the new disordered region OVT[1...$n-2$] and ordered region OVT[$n-1$...n] while still satisfying OVT[1...$n-1$] $\leq$ OVT[$n-1$...n].

The sorting process continues by repeating steps 3 and 4 until there is only one element OVT[1] in the disordered region. Sorting makes the data more coherent, which helps narrow the selection range of N values for the N-th root slant stack. This reduces the number of tests needed to select the N value, which greatly reduces the computation time and is especially helpful for processing 3D data. During the sequencing process, the relationship between a coherence value and its corresponding OVT gather must be clear to achieve noise suppression. Simply sorting the coherence values without considering the OVT gathers would be meaningless. In other words, the coherence values should be evaluated to determine the optimal N value for suppressing noise.

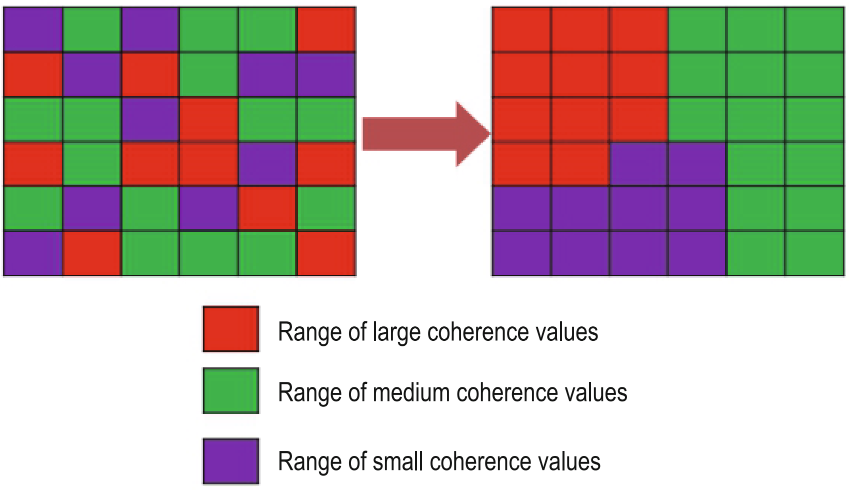

Figure 2. Sorting and grouping coherence values.

Noise suppression

Most methods for denoising seismic data are multi-domain techniques that consider the shot domain, receiver domain, offset domain, etc. Multi-domain denoising takes advantage of the different representation forms of noise in different domains. However, the conversion of data between different domains can cause distortion, which makes subsequent data processing difficult when a partial gather (or shot gather) is accompanied by a maximum value. Thus, alternative processes are being developed to suppress different types of noise in seismic data. Figure 3 shows the concept behind using the N-th root slant stack in the OVT domain to suppress noise in seismic data.

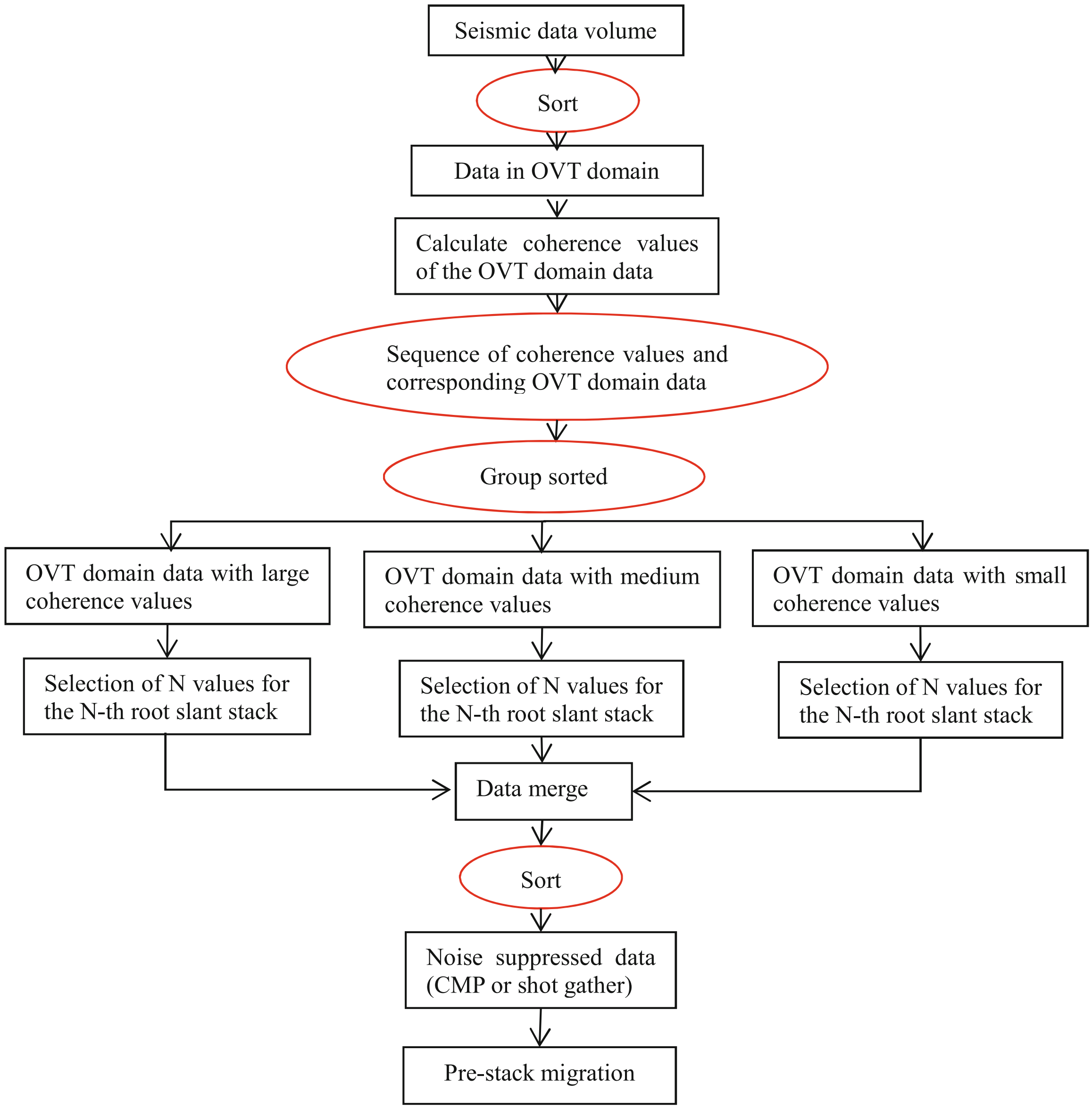

Figure 3. Noise suppression process with the N-th root slant stack.

The N-th root slant stack is mainly used to suppress random noise, and the parameters used to calculate the coherence values are designed for reflected waves. There is a large difference between the dips of surface waves and reflected waves, so some energy of the surface waves is suppressed by the denoising process. Therefore, the N-th root slant stack can not only improve the SNR of reflected waves but also suppress the energy of surface waves to some extent.

Application and results

In Western China, the surface topography is undulating, and the underground faults and steep structures are very developed, which produces complicated reflection paths and weak reflection signals. The complex surface results in a narrow frequency band and low SNR for the received data, which makes seismic imaging difficult to carry out effectively. To improve the image quality, noise suppression is necessary before prestack migration. The proposed N-th root slant stack method was applied to denoising prestack data of the W working area in Western China, and the performance was evaluated by comparison to a multi-domain denoising method.

Figure 4 shows the original data. The SNRs of the original shot gather (Figure 4a) and CMP gather (Figure 4b) were low, and the signal was completely submerged in the noise, which made it difficult to identify. In addition, the continuity and resolution of events in the original stacked section were poor (Figure 4c). Figure 5 shows the denoising results of the multi-domain denoising method, which improved the SNR. However, some noise was still present, and the reflection information was very vague, so the energy of the weak reflection signal needs to be further increased. Figure 6 shows the denoising results of the proposed method. Figure 6a shows the shot gather after coherent sequencing in the OVT domain and the N-th root slant stack with N = 2. In contrast with the original shot gather (Figure 4a) and the shot gather after processing by the multi-domain denoising method (Figure 5a), the weak signal was enhanced, and the SNR was greatly improved. The proposed method also obtained a much better CMP gather (Figure 6b) than the multi-domain

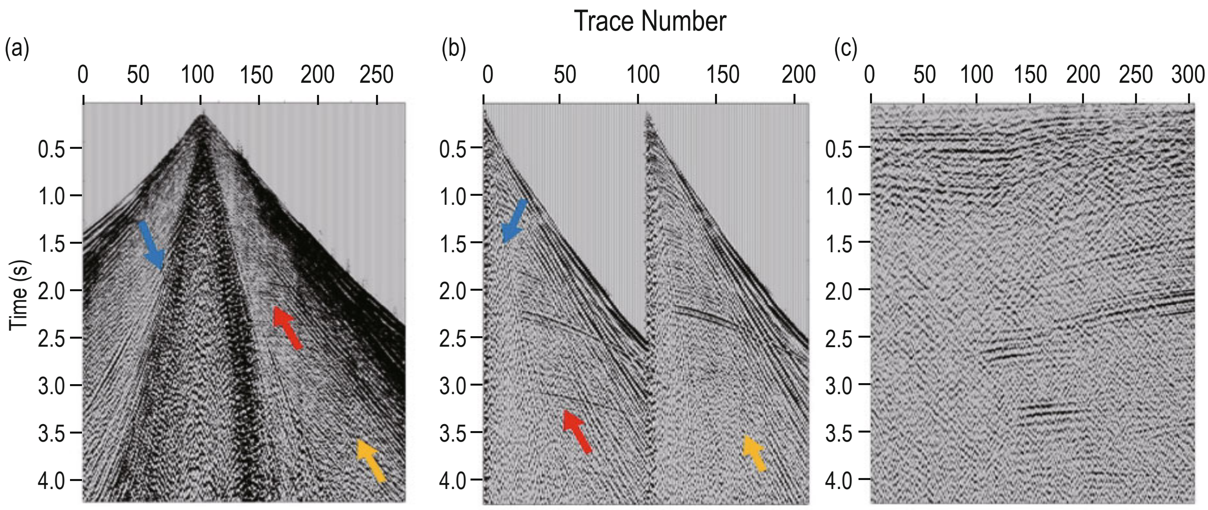

Figure 4. Original data: (a) shot gather, (b) CMP gather, and (C) stacked section. Blue arrows: surface waves. Red arrows: signals. Orange arrows: random noise.

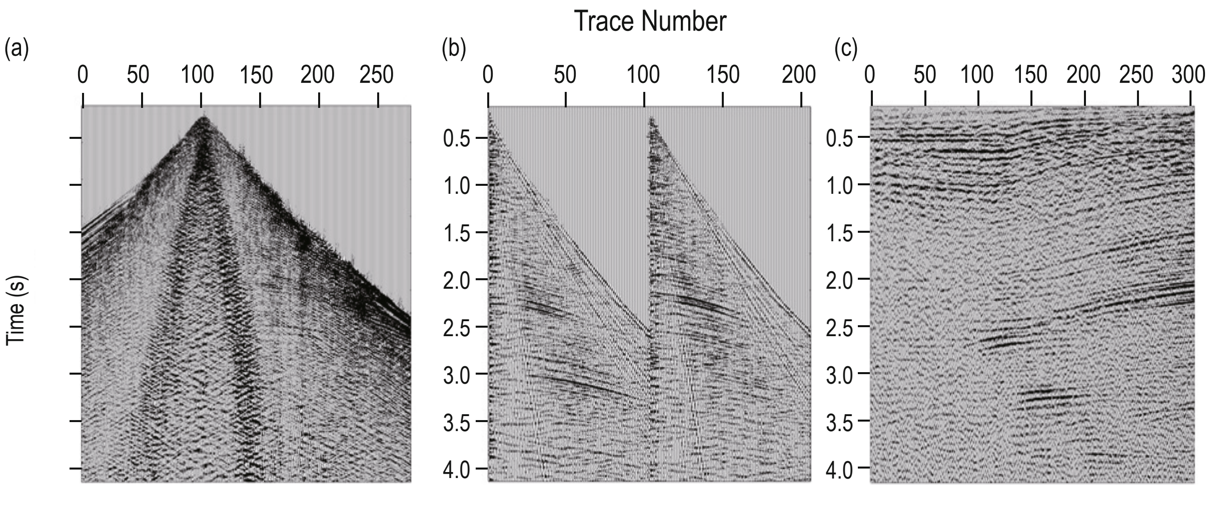

Figure 5. Multi-domain denoising results: (a) shot gather, (b) CMP gather, and (C) stacked section. Blue arrows: surface waves. Red arrows: signals. Orange arrows: random noise.

denoising method (Figure 5b). These results show that the proposed method improves the quality of low-SNR data, which will be helpful for processing seismic imaging data.

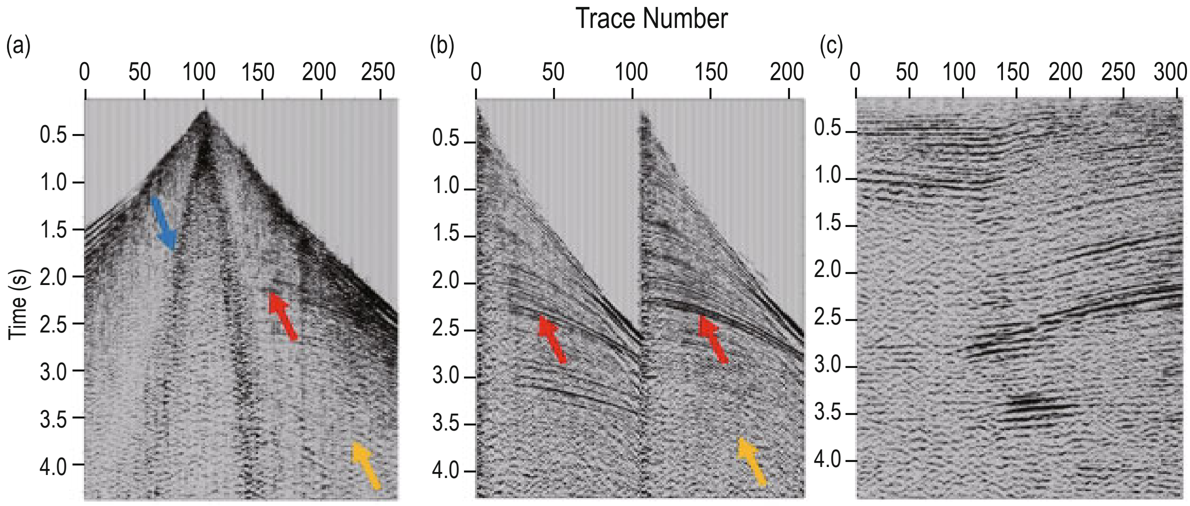

Figure 6. N-th root slant stack denoising results: (a) shot gather, (b) CMP gather, and (C) stacked section. Blue arrows: surface waves. Red arrows: signals. Orange arrows: random noise.

Conclusion

The proposed N-th root slant stack method effectively enhances a weak seismic signal while suppressing random noise. OVT domain data and the corresponding coherence values are sorted to select the optimal N value for the N-th root slant stack to suppress noise. Critically, the sequencing process considers the OVT gather and corresponding coherence value as a new data body. The proposed method can satisfy the unsteady and nonlinear characteristics of seismic imaging signals by using different N values to suppress the maximum values and strengthen a weak seismic signal. If a suitable N value is not chosen, the signal becomes distorted.

Reference

Donoho, D.L.,1993, Progress in Wavelet Analysis and Application :Ten-minute tour, **338**(2), 639−654.

Donoho,D.L.,2002,De-noising by soft-thresholding:IEEE Transactions on Information Theory, **41**(3), 613−627.

Fan, J., 2008, Research on data sorting of massive two-dimensional table: Zhejiang Normal University.

Gao, J. H., Mao, J., Man, W. S., et al.,2006,On the denoising method of pre-stack seismic data in wavelet domain :Chinese Journal of Geophysics, **49**(4), 1155−1163.

Guo, H.N., Tang, H.C., Xu, L., et al,2013, Research and application of high-precision 3D seismic acquisition method under double complex conditions: a case study of HM exploration area in Gaoyou sag, Subei Basin : Acta petroleum and natural gas, **35** (9), 72−75.

Hu, G.B., and Wang, Z.J.,2016, Discussion on several surface wave suppression methods:Petrochemical technology, **23** (3): 73−74.

Hu, T., and White, R. E., 1993, Implementation of MVU beamformers: In: 55th EAEG Meeting, Stavanger, Norway, Extended Abstract ,

Hu, X. H., Ouyang, Y. L., Zeng, Q. C., et al.,2014,De-noising seismic data with pre-stack nonlocal means method: Coal Geology & Exploration, **42**(5), 87−91.

Li, C. Z., Wang, J. D., and Zhang ,L.,2007, Curvelet transform for image denoising based on Bayes shrink:Video En-gineering, **31**(6),14−16.

Li ,F., Duan, W, S., Zhao R. R., et al., 2015,Seismic signal to noise ratio improvement with volume τ-p transform in OVT domain: Oil Geophysical Prospecting. **50**(3), 418−423.

Li, J., Zhang, X. J., Zhu H., et al. , 2017,Person Re-identification via Multiple Confidence Re-ranking: Pattern Recognition and Artificial Intelligence, **30**(11), 995−1002.

Liu ,F.Q.,Zhang, G.Q., 1996, Appliation of wavelet transform and F-K algorithm in filtering:Oil Geophysical Prospecting, **31**(6),782−792.

Liu,T., Li, L.H., Ran, X.Y., et al.,2018 Application of fractional frequency ant tracking method based on deconvolution generalized S transform: Liaoning chemical engineering, **47** (3), 214−216.

Liu, Z. G.,Xie, Y.G., and Chen ,F.,2009,Discussion on noise attenuation methods in seismic data processing:Oil Geophysical Prospecthing, **44**(S1). 67−71.

Lu,W.K.,2006,Adaptive noise attenuation of seismic images based on singular value decomposition and texture direction detection: Journal of Geophysics and Engineering, **3**(3), 28−34.

Mallat , S.,1989, A theory for multiresolution signal decomposition on Pattern: The wavelet representation :IEEE Tran-sections on Pattern Analysis and Machine Intelligence, **11**(7), 674−693.

Kanasewich, E .R., 2012, Nth-root stack nonlinear multichannel filter: Geophysics, **38**(2), 327−338.

McFadden, P. L., Drummond, B. J., and Kravis S.,1986,The N-th root stack: theory, applications and examples :Geophysics, **51**(10): 1879−1892.

Mu X., 2012, Seismic weak signal separation based on blind signal process: Petroleum Geology and Recovery Efficiency, **19**(5), 47−49.

Muirhead, K. J., and Datt ,R., 1976, The N-th root process applied to seismic array data:Geophys. J. R. Astr. Soc., **47**(1), 197−210.

Peng ,C., Zhu, S. J., Sun, J. K., et al.,2007,K-L transformation in wavelet conversion domain the analysis of de-noise effect:Geophysical Prospecting for Petroleum, **46**(2), 112−114.

Wu, Z.C., and Liu ,T.Y. ,2008,Wavelet method in seismic data denoising: Progress in geophysics, **23**(2): 199−205.

Xue ,C.,2019, Research and application of cross array denoising method: Inner Mongolia petrochemical industry, (7), 11−16.

Yang L., Chen W., Liu W., Zha B., Zhu L. Q. 2020, Random Noise Attenuation Based on Residual Convolutional Neural Network in Seismic Datasets[J]. IEEE Access, 8:30271−30286.

Yang L. Q., Chen W., Wang H., Chen Y. K. 2021, Deep Learning Seismic Random Noise Attenuation via Improved Residual Convolutional Neural Network[J]. IEEE Transactions on Geoscience and Remote Sensing, **59**(9): 7968−7981.

Yu, S.J., Gong, X. Q., Zhu, J., et al., 2016,Sorting algorithm analysis of distributed data based on Map/Reduce: Journal of East China Normal University(Natural Science), **14**(5), 121−130.

Zang, S. X., and Zhou, Y.Z., 2002, Nth root slant stack and its application in study of mantle discontinuities: Chinese Journal of Geophysics-Chinese Edition, **45**(3), 407−415.

Zhao, D. B., Huang, Z. P., and Wang,C.M.,1998,Random noise attenuation using predictive filtering in F-X domain by singular value decomposition :Geophysical Prospecting for Petroleum, **37**(3), 29−33.

Zheng J., Jiang T. Q., Wu Z. X., Sun Y. 2021, Application of residual learning to microseismic random noise attenuation[J]. Acta Geophysica, **69**(4): 1151–1161.

Zhong, W., Yang, B. J., and Zhang ,Z.,2006,Research on application of polynomial fitting technique in highly noisy seismic data :Progress in Geophysics, **21**(1),184−189.

Resume of the Author

Li Fei is a CPC member and senior engineer with a doctoral degree in geophysics. He is currently working at the Changqing Oilfield Research Institute. He is engaged in seismic imaging and seismic interpretation research.