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Inversion-based data-driven time-space domain random noise attenuation method*

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Abstract: Conventional time-space domain and frequency-space domain prediction filtering methods assume that seismic data consists of two parts, signal and random noise. That is, the so-called additive noise model. However, when estimating random noise, it is assumed that random noise can be predicted from the seismic data by convolving with a prediction error filter. That is, the source-noise model. Model inconsistencies, before and after denoising, compromise the noise attenuation and signal-preservation performances of prediction filtering methods. Therefore, this study presents an inversion-based time-space domain random noise attenuation method to overcome the model inconsistencies. In this method, a prediction error filter (PEF), is first estimated from seismic data; the filter characterizes the predictability of the seismic data and adaptively describes the seismic data's space structure. After calculating PEF, it can be applied as a regularized constraint in the inversion process for seismic signal from noisy data. Unlike conventional random noise attenuation methods, the proposed method solves a seismic data inversion problem using regularization constraint; this overcomes the model inconsistency of the prediction filtering method. The proposed method was tested on both synthetic and real seismic data, and results from the prediction filtering method and the proposed method are compared. The testing demonstrated that the proposed method suppresses noise effectively and provides better signal-preservation performance.
Keywords: Random noise attenuation, prediction filtering, seismic data inversion, regularization constraint

Introduction

Seismic data are unavoidably mixed with various types of noise, and reflection energy is reduced because of earth filtering and absorption (Futterman, 1962; Li et al., 2015; Li et al., 2016a; Li et al., 2016b); both enhance the contamination of seismic signals with ndom noise. Prediction filtering is a classic and commonly used random noise attenuation method. It assumes that seismic data is predictable; seismic data can be expressed as the convolution of its nearby traces and a prediction filter (PF) (Hornbostel, 1991). The method can be realized in both time-space and frequency-space

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domains and are called t-x and f-x prediction filtering, respectively.

The f-x prediction filtering was first introduced by Canales (1984) and further developed by Gulunay (1986) into f-x deconvolution. Chase (1992) extended the method to a 3-D version and realized f-x-y domain random noise attenuation. However, prediction filtering harms weak signals, to some extent, while suppressing noise in practical applications. Soubaras (1994, 1995) believed that signal damage was caused by model inconsistency before and after filtering and devised the f-x projection filtering method. Li (1995) replaced the 1-D prediction filter, commonly used for conventional random noise attenuation, with a 2-D rectangular PF and realized 3-D seismic data random noise attenuation. Soubaras (2000) extended f-x projection filtering to an f-x-y version, which further improved the signal-preservation ability of the method. Sacchi and Kuehl (2001) estimated a prediction error filter (PEF) and noise using an autoregressive moving average (ARMA) of the signal. They claimed that the projection filter can be estimated by solving the original ARMA problem without introducing the conception of quasi-prediction. Liu et al. (2009) solved the noncausal PEF problem by constructing an ARMA model and solved the additive noise via self-deconvolved projection filtering; this filtering avoided model inconsistency before and after denoising. The 1-D ARMA model was then extended to a 2-D ARMA model, and the 2-D ARMA model was applied to 3-D seismic data random noise attenuation. In addition, the rank attenuation method, which includes the eigen-image and the SSA method, can attenuate random noise (Sacchi, 2009; Oropeza and Sacchi, 2011). The SSA method is also called the Cadzow filtering method (Trickett, 2008) and is considered as another form of f-x prediction filtering (Chen and Sacchi, 2013).

Abma and Claerbout (1995) compared f-x and t-x prediction filtering and proved that t-x prediction filtering attenuates more noise than f-x prediction filtering because the PEF for each frequency component is calculated for the f-x domain and the time length of the filter is similar to that of the seismic data when converting the filter to the time domain; this results in noise spreading throughout the whole time window (Abma and Claerbout, 1995). Conventional t-x domain prediction filtering can only process stationary seismic data. Patching is a commonly used method to handle nonstationary seismic data (Claerbout, 1992). Crawley et al. (1999) calculated a smooth and nonstationary PEF, which produced a better denoising result compared to a rectangular patching method. Sacchi and Naghizadeh (2009) proposed a method to calculate a time-space varying PEF, which continuously adapts to the variation of seismic data. Liu et al. (2015) calculated a prediction filter (PF) based on a nonstationary, auto-regularization process, which improved the accuracy of the predicted seismic data.

Yuan et al. (2012) proposed a t-x domain edge-preserving random noise attenuation method based on Bayesian inversion theory, called Bayesian inversion filtering. In this method, a blocky earth model is introduced as a regularization constraint to the inversion system, and denoised seismic data is calculated directly from noisy data. As a result, the noise attenuation problem is converted into an inversion problem; therefore, this method is of great theoretical significance. However, when the signal-to-noise ratio (SNR) of seismic data is very low and the structure is very complex, this method cannot predict denoised seismic data accurately.

To better preserve signal and overcome the problem of model inconsistency before and after prediction filtering, an inversion-based t-x domain method for random noise attenuation and signal preservation is proposed herein. In this method, a PF is initially calculated in the t-x domain, which describes the predictability of seismic data in space. After the prediction filter is calculated, its corresponding PEF is used as a regularization constraint in the inversion procedure to suppress noise and recover the signal instead of convolving with noisy data directly. The regularization operator is derived from the seismic data and is not given in advance, as in the method proposed by Yuan et al. (2012). Therefore, the proposed method possesses better adaptability for seismic data.

In this study, the prevailing problem of the basic theory of t-x domain prediction filtering is first analyzed and discussed. An inversion-based t-x domain random noise attenuation method, overcoming the model inconsistency problem before and after filtering in conventional methods, is then proposed. Furthermore, the method is extended to the 3-D case. Finally, real seismic data is processed with the proposed method using model tests.

Theory

The time-space domain prediction filtering problem

Hornbostel (1991) discussed and analyzed t-x domain prediction filtering; the basic theory of this method is discussed herein. Assuming 2-D seismic data with clean signal, $s(x, t)$, and noise, $n(x, t)$, we obtain

$$d(x,t) = s(x,t) + n(x,t). \quad (1)$$

Seismic data is predictable in the space domain; therefore, there exists a prediction filter, $h(x, t)$, which satisfies equation (2):

$$d(ix,it) = \sum_{jx=-mx,\, jx\neq 0}^{mx} \sum_{jt=-mt}^{mt} h(jx,jt)\, d(ix-jx, it-jt), \quad (2)$$

where ix and jx are spatial indices for seismic data; it and jt are temporal indices for seismic data; and mx and mt indicate the size of the 2-D prediction filter in the space and time directions, respectively. If $mx = 2$ and $mt = 2$, then the prediction filter has the following form (Abma and Claerbout, 1995):

$$\begin{matrix} h_{-2,-2} & h_{-1,-2} & 0 & h_{1,-2} & h_{2,-2} \\ h_{-2,-1} & h_{-1,-1} & 0 & h_{1,-1} & h_{2,-1} \\ h_{-2,0} & h_{-1,0} & 0 & h_{1,0} & h_{2,0} \\ h_{-2,1} & h_{-1,1} & 0 & h_{1,1} & h_{2,1} \\ h_{-2,2} & h_{-1,2} & 0 & h_{1,2} & h_{2,2} \end{matrix}. \quad (3)$$

The prediction filter, $h(x, t)$, can be calculated from noisy data by minimizing the following objective function:

$$J = \sum_{ix}\sum_{it}\left\| d(ix,it) - \sum_{jx=-mx,\, jx\neq 0}^{mx} \sum_{jt=-mt}^{mt} h(jx,jt)\, d(ix-jx, it-jt)\right\|^2. \quad (4)$$

Based on the prediction filter, $h(x, t)$, the denoised seismic data, $\bar{s}(x,t)$, is obtained by calculating the 2-D convolution of the prediction filter and noisy seismic data, $d(x, t)$:

$$\bar{s}(x,t) = d(x,t) * h(x,t), \quad (5)$$

where, the noise can be predicted as

$$\bar{n}(x,t) = d(x,t) * p(x,t), \quad (6)$$

where $p(x, t) = \delta(x, t) - h(x, t)$ represents the PEF.

This is the basic theory of the prediction filtering method and the basic procedure to estimate noise and signal. Further extending equation (6) provides

$$\begin{aligned} \bar{n}(x,t) &= [s(x,t) + n(x,t)] * p(x,t) \\ &= n(x,t) * p(x,t). \end{aligned} \quad (7)$$

Equation (7) shows that the noise estimated by prediction filtering is not true noise; it is the convolution of the PEF and noise. This is because in equation (1) the noisy data consists of signal and noise (additive-noise model), whereas noise is considered as the convolution of the PEF and the noisy data in equation (6) (source-noise model). Thus, the two noise models are inconsistent before and after filtering. Noise model inconsistency not only decreases the accuracy of the prediction filter but also decreases the method's noise attenuation and signal-preservation performances.

Inversion-based t-x domain random noise attenuation

The prediction filter, $h(x, t)$, can't be used to obtain denoised results by convolving with noisy data calculated by minimizing equation (4). For this problem, the corresponding PEF of the prediction filter should be calculated and used as a regularization constraint in the inversion procedure; the inversion result is the denoised result. The objective function of the inversion problem can be expressed:

$$\begin{aligned} J = &\sum_{ix}\sum_{it}\left[d(ix,it) - \bar{s}(ix,it)\right]^2 \\ &+ \lambda \left\| \sum_{jx}\sum_{jt} p(jx,jt)\, \bar{s}(ix-jx, it-jt)\right\|_L^k, \end{aligned} \quad (8)$$

where L is norm and k is index. When $L = 1$, the second term is l_1 norm, and when $L = 2$, the second term is l_2 norm (λ is a trade-off parameter). The first term describes the misfit between estimated and original seismic data, which can also be taken as the energy of the estimated noise. The second term describes the predictability of the seismic data. When $L = 2$, rewriting equation (8) to matrix form results in

$$J = \|\mathbf{d} - \bar{\mathbf{s}}\|^2 + \lambda \|\mathbf{p}\bar{\mathbf{s}}\|^k, \quad (9)$$

and making $k = 2$, equation (9) becomes

$$J = \|\mathbf{d} - \bar{\mathbf{s}}\|^2 + \lambda \|\mathbf{p}\bar{\mathbf{s}}\|^2, \quad (10)$$

by taking the derivative of equation (10) with regard to $\bar{\mathbf{s}}$ and making its derivative equal to 0. The estimated signal can then be expressed by equation (11):

$$\bar{\mathbf{s}} = \left(\mathbf{I} + \lambda \mathbf{p}^T \mathbf{p}\right)^{-1} \mathbf{d}. \quad (11)$$

When applied, the PEF, $p(x, t)$, is calculated from the noisy data. However, it cannot predict the signal perfectly. Thus, by first calculating a PEF from the noisy seismic data, first-estimated denoised seismic data, $\overline{s}_1(x,t)$, can then be obtained by solving equation (10). A new PEF, $p_1(x, t)$, can then be recalculated from $\overline{s}_1(x,t)$, and second-estimated denoised seismic data, $\overline{s}_2(x,t)$, can be obtained. After several iterations, the final denoised seismic data, $\overline{s}(x,t)$, is acquired.

Yuan et al. (2012) proposed a Bayesian inversion filtering method to suppress random noise. The basic theory assumes that seismic data is blocky in space, which means the first derivatives of seismic data along both space and time directions are sparse. In this case, the objective function is

$$J = \left\|\mathbf{d} - \overline{\mathbf{s}}\right\|^2 + \lambda\left\|\nabla\overline{\mathbf{s}}\right\|_1, \tag{12}$$

where $\nabla = (-1,1)$ is the difference operator in the space or time direction. Comparing equations (9) and (12), when the 2-D PEF degenerates to 1-D and $p(x)= (-1, 1)$, the two filters exhibit exactly the same formation; Equation (12) can then be considered as a special case of equation (9). The method prescribed by Yuan et al. (2012) is based on the blocky model, and the PEF is given before filtering and its value is fixed; therefore, their method is an inversion-based model-driven random noise attenuation method. In contrast, the proposed method directly calculates the PEF, $p_1(x, t)$, from seismic data. Thus, the proposed method is an inversion-based data-driven random noise attenuation method, and the PEF describes the space variation of the seismic reflection structure. The filter driven mode is the essential difference between the two methods.

For 3-D seismic data, the PEF exhibits the form shown Figure 1, where the dark-shadowed area in the center is the position of the seismic data being predicted, and the other data samples, except for the light-shadowed parts, are used to predict the data sample in the center. After the 3-D prediction filter is calculated, for a value in the center equal to −1 and the other values unchanged, the 3-D PEF can be obtained. The filter describes the predictability of seismic data in 3-D space. It can suppress noise and recover 3-D seismic data signal when used as a constraint in equation (9).

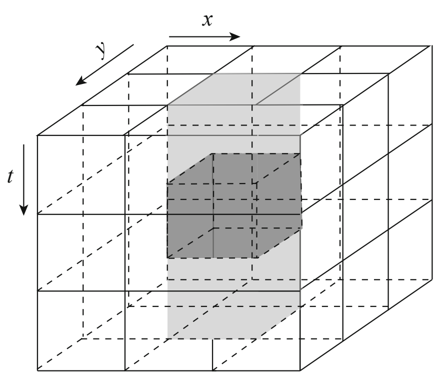


Fig.1 3-D prediction error filter (PEF).

Synthetic model tests

To test the validity of the inversion-based random noise attenuation method, 2-D and 3-D model tests were performed. The synthetic models were composed of lateral, tilt, and curved events, which are common event types in real seismic data. The determination of filter size and the selection of trade-off parameters are briefly discussed.

2-D synthetic data random noise attenuation test

The synthetic data shown in Figure 2a consisted of three linear events and two curved events and each event consisted of a different dip. A 40-Hz, zero-phased Ricker wavelet was used with a time sample interval of 4 ms. The seismic data comprised 200 traces with 501 samples in each trace. Figure 2b shows the noisy seismic data; the SNR of the noisy data is 1. Figures 2c and 2d represent the denoised result from f-x prediction filtering and its corresponding removed noise section, respectively. Figures 2e and 2f denote the denoised result of t-x prediction filtering and its corresponding removed noise section, respectively. Figures 2g and 2h illustrate the denoised result of inversion-based t-x domain random noise attenuation and its corresponding removed noise section, respectively. These three methods effectively removed the noise and preserved the signal of the linear events with small dips (Figures 2c, 2e, and 2g). However, prediction filtering reduces the energy of linear events with big dips and curve events. The energy of the denoised seismic data was very weak, and obvious signals existed in the removed noise sections (Figures 2d and 2f). In contrast, the proposed inversion-based t-x domain random noise attenuation method not only preserved the energy of the linear events with big dips but also preserved the energy of the curved events; there was very little signal in the removed noise section (Figure 2h).

For inversion-based t-x domain random noise attenuation, it is important to verify the methods to determine the filter size and select a trade-off parameter so that a relatively accurate PEF can be calculated. To

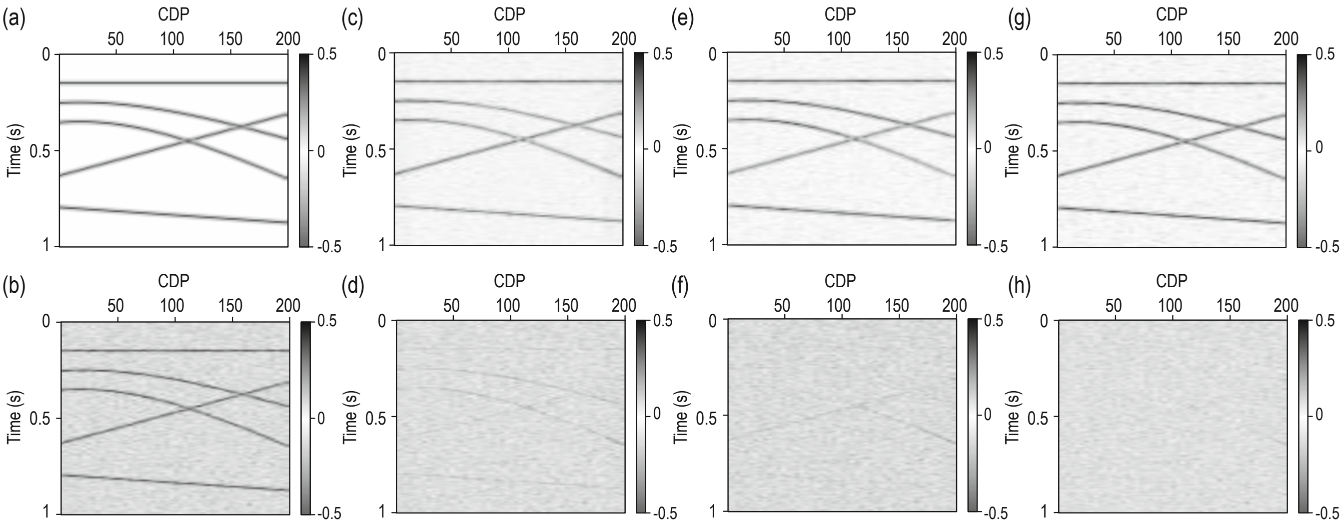


Fig.2 2-D synthetic seismic data random noise attenuation test.
(a) Clean synthetic seismic data; (b) noisy synthetic seismic data (SNR = 1); (c) denoised result from f-x prediction filtering; (d) removed noise section corresponding to (c); (e) denoised result from t-x prediction; (f) removed noise section corresponding to (e); (g) denoised result from inversion-based t-x domain random noise attenuation; and (h) removed noise section corresponding to (g).

determine the filter size, usually enough seismic traces in the space direction are chosen so that the signal can be well predicted. Filter size in the time direction should be determined by the dip of the seismic events. A larger-sized filter is needed in the time direction if the dip of the event is large. If the dip of the event is not very steep, the denoised result is not sensitive to the size of the filter in the time direction (Abma and Claerbout, 1995). The trade-off parameter, λ, should not be too large or too small; if it is too large, the function of the PEF is stronger and the signal is easily harmed; if λ is too small, it cannot remove noise effectively. In addition, λ should be increased with decreasing SNR. In this synthetic data example, the filter sizes chosen were 8 and 5 in the time and space directions, respectively, and the value of λ was 1.

To determine the iteration number, after a few tests, it was found that the value of the cost function converged for general SNRs normally after 10 iterations. Figure 3 demonstrates the changes of the objective function values with the number of iterations for this example.

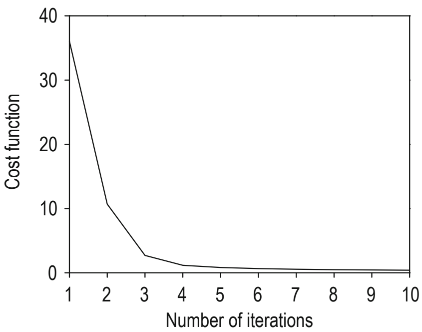


Fig.3 Changes of objective function values with number of iterations.

3-D synthetic data random noise attenuation test

A 3-D model was used to test the inversion-based t-x-y domain random noise attenuation. The model consisted of two linear events and a curved event, and each event comprised different dips. A 40-Hz, zero-phased Ricker wavelet was used with a time interval of 2 ms to synthesize the clean data shown in Figure 4a. Each trace consisted of 501 samples, and the seismic data consisted of $Nx \times Ny$ traces, where $Nx = 20$ and $Ny = 20$. Figure 4b shows the noisy data with a SNR of 1, which was obtained by adding Gaussian noise to the clean data shown in Figure 4a. For this test, the size of the chosen prediction filter was 5, 3, and 3 in the time, xline, and inline directions, respectively, and the value of the trade-off parameter was 5. Figure 4c shows the denoised result from t-x-y prediction filtering, whereas Figure 4d demonstrates the denoised result from inversion-based t-x-y domain random noise attenuation. Figures 4c and 4d indicate that both of these methods removed noise effectively; however, the prediction filtering method also removed some signal, and a small amount of random noise remained, as shown in Figure 4c. In contrast, the signal energy was strong and the whole profile was very clean in Figure 4d.

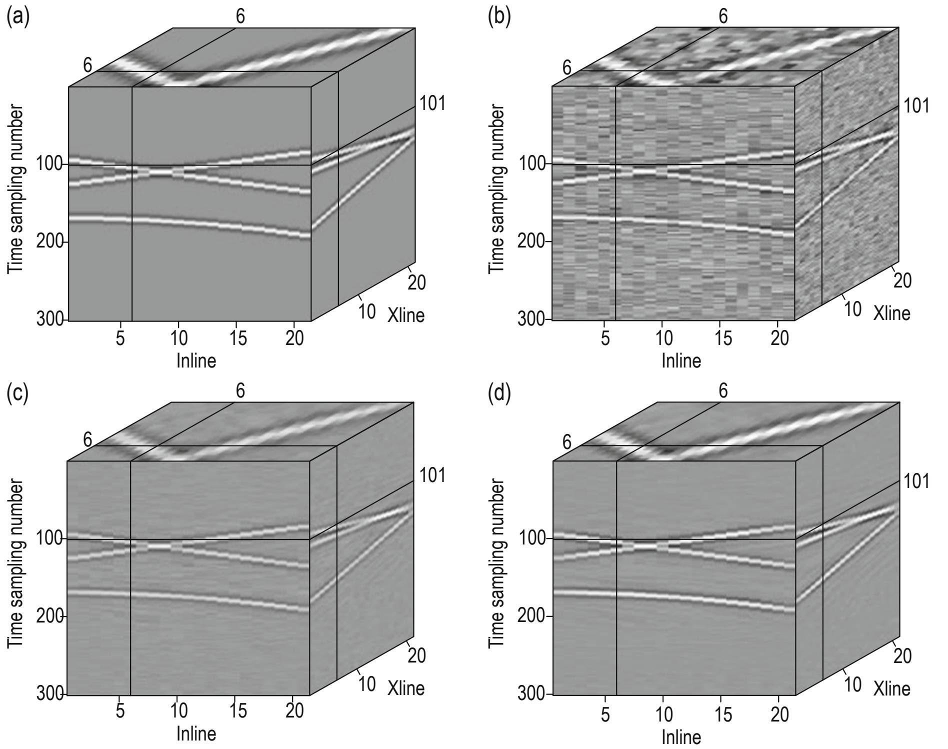
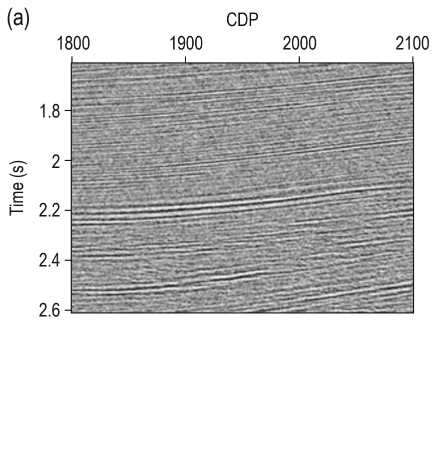


Fig.4 3-D synthetic seismic data random noise attenuation test.
(a) Clean synthetic seismic data; (b) noisy synthetic seismic data (SNR = 1); (c) denoised result from t-x-y prediction filtering; and (d) denoised result from inversion-based t-x-y domain random noise attenuation.

Field data examples

2-D field data example

The proposed method was applied to field data acquired from oilfields located in East China (Figure 5a). The field area is located in a depression zone between two faults and contained strong random noise, indicating a low SNR. There were 200 traces with 501 samples in each trace with a time interval of 2 ms. The events' dips were relatively small in comparison with the dip events in the synthetic data; therefore, the filter sizes were 5 and 3 in the time and space directions, respectively, and the trade-off parameter, λ, was 2. Figures 5b and 5c show the denoised result from f-x prediction filtering and its corresponding removed noise section, respectively. Figures 5d and 5e demonstrate the denoised result from t-x prediction filtering and its corresponding removed noise section, respectively. Figures 5f and 5g illustrate the denoised result from inversion-based t-x domain random noise attenuation and its corresponding removed noise section, respectively. Comparing the denoised results and the removed noise sections from these three methods, the prediction filtering method removed the noise effectively, but it also removed some signals; there were some significant signals in the removed noise section (Figures 5c and 5e). Comparing Figures 5b, 5d, and 5f, it was observed that the denoised result from the proposed method exhibited the highest resolution (indicated by the red arrows in Figures 5b, 5d, and 5f). The f-x and t-x prediction filtering methods smoothed the subtle structures, which decreased the resolution of the seismic data. The proposed method achieved a superior trade-off between random noise attenuation and amplitude preservation for effective signals. Therefore, the proposed method was better able to preserve the weak signals and subtle structures of the noisy seismic data, and it demonstrated better signal-preservation ability and fidelity than the other methods.



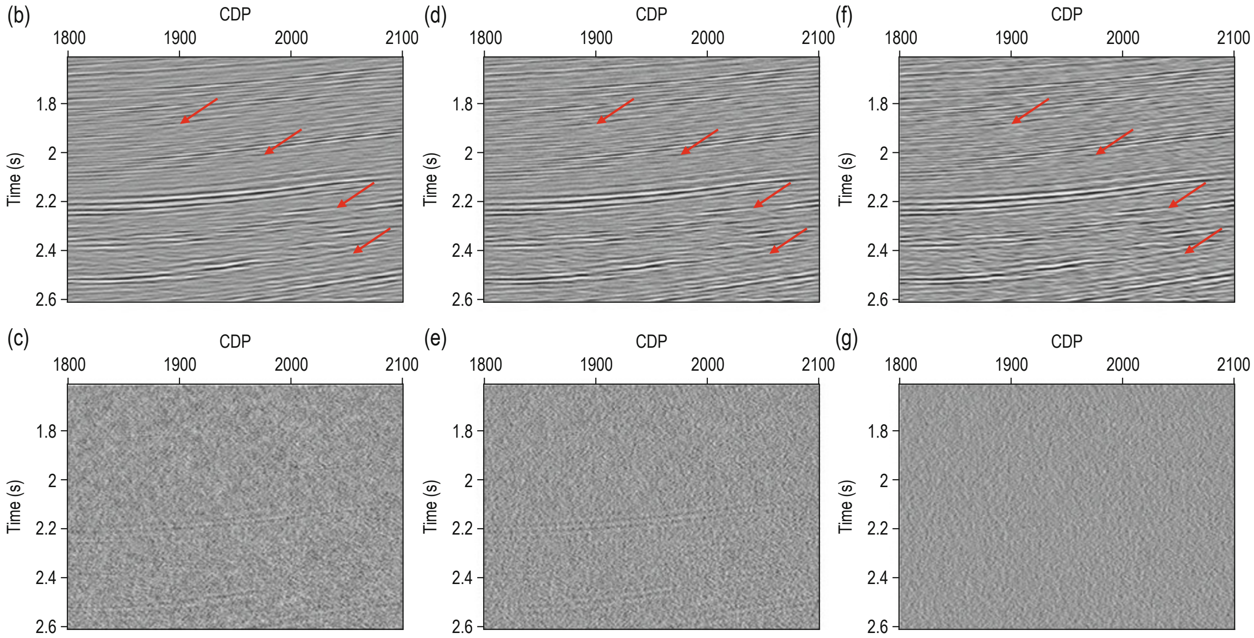


Fig.5 2-D seismic field data processing example.

(a) 2-D seismic field data; (b) denoised result from f-x prediction filtering; (c) removed noise section corresponding to (b); (d) denoised t-x prediction filtering; (e) removed noise section corresponding to (d); (f) denoised result from inversion-based t-x domain random noise attenuation; and (g) removed noise section corresponding to (f).

3-D field data example

Figure 6a shows the 3-D seismic field data. The events were discontinuous, and there were many weak signals that overwhelmed by random noise. There were 1001 samples in each trace, and the time interval was 2 ms. The trace numbers were $Nx \times Ny = 1701$, where $Nx = 81$ and $Ny = 21$. The filter sizes were 5 for the time direction and 3 in the both xline and inline directions. Figures 6b and 6c demonstrate the denoised inversion-based t-x domain random noise attenuation result and its corresponding removed noise section, respectively. The proposed method improved the continuity of the seismic events and preserved the weak signals (Figure 6b). The denoised result exhibited higher resolution and fidelity, and effective signals were barely seen in Figure 6c.

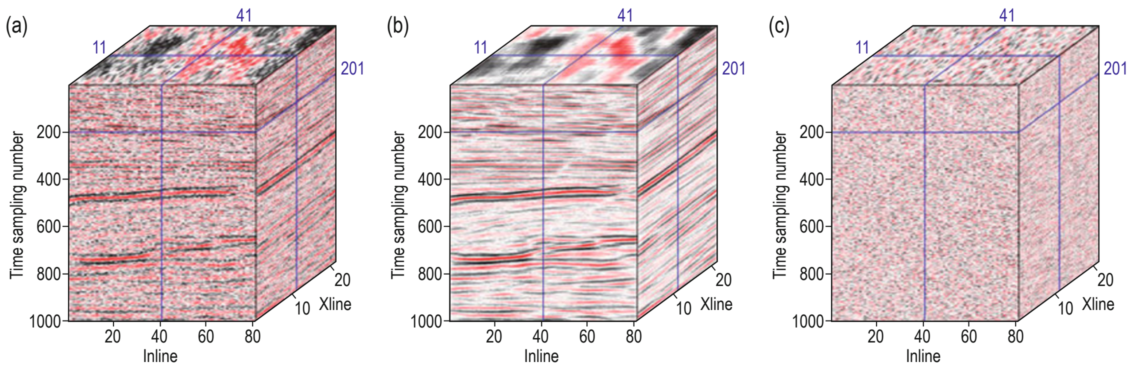


Fig.6 3-D seismic field data processing example.

(a) 3-D seismic field data; (b) denoised inversion-based t-x-y domain random noise attenuation; and (c) removed noise section corresponding to (b).

Conclusions

When suppressing noise, seismic signal damage should be reduced as much as possible and the kinetic characteristics of the seismic reflection kept relatively intact so that high quality data is available for attribute analysis and seismic inversion processes. In line with the basic idea of random noise attenuation, an inversion-based data-driven time-space domain random noise attenuation method was proposed. Differing from conventional prediction filtering methods, a prediction filter was used to describe signal predictability and bring its corresponding PEF to the seismic data inversion system and inverse the signal directly from the noisy data; the prediction filter improved signal preserving performance. In addition, compared with model-driven seismic data inversion, the proposed method calculates a prediction filter from seismic data adaptively and uses it to characterize the space structure of seismic data; therefore, this proposed method performs better for complex seismic data. Because the proposed method requires simultaneous retrieval of all sample points in a 3-D time window, it improves denoising accuracy but concurrently reduces computational efficiency to some extent. Therefore, low computational efficiency is the method's main obstacle to practical application. Finding methods to improve computational efficiency is the key for our future research.

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