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Digital core based transmitted ultrasonic wave simulation and velocity accuracy analysis*

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Abstract: Transmitted ultrasonic wave simulation (TUWS) in a digital core is one of the important elements of digital rock physics and is used to study wave propagation in porous cores and calculate equivalent velocity. When simulating wave propagates in a 3D digital core, two additional layers are attached to its two surfaces vertical to the wave-direction and one planar wave source and two receiver-arrays are properly installed. After source excitation, the two receivers then record incident and transmitted waves of the digital rock. Wave propagating velocity, which is the velocity of the digital core, is computed by the picked peak-time difference between the two recorded waves. To evaluate the accuracy of TUWS, a digital core is fully saturated with gas, oil, and water to calculate the corresponding velocities. The velocities increase with decreasing wave frequencies in the simulation frequency band, and this is considered to be the result of scattering. When the pore fluids are varied from gas to oil and finally to water, the velocity-variation characteristics between the different frequencies are similar, thereby approximately following the variation law of velocities obtained from linear elastic statics simulation (LESS), although their absolute values are different. However, LESS has been widely used. The results of this paper show that the transmission ultrasonic simulation has high relative precision.

Keywords: digital rock, transmitted ultrasonic wave simulation, velocity, relative precision

Introduction

A considerable amount of research has been conducted on the simulation of physical processes within a digital core, but research has only been conducted over the past 10 years on transmitted ultrasonic wave simulation (TUWS) in a digital core. TUWS was first developed by Saenger (2004), Saenger et al. (2007), and Saenger (2008), where the rotated staggered grid finite difference method (RSGFDM) was used to calculate the equivalent velocity of a digital core saturated with non-viscous fluid. This was subsequently upgraded by Saenger et al. (2005) and Saenger et al. (2011), where the constitutive relation of Newtonian fluid was approximately expressed as that of a generalized Maxwell body, which improved the ability to simulate a wave in realistic media. Zhang et al. (2010) then directly solved a coupled system of a

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Naiver–Stokes equation and an elastic wave equation, which avoided excessive energy dissipation due to utilization of the generalized Maxwell body. Madonna et al. (2012) calibrated velocity from TUWS to that of experimental velocity by changing the elastic moduli of the grain contact boundary, and Wang et al. (2015) used TUWS to study the relationship between velocity and pores in a carbonate rock.

Whether the transmission ultrasonic simulation based on digital core can be used in rock physics depends on its simulation precision. Although the simulated velocity determined in Madonna et al. (2012) are basically the same as the measured velocity, the millimeter-sized digital core is generally much smaller than the centimeter-sized rock sample used in ultrasonic testing. The simulation velocity and measured velocity actually correspond to different physical models, and calibrating velocities by adjusting the elastic moduli of a grain contact boundary is not a unique method. Thus, a simple comparison of the two velocities will not give a reasonable evaluation of the accuracy of TUWS. However, linear elastic statics simulation (LESS) of a digital core is a widely applied method for use in calculating equivalent elastic moduli and velocities (Arns et al., 2002; Knackstedt et al., 2003; Liu et al., 2009; Liu, 2010; Andrä et al., 2013; Sun et al., 2014; Zhao et al., 2014; Zhu and Shan, 2014). Therefore, by making a comparison between the results of TUWS and LESS when the same digital core is used, it may be possible to determine the accuracy of TUWS. Although, Andrä et al.(2013) have done several numerical experiments including LESS and TUWS to compare their results, they did not analyze the effect of an factor-source frequency on the result of TUWS.In this study, TUWS is used to calculate velocities of a fluid saturated digital core at different source frequencies, and the precision of TUWS is evaluated by analyzing the variation law of resulting velocities and comparing with that of LESS, so that the cause of velocities variation is found out from the view point of source frequency .

Principle of transmitted ultrosonic wave simulation

Construction of forward model

It is not possible to remove the boundary effect on the wave field in finite-sized numerical models. Therefore, to avoid disturbance from boundary reflection, the digital core is pre-processed to construct a forward model using the following methods. Firstly, a homogeneous solid layer (known as "additional layers" below) are added separately to two surfaces of the digital core, perpendicular to the wave propagating direction, where the elastic property of the additional layers is identical to the matrix mineral of the digital core, and their thickness are related to source parameters, they can be half the length of the source waveform in space domain. Secondly, the periodic boundary condition is applied on the surfaces of the digital core and additional layers, parallel to the wave propagating direction. Thirdly, a planar wave source is then located at one end-surface of the model while the other end-surface receives no special treatment when the additional layers are thick enough. Fourthly, two receiver-arrays are then installed at interfaces between the digital core and additional layers, where the one located next to the source records the incident wave and the other one records the transmitted wave. A forward model constructed using these steps is shown in Figure 1.

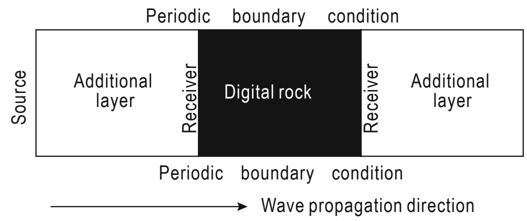

Fig.1 Sketch map of forward model.
The digital core (light black region) is in the center of the forward model and the white regions at both sides are the additional layers. The planar wave source is located at one end-surface of the model. An excited planar wave propagates along with the direction indicated by the black arrow. Two receiver-arrays are located at the interfaces between the digital core and additional layers. The periodic boundary condition is applied on the model's surface parallel to the wave propagating direction.

The periodic boundary condition is applied on the surfaces parallel to the wave propagating direction, and the model is thus infinite in a direction vertical to the wave propagating direction. Therefore, the velocity calculated by TUWS is the P-wave velocity or S-wave velocity.

Constitutive equations of matrix mineral and pore fluids

The solid matrix of the digital core and additional layers are made up of homogeneous, isotropic, and linear elastic mineral, and their constitutive equation can be expressed as

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij} \quad i,j,k = 1,2,3, \tag{1}$$

where σ_{ij}, ε_{ij} are stress and strain components, the repeated subscripts represents summation. λ and μ are láme constants; $\mu = G$, $\lambda = K-2\mu/3$, and G, K are the shear modulus and bulk modulus, respectively.

The pore space is saturated with Newtonian fluid, and its constitutive equation can be written as

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + \eta_{\lambda}\delta_{ij}\dot{\varepsilon}_{kk} + 2\eta_{\mu}\dot{\varepsilon}_{ij} \quad i,j,k = 1,2,3, \tag{2}$$

where $\dot{\varepsilon}_{ij}$ are strain rate components, η_λ and η_μ are viscosity, and where we assume that $\eta_\lambda = \eta_\mu$, $\lambda = K$.

Rotated staggered grid finite difference method

The staggered grid finite difference method (SGFDM) is one of the most commonly used methods in seismic wave modeling. However, as elastic moduli, density, stress, velocity, and displacement are placed at different positions within grids, SGFDM is inappropriate for modeling wave propagation in a strongly heterogeneous digital core. RSGFDM is a special SGFDM, where the elastic moduli, density, and stress are all located at central of grid, and velocity and displacement are placed at the grid corners. Therefore, when updating velocity, displacement, and stress, the only averaged argument between different grids is density. Differentiation computation involves two steps. Firstly, differentiation is computed along four diagonal directions of cubic grids and it is then converted to the differentiation of coordinate directions. The interface between the solid matrix and pore fluid is handled automatically by RSGFDM, and a detailed description of RSGFDM can be found in Saenger et al. (2000).

Method of calculating ultrasonic wave propagating velocity

The propagating velocity of ultrasonic wave in a digital core, which is the velocity of digital core, is the ratio of propagated distance to propagated time and can be expressed as

$$V = \frac{L}{T_t - T_i}, \tag{3}$$

where V presents the wave propagating velocity, L represents propagated distance (which is the distance between the two receiver-arrays), T_i represents the arrival time of the incident wave, and T_t represents the arrival time of the transmitted wave. The type of V is determined by source polarization; it is a P-wave velocity if the polarization direction is consistent with the wave propagating direction and is an S-wave velocity if the polarization direction is perpendicular to the wave propagating direction.

Parallel computation based on GPU

When simulating an ultrasonic wave propagating in a 3D digital core, the dominant wavelength (wavelength corresponding to the dominant frequency) of the source should be much larger than the heterogeneity scale of the digital core. In addition, to avoid interference from boundary reflection, the additional layers need to be thick. As a result, the grid amount of the forward model is huge, the computational load is very heavy, and parallel computation is imperative.

Parallel computation based on a graphic process unit (GPU) can be easily realized at a low cost, and a PC with mid- and high-end graphic cards for gaming can be used as a high performance parallel computation device. A comparison between a GPU based parallel C-code and a CPU based serial MATLAB-code shows that the parallel one is at least five times faster than the serial one.

Numerical experiments

Numerical experiments design

The digital core is the foundation for simulating ultrasonic waves in computers and a large number of mathematical and experimental methods have been developed to construct digital cores. The digital core in Figure 2 is an output from a CT imaging method, where all isolated pore and matrix voxels are deleted. The edge

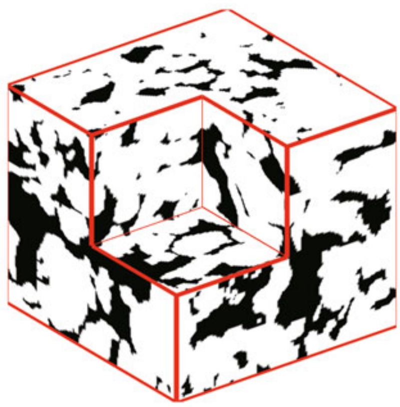

Fig.2 Digital core.
The white region in the digital core represents the solid matrix and the black region represents pore space in the figure. The solid matrix is made up of homogeneous, isotropic, and linear elastic minerals, and the pore fluids are gas, oil, or water.

length of the cubic digital core is 150 voxels (grid points, gps), and the edge length of each cubic voxel is 5 μm.

To simulate wave propagation in the digital core, the elastic moduli, viscosity, and density must be assigned to every voxel in advance. The solid matrix is an homogeneous, isotropic, and linear elastic mineral and the gas, oil, or water in the pore is a Newtonian fluid. Details pertaining to the matrix and fluids parameters are shown in Table 1.

Table 1 Physical parameters of matrix and pore fluids of digital core

Mineral/fluid	Bulk modulus (GPa)	Shear modulus (GPa)	Viscosity (Pa.s)	Density (kg/m^3)
Matrix	37.0	44.0	—	2650.0
Gas	0.022	0.0	2.0×10^{-5}	100.0
Oil	1.30	0.0	1.0×10^{-2}	800.0
Water	2.20	0.0	1.0×10^{-3}	1000.0

To determine the velocity corresponding to the domain of the effective medium, the maximum frequency used in the simulation needs to be restricted. In this respect, we designed three groups of numerical experiments (based on the digital core in Figure 2) to observe variations in the results. Details of these numerical experiments are shown in Table 2.

Table 2 Schemes used for ultrasonic wave simulation in 3D digital core

Experiment	Dominant frequency (MHz)	Additional layer thickness (gps)	Simulation time (μs)	Snapshot time (μs)
A	3.5	350	1.0	0.65
B	2.5	500	1.4	—
C	2.0	500	1.6	—

Wave field snapshots

The dominant wavelength of the planar wave is much longer than the heterogeneity scale of the digital core. Under such a condition, the displacement component vibrating in the wave propagating direction should reflect the characteristics of the wave field. The snapshots shown in Figure 3 correspond to a dominant frequency of 3.5 MHz. The snapshots of the oil-saturated and water-saturated digital core are similar and the space-variation of the displacement is smooth. However, the snapshot of the gas-saturated digital core is quite different from that of the oil- and water-saturated core and the space-variation of displacement is strong. This phenomenon indicates variation in the heterogeneity of

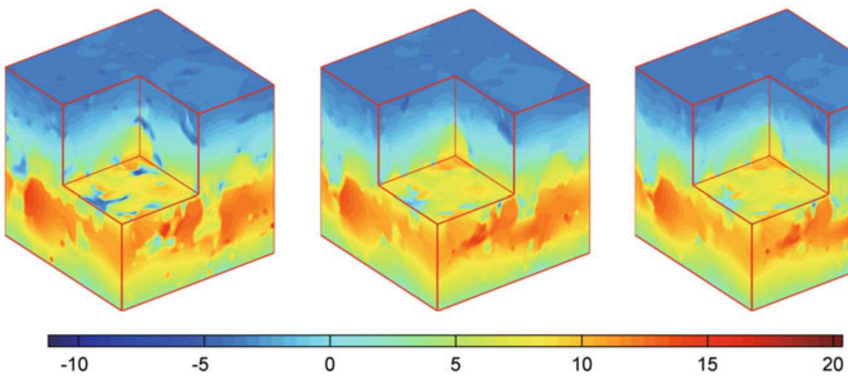

Fig.3Wave field snapshots of saturated digital core.

Top-left: gas-saturated; Top-middle: oil-saturated; Top-right: water-saturated.Bottom: color bar of wave field snapshots. Numbers in the color bar represent relative wave field value that has no unit. Dominant frequency of the simulating source is 3.5MHz, thickness of the additional layers is1.75 mm, and the snapshot time is 0.65 μs.

the digital core. The modulus of gas is far smaller than that of oil and water, and as a result the heterogeneity of the gas-saturated digital core is stronger.

Incident and transmitted waves

Two receiver-arrays are placed at two sides of the digital core, and the receiver next to the source records the incident wave and the other array records the transmitted wave. The waveforms recorded by the different receivers within an array are different from each other and they are thus averaged to one waveform (the average incident and transmitted waveforms are shown in Figure 4). The transmitted waveforms in different saturation conditions are found to be almost the same, and although it is difficult to determine differences between first-arrivals, the differences between the peaks are comparatively obvious. The peak-time of the gas-saturated is the smallest and that of the oil-saturated is the largest, whereas incident waves of both the gas-, oil-, and water-saturated are identical. However, the recorded incident waveforms are contaminated by reflection and scattering from interior interferes of the digital core, and thus do not exactly reflect the true ones. Therefore, the incident waveforms shown in Figure 4 are those recorded when the digital core was replaced using an homogeneous medium.

Ultrasonic wave propagating velocity

The propagating velocity of the ultrasonic wave is the ratio of propagated distance to propagated time, where the propagated distance is the length of the digital core. According to the above section, the propagated time is the difference between the peak-times of the transmitted and incident waves. The velocities (known as "dynamic velocities" below) are calculated by Formula (3) and are shown in Figure 5, where two important features can be seen. The first is that the dynamic velocities at different frequencies exhibit similar variation laws: the velocities of the gas-saturated condition are larger than that of the water-saturated, which are larger than the oil-saturated. The second feature is that the velocity is larger at a lower frequency within identical saturation conditions. The velocities from LESS (known as "static velocities" below) are also shown in Figure 5, where it can be seen that although the variation laws of dynamic and static velocities are similar, those of static velocities are larger. The relative differences between the dynamic velocities at a dominant frequency of 2.0 MHz and the static velocities of gas-, oil- and water-saturated digital core are 2.10%, 2.17%, and 2.18%, respectively.

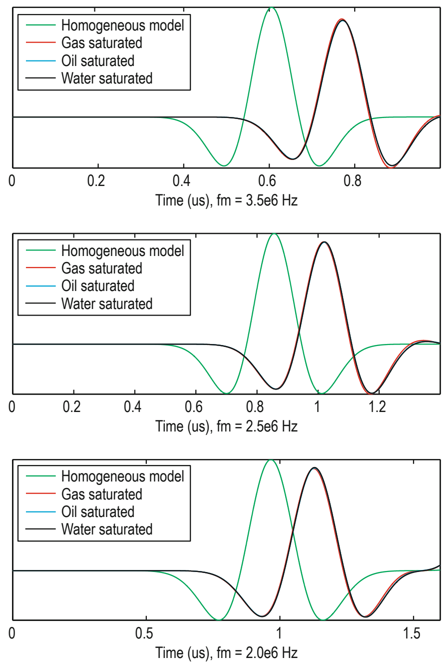

Fig.4 Incident and transmitted waveforms of digital core. Top: Dominant frequency is 3.5 MHz; Middle: Dominant frequency is 2.5 MHz; Bottom: Dominant frequency is 2.0 MHz. The waveform labeled "homogeneous model" represents the condition when the digital core was replaced using homogeneous media, and the other curves represent transmitted waves.

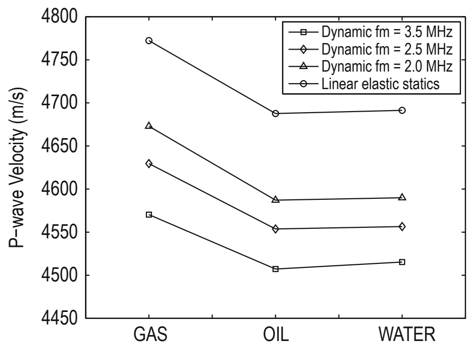

Fig.5 Dynamic and static velocities of saturated digital core. In the legend "dynamic" represents dynamic velocities, "fm" represents the dominant frequency, and "linear elastic statics" represents static velocities. Data points linked by the same solid lines correspond to same dominant frequency.

Precision analysis

We have found that velocities from TUWS and LESS vary in approximated laws, and thus TUWS would be expected to have a high relative precision compared to LESS. However, the phenomena whereby the static velocities are larger than the dynamic velocities, and where the dynamic velocities decrease with increasing frequency require further analysis.

The static velocities are essentially dynamic velocities of the zero frequency limit, which correspond to an infinite long wavelength. The order of magnitude of the frequencies of dynamic velocities is 1 MHz, which corresponds to a much shorter wavelength. However, wave scattering will be strong enough when the ratio of the wavelength to the heterogeneity scale decreases. The ultrasonic wave is not a sine function, but it spans a frequency band and the high frequency content is scattered more seriously than the low frequency content. In the time domain, the oscillating time of the transmitted wave is longer than that of the incident wave and the peak-time is delayed. The dynamic velocities are calculated from the peak-time difference between the transmitted and incident waves, and as the frequency decreases the peak-time delay is reduced; therefore the dynamic velocities increase (the velocities from TUWS tend to decrease when the source frequency is high). In contrast, according to scattering theory (Mavko et al., 2009), velocity decreases with increasing frequency from the domain of an effective medium to Rayleigh scattering, and thus velocities that are calculated from the difference of the first-arrival-time may still be lowered. This indicates the complexity of utilizing TUWS when studying the physical properties of porous rock.

To maintain high precision, the time difference between the transmitted and incident waves needs to be sufficiently large. For two digital cores with similar physical properties there will not be obvious time difference unless the digital core is long. On the other hand, the wavelength may affect determination of the first-arrival-time, and thus influence the accuracy.

If the source frequency is reduced then the additional layers need to be thickened, and if the wavelength is elongated then the length of digital core must be extended to retain accuracy. However, if additional layers are thickened, or the digital core size is extended, then the total grid size and length of simulation time needs to increase, which would thus result in an unacceptable computational load.

Conclusions

To obtain the ultrasonic wave propagating velocity in a 3D digital core, we attached additional layers and installed source and receiver-arrays onto the digital core to construct a forward model. After exciting the source, the incident and transmitted waves were recorded and the velocity was calculated from the picked peak-time difference of the two waves and the length of the digital core. Velocities at three different frequencies from TUWS were compared with those of LESS. The variation laws of the velocities between different frequencies were found to be similar, which indicates that TUWS has a high relative accuracy. In addition, velocities from different frequencies were different, and when the frequencies were higher the velocities were lower; this is considered to result from wave scattering in the digital core.

Scattering interferes with the velocity from the digital core when based on TUWS. Therefore, to study the physical properties of rock using this method, it is necessary to use sufficient consideration to avoid unreasonable conclusions, and to balance numerical precision with the computational load. Due to its flexibility, TUWS could then assist geophysicists when designing and interpreting laboratory experiments.

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