APPLIED GEOPHYSICS, Vol.12, No.4 (December 2015), P. 523-532, 15 Figures.
DOI:10.1007/s11770-015-0523-z

Reservoir parameter inversion based on weighted statistics*

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Abstract: Variation of reservoir physical properties can cause changes in its elastic parameters. However, this is not a simple linear relation. Furthermore, the lack of observations, data overlap, noise interference, and idealized models increases the uncertainties of the inversion result. Thus, we propose an inversion method that is different from traditional statistical rock physics modeling. First, we use deterministic and stochastic rock physics models considering the uncertainties of elastic parameters obtained by prestack seismic inversion and introduce weighting coefficients to establish a weighted statistical relation between reservoir and elastic parameters. Second, based on the weighted statistical relation, we use Markov chain Monte Carlo simulations to generate the random joint distribution space of reservoir and elastic parameters that serves as a sample solution space of an objective function. Finally, we propose a fast solution criterion to maximize the posterior probability density and obtain reservoir parameters. The method has high efficiency and application potential.
Keywords: Reservoir parameters, inversion, weighted statistics, Bayesian framework, stochastic simulation

Introduction

Reservoir parameters such as gas saturation and porosity are important to reservoir evaluation and well site selection. Seismic amplitude contains abundant reservoir information. The use of seismic amplitude to obtain reservoir parameters is a hot topic in the field of reservoir prediction. Currently, the most common method for obtaining reservoir property parameters is to use seismic amplitude to obtain elastic parameters using various prestack seismic inversion methods; notably, the prestack seismic inversion technology has matured (Russell et al., 2011). The obtained elastic parameters are inverted to reservoir parameters via some transformation methods (Kabir et al., 2000).

To transform the elastic parameters to reservoir parameters, several methods such as multivariate statistics (Doyen, 1998; Fournier, 1989) and deterministic rock physics modeling (Blangy, 1992; Marion and Jizba, 1997) are used. Multivariate statistical techniques are used to obtain the relation between the elastic parameters and physical properties of reservoirs. However, the obtained relation is purely mathematical with unclear

Manuscript received by the Editor June 8, 2015; revised manuscript received November 11, 2015.
*This research work is supported by the National Science and Technology Major Project (No. 2011 ZX05007-006), the 973 Program of China (No. 2013CB228604), and the Major Project of Petrochina (No. 2014B-0610).
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physical meaning; moreover, the accuracy of the relation depends on the number of samples. Deterministic rock physics modeling is based on the Gassmann equation for fluid replacement combined with various equivalent medium models to establish the deterministic transform relation between the elastic and reservoir parameters. In fact, in complex reservoir environments, deterministic rock physics models are often very ideal and with large errors. Reservoir parameter inversion methods based on statistical rock physics, having both the advantages of multivariate statistics and deterministic rock physics modeling, have attracted increasing interest. In these methods, random errors are added to a deterministic rock physics model to simulate the deviation between the model and actual data and establish the statistical relation between the elastic and reservoir parameters. Then, reservoir parameters are derived by the inversion of an objective function based on Bayesian statistics. Finally, Markov chain Monte Carlo (MCMC) techniques are used to calculate the maximum a posteriori probability density in the final inversion results. In these methods, the uncertainty and randomness of rock physics relations are effectively combined, and uncertainties in reservoir parameters are considered. Thus, the inversion precision is high (Bachrach, 2006; Spike et al., 2008; Grana and Rossa, 2010; Yin et al., 2014). However, such methods are time consuming, especially for 3D data. Moreover, elastic parameters obtained by prestack seismic inversion contain errors of variable magnitude. Generally, the accuracy of common elastic parameters such as P- and S-impedance and density decreases from P-impedance to density. If elastic parameters with different uncertainties are treated equally in inversion, this will likely affect inversion results. In this study, we propose a method to invert reservoir parameters based on traditional statistics methods. We consider uncertainties in elastic parameters and use weighting coefficients to treat them. We also propose an objective inversion function and a solving strategy. The proposed method is tested using model and actual data with good results.

Deriving the objective inversion function

We use $\mathbf{m}$ to denote elastic parameters such as P-impedance, S-impedance, density, and Poisson's ratio. The elastic parameters are closely related to the prestack seismic amplitude obtained by various prestack seismic inversion methods. We also use $\mathbf{R}$ to represent inverted reservoir parameters such as gas saturation, porosity, and clay content. The reservoir parameters are divided into N classes, such as $\mathbf{R} = [\mathbf{R}_1, \mathbf{R}_2, \ldots, \mathbf{R}_N]$. For example, if gas saturation is 0–1, then the saturation can be divided into 11 classes at intervals of 0.1, e.g., $\mathbf{R} = [0, 0.1, 0.2, \ldots, 1]$.

Bayesian statistics provides the framework for probabilistic inversion, in which the inversion parameters have a specific probability distribution and can be predicted by probability characteristics and statistics of known samples (Tarantola, 2005). Consequently, objective reservoir parameters $\tilde{\mathbf{R}}$ can be obtained by looking for the maximum a posteriori probability density for a set of elastic parameters $\mathbf{m}$ (Bachrach, 2006)

$$\begin{aligned}\tilde{\mathbf{R}} &= \arg \underset{\mathbf{R}_i \in \mathbf{R}}{Max}\, P(\mathbf{R}_i \mid \mathbf{m}) \\ &= \arg \underset{\mathbf{R}_i \in \mathbf{R}}{Max} \left\{ \frac{P(\mathbf{m} \mid \mathbf{R}_i) \times P(\mathbf{R}_i)}{P(\mathbf{m})} \right\},\end{aligned} \tag{1}$$

where $P(\mathbf{R}_i)$ is the a priori probability density function (PDF) of the ith class, and the conditional PDF $P(\mathbf{m}|\mathbf{R}_i)$ is known as the likelihood function; $P(\mathbf{m})$ is a normalization parameter that can be omitted.

Solving the objective inversion function

In equation (1), the a priori PDF $P(\mathbf{R}_i)$ can be approximated by a Gaussian mixture model (GMM) based on the statistical characteristics of well log data (Hastie et al., 2002; Grana and Rossa, 2010)

$$P(\mathbf{R}_i) = \sum_{n=1}^{Nr} \alpha_n N(\mathbf{R}_i;\, \mu_{\mathbf{R}_i}^n,\, \textstyle\sum_{\mathbf{R}_i}^n), \tag{2}$$

where $N(\cdot)$ denotes the PDF of the Gaussian distribution. Unit number N_r, weight coefficient α_n, mean $\mu_{\mathbf{R}_i}^n$. and covariance $\sum_{\mathbf{R}_i}^n$ all are relative to $\mathbf{R}_i$ Gauss distribution. The biggest advantage of the GMM is that as long as parameters are selected appropriately, it can model any distribution pattern.

The difficulty in solving the objective inversion function is the determination of the likelihood function $P(\mathbf{m}|\mathbf{R}_i)$ that affects the inversion accuracy and efficiency. We use $f_d(\mathbf{R})$ to denote the deterministic rock physics model and $\boldsymbol{\varepsilon}$ to denote the deviation between the model and actual data. Then, the relation between the elastic and reservoir parameters is expressed as (Bachrach, 2006)

$$\mathbf{m} = f_d(\mathbf{R}) + \boldsymbol{\varepsilon}. \tag{3}$$

Equation (3) is the expression of the statistical rock physics model. From this expression, it can be found that the statistical rock physics model not only reflects the relation between the elastic and reservoir parameters but also highlights noise randomness; it is a rock physics model with both deterministic and stochastic characteristics.

Conventional methods combine statistical rock physics modeling and MCMC stochastic simulation to obtain the random joint distribution of samples of elastic and reservoir parameters $\{(\mathbf{m}_k, \mathbf{R}_k)\}_{k=1,2,...Ns}$ as training samples and then derive the likelihood function (Hastie et al., 2002)

$$P(\mathbf{m}_j \mid \mathbf{R}_i) = \frac{n(\mathbf{m}_j)}{n(\mathbf{R}_i)}, \tag{4}$$

where $n(\mathbf{R}_i)$ represents the number of samples of reservoir parameter $\mathbf{R}_i$ and $n(\mathbf{m}_j)$ represents the number of samples of elastic parameter $\mathbf{m}_j$. The training samples obtained by statistical rock physics modeling and MCMC stochastic simulation contain all the log curve data and can generate samples that are not present in the log curves. Thus, they can effectively solve the problem of the dependence of traditional multivariate statistics on the number of training samples. However, in general, multiple elastic parameters are used to reduce the number of solutions in the reservoir parameters inversion. For N elastic parameters, equation (4) is transformed to

$$\begin{aligned} P(\mathbf{m}_j \mid \mathbf{R}_i) &= P(m_j^1, m_j^2, ...m_j^{Nm} \mid \mathbf{R}_i) \\ &= \frac{n(m_j^1, m_j^2, ...m_j^{Nm})}{n(\mathbf{R}_i)}. \end{aligned} \tag{5}$$

To obtain reasonable results with equation (5), samples with elastic parameters $[m_j^1, m_j^2, ...m_j^{Nm}]^T$ should be present multiple times in the training data. This requires a large size of training samples and, consequently, data processing is time consuming.

In this study, based on naive Bayes classification, we assume that the elastic parameters are conditionally independent (Friedman et al., 1997) and then

$$P(m_j^1, m_j^2, ...m_j^{Nm} \mid \mathbf{R}_i) = \prod_{k=1}^{Nm} P(m_j^k \mid \mathbf{R}_i). \tag{6}$$

In equation (6), the conditional PDF of the multiple elastic parameters can be computed by multiplying a single elastic parameter. Each elastic parameter needs to be present independently in the training samples fewer times to satisfy the statistical requirements. For example, assuming that the range of each reservoir parameter is divided into ten classes and there are three elastic parameters each with j different values, the total number of samples N is smaller than the number of samples before the independence assumption, as shown in Figuer 1. Thus, the statistical efficiency of the likelihood function is improved.

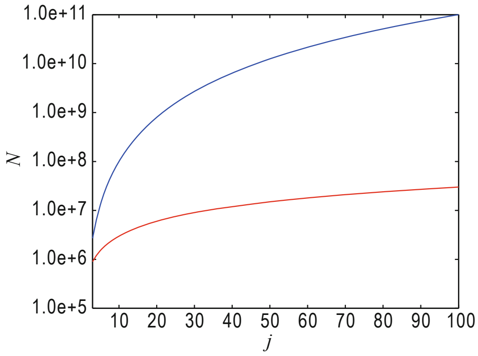

Fig.1 Total number of samples N vs j.
The blue and red curves represent the results before and after the independence assumption.

In actuality, the assumption that the elastic parameters are conditionally independent is not likely satisfied because of the correlations among the elastic parameters. However, the naive Bayesian classification shows that even the independence assumption is not satisfied, and the classification performance is equivalent to classical classification algorithms, such as the decision-tree and k-nearest neighbor algorithms, and close to the case of no independence assumption (Friedman et al., 1997; Fan and Liu, 2008). Furthermore, according to the statistical characteristics of $P(m^k|\mathbf{R}_i)$, we use the GMM to approximate the distribution characteristics of $P(m^k|\mathbf{R}_i)$ (Figure 2) and obtain the analytical expression

$$P(m^k \mid \mathbf{R}_i) = \sum_{\pi=1}^{Nc} \alpha_\pi N(\mathbf{R}_i; \mu^\pi_{m^k|\mathbf{R}_i}, \Sigma^\pi_{m^k|\mathbf{R}_i}), \tag{7}$$

where $N(\cdot)$ is the Gaussian PDF, $N_c, \alpha_\pi, \mu^\pi_{m^k|\mathbf{R}_i}$, and $\Sigma^\pi_{m^k|\mathbf{R}_i}$ is the number of components in the GMM, the weighting coefficients, and the mean and variance related to m^k and $\mathbf{R}_i$. Thus, we obtain the likelihood function.

When m_j^k is obtained by prestack seismic inversion, we do not need to repeat the statistical processing and, thus, the efficiency sharply increases. For example, as shown in Figure 2, the histogram represents the statistical characteristic of P-impedance I_p, while the corresponding porosity Por is known as 0.2, we obtain the analytical expression of the likelihood function $P(I_p|Por = 0.2)$ by using a GMM that consists of three Gaussian distribution functions (red, green, and yellow curves in Figure 2) to fit I_p.

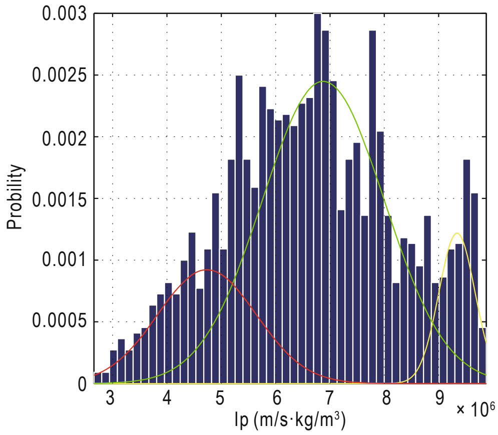

Fig.2 Determination of the likelihood function.

Then, equation (1) is rewritten as

$$\tilde{\mathbf{R}}=\arg\underset{\mathbf{R}_i\in\mathbf{R}}{Max}\left\{\sum_{n=1}^{Nr}\alpha_n N(\mathbf{R}_i;\mu_{\mathbf{R}_i}^n,\sum\nolimits_{\mathbf{R}_i}^n)\right.$$
$$\left.+\prod_{k=1}^{Nm}\sum_{\pi=1}^{N_c}\alpha_\pi N(m^k|\mathbf{R}_i;\mu_{m^k|\mathbf{R}_i}^\pi,\sum\nolimits_{m^k|\mathbf{R}_i}^\pi)\right\}. \quad (8)$$

It is well known that the elastic parameters obtained by prestack seismic inversion contain errors and the error magnitude is different for each elastic parameter. Thus, the weights for the different elastic parameters in the objective function have to be different. We use W to denote the weighting coefficients of the elastic parameters. We obtain the weighting coefficient by calculating the correlation coefficients between borehole and actual log data

$$W=\frac{\sum_{i=1}^{N}(x_{pi}-\bar{x}_p)(x_{wi}-\bar{x}_w)}{\sqrt{\sum_{i=1}^{N}(x_{pi}-\bar{x}_p)^2\sum_{i-1}^{N}(x_{wi}-\bar{x}_w)^2}}, \quad (9)$$

where x_{pi} and x_{wi} are the values of the ith sample in the borehole and actual well data (i = 1, 2, 3…, N), and $\bar{x}_p$ and $\bar{x}_w$ are the means of the borehole and actual well data.

We perform a log transformation to equation (8) and use W to modify the elastic parameters information

$$\tilde{\mathbf{R}}=\arg\underset{\mathbf{R}_i\in\mathbf{R}}{Max}\left\{\ln\sum_{n=1}^{Nr}\alpha_n N(\mathbf{R}_i;\mu_{\mathbf{R}_i}^n,\sum\nolimits_{\mathbf{R}_i}^n)\right.$$
$$\left.+\sum_{k=1}^{Nm}W_k\ln\sum_{\pi=1}^{Nc}\alpha_\pi N(m^k\,|\,\mathbf{R}_i;\mu_{m^k|\mathbf{R}_i}^\pi,\sum\nolimits_{m^k|\mathbf{R}_i}^\pi)\right\}, \quad (10)$$

where W_k represents the weighting coefficient of the kth elastic parameter. The final inversion results are the reservoir parameters and equation (10) is the maximum a posteriori probability density.

Reservoir parameters inversion

The entire inversion process proposed in this study focuses on solving the objective inversion function and is divided into five steps.

1) Determine the a priori distribution density function. Use equation (2) to fit the distribution of the reservoir parameters from the logging curves in the target layer.

2) Determine the rock physics model according to experimental or empirical relations. Set the relation between reservoir and elastic parameters $f_d(\mathbf{R})$. Then, the random error $\boldsymbol{\varepsilon}$ is determined by analyzing the distribution characteristics of the error between the model and actual curves. Lastly, substitute $f_d(\mathbf{R})$ and $\boldsymbol{\varepsilon}$ into equation (3) to obtain the statistical rock physics model.

3) Determine the likelihood function. Based on the statistical rock physics model of step 2, use MCMC stochastic simulation to generate the joint sampling space of reservoir and elastic parameters. After applying equation (6) to analyze the distribution of the likelihood function, use the equation (7) to obtain the analytical expression of the likelihood function.

4) Determine the weighting coefficients. In conventional prestack seismic inversion, many elastic parameters can be obtained. First, elastic parameters

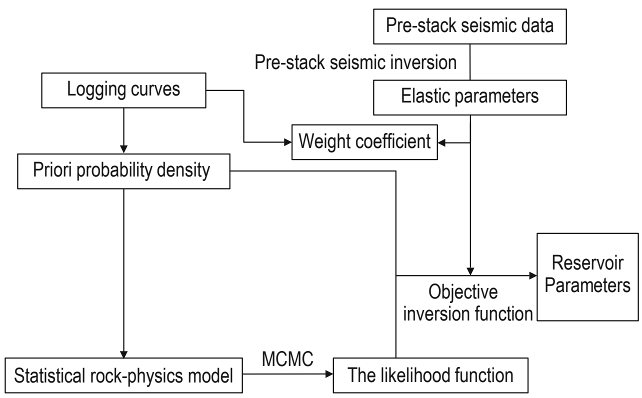

Fig.3 Reservoir parameters inversion flowchart.

of high sensitivity are selected as input data in the reservoir parameters inversion. Second, extract the prestack seismic inversion results for the target zone near the borehole as pseudo well curves. Finally, the sampling points of the pseudo well and actual curves are substituted into equation (9) to calculate the weighting coefficients of the elastic parameters.

5) Solve the objective inversion function. Input the a priori PDF and the likelihood function obtained by steps 1–4 into equation (10). Then, input the elastic parameters data and obtain the maximum posterior probability density solution of the objective inversion function. The inversion flowchart is shown in Figure 5.

Model analysis

We use the shale content V_{sh}, porosity Por, and gas saturation S_g curves proposed by Grana et al. (2010) as model curves (Figure 4). We use the reservoir parameter curves and the Hertz–Mindlin particle contact theory (Mavko et al., 2003) to synthesize the P-impedance I_p, the S-impedance I_s, and density ρ curves, and respectively add 3%, 6%, and 9% Gaussian noise to simulate the elastic parameters with variable precision obtained by prestack seismic inversion, as shown in Figure 5. We then use the elastic parameter curves as input data, $\mathbf{m} = [I_p \;\; I_s \;\; \rho]^T$. The shale content, porosity, and gas saturation serve as the target inversion parameters, $\mathbf{R} = [V_{sh} \;\; S_g \;\; Por]$. Then, we test the validity of the proposed method.

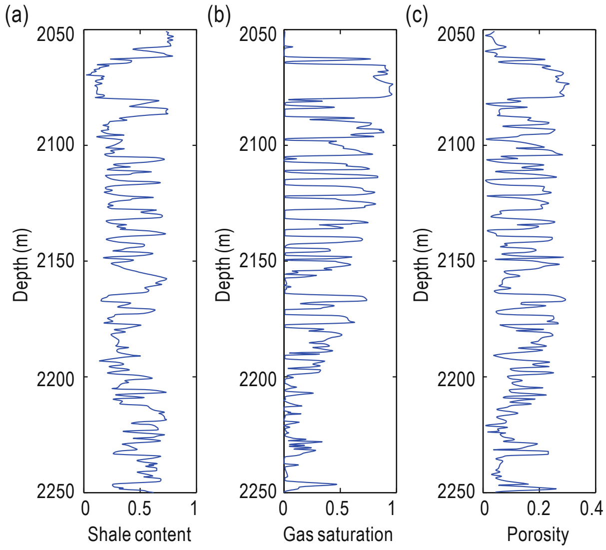

Fig.4 Reservoir parameter curves: (a) shale content, (b) gas saturation, and (c) porosity.

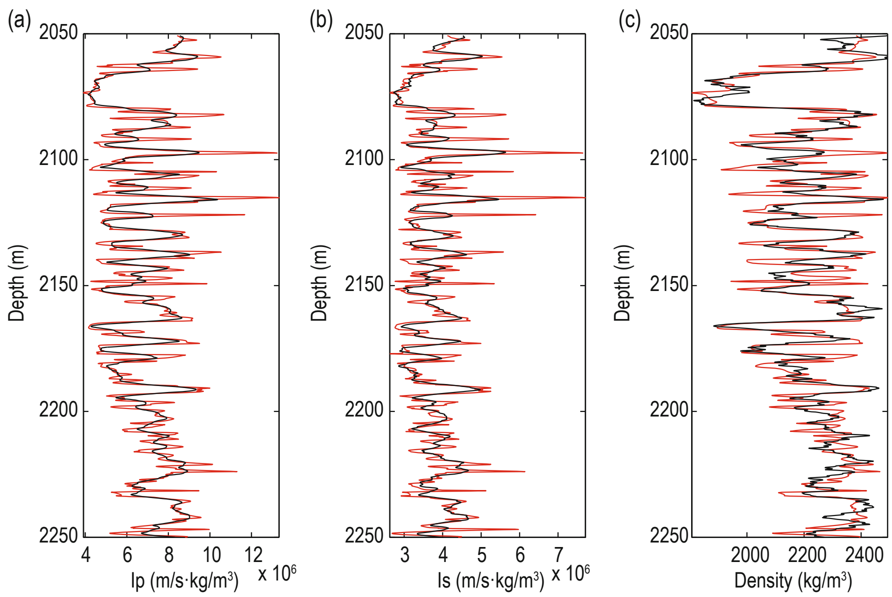

Fig.5 Elastic parameter curves.

The black curves represent the model curves without noise and the red ones represent the model curves with noise: (a) P-impedance, (b) S-impedance, and (c) density.

Figure 6 shows the a priori distribution obtained by the Gauss mixture PDF and the actual distribution. We can see that the distribution of the reservoir parameters has two peaks, and the peak regions are consistent with the actual reservoir parameters of gas-bearing sand and water-bearing sand. Hence, the a priori PDF obtained by the GMM distribution accurately reflects the a priori distribution of the log samples. Furthermore, we generate samples that are not present in the well data by combining the Gaussian mixture distribution with MCMC (Figure 6).

We select the low, medium, and high values of the reservoir parameter samples and obtain the corresponding conditional probability density distribution. In addition, we obtain the likelihood function using equation (7). We can see that the distribution of the likelihood function is similar to the Gaussian mixture distribution. Therefore, the analytical expression for the likelihood function can be obtained by using equation (9), which reduces the repetition of sample statistics.

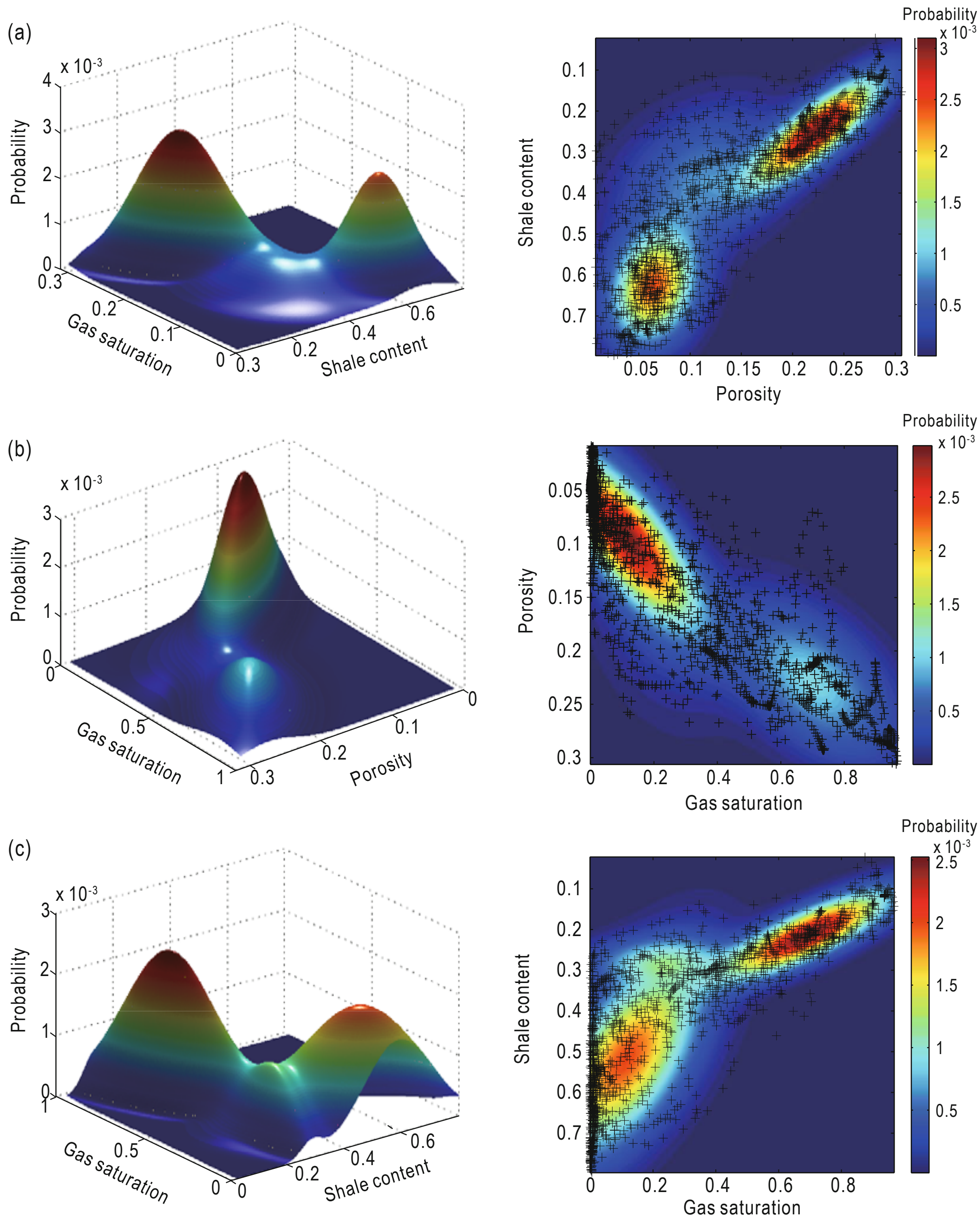

Fig.6 Priori distribution characteristics of reservoir parameter: Left panels are the three-dimensional distribution of prior probability density; right panels are the log samples projection on 2D distribution of the prior probability density. (a) Porosity and clay content. (b) Porosity and gas saturation. (c) Shale content and gas saturation.

From the distribution of $P(\mathbf{m}|\mathbf{R})$, we obtain the reservoir data associated with the elastic parameters. As shown in Figure 7, for porosity of 0.05, 0.15, and 0.25, the corresponding probability distributions of the elastic parameters overlap and the degree of overlapping varies. The larger the overlapping regions are, the lesser the reservoir information is as well as the sensitivity of the elastic parameters.

Similarly, different elastic parameters have different sensitivities to gas saturation, as shown in Figure 8. For gas saturation of 0.1, 0.4, and 0.7, the overlapping regions in Figure 8 are larger than those in Figure 7,

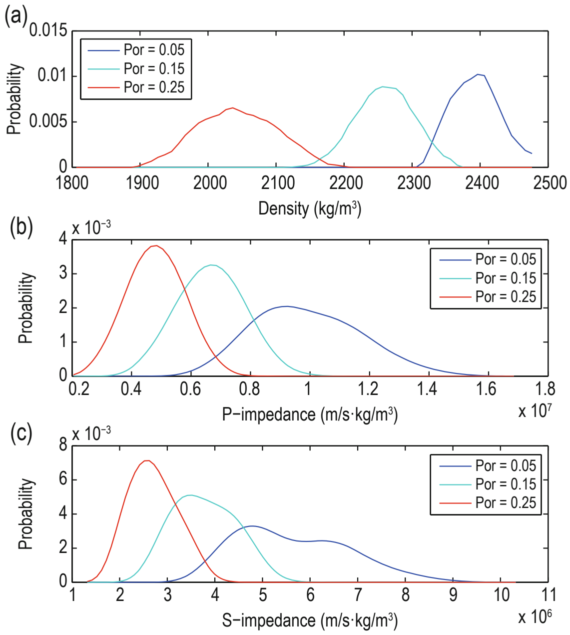

Fig.7 Probability density distribution of elastic parameters for different porosity values: (a) density, (b) P-impedance, and (c) S-impedance.

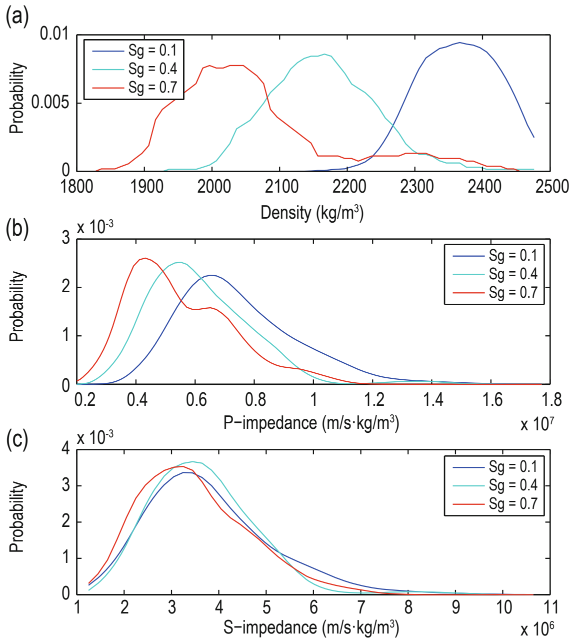

Fig.8 Probability density distribution of elastic parameters at different gas saturation values: (a) density, (b) P-impedance, and (c) S-impedance.

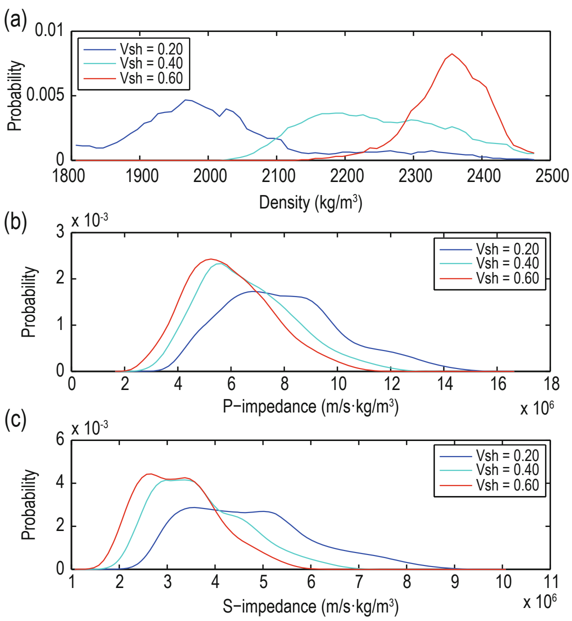

Fig.9 Probability density distribution of elastic parameters at different shale content values: (a) density, (b) P-impedance, and (c) S-impedance.

whereas the S-impedance distribution curves almost coincide. The latter means that the S-impedance carries lesser gas saturation information.

For shale content of 0.2, 0.4, and 0.6, we can see that, in addition to density, the overlapping regions of P- and S-impedance are large. By analyzing Figures 6−8, we see that the total sensitivity of the P-impedance, S-impedance, and density to porosity, gas saturation, and shale content in sand (V_{sh} < 0.6) decreases. In practice, we can choose elastic parameters with smaller overlapping regions as input parameters.

The same weighting coefficients $\mathbf{W}$ = [1 1 1] are used for the P-impedance, S-impedance, and density and the inversion results are shown in Figure 10. The elastic parameters are treated similarly in the inversion procedure but the inversion results differ from the actual model and the correlation coefficients are respectively 0.37, 0.79, and 0.88. Based on equation (9), we use the model curves with noise and the actual model curves to calculate the weighting coefficients. The weighting coefficients for the P-impedance, S-impedance, and density are 0.97, 0.89, and 0.7, respectively, and the inversion results are shown in Figure 11. The inversion results are in good agreement with the model, and the correlation coefficients are 0.69, 0.89, and 0.93, respectively. The weighted statistical relation between the elastic parameters and reservoir parameters is more reasonable when the input data are multiple reservoir parameters of variable precision. The inversion precision for porosity, gas saturation, and shale content decreases. This is consistent with the analytical results in Figures 7–9.

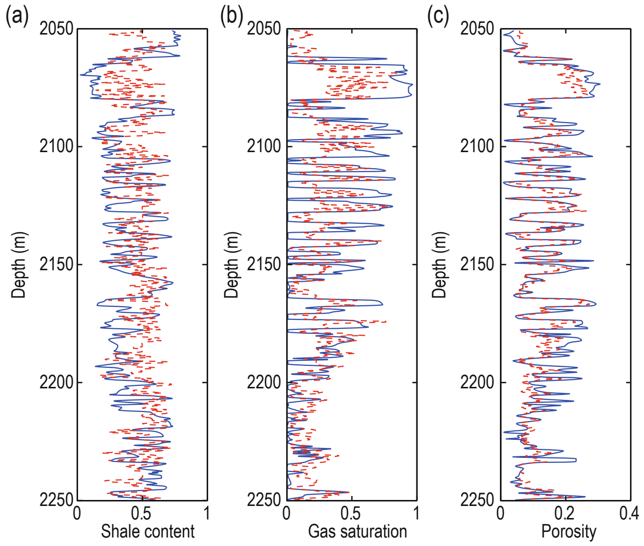

Fig.10 Inversion results when the weight coefficients of P-impedance, S-impedance and density are the same.
The blue and red curves respectively represent the inversion results with and without noise: (a) shale content, (b) gas saturation, and (c) porosity.

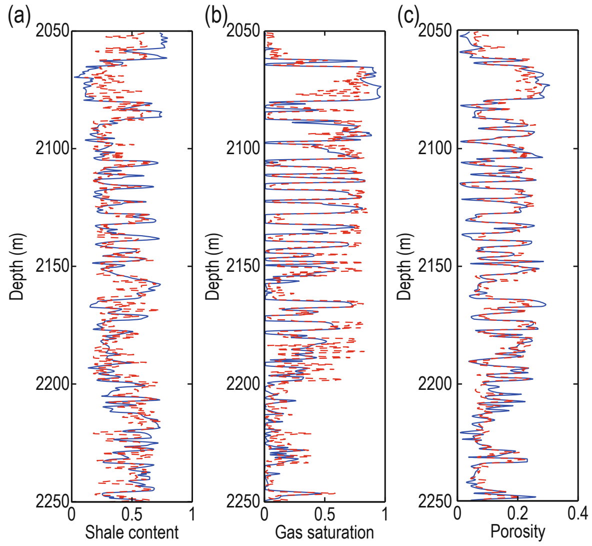

Fig.11 The inversion results when the weight coefficients of P-impedance, S-impedance and density are different.
The blue and red curves respectively represent the inversion results with and without noise: (a) shale content, (b) gas saturation, and (c) porosity.

Real data application

We applied the proposed method to a gas-bearing sand reservoir. The distribution of sand layers in this area is clear but the main exploration problem is the similarity between the reservoirs with low gas saturation and low porosity and the reservoirs with high gas saturation and high porosity in the seismic section, both show as bright spots. This leads to high drilling risks. Therefore, high-precision inversion results for gas saturation and porosity can reduce the exploration risk in this area.

There are two gas-producing well in this area, well A and well B. We use well A as the a priori information in the inversion process and well B as the verification well. The gas saturation and porosity inversion results are shown in Figure 12. The inversion results are in good agreement with the actual logging data and the reservoirs with different gas saturation and porosity are clearly distinguished in the inversion section. Figures 13a and 13b show the gas saturation and porosity inversion for well B.

To further demonstrate the effectiveness of the method, we extract the borehole traces near well B and compare them with the actual logging curves, as shown in Figure 14. The inversion results are in good agreement with the actual logging data.

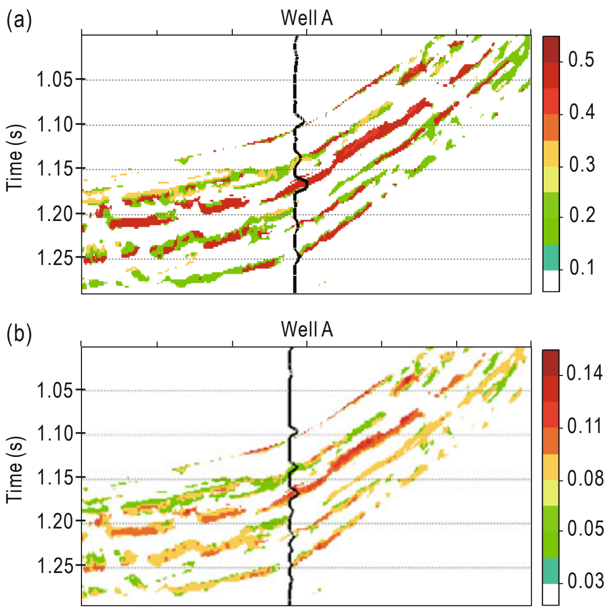

Fig.12 Cross-sections of inversion results and logging data for well A: (a) gas saturation and (b) porosity.

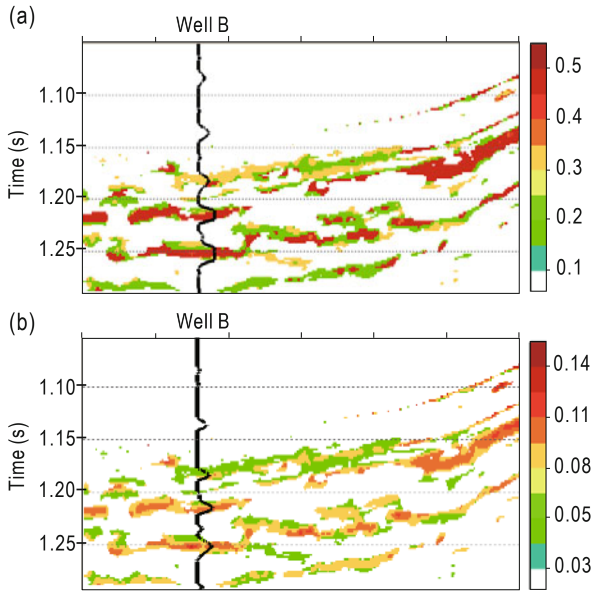

Fig.13 Cross-sections of inversion results and logging data for well B: (a) gas saturation and (b) porosity.

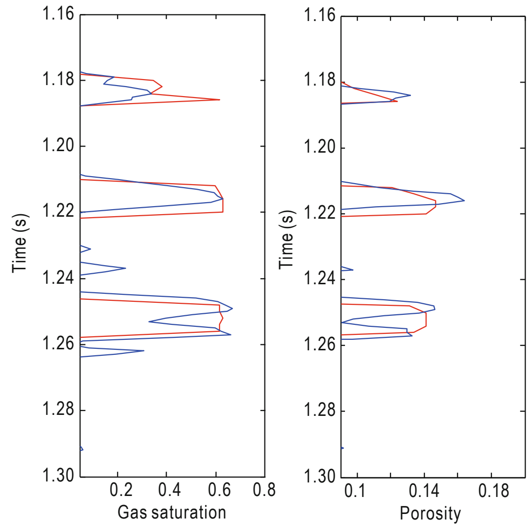

Fig.14 Comparison of inversion results and logging data in boreholes near well B: (a) gas saturation and (b) porosity. The blue and red curves respectively represent the actual logging data and inversion results.

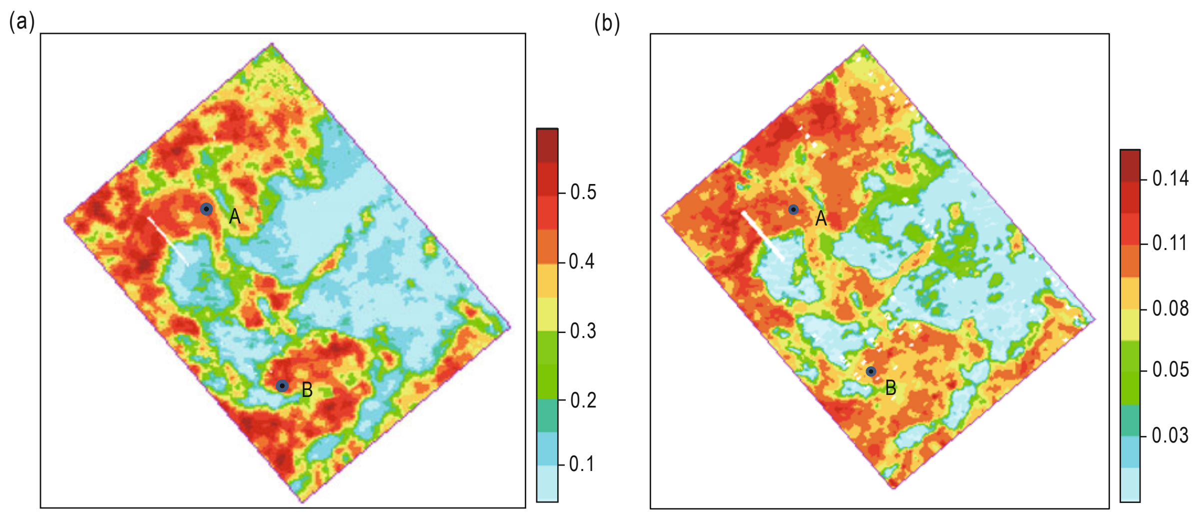

Fig.15 Horizontal slices of the inversion results: (a) gas saturation and (b) porosity.

Figure 15 shows the gas saturation and porosity along a section of the layers in this area. Wells A and B are all located in gas-rich and high-porosity areas. Logging and drilling data are consistent in both wells.

Conclusions

The prestack seismic inversion precision of different elastic parameters is variable. If the precision variability is neglected, this will adversely affect the inversion results. We use weighting coefficients to minimize the effect of the precision variability and improve the inversion results.

The total number of sampling points substantially decreases if we assume independent elastic parameters. Consequently, the analytical expression of the likelihood function is obtained by fitting the distribution of the likelihood function with a Gaussian mixture of distribution PDFs. Thus, the unnecessary repetition of statistics is minimized or avoided, and the efficiency of the objective function is improved.

The weighted statistics minimize the effect of precision variability of the elastic parameters but that does not necessarily make the method suitable for the areas with low-precision elastic parameters. The proposed high-precision inversion method and appropriate rock physics models ensure the adaptability of the method.

Acknowledgements

We thank the anonymous reviewers for their constructive comments.

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