Seismic data denoising based on mixed time-frequency methods*

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Abstract: Deconvolution denoising in the f-x domain has some defects when facing situations like complicated geology structure, coherent noise of steep dip angles, and uneven spatial sampling. To solve these problems, a new filtering method is proposed, which uses the generalized S transform which has good time-frequency concentration criterion to transform seismic data from the time-space to time-frequency-space domain (t-f-x). Then in the t-f-x domain apply Empirical Mode Decomposition (EMD) on each frequency slice and clear the Intrinsic Mode Functions (IMFs) that noise dominates to suppress coherent and random noise. The model study shows that the high frequency component in the first IMF represents mainly noise, so clearing the first IMF can suppress noise. The EMD filtering method in the t-f-x domain after generalized S transform is equivalent to self-adaptive f-k filtering that depends on position, frequency, and truncation characteristics of high wave numbers. This filtering method takes local data time-frequency characteristic into consideration and is easy to perform. Compared with AR predictive filtering, the component that this method filters is highly localized and contains relatively fewer low wave numbers and the filter result does not show over-smoothing effects. Real data processing proves that the EMD filtering method in the t-f-x domain after generalized S transform can effectively suppress random and coherent noise of steep dips.

Keywords: Empirical Mode Decomposition, generalized S transform, coherent noise, random noise, noise suppression

Introduction

In seismic data acquisition, interference factors can lead to both random and coherent noise. Besides, data processing, such as deconvolution and migration, can also introduce interference noise. Noises can greatly disturb the identification of geological information, so denoising is a crucial step in seismic data processing, especially crucial to seismic data interpretation and analysis (Shen et al., 2010). There are many methods for seismic data denoising. For surface waves, Roohollah and Siahkoohi (2008) noticed the distribution difference between surface waves and effective reflection waves in the f-k domain and suggested that S transformation and x-f-k transformation can be used to suppress surface waves. Taking advantage of the low frequency and strong energy characteristics of surface waves, Ba et al (2007) designed a zero-crossing filter to suppress surface waves through approximation coefficients after

Manuscript received by the Editor March 11, 2011; revised manuscript received August 15, 2011.

^{*}This work was sponsored by the National Natural Science Foundation of China (Grant No. 41174114) and the National Natural Science Foundation of China and China Petroleum & Chemical Corporation Co-funded Project (No. 40839905)

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wavelet decomposition. For random noise suppression, spatial predictive filtering in the f-k domain, which make use of the predictive signal characteristics in the spatial direction, is an effective method (Canales, 1984; Galbraith, 1991; Harris, and White, 1997).

When dealing with non-linear and non-stationary signals using standard spatial filtering techniques, a piecewise stationary and linear assumption is often used and short spatial windows are used to filter the data. To meet special processing demand, finding the best window size and filter length is the key technique. In ideal situations, these parameters depend on the smoothness and frequency of the data. Time-frequency analysis provides a way to analyze non-stationary data and it also can be used to suppress noise. Gabor (1946) introduced the short period Fourier transform to improve the Fourier transform drawbacks in local time-frequency analysis but its time window is constant. To meet the demand that higher frequencies need shorter time windows and lower frequencies need longer time windows, Morlet (1984) introduced wavelets into seismic data analysis. The time window in wavelet analysis can vary with scale factor and is dependent on time-frequency scale. However, wavelets must satisfy wavelet admissible conditions.

To overcome this restriction, Stockwell et al. (1996) proposed the S transform that focuses on detailed analysis of local time-frequency characteristics. The S transform time window length can be automatically added with increasing time windows and the time window doesn't need to satisfy the wavelet admissible conditions. Besides, the S transform computation is directly related to Fourier transform. Huang et al (1998) believed that any complicated data can be decomposed into limited Intrinsic Mode Functions (IMFs) and proposed the Empirical Mode Decomposition (EMD) method which focuses on non-linear and non-stationary data. The EMD method can effectively analyze irregular signals (Hassan, 2005; Battista et al., 2007).

Considering the time-frequency characteristics and decomposition methods of seismic data, self-adaptive filters can be designed to suppress random and coherent noise. We propose a new method which uses the generalized S transform which has good time-frequency criterion to transform seismic data from the time-space domain to the time-frequency-space (t-f-x) domain, then, in the t-f-x domain apply the EMD method to suppress noise. In this paper, we first analyze the generalized S transform principle and EMD method, elaborate how this method can suppress coherent and random noise, and then analyze the advantages and drawbacks of the generalized S transform and EMD method through model studies. Finally, this method is used in real seismic data processing and its effectiveness proved.

Method theory

Generalized S transform based on timefrequency concentration criterion

Stockwell et al. (1996) introduced a new kind of short-time Fourier transform, called the S transform. The S transform is an extension of the wavelet and Gabor transforms. The S transform of signal x(t) is defined as:

$$S(f,\tau) = \int_{-\infty}^{\infty} x(t) \frac{|f|}{\sqrt{2\pi}} \exp\left(-\frac{\left(t-\tau\right)^2 f^2}{2}\right) \exp\left(-j2\pi ft\right) dt.$$
(1)

The S transform window function length can selfadaptively vary with frequency. The time-frequency window function is self-adaptive and does not need to satisfy the wavelet admissible conditions and is directly related to the DFT. However, the S transform definition in equation (1) has unnecessary restrictions on window function: ① Only the Gaussian window function is considered; ② Can't adjust time and frequency in the window function; and ③ The window function used in equation (1) does not have complex conjugate characteristics. To overcome this restriction, using the time-frequency concentration criterion suggested by Jones and Parks (1990), we rewrite the definition equation as:

$$S(f,\tau) = \frac{\left|f\right|^p}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t) \exp\left(-\frac{(t-\tau)^2 f^{2p}}{2}\right) \exp\left(-j2\pi ft\right) dt \quad (p>0).$$
(2)

Equation (2) is the generalized S transform. Parameter p is used for adjusting the wavelet variation trend with frequency scale based on the frequency distribution characteristics of the non-stationary signal. It can speed up or slow down the variation speed of time window length with signal frequency and also can diversify the wavelet amplitude for best analyzing a specific signal. The parameter is not fixed and is relevant to frequency. Determination of this parameter requires the time-frequency concentration criterion suggested by Jones and Parks (1990). Performing the inverse Fourier transform on equation (2), we have:

$$x(t) = \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} S(f,\tau) d\tau \right\} \exp(j2\pi ft) df.$$
 (3)

Equation (3) shows that a lossless inverse S transform can be realized by inverse Fourier transform. What's more important, equation (3) correlates the S transform directly with the Fourier transform, which provides a rapid way to implement S transform and inverse S transform using FFT and improve computation speed.

Empirical mode decomposition (EMD)

EMD decomposes data into a series of signals called intrinsic mode functions (IMFs). Different IMFs represent information of different frequency or wave number. An IMF must satisfy two conditions: (1) the number of poles and zeros must be equal or differ by no more than one and (2) at any local point, the maximum envelope and minimum envelope have a zero average. These two conditions prevent frequency dispersion caused by waveform asymmetry and ensure each IMF has one local frequency. Fourier transforms decompose data into harmonic waves of a single frequency and constant amplitude, while IMFs are frequency modulation (FM) or amplitude modulation (AM) signals and can capture non-stationary and nonlinear data variations. IMFs are computed by iteration. The local maximums and minimums envelope are used in data decomposition. Once the extreme values are determined, we do a cubic spline interpolation on the local maximums and get the upper envelope and then do cubic spline interpolation on the local minimums and get the lower envelop. In each data sequence sample point, we compute the average of the upper and lower envelopes and subtract it from the initial data. We then do this interpolation method on the remaining data iteratively. When the envelop average of each sample point approaches zero to within a tolerance value, the iteration process is stopped and the signal yield is called the first IMF (IMF1). We then subtract IMF1 from the original signal to get a new signal and we get the next IMF from this new signal using the same process. Decomposition continues until the last IMF becomes constant or very close to zero.

EMD becomes a signal analysis tool for its special characteristic. It can completely decompose data and losslessly reconstruct data through summation of all IMFs. Since the cross correlation function of different IMFs is very close to zero, the EMD process is close to orthogonal. What's more important, many important characteristics of EMD are different from Fourier, wavelet, and S transforms. EMD does not need to provide basis functions, while other decomposition methods need to provide a basis function (such as sine function, cosine function, mother wavelet, and so on). EMD adopts cubic spline interpolation and does not require a constant sample rate, while Fourier, wavelet, and S transforms need a constant sample rate to effectively decompose signal (Bekara, and van der Baan, 2007; Flandrin et al., 2005; Bekara and van der Baan, 2009; Rilling, and Flandrin, 2009).

Denoising principle and realization steps

Ideally, seismic sections can be created by convolving a seismic wavelet and the reflection coefficient series. The sedimentary effect is laterally the same over a particular range, so in sedimentary strata, the wave impedance has good comparability in the lateral direction and so does the reflection coefficient. When no noise is present, the seismic amplitudes show constant or slowly characteristics in the lateral direction and show low wave numbers in the wave number domain. When steep dip coherent noise, such as diffraction waves from fault break points, exist in the seismic section in the time direction, it shows the same or a close frequency band with the reflection waves so it's hard to eliminate coherent noise in the frequency domain. In the lateral direction, coherent noise causes drastic lateral amplitude changes and shows high wave number characteristics in the wave number domain. Random noise shows a white spectrum in both frequency and wave number domains. So for most seismic data, steep dip angle coherent noise and random noise have a large contribution to high wave number energy (Bekara, and van der Baan, 2007).

Based on the characteristics, suppressing high wave number energy can suppress steep dip angle coherent noise and random noise in seismic data. The coherent and random noise disturbances occur at different frequencies in seismic data. Therefore, it's necessary to perform noise suppression on different frequency components. Based on the characteristics of this method and the noise distribution, we first divide seismic data into volumes of equal frequency bands, transform the seismic volumes from the t-x domain into the t-f-x domain, apply EMD in the x direction, eliminate the IMFs that represent high wave number energy, and then the S/N ratio can be improved. The first IMF represents the signal component with the highest wavenumbers and as the IMFs order increases, the wavenumbers that the IMF represents decreases, and the effective information increases. So in most situations, eliminating IMF1 (or other IMFs, depending on the data characteristics) can suppress noise. Considering that the generalized S transform can represent the local characteristics of nonstationary signals, its computation uses the FFT, so the frequency sample interval is inversely proportional to the

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number of sample points chosen to do the S transform. In other words, this means that the more sample points, the more data volumes of the same frequency band, and this obviously increases computation time and storage memory. In addition, the EMD method needs to find local maximums and minimums and needs to compute cubic spline interpolations, so longer data sequences result in lower computation efficiency. To handle a complete seismic section, we suggest these process steps that resemble f-x deconvolution to perform seismic data noise suppression in the t-f-x domain:

First: choose window lengths for the time direction (T) and space direction (X) and transform the seismic data into equal frequency bands in the time-frequency-offset (t-f-x) domain using the generalized S transform.

Second: apply EMD along the x direction on each equal frequency band.

(1) Separate the real and imaginary parts of the data generated in the t-f-x domain from the first step.

(2) Apply EMD on the real part, compute each IMF component, and then subtract the IMF components dominated by noise, and store the real part data after noise suppression.

(3) Apply EMD on the imaginary part, compute each IMF component, and then subtract the IMF components dominated by noise, and store the imaginary part data after noise suppression.

(4) Construct a complex signal after noise suppression in the t-f-x domain from the real and imaginary parts after denoising.

Third: perform an inverse S transform on the complex signal in the t-f-x domain and transform it to the t-x domain to complete the seismic data denoising in the given time-space window.

Fourth: Choose another time-space window and

repeat the procedure until the entire seismic section is processed.

Though these steps, random and coherent noise in seismic data can be suppressed using the mixed timefrequency method. The process steps show that the key point of this method is to eliminate noise in the x direction and preserve the high and low frequency components of the seismic data in the time direction and this can largely preserve the vertical seismic data resolution.

Result analysis

First, we analyze the advantage of time-frequency decomposition using the generalized S transform based on the mixed time-frequency method. To evaluate the method effectiveness, we designed a signal shown in Figure 1a. The signal is constructed from two linear FM signals that cross each other and two short cosine signals (one with higher frequency and the other with lower). The S transform time-frequency and generalized S transform sections are shown in Figures 1b and 1c. The S transform spectrum shows the four components clearly but at the crossover frequency there are overlaps and the resolution is not good enough. The generalized S transform based on time-frequency concentration (Figure 1c) has better performance. Its time-frequency decomposition not only shows the four signals in the test signal but also preserves the time axis correspondence between wave peak and energy peak. The result shows that this method has good time-frequency concentration ability and can divide data into different frequency slices for data processing.



transform based on time-frequency concentration

Fig. 1 Test signal and its time-frequency spectra.

Next we apply the EMD method on the signal shown in Figure 2a, Figures 2b to 2j are the EMD results and represent IMF1 to IMF9, respectively. The initial data in Figure 2a is non-stationary. In the IMFs computed by iteration, IMF1 changes fastest and IMF9 changes slowest. IMF1 contains the high wave number information, while IMF9 represents the data variation trend. The IMF2 and IMF3 waveforms show that, although they overlap in wave number, they are separated in spatial location. So the EMD method is different from a band pass filter.



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The main difference between EMD and f-x deconvolution is that f-x deconvolution needs a constant filter length for all frequencies, while EMD can selfadaptively decompose the data based on the data smoothness. For different frequencies or wave numbers, the EMD method provides different filtering choices. Note that in most situations, merely eliminating IMF1 can achieve the denoising goal (later the model analysis and application example can prove its effectiveness). For example, the IMF1 (Figure 2b) computed by applying EMD on the signal in Figure 2a shows a rapid vibrating characteristic, which is the characteristic of noise. In IMF2, although it's still vibrating fast, it contains a lower wave number component that represents effective information (for example, the signal at about 2000 m).

From the processing steps we see that the noise suppression procedure is accomplished by applying EMD in the x direction in t-f-x domain and this will indirectly change the frequency distribution in the time direction. For random noise suppression, an autoregression filter in the f-x domain is a good tool, so we apply an auto-regression filter and an EMD filter on the signal shown in Figure 3a to test the effectiveness and advantage of the EMD denoising method. Figure 3a is the real part of a 35 Hz signal sequence computed using the generalized S transform and it is fixed in time t in the f-x domain (the horizontal and vertical coordinates in Figure 3 are the same as in Figure 4). Figure 3 shows the comparison of the filtering results using secondorder and tenth-order auto-regression linear predictive filters with the EMD filtering result that just eliminated



Fig. 3 Comparison of filtering results by the EMD and AR model prediction filtering methods.

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The lower order predictive filter will filter data more smoothly and can only recover the main spatial changes (Figure 3a). From the difference figure (Figure 3b) and Figure 4, we see that the filtered noise in the wave number map is very clear and it contains low wave number components (from about 2000 m). As the filter order increases, this filter can handle rapid changing signals more accurately (Figure 3c) and better predict the lower wave number component trend. As the filter order increases, the data variation can be better predicted but the ability to filter noise will decrease. Figure 3e shows the signal after applying EMD and eliminating IMF1 on the initial data. From this figure we see that the filter result captured the initial data low wave number components. To compare the filtered noise difference between the EMD method and AR prediction filter, we plot the IMF1 component in Figure 3f. The AR prediction filter result is not as smooth as the EMD filter result and the reason is that the AR model is fit in a short window and then uses the fit coefficients to predict at least one sample point ahead. The comparison of AR prediction filtering results (Figures 3b and 3d) and the EMD filtering result (Figure 3f) and Figure 4 shows that the EMD method filtered component based on the t-f-x domain is highly localized and contains relatively small low wave numbers and the filter result does not show an oversmoothed characteristic. The similar results can also be verified on the imaginary part.



Fig. 4 Comparison of wave numbers of the filtered noise by the EMD and AR model prediction filtering methods.

Following is an analysis of effectiveness and performance for seismic noise suppression in the mixed domain. We first analyze the suppression effect on coherent noise. We used wave equation forward modeling to compute the wave field shown in Figure 5a. In this model, there are diffracted waves and primaries. We consider diffracted wave as coherent noise. Figure 5b is the section after coherent noise suppression using the method mentioned in this paper. Comparing Figures 5b and 5a, Figure 5b shows that the diffracted waves were suppressed to some extent and effective signal was well preserved. Figure 5c is the difference between the original section and the denoised section. This figure shows that the higher dip angles has been better suppressed. This means the method is better for removing coherent noise generated by higher dip angles than lower dip angle (yellow arrows).



Fig. 6 Coherent noise and random noise suppression analysis (added 15% Gaussian random noise to the initial data).

To analyze the random noise suppression of this method, we add 15% Gaussian random noise to the model data shown in Figure 5a. After applying the EMD method in the t-f-x domain using the generalized S transform and eliminating IMF1, the noise suppression section is shown in Figure 6b and the difference section is shown in Figure 6c. These two figures show that this method can effectively suppress coherent and random noise at the same time.

Figure 7 is the model data in Figure 5a plus 30%

Gaussian noise. The Gaussian noise energy is stronger than the diffracted waves and the diffracted waves show random noise characteristics (red arrows). We apply the EMD denoising method and the results are shown in Figure 7b. The reflection events continuity in Figure 7b is improved over Figure 7a (see the red arrows) and the section quality is enhanced. The result in Figure 7b shows that, even with strong random noise, this method is still effective in suppressing noise.



Fig.7 Coherent noise and random noise suppression effect analysis (added 30% Gaussian random noise to the initial data).

Applications

To verify the practicality of this method, it was applied to marine seismic data from the Jinzhou area. Figure 8 shows a stacked seismic profile with uneven trace interval. This data shows strong acquisition footprints and random and coherent noise. The coherent noise is caused by errors in migration velocity and the profile shows residual arcs. The random noise blurs the reflection events. In the t-f-x domain generated by the generalized S transform, we eliminate each IMF1 in every equal frequency band data volume after EMD and generate the noise suppressed (Figure 8b) and difference profiles (Figure 8c). The coherent noise indicated by red arrows in Figure 8a is suppressed, the random noise is also suppressed, and the reflection event continuity is enhanced. The difference section (Figure 8c) shows the EMD filtering method can not only suppress background noise but also has low amplitude distortion. What's more important, this method can effectively suppress coherent noise and falsely crossing events, and has some effect on suppressing acquisition footprints. The noise energy distribution in the difference section shows that for coherent noise caused by steep dips, the higher dips are better suppressed.



Fig. 8 Denoising analysis on marine data from the Jinzhou area.

To test the universality of this method, it was also applied to Sichuan land seismic data. The original data is shown in Figure 9a. There are many random noises in the seismic data, the event continuity is poor, and the fault boundaries are blurred, which adds much difficulty to horizon tracing and fault

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interpretation. The denoising result from this method by only removing IMF1 to suppress noise is shown in Figure 9b. Compared with Figure 9a, the wave group relation is clearer and the reflection event continuity is enhanced, which improves horizon tracing ability for interpretation (blue arrows).



Fig. 9 Denoising analysis on land data from Sichuan.

The EMD noise suppression method was applied on two seismic data sets acquired in different environments and the results show that this method can be a good choice for suppressing random and coherent noise. The EMD in the t-f-x domain generated by the generalized S transform works as a filter that self-adaptively filters out high wave number components in the x direction.

The cutting wave number can be self-adaptively determined from the data and changed as a function of frequency. This method does not require convolution, so the sample interval between seismic traces does not need to be equal. This method has low amplitude distortion and can suppress major background noise. However, not all wave energy from steep dip angles is unwanted and the removal of IMF1 may remove some reflection energy that we wanted (for example, red arrows at 1.6 s in Figure 8c and 3.1 s in Figure 9c pointed out strong energy that contains reflection energy from the steep dip angle interface), so applying this method may reduce some portion of the reflection wave energy and this still needs improvement. The high wave number component is related to random noise and coherent noise from steep dips, but in other wave number components there are also some interference and noise. So we suggest using the signal difference sections to estimate if useful signals are over-suppressed or to determine which IMFs should be removed to improve data quality.

Conclusions

The real data study shows that in the t-f-x domain generated by the generalized S transform, applying the EMD method on each equal frequency band data volume can self-adaptively determine the cutting wave number of data and suppress random and steep dip angle coherent noise. Comparing with traditional denoising tools, the proposed method applies the generalized S transform in the time direction to analyze the timefrequency distribution at each time and does not need the piecewise stationary assumption. So this method can deal with problems like complicated structure and coherent noise pollution. What's more important, this EMD method can be applied on irregularly sampled data. In most situations, denoising only needs to remove the IMF1 component, does not need to consider parameter adjustment, and can also adjust the denoising scheme freely. However, care must be taken because this method is affected by extreme value picking accuracy and the terminal process method.

Acknowledgments

Sincere thanks to Qin Si from the Xi'an Research Institute of China Coal Technology & Engineering Group Corporation for his careful help and valued suggestions. We also want to thank the anonymous reviewers for their thoughtful comments and valuable suggestions, which greatly improved the manuscript.

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