APPLIED GEOPHYSICS, Vol.8, No.4 (December 2011), P. 363 - 369, 5 Figures.
DOI:10.1007/s11770-011-0299-8

Study on the scales of heterogeneous geologic bodies in random media*

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Abstract: In order to study the scale characteristics of heterogeneities in complex media, a random medium is constructed using a statistical method and by changing model parameters (autocorrelation lengths *a* and *b*), the scales of heterogeneous geologic bodies in the horizontal and the vertical Cartesian directions may be varied in the medium. The autocorrelation lengths *a* and *b* represent the mean scale of heterogeneous geologic bodies in the horizontal and vertical Cartesian directions in the random medium, respectively. Based on this model, the relationship between model autocorrelation lengths and heterogeneous geologic body scales is studied by horizontal velocity variation and standard deviation. The horizontal velocity variation research shows that velocities are in random perturbation. The heterogeneous geologic body scale increases with increasing autocorrelation length. The recursion equation for the relationship between autocorrelation lengths and heterogeneous geologic body scales is determined from the velocity standard deviation research and the actual heterogeneous geologic body scale magnitude can be estimated by the equation.
Keywords: random medium, autocorrelation length, velocity standard deviation, heterogeneous geologic body scale

Introduction

Seismic exploration subjects have been complex heterogeneous media and seismic wave propagation in actual media is affected by geologic heterogeneities, especially the heterogeneous geologic body scale. According to scattering theory (Wu et al., 1993), during seismic wave propagation, when the geologic bodies scale can be comparable to the seismic wavelength, the seismic wave will be scattered when it encounters geologic bodies. The scattering research in some fields has made progress (Aki, 1969; Aki and Chouet, 1975; Aki and Richards, 1980; Wu and Aki, 1985; Wu, 1985; Wu, 1989; Zeng et al., 1991; Eaton, 1999; Hu et al., 2010; Liu, 2010; Chen, 2011; Lei et al., 2011). Different scales and constituents heterogeneous geologic bodies may produce different seismic wave scattering. Consequently, the distribution and character of the heterogeneities can be inferred from the scattering. Furthermore, heterogeneity is usually associated with more geologic structure and oil, gas, and ore resources (Wu et al., 1993). Therefore, it is widely and practicably valuable to study the scales of heterogeneous geologic bodies in complex heterogeneous media.

For anomalies which can’t be ignored in complex heterogeneous media, traditional homogeneous medium

Manuscript received by the Editor September 1, 2010; revised manuscript received July 7, 2011.
*This research is sponsored by the 973 Program (No. 2009CB219505) and the Talents Introduction Special Project of Guangdong Ocean University (No. 0812182).
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or layered homogeneous medium theory has difficulty describing them completely. Statistical methods can describe medium heterogeneities flexibly, conveniently, and completely (Xi and Yao, 2001). Using a statistical representation of inhomogeneities, we constructed a model consisting of large and small scale inhomogeneities. The large scale inhomogeneities were the mean characteristics of the earth while the small scale inhomogeneities were fluctuations from these mean values (Xi and Yao, 2005). The 2-D random medium with an isotropic auto-correlation function has also been used to describe the small scale inhomogeneities (Frankel and Clayton, 1984). Here the inhomogeneities are isotropic and do not have any preferred orientation. Ikelle et al. (1993) proposed a random medium with small scale inhomogeneities as a random process in space, regarded a complex heterogeneous media containing a large number of random distributions and small-scale inhomogeneities as a random media, and also considered the inhomogeneity orientations and constructed a 2-D random medium with an ellipsoidal autocorrelation function.

The previous research only constructed random medium models for describing complex heterogeneous media, but the relationship between autocorrelation lengths of the random medium and the heterogeneous geologic body scales in the complex medium was not studied.

In this paper, we construct a random medium model and the relationship between model autocorrelation lengths and heterogeneous geologic body scales was studied by horizontal velocity variation and standard deviation based on the random model. The qualitative knowledge was obtained from the horizontal velocity variation research. The randomly perturbed velocities have characteristics of mean and variance and the heterogeneous geologic body scale increases with increasing autocorrelation length. From the velocity standard deviation, we derive the recursion equation between autocorrelation length and heterogeneous geologic body scale. The actual magnitude of the heterogeneous geologic body scales was achieved quantitatively by model autocorrelation lengths using this equation. It has actual significance for seismic exploration in complex areas with heterogeneous media.

Random medium models

Basic concept of a random medium

The random medium model is based on the concept that inhomogeneities in a complex heterogeneous medium are irregularly distributed small scale anomalies that can be described using the statistical method (Korn, 1993). The random medium model consists of various scale inhomogeneities. The large scale inhomogeneities represent the mean characteristics of the medium, that is, the traditional geologic model. The small scale inhomogeneities are fluctuations from these mean values and are usually expressed by a random process in space which has zero mean and is second order and steady. The small scale fluctuations in space may be described by a few statistical property parameters that are spatial autocorrelation functions, autocorrelation lengths, mean values, and variance (Ikelle et al., 1993; Xi and Yao, 2001).

Taking a two-dimensional (2-D) sound-wave random medium as an example, we decompose the velocity $v(x, z)$ and density $\rho(x, z)$ in spatial coordinates (x, z) into:

$$v(x,z) = v_0 + \delta v(x,z), \tag{1}$$

$$\rho(x,z) = \rho_0 + \delta\rho(x,z), \tag{2}$$

where v_0 and ρ_0 represent the velocity and density of large scale inhomogeneities, which we assume to be homogeneous or vary with (x, z) coordinate slowly, and $\delta\rho$ and $\delta\rho$ represent the velocity and density of small scale inhomogeneities. According to the Birch theory (Birch, 1961), the relative fluctuation of density and velocity are assumed to be linear. Therefore, one fluctuation parameter (velocity or density) can be used to describe the small scale inhomogeneities in the random medium, for example, velocity fluctuation, which is expressed as

$$\sigma(x,z) = \delta v / v_0 = K^{-1}\delta\rho / \rho_0, \tag{3}$$

where $\sigma(x, z)$ is the relative fluctuation in space and K is the ratio constant with a range of 0.3 to 0.8. Supposing $\sigma=\sigma(x, z)$ is second order and a steady spatial random process with zero-mean and an autocorrelation function and variance. From equations (1), (2), and (3) we get:

$$v(x,z) = v_0 + \delta v(x,z) = v_0(1+\sigma), \tag{4}$$

$$\rho(x,z) = \rho_0 + \delta\rho(x,z) = \rho_0(1+\sigma). \tag{5}$$

The common autocorrelation functions describing random media have Gaussian, exponential, and Von Karm types. They have their own characteristics, which are suitable for different geological cases. The exponential autocorrelation function has the characteristics of multiple scales and self-similarity and it simulates real media conveniently. In this paper, we use the exponential elliptical autocorrelation function to describe the random medium. Its expression is:

$$\varphi(x,z)=\exp\sqrt{-\left(\frac{x^2}{a^2}+\frac{z^2}{b^2}\right)}, \quad (6)$$

where a and b are the autocorrelation lengths in the x and z directions, respectively.

Random medium model algorithm

Based on random process theory, the autocorrelation function Fourier transform is expressed as the random process power spectrum. The random fluctuations are simulated from the power spectrum function based on the random process spectrum expansion theory (Xi and Yao, 2001). Taking the 2-D random medium for an example, the random medium model is generated by the chosen autocorrelation function $\varphi(x, z)$ (Ikelle et al., 1993; Zhang and Zeng, 2003; Wu et al., 2008), the specific modeling steps are:

First, the power spectrum function of the random process is generated. The 2-D Fourier transform is carried out to the exponential autocorrelation function as $\varphi(x, z)$

$$\Phi(k_x,k_z)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\varphi(x,z)\mathrm{e}^{-\mathrm{i}(k_x x+k_z z)}\mathrm{d}x\mathrm{d}z. \quad (7)$$

where k_x and k_z are the wavenumbers corresponding to the x and z coordinates, respectively.

Second, the 2-D random field is generated. The discrete random field $\theta(k_x, k_z)$ is generated by the random reactor, distributed evenly on the interval $[0, 2\pi]$. The random reactor is a function or subroutine generating random numbers in computer program.

Third, the random spectrum function is generated. The random power spectrum is the product of the power spectrum and 2-D random field $\theta(k_x, k_z)$ based on the random process method. The autocorrelation function and random numbers are all continuous. However, the computations are performed discretely and, as a result, some errors are introduced in the process, resulting in the random fluctuation obtained no longer agrees with the assumptions that the random medium is stationary with a constant mean value. These errors are weakened by smoothing. Finally, a random spectrum function is derived as:

$$\Psi(k_x,k_z)=\sqrt{\Phi(k_x,k_z)}\cdot\exp[\mathrm{i}\theta(k_x,k_z)]. \quad (8)$$

Fourth, by 2-D inverse Fourier transform of the random spectrum function $\Psi(k_x, k_z)$, we get the random fluctuation $\psi(x, z)$ as:

$$\psi(x,z)=\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Psi(k_x,k_z)\mathrm{e}^{\mathrm{i}(k_x x+k_z z)}\mathrm{d}k_x\mathrm{d}k_z. \quad (9)$$

Fifth, the mean value and variance of the random fluctuation are calculated by

$$\mu=\langle\psi(x,z)\rangle, \text{ and } d^2=\langle(\psi(x,z)-\mu)^2\rangle. \quad (10)$$

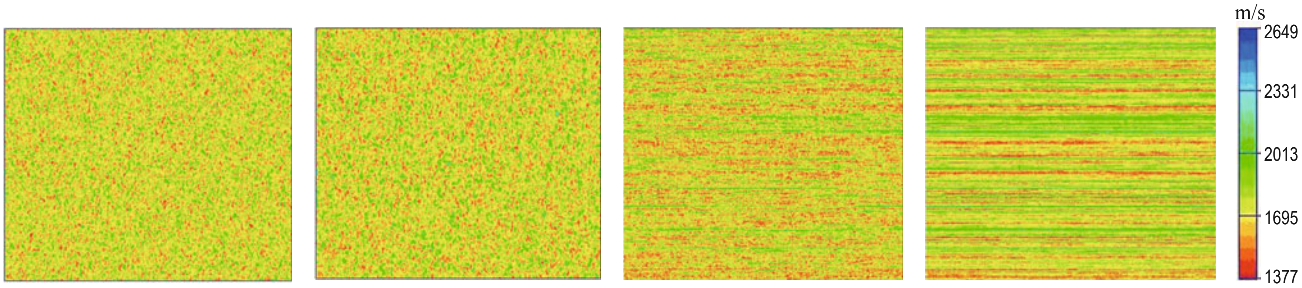
Sixth, the random fluctuation $\psi(x, z)$ is normalized to zero mean and desired variance ε^2, by the exponential autocorrelation function $\varphi(x, z)$. The random fluctuation is rewritten as

$$\sigma(x,z)=\frac{\varepsilon}{d}\cdot[\psi(x,z)-\mu]. \quad (11)$$

The random fluctuation $\sigma(x, z)$ is substituted into equations (4) and (5) and the velocity and density are derived, from which the random medium model is built.

Description and building of the random medium model

We have built the random media with the exponential ellipsoidal autocorrelation function (equation (6)) based on the model generating steps. Figure 1 shows the models with 500 m in the horizontal direction and 250 m in the vertical direction. The four models contain various scale inhomogeneities generated by four particular autocorrelation lengths pairs (a, b): (1, 1), (5, 5), (1000, 1) and (∞, 1) (units are meters). The grid step is 1 m in the horizontal and the vertical directions. The mean velocity $\bar{v}$ is 1700 m/s, the mean density $\bar{\rho}$ is 2.1 g/cm^3 and the standard deviation ε is 10 %. The ratio constant K is 0.5.



(a) Random medium with a = 1 m and b = 1 m.
(b) Random medium with a = 5 m and b =5 m.
(c) Random medium with a = 1000 m and b=1 m.
(d) Random medium with a = ∞ m and b = 1 m.

Fig. 1 Random media with an exponential autocorrelation function.

From Figure 1 we see that the random medium is very similar to core slices. With autocorrelation lengths varying, the inhomogeneity scales will change in the horizontal and vertical directions. From Figures 1c and 1d, fixing autocorrelation length *b*, when the autocorrelation length *a* increases, the model layer characteristics becomes distinct. Especially when *a* approaches infinity, a 2-D random medium becomes a 1-D random medium. This shows that the 2-D random medium can better describe terrestrial thin formation heterogeneities. Therefore, model parameter autocorrelation lengths *a* and *b* can describe the mean inhomogeneity scale in random media in both the horizontal and vertical directions. A random medium model can simulate a realistic subsurface medium.

The relationship between inhomogeneity scales and autocorrelation lengths

From Figure 1 we see that with increasing autocorrelation lengths, the inhomogeneity scales in models will also enlarge and when autocorrelation lengths are very large, the model layer characteristic is obvious. However, it is only a qualitative knowledge. We will research the relationship also from a quantitative angle and show how actual heterogeneity scales are derived from model autocorrelation lengths.

We study the relationship using velocity horizontal variation and statistical properties. From the exponential autocorrelation function in equation (6), we see that the horizontal and vertical autocorrelation lengths are symmetrical. Hence, only the relationship between the horizontal autocorrelation length and the horizontal inhomogeneity scale is studied and the result can also be adapted to the relationship between the vertical autocorrelation length and the vertical inhomogeneity scale.

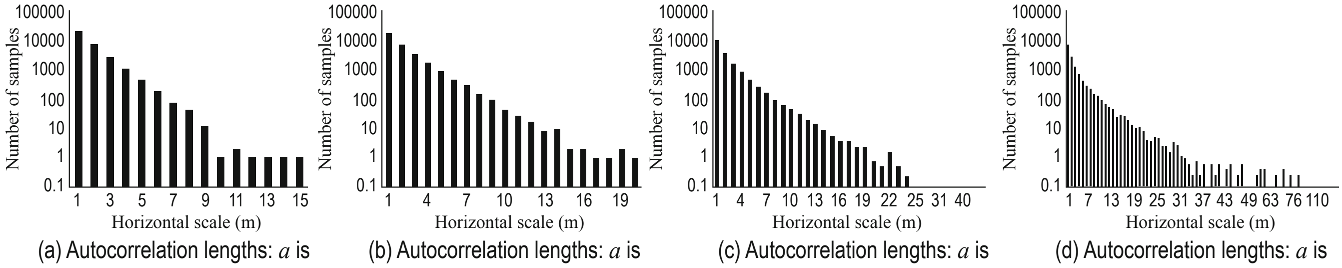
The horizontal velocity variation

The model parameters used in Figure 2 are the same as in Figure 1 and the grid step is 1 m in the horizontal and vertical directions. Nevertheless, the autocorrelation lengths are different. The vertical autocorrelation length *b* is 1 m and the horizontal autocorrelation length has four different parameters: 1 m, 10 m, 100 m, and 1000 m in building four models. The relationship between autocorrelation lengths and inhomogeneity scales is studied using algorithms from Li (2006), the special realization steps are:

First, the velocities in models subtract the background velocity 1700.0 m/s and velocity variations δv are derived from equation (4). Because δv reflects the difference between the velocity at every position and the background velocity and also produces the seismic response, δv will be used in the following computation.

Second, the chosen threshold velocity vv is 51.0 m/s and δv is classified. When $|\delta v|<vv$, the velocity variant in this position is assumed to be zero, hence the velocity fluctuation is small and the seismic response is not distinct. When $\delta v<-vv$, the velocity variant in this position is assumed to be -1 and the velocity is smaller than the background velocity, yet the velocity difference between them is large and the seismic response can be produced. When $\delta v>vv$, the velocity variant is assumed to be +1 and the velocity is bigger than the background velocity, the velocity difference between them is big, and the seismic response can also be produced. Through this classification scheme, the velocity data body contains -1, 0, and +1.

Third, scanning the classified data body at every line, if adjacent data values are equal, the number or width of the equal values will be recorded. The width is the number of equal values subtracted by 1 and multiplied by the horizontal grid interval. The number and the corresponding width of the adjacent equal data are calculated on every line and the width is a measure of the horizontal inhomogeneity scale. The number of each scale is counted and the histogram of the horizontal



(a) Autocorrelation lengths: *a* is 1 m and *b* is 1 m.
(b) Autocorrelation lengths: *a* is 10 m and *b* is 1 m.
(c) Autocorrelation lengths: *a* is 100 m and *b* is 1 m.
(d) Autocorrelation lengths: *a* is 1000 m and *b* is 1 m.

Fig. 2 Probability distribution of different horizontal scales.

scales is derived, as shown in Figure 2. We see that the probability distribution of the horizontal inhomogeneity scales is the same as the different horizontal autocorrelation lengths and the number of samples is largest when the horizontal scale is 1. With increasing horizontal scale, the number of samples decreases, showing that the model velocity varies greatly in the horizontal direction. However, when the horizontal autocorrelation length increases, the number of samples will decline significantly with increasing horizontal scale. At the same time, as the number of horizontal scales increases, the largest horizontal scale also increased. This shows that the horizontal inhomogeneity scale increases with increasing horizontal autocorrelation length.

Therefore, the relationship between autocorrelation lengths in random media and inhomogeneity scales is studied through the horizontal velocity variation, from which we deduce the qualitative knowledge that velocities in random medium vary acutely in the horizontal direction and horizontal inhomogeneity scale increases with increasing horizontal autocorrelation length. This conclusion accords with the results from Figure 1.

The velocity standard deviation

The model size is 1280 m in the horizontal direction and 1280 m in the vertical direction and the grid step is 5 m both directions. The vertical autocorrelation length *b* is fixed at 10 m and the horizontal autocorrelation lengths are 1, 5, 10, 20, 50, 80, 100, 200, 300, 400, 500, 600, 700, 800, 900, and 1000 m. So there are sixteen models. Other parameters are the same as in Figure 1.

The velocity standard deviation describes the degree of velocity deviation and the background velocity is a mean velocity. When the models are built, the velocity standard deviation is normalized to 10 percent. Nevertheless, the velocity standard deviations in a small area around any position are different and the standard deviations are related to the inhomogeneity scales. The algorithms (Li, 2006) for studying the relationship between autocorrelation length and inhomogeneity scale using velocity standard deviation are:

First, for models with different horizontal autocorrelation lengths, the total model is divided into small time windows with the same length and width. The time window length is fixed at 25 m (5 grid nodes). The time window width changes continuously from one to 30 grid nodes with an interval of 1 grid spacing. Based on the actual model scale, the time window width is assumed to be the corresponding horizontal inhomogeneity scale.

Second, changing the time window width, the standard deviation is calculated in each model. The velocity standard deviation of each small time window is first computed, then standard deviations of all time windows are averaged, and, finally, the standard deviation mean is considered as the standard deviation corresponding to the time window width.

Figure 3 shows the relationship between the velocity standard deviations and the time window widths for different horizontal autocorrelation lengths. Owing to so many models, only five curves are displayed. From Figure 3 we see that the velocity standard deviation increases with increasing time window width and it illuminates the fluctuation range as velocities increase. For the same time window width, the velocity standard deviation decreases with increasing horizontal autocorrelation length and it shows that the velocity fluctuation range gets smaller. For different autocorrelation lengths, the velocity standard deviation variation is the same as the time window width

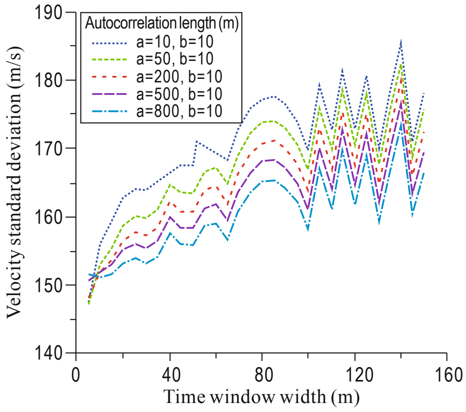


Fig. 3 Relationships of velocity standard deviation and time window width.

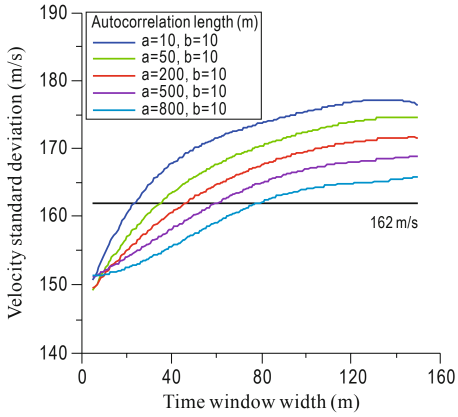


Fig. 4 Regression curves of velocity standard deviations corresponding to different time window widths.

increases. Therefore, the velocity standard deviations curves corresponding to different time window widths are derived by polynomial fitting. Figure 4 displays only five curves and from the figure we see that the velocity standard deviations increased with increasing time windows widths, whereas the velocity standard deviation tends to stabilize near 170 m/s when the time windows widths reach a scale of 120 m. For some velocity standard deviations, the time window widths are different for different horizontal autocorrelation lengths which shows that the corresponding inhomogeneity scales are different. Therefore, a fixed velocity standard deviation is intersected with the deviation curves with different autocorrelation lengths and the horizontal coordinates intersection points corresponds to different time window widths. It also corresponds to different horizontal inhomogeneity scales. The fixed velocity standard deviation is 162.0 m/s in Figure 4. The horizontal inhomogeneity scales corresponding to the horizontal autocorrelation lengths are extracted, the relationship between them is derived and shown as the green line in Figure 5. The horizontal autocorrelation length and horizontal inhomogeneity scale curve is obtained by polynomial regression and is shown as the red line in Figure 5. The regression equation is

$$y=20.2226+0.2514x-7.1893\times10^{-4}x^2+9.4781\times10^{-7}x^3-4.0856\times10^{-10}x^4$$

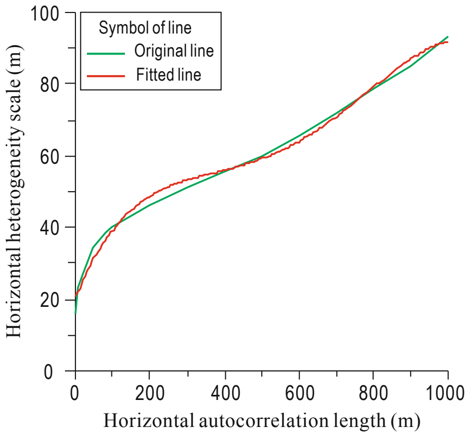


Fig. 5 The relationship of horizontal autocorrelation length and horizontal heterogeneity scale and its regression curve.

From Figure 5 we see that horizontal autocorrelation lengths and actual horizontal inhomogeneity scales display a tendency to increase with autocorrelation length, meaning that horizontal inhomogeneity scale also increases when horizontal autocorrelation length increases. This agrees with the conclusion derived from previous random medium models. Furthermore, based on the regression equation, the actual inhomogeneity scale magnitude was obtained quantitatively by the horizontal autocorrelation length. However, the method needs additional verification that the relationship between autocorrelation length and inhomogeneity scale in random media is derived by velocity standard deviation.

Conclusions

Random medium models are built based on the statistical method in this paper and the model parameter autocorrelation lengths *a* and *b* describe the mean scale of inhomogeneities in horizontal and vertical position, respectively. The relationship between model autocorrelation length and inhomogeneity scale in random media was studied qualitatively and quantitatively by horizontal velocity variation and standard deviation.

The relationship between autocorrelation length and inhomogeneity scale was studied by horizontal velocity variation in models. The qualitative knowledge is obtained that velocities in random media vary acutely and horizontal inhomogeneity scale will increase with increasing horizontal autocorrelation length.

The relationship between autocorrelation length and inhomogeneity scale is also studied by velocity standard deviation. The same conclusion is reached that horizontal inhomogeneity scale will increase with increasing horizontal autocorrelation length. The recursion equation is derived and the actual inhomogeneity scale magnitude is obtained quantitatively by the horizontal autocorrelation length.

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