APPLIED GEOPHYSICS, Vol.8, No.3 (September 2011), P. 217 - 224, 9 Figures.
DOI:10.1007/s11770-011-0286-0

Shifted first arrival point travel time NMO inversion

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Abstract: Serious stretch appears in shallow long offset signals after NMO correction. In this article we study the generation mechanism of NMO stretch, demonstrate that the conventional travel time equation cannot accurately describe the travel time of the samples within the same reflection wavelet. As a result, conventional NMO inversion based on the travel time of the wavelet's central point occurs with errors. In this article, a travel time equation for the samples within the same wavelet is reconstructed through our theoretical derivation (the shifted first arrival point travel time equation), a new NMO inversion method based on the wavelet's first arrival point is proposed. While dealing with synthetic data, the semblance coefficient algorithm equation is modified so that wavelet first arrival points can be extracted. After that, NMO inversion based on the new velocity analysis is adopted on shot offset records. The precision of the results is significantly improved compared with the traditional method. Finally, the block move NMO correction based on the first arrival points travel times is adopted on long offset records and non-stretched results are achieved, which verify the proposed new equation.
Keywords: long offset, NMO stretch, first arrival point, travel time equation, NMO inversion

Introduction

Conventional normal moveout corrections can only level a short offset phase axis without stretch. However, a shallow long offset phase axis can't be leveled correctly and exhibits serious sample stretch after NMO correction. Geophysicists have proposed various methods to improve travel time equation accuracy in order to eliminate NMO stretch. Taner and Koehler (1969) proposed a 4th order NMO equation, Castle (1994) proposed a shifted hyperbolic equation, Al-Chalabi (1973) proposed a series expansion approximation of the multi-layer travel time equation, Hake (1984) proposed a third-order Taylor series expansion for P and SV waves, and Xue et al (2003) studied the accuracy of high-order travel time equations. On the other hand, more geophysicists realize that NMO corrections for all the same wavelet's samples should be constant. Rupert and Chun (1975) proposed the block move sum normal moveout correction, Shatilo and Aminzadeh (2000) proposed a constant normal-moveout (CNMO) correction, and Trickett (2003) proposed stretch-free stacking; all of which eliminate the long-offset NMO stretch by block moving entire wavelet signal at the same time.

Semblance coefficient velocity analysis (Taner and Koehler, 1969) has always been the standard velocity analysis method and is widely used in converted wave velocity analysis (Thomsen, 1999; Yuan and Li, 2001) and VTI media velocity analysis (Alkhalifah and Tsvankin, 1995; Alkhalifah, 2000; Tsvankin, 1996; You et al., 2006). Up to now, the core algorithms to extract velocity information are all based on the travel time of wavelet's central point. However, our research shows that the conventional travel time equation can not accurately describe the travel time of the wavelet'

Manuscript received by the Editor December 24, 2010; revised manuscript received April 29, 2011.
*The research is sponsored by the National Natural Science Foundation of China (No. 41074075).
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s central point, so the velocity information described in the conventional equation is not accurate. On the basis of previous research, we reconstructed the travel time equation of samples within the same wavelet, proposed a NMO correction method based on the travel time of the first arrival point, modified the semblance coefficient algorithm, and improved the inversion accuracy dramatically. Finally, the block moving reflection wavelet signal based on the first arrival point travel time is performed on long offset records and non-stretched results are achieved.

Theory derivation

The Dix travel time equation is:

$$t = (t_0^2 + \frac{x^2}{V_{nmo}{}^2(t_0)})^{\frac{1}{2}}, \tag{1}$$

where t represents the reflection travel time, t_0 is the vertical two-way travel time, x is offset, and V_{nmo} (t_0) is the NMO velocity at t_0. The conventional travel time equation can "directly" depict a hyperbolic travel time curve for every sample in a zero-offset trace. As shown in Figure 1, the reflection wavelet was recorded from $t_0(a)$ to $t_0(b)$ in the zero-offset trace and the i-th sample is recorded at t_0 (i). The time interval between $t_0(i)$ and $t_0(a)$ can be calculated as $\Delta T(i)=i \cdot dt$. Based on the travel time equation, at offset x their corresponding positions should be:

$$t(a) = (t_0^2(a) + \frac{x^2}{V_{nmo}{}^2(t_0(a))})^{\frac{1}{2}}, \tag{2}$$

$$t(i) = (t_0^2(i) + \frac{x^2}{V_{nmo}{}^2(t_0(i))})^{\frac{1}{2}}. \tag{3}$$

Comparing equations (2) and (3), it is obvious that the shape of the travel time curves is different and the wavelet was stretched at long offset after NMO correction. This means the sample moveout within the same wavelet is different, which creates the NMO stretch. Based on this, Shatilo and Aminzadeh (2000) proposed a Constant Normal Moveout Correction (CNMO). In the CNMO correction, the normal moveout of all samples within the same wavelet is calculated using equation (1) based on the wavelet travel time at the central point which is held constant during the NMO correction. Although the examples given in Shatilo and Aminzadeh (2000) show significant improvement, in our research we found that the travel time of wavelet's central point calculated from equation (1) is not accurate.

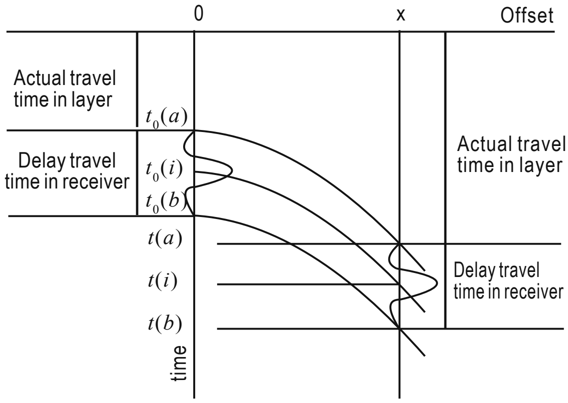


Fig.1 Travel time of first arrival point.

Reflection wavelets have a certain time length, so their signal points reach a receiver at different times. In Figure 1, at the zero-offset trace $t_0(a)$ is the earliest wavelet arrival time. After then, the earliest arrival signal will take a time delay for the subsequent signals to arrival in order. Finally the total travel time of all sample arrivals will be recorded at $t_0(b)$ after the wavelet time length. So the travel time of the signal finally recorded at the zero-offset trace is the sample point travel time at $t_0(i)$, which can be divided into two parts: the vertical two-way travel time in the layer $t_0(a)$ and the delay travel time at the receiver $\Delta T(i)$:

$$t_0(i) = t_0(a) + \Delta T(i). \tag{4}$$

This is an interesting result, which means the vertical two-way travel times of all samples within the same wavelet are the same constant value $t_0(a)$, instead of variable $t_0(i)$. Similarly at non-zero offset traces, the corresponding record time $t(i)$ should also be divided into two parts for processing. $t(a)$ is the first wavelet arrival point corresponding to $t_0(a)$ recorded at offset x which directly indicates the travel time of the wavelet in the layer, hence its time travel can be "directly" described by equation (2). The time interval between the identical sample and the first arrival point of the wavelet recorded at a different offset is constant, so the travel time $t_0(i)$ at the offset x should be described by first arrival travel time "indirectly":

$$t(i) = t(a) + \Delta T(i) = (t_0^2(a) + \frac{x^2}{V_{nmo}(t_0(a))^2})^{\frac{1}{2}} + \Delta T(i). \tag{5}$$

This is the reconstructed travel time equation for each sample within the same wavelet. Equation (3) mixes the travel time in layer and the delay travel at the receiver, and, therefore, is not accurate. Equation (5) describes the travel time of these samples as the same shape hyperbolas with the first arrival point, which corresponds to shifting the first arrival point travel time curve to the other samples of the wavelet based on the corresponding interval $\Delta T(i)$. So we call equation (5) the shifted first arrival point travel time equation. While $\Delta T(i)$ is zero, equation (5) reduces to equation (2), which means the traditional equation can only accurately describe the wavelet's first arrival travel time. Now we are able to clearly understand the generation mechanism of NMO stretch: when the reflection wavelet appears and traditional NMO correction moves the samples of the wavelet according to an inaccurate equation. This error finally results in increasingly serious NMO stretch at long offsets.

While dealing with multilayer reflected wavelet travel times, a high-order equation should be adopted (Taner and Koehler, 1969). The traditional equation is:

$$t = (c_1 + c_2x^2 + c_3x^4 + c_4x^6 + ...)^{\frac{1}{2}},$$

$$c_1 = t_0^2, \quad c_2 = \frac{1}{\mu_2}, \quad c_3 = \frac{\mu_2^2 - \mu_4}{4t_0^2\mu_2^4},$$

$$c_4 = \frac{2\mu_4^2 - \mu_2\mu_6 - \mu_2^2\mu_4}{t_0^4\mu_2^7}, \quad \mu_j = \frac{\sum_{k=1}^{N}\Delta t_k v_k^j}{\sum_{k=1}^{N}\Delta t_k}, \tag{6}$$

where c is the corresponding coefficient, Δt_k is the wavelet vertical two-way travel time in the k-th layer, v_k is the interval velocity, and μ_j is the weight of v_k with Δt_k. We combined the 4[th] order truncated equation with equation (5) to obtain the shifted first arrival point 4[th] order travel time equation:

$$t(i) = t(a) + \Delta T(i) = (c_1' + c_2'x^2 + c_3'x^4)^{\frac{1}{2}} + \Delta T(i),$$

$$c_1' = (t_0(a))^2, \quad c_2' = \frac{1}{\mu_2}, \quad c_3' = \frac{\mu_2^2 - \mu_4}{4(t_0(a))^2\mu_2^4},$$

$$\mu_j = \frac{\sum_{k=1}^{N}\Delta t_k v_k^j}{\sum_{k=1}^{N}\Delta t_k}. \tag{7}$$

where $t(a)$ is the travel time of wavelet's first arrival point, $t_0(a)$ is vertical two-way travel time, and the c's are the corresponding coefficient.

Shifted first arrival travel time NMO inversion

Conventional hyperbolic NMO inversion is based on the reflection wavelet's central point travel time. A time window with time length T is moved along the zero-offset trace and the central point is chosen as the vertical two-way travel time for calculation. Next we compute a hyperbolic travel time curve corresponding to a particular velocity in a given range and the coherent energy is calculated. The value of V_{nmo} and t_0 corresponding to the highest points of the velocity spectrum are the best inversion results:

$$S(t_0, V_{nmo}) = \frac{\sum_{t_0'=t_0-T/2}^{t_0+T/2}\left[\sum_{x=x_{\min}}^{x_{\max}} F(x,t)\right]^2}{M\sum_{t_0'=t_0-T/2}^{t_0+T/2}\sum_{x=x_{\min}}^{x_{\max}} F^2(x,t)}, \tag{8}$$

where S is the semblance coefficient, $F(x, t)$ is the amplitude of seismic event along the travel time, and M is the trace number. As shown in Figure 2, when the time window coincides with the effective wavelet, the central point $t_0(2)$ is chosen as its vertical two-way travel time:

$$t(2) = (t_0^2(2) + \frac{x^2}{(V_{nmo}')^2})^{\frac{1}{2}}, \tag{9}$$

where V_{nmo}' is the velocity corresponding to the extreme point. However the problem is: $t_0(2)$ is not the actual vertical two-way travel time of this wavelet according to our research, which means the travel time of this wavelet's central point described by equation (9) is not accurate, thus the NMO velocity V_{nmo}' is not the expected one.

Only the travel time of wavelet's first arrival point represents the real travel time in the layer, which can be "directly" described by traditional equation. As shown in Figure 3, $t_0(1)$ should be chosen as the vertical two-way travel time for the wavelet's travel time calculation:

$$t(1) = (t_0^2(1) + \frac{x^2}{V_{nmo}^2})^{\frac{1}{2}}. \tag{10}$$

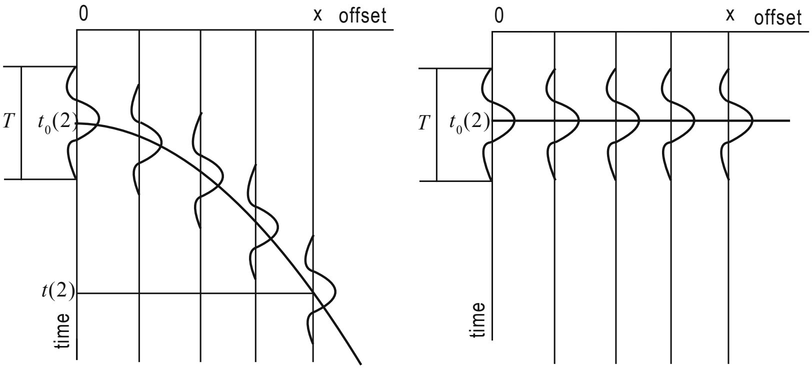


Fig. 2 Conventional velocity analysis.

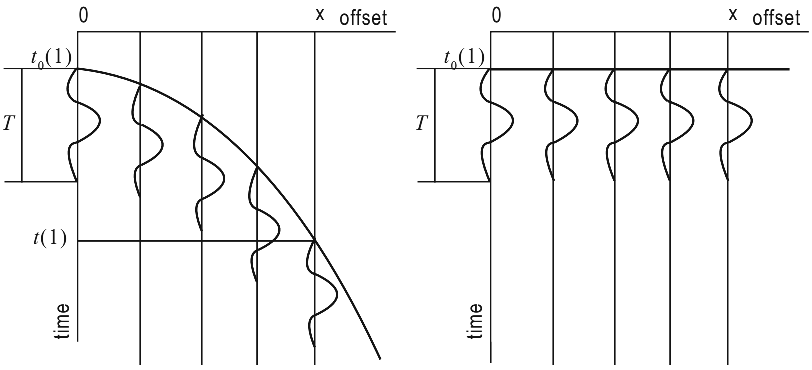


Fig. 3 Proposed velocity analysis method.

However, it is difficult to identify the reflection wavelet's first arrival point whose energy is usually zero. In order to obtain its accurate position and extract accurate NMO velocity, the conventional semblance coefficient equation should be modified as:

$$S(t_0, V_{nmo}) = \frac{\sum_{t_0'=t_0}^{t_0+T}\left[\sum_{x=x_{\min}}^{x_{\max}} F(x,t)\right]^2}{M\sum_{t_0'=t_0}^{t_0+T}\sum_{x=x_{\min}}^{x_{\max}} F^2(x,t)}. \quad (11)$$

Thus, when the window moves along the zero-offset trace, its top point always is chosen as the vertical two-way travel time for calculating the corresponding travel time curve. If the window time length T is set equal to the wavelet length, then they should overlap and the top point will exactly coincide with the first arrival time $t_0(1)$. So we can get accurate NMO velocity and layer parameters.

Synthetic data test

The model parameters are shown in Table 1. Forward modeling was processed using the finite difference method with the Aku2d program. Figure 4 shows seismic records muted by different apertures.

Tabel 1 Model parameters

Layer number	V (m/s)	V_{nmo} (m/s)	t_0 (s)	c_3 ($\times10^{-16}$)	Depth (m)
1	2500	2500.0	0.480	0	600
2	2750	2560.5	0.626	-1.0638	800
3	3000	2642.8	0.759	-2.0376	1000
4	3250	2735.6	0.882	-2.5598	1200

First, while dealing with seismic records with aperture less than 1.5, we applied both traditional hyperbolic NMO inversion and our new method.

We illustrate in Figure 5 and Table 2 that the extreme velocity spectrum values calculated by the conventional velocity analysis all correspond to the wavelet center, thus both NMO velocity and vertical two-way travel time are inaccurate. The extreme points obtained by the new method accurately correspond to the first arrival time. As a result, we see from Table 2 that NMO velocity and vertical two-way travel time are much more accurate than from the conventional method. The layer velocity and depth can be calculated by the equation:

$$V_i = \sqrt{[V_{nmo}^2(i)t_0(i) - V_{nmo}^2(i-1)t_0(i-1)]/[t_0(i) - t_0(i-1)]}, \tag{12}$$

then the fourth-order coefficient can be calculated using equation (6). After that we perform an error analysis for the inversion results.

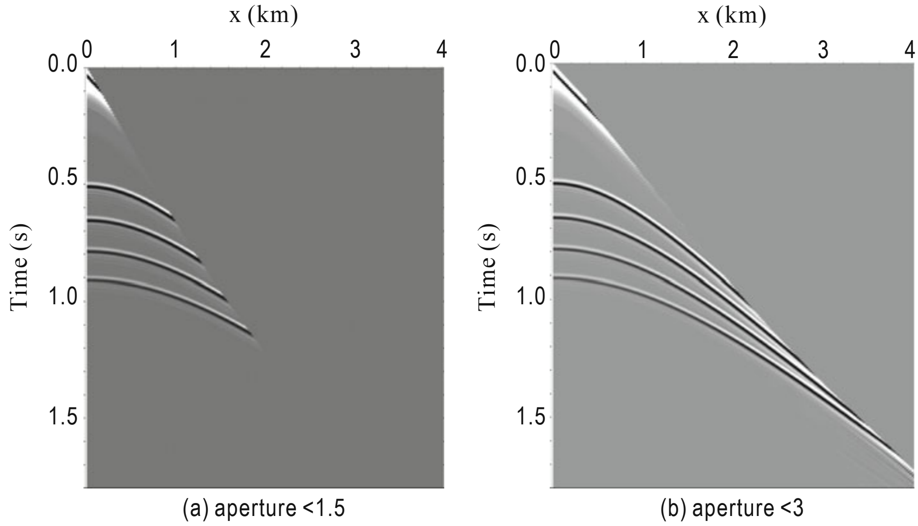


Fig. 4 CMP gather after aperture truncation.

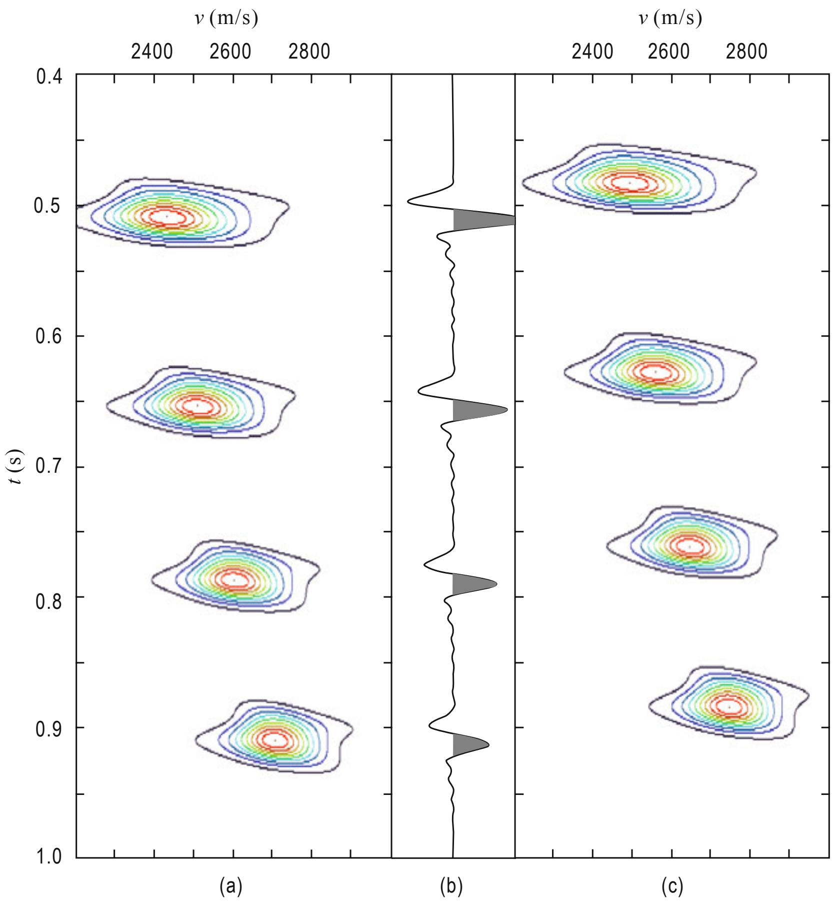


Fig. 5 (a) Conventional velocity analysis result. (b) Zero offset record. (c) Proposed velocity analysis result.

Travel NMO inversion

Table 2 Inversion results

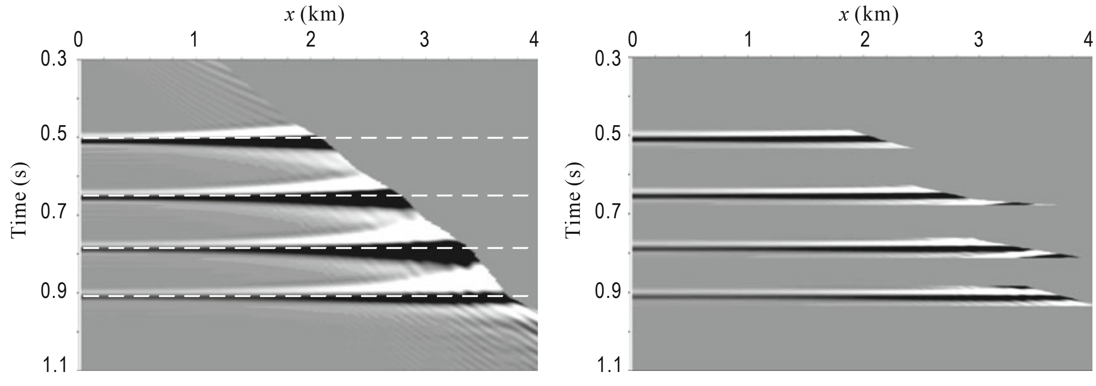
Layer number	Traditional method					Proposed method				
	t_0 (s)	V_{nmo} (m/s)	Depth (m)	V (m/s)	c_3 ($\times10^{-16}$)	t_0 (s)	V_{nmo} (m/s)	Depth (m)	V (m/s)	c_3 ($\times10^{-16}$)
1	0.509	2434	619	2434	0.540	0.483	2492	602	2492	0.575
2	0.654	2511	821	2764	-1.650	0.628	2559	804	2771	-1.082
3	0.787	2605	1025	3025	-2.854	0.761	2645	1006	3018	-2.152
4	0.910	2709	1233	3298	-3.375	0.884	2745	1213	3297	-2.793

As shown in Table 3, the conventional method can only provide a generally accurate NMO velocity. The 4th order coefficient enlarged the inversion errors to more than 30%. So inversion results are basically unreliable. However, our new method reduced all inversion errors by about one order of magnitude. This shows that the NMO inversion based on the travel time of first arrival time can significantly improve the inversion accuracy.

Table 3 Inversion errors (%)

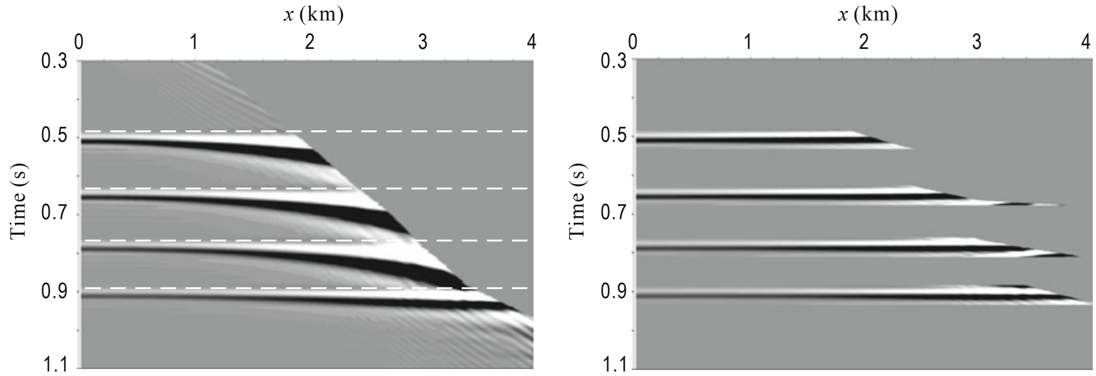
Layer number	Traditional method					Proposed method				
	t_0 (s)	V_{nmo} (m/s)	Depth (m)	V (m/s)	c_3 ($\times10^{-16}$)	t_0 (s)	V_{nmo} (m/s)	Depth (m)	V (m/s)	c^3 ($\times10^{-16}$)
1	6.04	-2.64	3.17	-2.64	-	0.63	-0.32	0.33	-0.32	-
2	4.47	-1.93	2.62	0.51	55.16	0.32	-0.06	0.50	0.76	1.70
3	3.69	-1.43	2.50	0.83	40.05	0.26	0.08	0.60	0.60	5.63
4	3.17	-0.97	2.75	1.48	31.84	0.23	0.34	1.08	1.45	9.12

Finally, a long offset record with aperture range of less than 3 is NMO corrected using the inversion parameters of the two methods. Figure 6a shows the normal fourth-order NMO correction result using traditional inversion results. We see that the wavelet central points (white dotted lines) are flat. Figure 6b is the CNMO correction result. Figure 7a shows the conventional fourth-order NMO correction result using the parameters from our new method. It's obvious that first arrival times (white dotted lines) are flattened. Figure 7b shows the block move correction result based on first arrival fourth-order travel time. Comparing Figures 6a and 7a, we see that the different mechanism between the traditional method and our new method is clear. Conventional velocity analysis is based on the wavelet's central travel times, so the NMO velocity obtained is not accurate. Inversion parameter errors are large. The CNMO correction gives non-stretched flattened NMO results but is also based on the wavelet's central time and the inversion results are not credible. Our new method can get much higher accuracy inversion results and correct NMO results without stretch, which verifies the proposed new equation.



(a) Conventional NMO correction (b) CNMO correction

Fig. 6 NMO correction using traditional inversion results.



(a) Conventional NMO correction (b) Shifted first arrival point travel time NMO

Fig. 7 NMO correction using our inversion results.

From Table 3 we still see that there is some slight errors in our inversion results. This mainly is caused by the window's chosen time length is not completely identical with the actual wavelet. For actual seismic data, the length of the wavelet varies with the depth of the layer, so it's impossible that the length of the time window will coincide with the wavelet length during the entire NMO inversion procedure. However in shallow layers, where the wavelets suffer severe stretch after NMO correction, the wavelet lengths usually remain the same. So we can still get accurate inversion results and non-stretch NMO correction results as long as the length of time window is chosen close to the wavelet length.

Conclusions

The traditional travel time equation "directly" describes the travel time point by point for all the samples. However, when the reflection wavelet appears in the seismic records, the vertical two-way travel time of each sample within it is no longer the corresponding zero-offset time but its first arrival time point. So the calculated sample travel times in the wavelet based on the traditional equation is not accurate. First arrival time points represent the actual travel time in the layer and the shifted travel time should be used to describe the other sample travel times indirectly.

Conventional NMO inversion and constant normal-moveout correction are all based on the travel time of the wavelet's central points. Although we can obtain the non-stretched flattened NMO result, the inversion results are not accurate. In this paper, both the proposed NMO inversion and block move NMO are based on the wavelet's first arrival times, which cannot only achieve correct NMO results but also higher accuracy inversion results.

Anisotropy widely exists in actual layers and anisotropic parameters are very sensitive to velocity. Subtle velocity error can lead to great anisotropic parameter inversion error. So it is urgent to improve the precision of NMO inversion. The modified travel time equation can be used directly for two parameter inversion of VTI media.

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