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Attenuation compensation method based on inversion*

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Abstract: Attenuation compensation, which corrects the attenuation and dispersion of seismic waves, is one of the effective methods for improving seismic data resolution. In general, the attenuation compensation is achieved by an inverse Q-filter based on wave field continuation. In this paper, using the Futterman attenuation model, a method to compute synthetic seismogram is derived for an attenuation medium. Based on the synthetic method, the attenuation compensation problem is reduced to an inversion problem of the Fredholm integral equation and can be achieved by inversion. The Tikhonov regularization is used to improve inversion stability. The processing results of numerical simulation and real data show the effectiveness of the method.
Keywords: Attenuation compensation, inversion, Fredholm integral equation, inverse Q-filter, regularization

Introduction

Absorption attenuation of subsurface media is one of the important factors which reduces seismic data resolution, attenuates the high frequency components of seismic data, and distorts the wavelet phase characteristics. There are many models which can describe the subsurface absorption attenuation. The most well-known model is the Futterman model (Varela et al., 1993). Toverud and Ursin (2005) contrastingly analyzed eight attenuation models using VSP data. The results demonstrated that the Futterman model can better describe the subsurface seismic wave propagation. The subsurface attenuation is mainly in high-frequency attenuation and velocity dispersion. Attenuation correction can improve seismic data resolution and involves amplitude and dispersion corrections. In general, the dispersion correction process is stable and the amplitude correction process is unstable. The inverse Q-filter is an attenuation correction method. Hargreaves and Calvert (1991) give an inverse Q-filter method in the frequency domain. In their method, the attenuation is seen as a Q-filter and the inverse Q-filter is seen as wave field back-propagation. The phase distortion can be better corrected by the method and the amplitude correction is ignored to avoid the instability. Hargreaves (1992) showed an inverse Q-filter method based on Pareto-Levy stretch, which can better deal with the instability of amplitude compensation. Wang (2002; 2003; and 2006) give an inverse Q-filter method based on wave field continuation theory which can perform the amplitude compensation and phase correction simultaneously. The amplitude limit compensation method is used to solve the amplitude compensation instability. For the multi-layer case, the compensation of absorption and attenuation is achieved by applying the method layer by layer. Zhang et al. (2007) give another inverse Q-filter method for a layered Q-value model based on wave field continuation theory. They use Gabor spectral analysis on the signals to pick time-variant gain-constrained frequencies and then deduce

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the corresponding gain-constrained amplitudes to stabilize the inverse Q-filter algorithm. Zhang and Ulrych (2007) formulate the compensation of absorption and attenuation as a least squares problem and impose regularization by means of Bayes' theorem. Yan and Liu (2009) introduce an inverse Q-filter to converted wave processing and extend a stable and effective poststack inverse Q filtering method to prestack data which uses wave field continuation along the ray path to compensate for attenuation in prestack common shot PP- and PS-waves.

In this paper, I give a new attenuation compensation method based on inversion theory. The Futterman absorption model is used to describe the attenuation of seismic waves propagating in the subsurface. A synthetic method for poststack seismograms in absorption media is deduced by mean of the exploded reflector theory. The attenuation compensation can be achieved by inversion. The inverse problem boils down to solving the first kind Fredholm integral equation problem. Mathematically solving the first kind Fredholm integral equation is an instability problem which explains the mathematical nature of the attenuation compensation instability. The attenuation compensation can be achieved by the Tikhonov regularization method which is a stability method solving the first kind Fredholm integral equation. Since the forward model considers changes in Q value with depth, the compensation method is suitable for the changing Q value with depth. The amplitude attenuation and velocity dispersion can be corrected by the compensation method. The processing results for synthetic data and real data verify the effectiveness of the proposed method.

Forward model

The exploding reflector idea is from poststack migration and poststack seismograms can be simulated by the exploding reflector model. The poststack seismogram is zero offset data. By the exploding reflector model, poststack data can be regarded as that the seismic wavelet is excited simultaneously at every subsurface point with its reflection coefficient. The wavelet amplitude is proportional to the reflection coefficient, the wavelet propagates in the medium at half of the velocity, and the seismogram is the sum of the wavelets after propagation. For the 1D case, the exploding reflector model and the convolution model are consistent. Below we prove the consistency in the frequency domain because it is easy to add the attenuation in the frequency domain.

Assuming that the reflection coefficient in the depth domain is $r(z)$ where z is depth. The average velocity of the subsurface medium is $v(z)$. The relationship between the two-way travel time t', depth z, and average velocity is

$$t'(z) = \frac{2z}{v(z)}. \tag{1}$$

The relationship between the wavelet $w(t)$ and its frequency spectrum $\hat{w}(\omega)$ is

$$w(t) = \int_{-\infty}^{\infty} \hat{w}(\omega) e^{i\omega t} d\omega, \tag{2}$$

then $w(t)$ is decomposed into a series of simple harmonics $\hat{w}(\omega)e^{i\omega t}$. First, we study the synthetic seismic data for a certain frequency harmonic and we can obtain the synthetic seismogram for wavelet $w(t)$ by taking the integral with respect to frequency. The harmonic wave exciting at depth z is the product of the reflection coefficient at point z and $\hat{w}(\omega)e^{i\omega t}$:

$$s_1(z,\omega,t) = r(z)\hat{w}(\omega)e^{i\omega t}. \tag{3}$$

The harmonic waves propagating to the surface at half velocity forms the synthetic seismogram,

$$s_2(z,\omega,t) = r(z)\hat{w}(\omega)e^{i\omega t} e^{-i\omega \frac{2z}{v(z)}}. \tag{4}$$

With two-way travel time instead of depth z the expression (4) is rewritten as

$$s_3(t',\omega,t) = r(t')\hat{w}(\omega)e^{i\omega t} e^{-i\omega t'}. \tag{5}$$

The reflection coefficient $r(t')$ at each depth can produce the corresponding harmonic wave and the seismogram received at the surface is the summation of all harmonic waves corresponding to different reflection coefficients, the expression is

$$s(\omega,t) = \int_{-\infty}^{\infty} r(t')\hat{w}(\omega)e^{i\omega t} e^{-i\omega t'} dt'. \tag{6}$$

If the wavelet is $w(t)$, the equation is

$$s(t) = \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} r(t')\hat{w}(\omega)e^{i\omega t} e^{-i\omega t'} dt'. \tag{7}$$

This is the normal convolution model which can be used to compute the seismogram in media without attenuation and can be written as

$$s(t) = \int_{-\infty}^{\infty} r(t')w(t-t')dt'. \qquad (8)$$

In attenuation media, the wave generated by reflection coefficient is still $r(z)\hat{w}(\omega)e^{i\omega t}$ but the propagation rule is different than the media without attenuation. The method used to introduce attenuation is to use the complex velocity. There are many models which can describe propagation in attenuation media. We use the model described by Wang (2002), which replaces the velocity in equation (4) with the complex velocity

$$\frac{1}{v(z)} = \frac{1}{v(z,\omega)}\left(1-\frac{i}{2Q(z)}\right), \qquad (9)$$

where $v(z, \omega)$ is the frequency-dependent phase velocity and $Q(z)$ is the medium quality factor. We use a model for the phase velocity $v(z, \omega)$ defined by

$$v(z,\omega) = v(z,\omega_0)\left|\frac{\omega}{\omega_0}\right|^{\gamma}, \qquad (10)$$

where

$$\gamma = \frac{2}{\pi}\tan^{-1}\left(\frac{1}{2Q}\right) \approx \frac{1}{\pi Q}, \qquad (11)$$

and $v(\omega_0)$ is the reference velocity which generally is the phase velocity at dominant frequency ω_0. Combining equations (9) and (4), we get the function

$$s_2(z,\omega,t) = r(z)\hat{w}(\omega)e^{i\omega t}e^{-i\omega\frac{2z}{v(z,\omega)}\left(1-\frac{i}{2Q(z)}\right)}. \qquad (12)$$

Substituting equation (10) into (12), we get

$$s_2(z,\omega,t) = r(z)\hat{w}(\omega)e^{i\omega t}e^{-i\omega\frac{2z}{v(z,\omega_0)}\left|\frac{\omega_0}{\omega}\right|^{\gamma}\left(1-\frac{i}{2Q(z)}\right)}. \qquad (13)$$

Replacing the depth z with two-way travel time t', equation (13) can be rewritten as

$$s_3(t',\omega,t) = r(t')\hat{w}(\omega)e^{i\omega t}e^{-i\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}\left(1-\frac{i}{2Q(t')}\right)}. \qquad (14)$$

This is the seismic record of the harmonic wave propagation to the surface from depth z. It is the same as no attenuation and taking the integral with respect to two-way travel time t', we get

$$s(\omega,t) = \int_{-\infty}^{\infty} r(t')\hat{w}(\omega)e^{i\omega t}e^{-i\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}}e^{-\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}\frac{1}{2Q(t')}}dt'. \qquad (15)$$

Taking the integral with respect to frequency, equation (15) can be rewritten as

$$s(t) = \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} r(t')\hat{w}(\omega)e^{i\omega t}e^{-i\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}}e^{-\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}\frac{1}{2Q(t')}}dt'. \qquad (16)$$

In the frequency domain its expression is

$$\hat{s}(\omega)=\hat{w}(\omega)\int_{-\infty}^{\infty} r(t')e^{-i\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}}e^{-\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}\frac{1}{2Q(t')}}dt'. \qquad (17)$$

Now we get the model for the exploding reflection surface in the frequency domain in attenuating media and we can use the equation to compute poststack synthetic seismograms in attenuating media. The attenuation compensation can be achieved by inversion of equation (17).

The attenuation compensation method based on inversion

The forward problem is defined by equation (17). Knowing the reflection coefficients, the spectrum of the seismogram can be computed. The compensation problem is how to compute the reflection coefficient when the seismogram spectrum is known. Assuming that seismic deconvolution has been performed and the wavelet effect eliminated, equation (17) can be rewritten as

$$\hat{s}(\omega)=\int_{-\infty}^{\infty} r(t')e^{-i\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}}e^{-\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}\frac{1}{2Q(t')}}dt'. \qquad (18)$$

Based on this equation, we define

$$a(\omega,t') = e^{-i\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}} e^{-\omega t'\left|\frac{\omega_0}{\omega}\right|^{\gamma}\frac{1}{2Q(t')}}, \quad (19)$$

then equation (18) can be reduce to

$$\hat{s}(\omega)=\int_{-\infty}^{\infty} a(\omega,t')r(t')dt'. \quad (20)$$

Equation (20) is a standard Fredholm integral equation of the first kind, in which $a(\omega, t')$ is the integral kernel function. Its numerical solution is often unstable. For this problem, measures must be taken to get a stable solution. A more effective stabilization method is Tikhonov regularization. Using this method, the problem of solving equation (20) can be reduced to the minimization of the functional

$$OBJ = \left\| \int_{-\infty}^{\infty} a(\omega,t')r(t')dt' - \hat{s}(\omega) \right\|^2. \quad (21)$$

The process of minimizing the functional (21) is unstable. The Tikhonov regularization method adds a stability functional to functional (21) to construct the following functional

$$OBJ = \left\| \int_{-\infty}^{\infty} a(\omega,t')r(t')dt' - \hat{s}(\omega) \right\|^2 + \beta\Omega(r(t')), \quad (22)$$

where $\beta > 0$ is the regularization parameter, $\Omega(r(t'))$ is the stabilizing functional which can be

$$\Omega(r(t')) = \int_{-\infty}^{\infty} \left(r(t')\right)^2 dt', \quad (23)$$

$$\Omega(r(t')) = \int_{-\infty}^{\infty} \left(\frac{dr(t')}{dt'}\right)^2 dt' \quad (24)$$

or

$$\Omega(r(t')) = \int_{-\infty}^{\infty} \left(\frac{d^2r(t')}{d\left(t'\right)^2}\right)^2 dt'. \quad (25)$$

The three functionals are the function itself, the first derivative of the function, and the second derivative and are different assumptions of the reflection coefficient. The reflection coefficient $r(t')$ can be solved by minimizing the functional (22) by the numerical method and the compensation can be achieved.

Numerical method

We first discretize and suppose that the seismic data is s_i, $i = 1, 2, \ldots n$ with a sample interval of Δt. The Fourier transform of the seismic data is $\hat{s}_i$, $i = 1, 2, \ldots n$ with a sample interval of

$$\Delta\omega = \frac{2\pi}{n * \Delta t}. \quad (26)$$

Note that the frequency domain samples $\hat{s}_i$, $i = 1, 2, \ldots n$ are complex and symmetrical. In practical problems, the signal to noise ratio of the higher frequency components is low. Supposing that there are $m < n/2$ valid data points, equation (20) can be written as

$$\hat{s}_i = \int_{-\infty}^{\infty} a(\omega_i, t')r(t')dt', \quad i = 1,2,\cdots,m, \quad (27)$$

here t' is discretized at the same interval as the seismic data. Equation (27) can be written as

$$\hat{s}_i = \sum_{j=1}^{n} a(\omega_i, t'_j)r(t'_j)\Delta t', \quad i = 1,2,\cdots,m. \quad (28)$$

Note that $\hat{s}_i$ and $a(\omega_i, t'_j)$ are both complex and equation (28) can be written in matrix form

$$\mathbf{AR} = \hat{\mathbf{S}}, \quad (29)$$

where $\mathbf{A}$ is a $2m \times n$ matrix and $2m < n$. Equation (29) is an underdetermined equation and has infinitely many solutions. From equation (29) we can see the nature of the mathematical instability of the attenuation compensation method. To get a unique solution, we must utilize the stability method. We adopt equation (22) to stabilize the solving process and get

$$\left\|\mathbf{AR} - \hat{\mathbf{S}}\right\|^2 + \beta\left\|\Omega R\right\|^2, \quad (30)$$

where Ω is the matrix corresponding to equations (26) to (28). If equation (23) is used as the stabilizing functional,

$$\Omega = \begin{pmatrix} 1 & & & & & & \\ & 1 & & & & & \\ & & \cdots & & & & \\ & & & \cdots & & & \\ & & & & \cdots & & \\ & & & & & 1 & \\ & & & & & & 1 \end{pmatrix}. \quad (31)$$

If equation (24) is used as the stabilizing functional,

$$\Omega = \begin{pmatrix} 1 & -1 & & & & & \\ & 1 & -1 & & & & \\ & & \cdots & \cdots & & & \\ & & & \cdots & \cdots & & \\ & & & & \cdots & \cdots & \\ & & & & & 1 & -1 \\ & & & & & & 0 \end{pmatrix}. \tag{32}$$

If equation (25) is used as the stabilizing functional,

$$\Omega = \begin{pmatrix} 0 & 0 & & & & & \\ 1 & -2 & 1 & & & & \\ & 1 & -2 & 1 & & & \\ & & \cdots & \cdots & \cdots & & \\ & & & \cdots & \cdots & \cdots & \\ & & & & 1 & -2 & 1 \\ & & & & & 0 & 0 \end{pmatrix}. \tag{33}$$

These three stabilizing methods are called the zero-order, first order, and second order methods. To minimize equation (30), the reflection coefficient **R** can be obtained and the attenuation compensation can be achieved. Letting the derivative of the equation (30) be equal to zero, we get

$$\mathbf{A}^{\mathbf{T}}\left(\mathbf{AR}-\hat{\mathbf{S}}\right)+\beta\Omega^{T}\Omega R=0, \tag{34}$$

$$R=\left(A^{T}A+\beta\Omega^{T}\Omega\right)^{-1}A^{T}\hat{S}. \tag{35}$$

Equation (35) can be used to achieve attenuation compensation. The regularization parameter β can have a large influence on the inversion result. When the seismic data signal to noise ratio is high, β should be small. When the signal to noise ratio of seismic data is lower, β should be larger. When processing actual data, the value of β can be determined by testing.

Numerical experiments

This section illustrates the results obtained using the compensation method based on inversion of both synthetic and real data. The first test supposes that there are seven reflection coefficients in the medium. The location of the reflection coefficients are 100, 400, 700, 1000, 1300, 1600, and 1900 ms. Figure 1

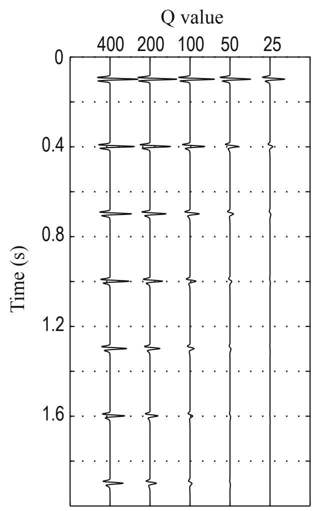
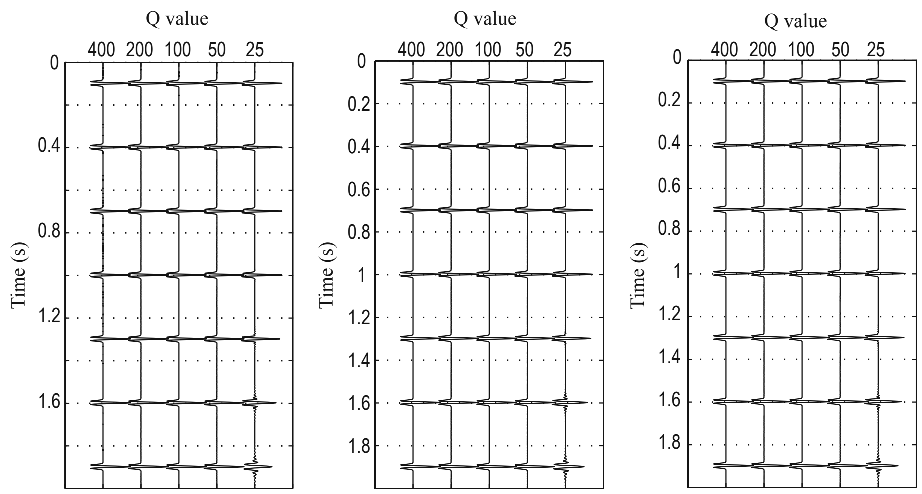


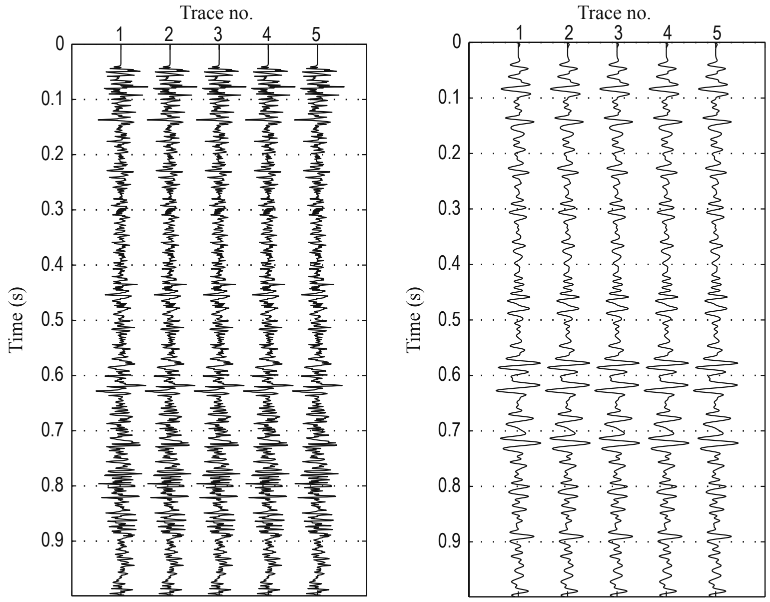
Fig. 1 Synthetic seismograms in 1D attenuation media.



(a) The result of zero-order stabilizing method. (b) The result of the first-order stabilizing method. (c) The result of the second-order stabilizing method.

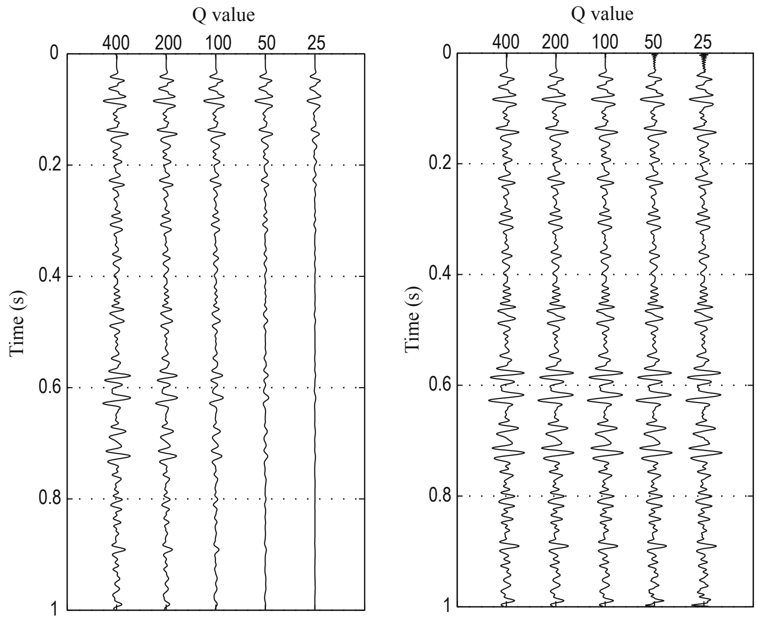
Fig. 2 The seismograms after attenuation compensation.

shows five synthetic traces with different Q values (Q = 400, 200, 100, 50, 25) constant with depth in each case. As the Q value decreases, the more seismic wave attenuation occurs and the deep seismic wave distortion is more serious. The results of applying this paper‘s compensation method to the synthetic seismic data is displayed in Figure 2. Figures 2a, 2b, and 2c correspond to the zero-, first-, and second-order stabilizing methods. The result shows that the three stabilizing methods can correctly compensate both amplitude and phase, especially for the Q value of 25. In this test the reflection coefficients are relatively sparse.



(a) Reflection coefficients. (b) Synthetic seismograms with no attenuation.

Fig. 3 Well log synthetic seismogram with no attenuation.



(a) The synthetic attenuation seismograms for the different Q values. (b) the seismograms after compensation.

Fig. 4 Seismograms before and after attenuation compensation.

For a more realistic case, we tested the reflection coefficients from real well log data. Figure 3a shows the reflectivity sequence obtained from the log data. The synthetic seismic data with no absorption is shown in Figure 3b. The synthetic seismic data for Q values of 400, 200, 100, 50, and 25 is shown in

Figure 4a. The results of applying the second-order stabilizing compensation method are shown in Figure 4b. Comparing Figures 3b and 4b, we see that the compensating results are consistent with the seismic data with no attenuation even for $Q = 25$. There is no instability and inadequate compensation in the compensating result.

We tested the processing of actual data acquired in an onshore oilfield in eastern China to verify the validity of the proposed method. The attenuation compensation experiment uses the second-order stable method for this study. From testing the quality factor Q is taken as 140 and the regularization factor is taken as 0.1. Figure 5 illustrates a migrated seismic section. The compensation results are shown in Figure 6. Comparing these two sections, we see that the migrated section resolution is improved. A number of reflections which are not separated in the section shown in Figure 5 can be observed after compensation and the lateral continuity can be tracked from trace to trace. The results demonstrate the viability of the new method.

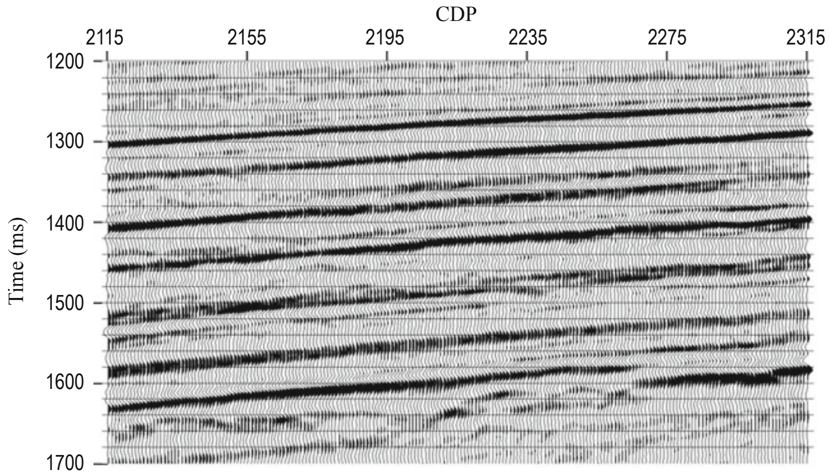


Fig. 5 A migrated seismic section before attenuation compensation.

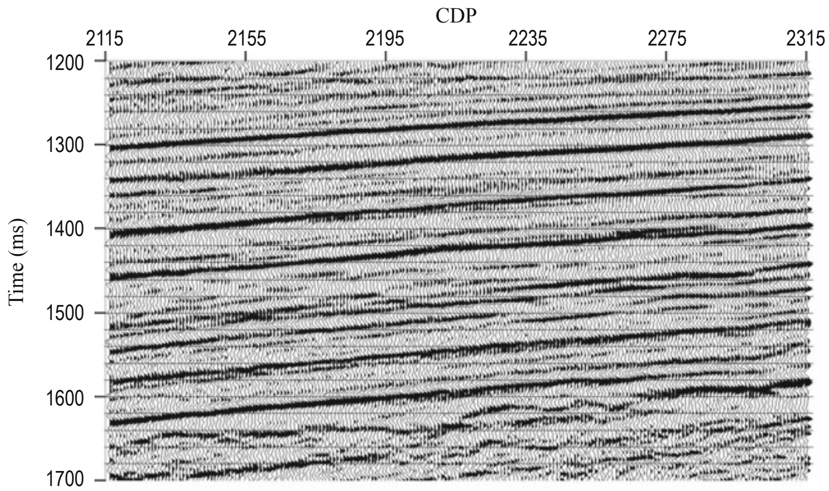


Fig. 6 The seismic section after attenuation compensation.

Conclusions

Attenuation compensation is an important method to improve seismic data resolution. The attenuation compensation problem is discussed in this paper from the forward modeling and inverse problem in attenuating media. The compensation problem can be transform into the solution of the Fredholm integral equation. The nature of compensation instability is also discussed mathematically. Based on the inversion, a compensation method using the Tikhonov regularization method to solve the attenuation compensation stability problem is presented in this paper. This method is suitable for continuously changing Q-value circumstances and can simultaneously correct the amplitude attenuation and velocity dispersion. The synthetic and real data results demonstrate that the attenuation compensation method has high stability and accuracy.

The presented compensation method is derived on the assumption that the medium is horizontally layered and the seismic data is stacked data and does not take into account the actual ray paths. It should apply to poststack seismic data with good signal to noise ratio.

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Wang Shou-Dong: See biography and photo in the Applied Geophysics June 2006 issue, P. 123.