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Gravity inversion using the frequency characteristics of the density distribution*

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Abstract: Three-dimensional gravity inversion based on the mass property model is very popular in recent years. The time and efficiency of inversion algorithms is relative to the magnitude of the target mesh. One approach is to search over the entire solution space for a more refined result. However, the inversion will be difficult with the increased parameters in the large search space and the number of computations increases exponentially. In this paper, we propose a novel approach based on the frequency characteristics of the density distribution over the mesh. The purposes of our study are to reduce the parameters of three-dimensional gravity inversion and to lighten the image quality of the inversion result. The results show that the new method can expedite the inversion processing and get a better geological interpretation than tradition methods.
Keywords: Gravity inversion, frequency decomposition, 3D density distribution, potential field

Introduction

Gravity anomaly data has good lateral resolution and is usually used in research of 3D density distribution problems. In particular, an important application is the use of density inversion to study regional tectonics and delineate geotectonic units (Parker, 1973; Oldenburg, 1974). Mass properties and geometry models are two popular inversion models for describing 3D density structure. In the mass properties model, the focus is to invert the density of each cell in order to constrain the anomaly shape. In theory, the mass properties model is easily used to invert very complex geo-models. Therefore, 3D gravity inversion based on the mass properties model has been very popular in recent years.

Intelligent inversion algorithms are being developed continuously. Shi (1992) gave an introduction to the application of geophysical inversion problems using the genetic algorithm (GA). Yao et al. (2003) discussed problems for gravity inversion with GA to enhance inversion efficiency.

Zhang et al. (2004) proposed a hybrid method using the finite element method (FEM) and GA to invert gravity anomaly data. Cai and Wang (2005) developed the FEM calculation method of complex heterogeneous regions for forward density modeling problems. For the mass properties model, we must grid the target region and search each mesh-cell's density in the inversion algorithm. However, if we want to describe the details of the density structure, the mesh magnitude must be increased, resulting in a large solution space to search. For a target mesh of 100 × 100 × 20 with a 100 × 100 measurement grid, the number of cells to be forward-modeled is 2×10^9. because there maybe are five variants on each mesh node, the search space is $200000^5 = 3.2\times$

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10^{26}.The complete workload is $2 \times 10^9 \times 3.2 \times 10^{26}$. If the computer can process at a rate of 109 times per second. it will take about 2×10^{19} years.

Due to the huge computational magnitude of inversion and the non-unique gravity inversion solutions, it is difficult to get reasonable geological results or the complete optimized solution in a practical time. Therefore, there have been many studies of how to optimize the process of gravity inversion in recent years. The inversion is based on the relationship between observed potential field data and density distribution. Pohanka (2001; 2003) utilized the integral transforms method to build a relationship between the gravity anomaly, g(x, y), and the 3D density structure, ρ(x, y, z), for the gravity inversion problem. He presented the characteristic density (Chi-density) concept. This method can get a smooth and simple solution and there is a linear relationship between the inverted density and the gravity anomaly.

In this paper, based on the frequency decomposition theory, we transform the mesh density inversion to a characteristic frequency inversion and use the GA to search the characteristic frequency and compute the 3D density structure. An initial density distribution model is built from the observed potential field data. We present a novel approach to reduce the parameters of three-dimensional gravity inversion and to finely depict the source density distribution for lightening the inversion image quality.

Methodology

Basic inversion idea

There is a traditional way of thinking that the potential field source is deduced from the frequency characteristics of anomalies. High frequency gravity anomalies are mostly interpreted as shallower density distributions, low frequency gravity anomalies are interpreted as deeper density distributions, and high frequency density fluctuations at greater depths are not likely to be recognized by the surface gravity observations. Using Fourier frequency analysis, any complex signal can be decomposed into several single frequency signals. Based on these basic ideas, we can divide an earth source into several layers. The mass property distribution in each layer can be regarded as a combination of stacked signals with single frequency characteristics. Thus, a combination of density distributions with different frequencies in every layer is searched instead of each mesh density. As a result, the combination of a few characteristic frequencies and amplitudes from each layer can be used as replacement for the mass magnitude of many mesh cells. Generally, the mass density distribution and anomalies observed on the surface are correlated. So the sources creating frequencies with different characteristics also can be estimated by frequency-decomposition inversion of the observed gravity data. If we can properly use a few frequency characteristics to substitute for each mesh density, we can achieve the aim of reducing the search space and gain a better geological solution for gravity inversion.

The core idea of frequency-decomposition inversion is to use a few combinations of characteristic frequencies and amplitudes for describing the layer density distribution in order to compress the solution space and highlight the overall characteristics of the density inversion result. The main techniques include the forward modeling of the gravity anomaly, global inversion, and transformation of the frequency components to the density distribution in each layer using an estimation of each mesh cell anomaly, applying a search algorithm for the inversion, and a genetic algorithm for estimating the density distribution with different frequencies. In numerical calculations, we use hexahedral cells for the regional division of source field and use an efficient storage technique (Yao et al., 2003) for enhancing the speed of computation. The GA with global search ability is used for searching the frequency parameters. Forward modeling results show that the new method

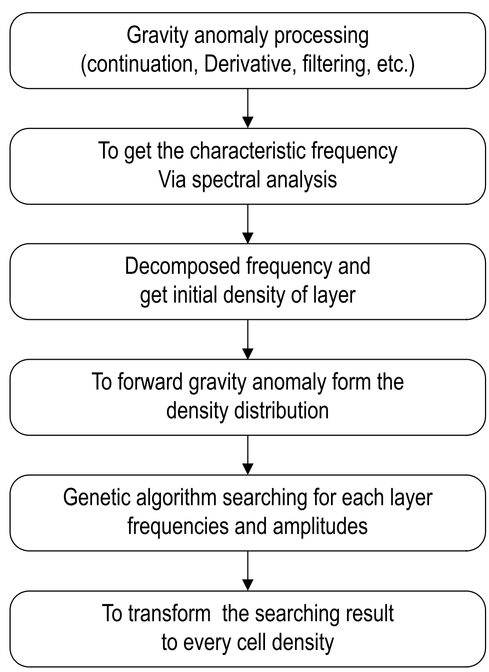


Fig. 1 The general workflow for frequency characteristics inversion.

can determine the density variation characteristics of each layer, accelerate the inversion processing, and get a better geological interpretation than tradition methods. The general workflow is shown in Figure1.

Forward modeling of the gravity anomaly

In this inversion method, a hexahedron cell is used for forward modeling. The computational mesh scheme is shown in Figure 2. The gravity anomaly of a hexahedron cell can be calculated from equation (1) and the gravity gradient is calculated from equation (2) (see Figure 2a). The target body can be divided into many hexahedron cells as shown in Figure 2b. The analytic gravity function of a hexahedron cell is the product of cell density σ and the cell geometry S, expressed in equations (1) and (2). Yao et al. (2003) greatly improved computational speed of the gravity anomaly forward modeling algorithm.

$$\begin{cases} \Delta g_z(x,y,z) = \sigma S_z(x,y,z) \\ S_z(x,y,z) = G\sum_{l=1}^{2}\sum_{m=1}^{2}\sum_{n=1}^{2} tg^{-1}\dfrac{(\zeta_l - z)R}{(\xi_n - x)(\eta_m - y)} \end{cases} \quad (2)$$

where $R = \sqrt{(\xi_n - x)^2 + (\eta_m - x)^2 + (\zeta_l - x)^2}$ and σ is density in the hexahedron. The gravity anomaly is equal to the sum of each cell's Δg for the entire anomaly body. The initial function of each hexahedron cell density can be given in equation (3):

$$\begin{cases} \sigma(x,y,z) = \{\sigma_1(x,y), \sigma_2(x,y), ..., \sigma_i(x,y), ..., \sigma_n(x,y)\} \\ \quad (i = 1, \ ..., n) \\ \sigma_i(x,y) = \sum_{j=1}^{m} A_i(a_j) f_i(\omega_j) \end{cases} \quad (3)$$

$$\begin{cases} \Delta g(x,y,z) = \sigma S(x,y,z) \\ S(x,y,z) = G\sum_{l=1}^{2}\sum_{m=1}^{2}\sum_{n=1}^{2}(-1)^{1+m+n} \\ \quad \{(\xi_n - x)\ln\left[(\eta_m - y) + R\right] + (\eta_m - y)\ln\left[(\xi_n - x) + R\right] + (\zeta_l - z)tg^{-1}\dfrac{(\zeta_l - z)R}{(\xi_n - x)(\eta_m - y)}\} \end{cases} \quad (1)$$

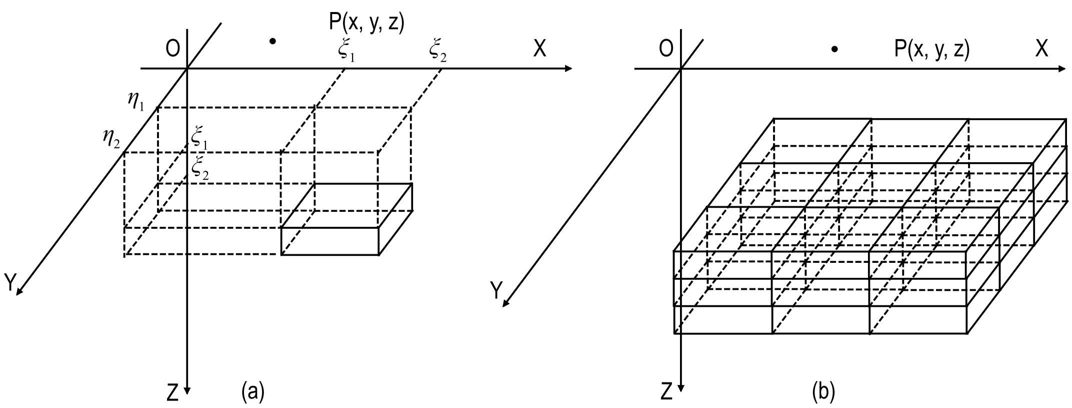


Fig.2 A schematic figure of the density forward model.

where $\sigma_i(x, y)$ is the ith layer density function in the Z direction, $A_i(a_j)$ is the response function of the j th amplitude a in the ith layer, and $f_i(\omega_j)$ is the response function of the jth frequency ω in the ith layer. Therefore, using the GA to search the parameters changes from searching each cell density, $\sigma(x, y, z)$, to searching each layer's a_j and ω_j.

In this paper, the main idea is to change the search target from each cell's density to each layer's characteristic frequency. Based on the frequency analysis of the gravity anomaly, we know that the low part of the power spectrum corresponds to the regional field and the high part of power spectrum corresponds to the local field. The middle range of the power spectrum is the most interesting and includes information of the target body's 3D density structure.

By extracting and combining the wave form of the mid-frequency range, the transformation processing between the frequency and density signals can be accomplished and the GA utilized to find the optimized solution. Forward modeling of one step with four thousand parameters takes about thirty seconds. We select a GA initial population with a size of 32. Thus, one evolution of the genetic algorithm needs about ten minutes on a PC and about two hundred steps can be computed in one day.

In the inversion, the magnitude and bounds of frequency parameters and the search space for optimized solutions are both key factors to determine the accuracy of the inversion results. In this paper, we estimate and test these inversion parameters based on the analysis and understanding of the observed anomaly spectrum. The transformation of the frequency signal into a density signal is realized through extracting the characteristic frequency using band-pass filter and wavelet transform techniques. Existing seismic or log data can be introduced to constrain the gravity inversion in depth in order for the inversion layers to correspond to real layers. Thus, we can solve the non-uniqueness of gravity inversion.

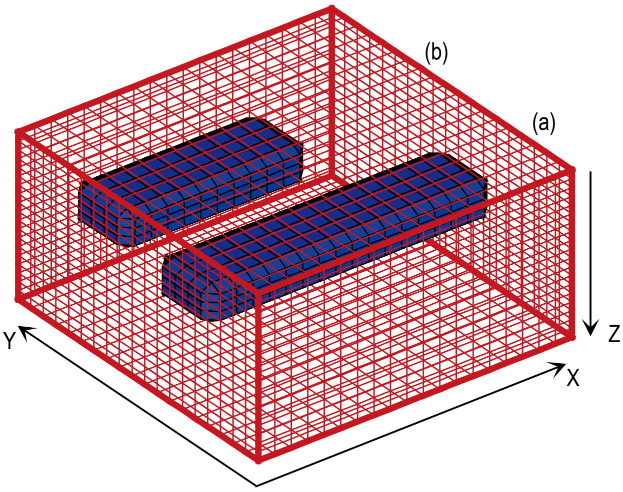


Fig. 3 The schematic density model
Density of body (a) is 0.9 g/cm^3
Density of body (b) is 0.8 g/cm^3

3D frequency-decomposition inversion

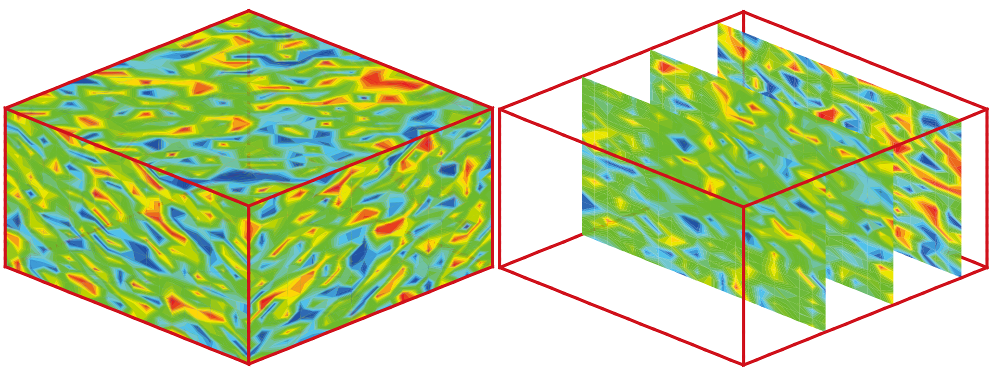
Numerical Model

Based on our inversion method, we built a numerical model shown in Figure 3. The inversion target body was 2 meters long, 2 meters wide, and 1 meter deep and was divided into 20 × 20 × 20 hexahedron mesh cells. Two anomaly bodies were inserted into the model with densities of 0.8 g/cm^3 and 0.9 g/cm^3, respectively. Each hexahedron cell was 0.1 m long, 0.1 m wide, and 0.05 m high. The boundary of anomaly body (a) is from 0.1 m to 1.8 m in the X direction, from 0.5 m to 0.9 m in the Y direction, and from 0.25 m to 0.45 m in the Z direction. The boundary of anomaly body (b) is from 0.2 m to 1.3 m in the X direction, from 1.3 m to 1.7 m in the Y direction, and from 0.25 m to 0.45 m in the Z direction. The measuring grid was 2 m long and 2 m wide and divided into 100 × 100 nodes. The distance between every measure point was 0.02 m. We used equations (1) and (2) to calculate the gravity and gravity gradient anomalies and used both the traditional and new methods to invert the field anomaly of the model.

The traditional 3-D inversion method uses 8000 (20 × 20 × 20) search parameters. The new inversion method, using frequency spectrum analysis, the anomaly density body is divided into 20 layers participating in this inversion and each layer's density distribution is replaced by a combination of 100 frequencies and 100 amplitudes. Thus, the new method's inversion parameters reduce to four thousand (100 + 100) × 20 from eight thousand.

Comparison of inversion results

We used two inversion methods on the model shown

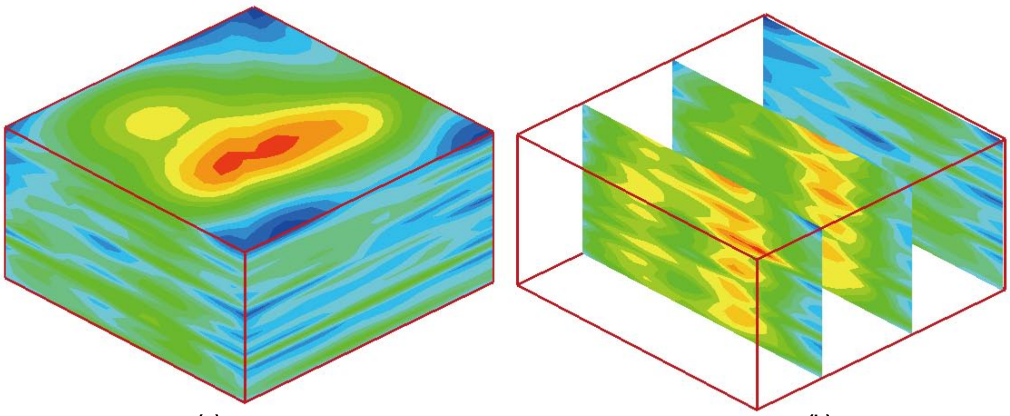


(a) (b)

Fig. 4 The 60 step result using the traditional method.
(a) Inverted density distribution from the gravity dataset. (b) Slices through the density body. The density anomaly is not apparent.

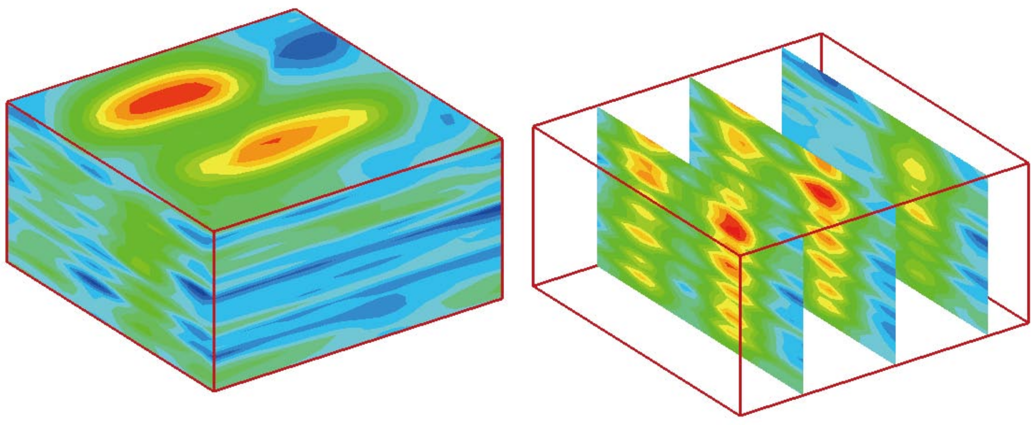
in Figure 3, both included 60 search steps using the GA. The inversion results of the tradition method show that there is no obvious anomaly apparent in the inverted density distribution and the density distributions appears random. The density slices indicate that the density changes greatly spatially and the density anomalies have no significance (see Figure 4).

However, the results of the frequency characteristics inversion method show both density anomalies (Figure 5). Their horizontal positions are similar to the anomaly distribution in the theoretical model. The entire anomaly structure is clear with a few random variations and the features make geological sense. However, the vertical resolution is not high, which is related to the non-uniqueness of gravity inversions. The resolution can be enhanced by introducing seismic or log data constraints.



(a) (b)

Fig. 5 The 60 step result using the new method.
(a) Inverted density distribution from the gravity dataset. (b) Slices through the density body. The density anomaly can easily be distinguished. The color variations express density difference. High values are red, middle values are yellow and green, and low values are blue.



(a) (b)

Fig. 6 The 60 step result of the new method for gravity gradient Inversion.
(a) Inverted density distribution from the gravity gradient dataset. (b) Slices through the density body. In the horizontal direction, the density anomalies are separated.

We also studied gravity gradient inversion using frequency characteristics. The results are shown in Figure 6, from which we can see that the horizontal resolution of gravity gradient inversion is higher than that of the gravity inversion. The two anomaly bodies are separated horizontally (Figure 6a).

The inversion results shown in Figures 4 to 6 indicate that the initial density distribution by frequency decomposition is based on the frequency characteristics of the observed anomaly and the inversion parameters are controlled by anomaly frequencies. This inversion method is able to be get reasonable results in the same time frame and the gravity gradient inversion resolution is better. One of the disadvantages of gravity inversion is the low vertical resolution when only using a few vertical constraints.

Spectrum comparison

We know that observed gravity anomalies are a synthesis of multiple of mass bodies in the subsurface. If there is no prior information, it is very difficult to accurately separate the observed gravity field. From the power spectrum, we can know more information about the field features. Based on power spectrum analysis, we can estimate the anomaly frequency characteristics and its distribution. Figure 7a is the theoretical gravity anomaly and its frequency spectrum. Figure 7b is the inverted gravity anomaly from the frequency characteristics inversion and the frequency spectrum. The red circles in (a) and (b) show that the frequency characteristics of the observed and theoretical gravity anomaly spectra are basically similar.

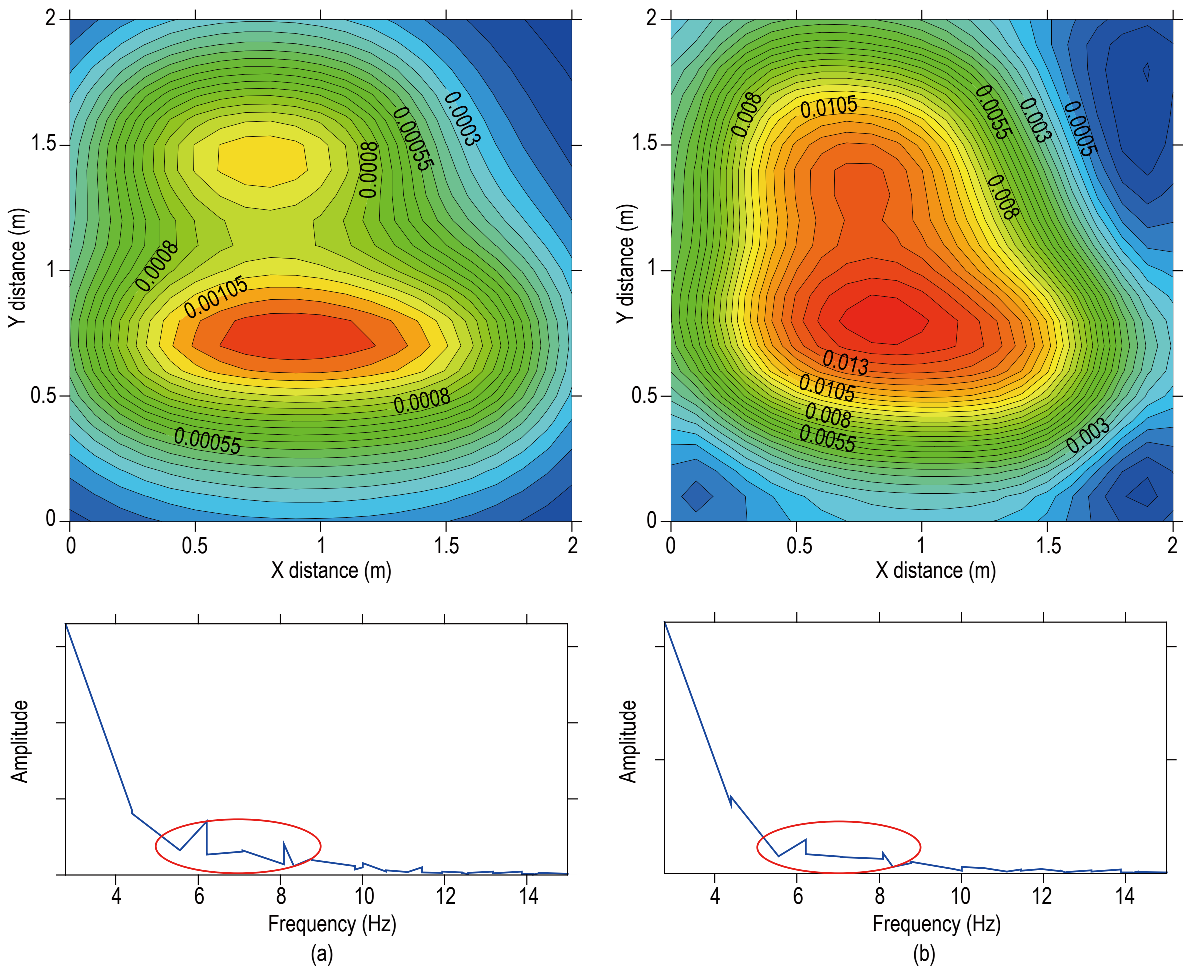


Fig.7 Theoretical and inverted gravity anomalies and their frequency spectra.
(a) The theoretical gravity anomaly (mGal) (Upper) and its spectrum (Lower). (b) Forward model for the gravity anomaly inversion results (mGal) (Upper) and its spectrum (Lower). The two red circles show that the theoretical and inversion spectra have similar frequency features.

In the same way, we analyze the theoretical and inversion results for gravity gradient data. Figure 8a shows the theoretical gravity gradient anomaly and its frequency spectrum. Figure 8b shows the inversion results. From the spectrum panel in Figures 8a, we see that the gravity gradient inversion exceeded the gravity inversion in the frequency characteristics. Meanwhile, we also see that the two inverted and theoretical spectrum characteristics are similar. This demonstrates that the horizontal resolution of gravity gradient inversion is better than gravity inversion.

Constrained inversion

The application of proper constraints in inversion processing is very important to get a reasoned result.

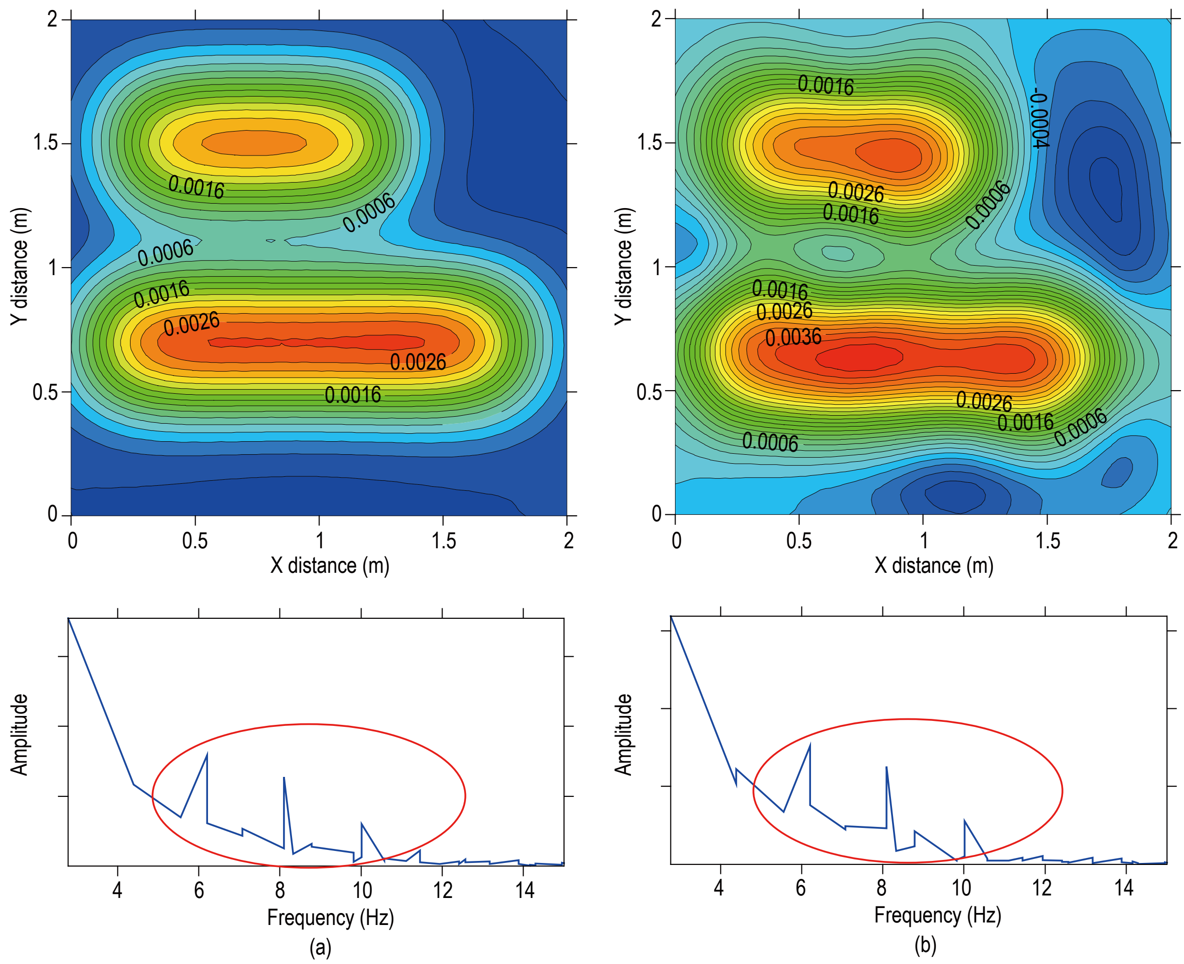


Fig. 8 Theoretical and inverted gravity gradient anomalies and frequency spectra.

(a) The theoretical gravity gradient anomaly (Upper) and its spectrum (Lower). (b) The inversed gravity anomaly (Upper) and its spectrum (Lower). The two red circles show that the theoretical and inversion spectra have very similar frequency features.

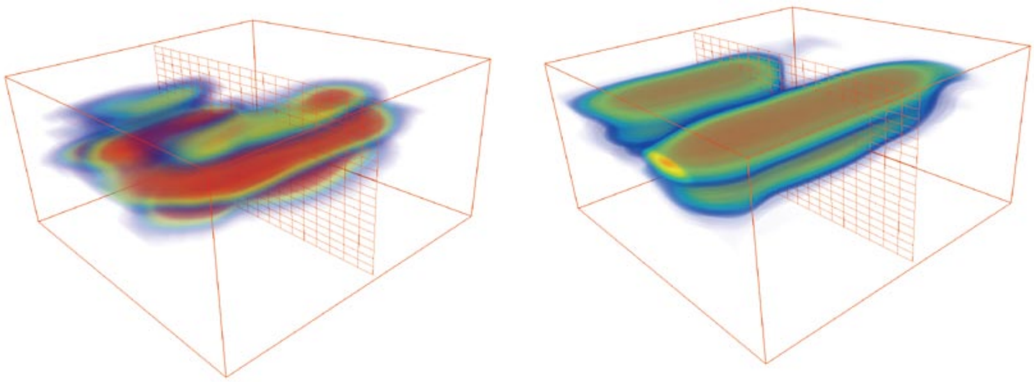
In geophysical inversion, prior information, such as seismic and log data, is usually used as a reference. Based on these data, we can understand general rock lithology in the earth. In this constrained inversion, we used an approximate vertical density structure. The search bounds for inversion are from 0.15 m to 0.55 m in depth. The search bounds for the theoretical model are from 0.25 m to 0.45 m. The inverted gravity result is shown in Figure 9a and the inverted gravity gradient result is shown in Figure 9b. In the figure 9 the inverted density values greater than 0.5 g/m^3 are shown using 3D volume rendering. The color changes show the variation of inverted density value. The red color indicates higher density values and the blue color represents lower density values. We also can see that the 3-D forms are similar to the theoretical density distribution shown in Figure 3. The resolution of gravity gradient inversion is higher than that of gravity inversion horizontally. The inverted density changes slowly and evenly. This demonstrates that suitable constraints can improve the vertical resolution and reduce the non-uniqueness of potential field inversion problems.

Conclusions

We present a novel inversion method based on the frequency characteristics of the gravity field. The new method remarkably improves the inversion speed due to fewer inversion parameters and the searched solution space is greatly reduced. Since the inverted parameter is frequency character, the complete inverted density shapes are smooth and have good continuity which is easier for geological interpretation.

However, the vertical resolution of the inversions is not satisfactory if we do not use vertical constraints.

In our numerical model, forward modeling one



(a) Gravity inversion result (b) Gravity gradient inversion result

Fig. 9 Volume rendering images of 3D density structure from constrained inversion.

step with four thousand parameters takes about thirty seconds. We select a GA initial population with a size of 32. Thus, one evolution of the genetic algorithm needs about ten minutes on a PC and about two hundred steps can be computed in one day. Therefore, parallel computation should be developed for large-scale gravity inversions.

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References

Cai, Y. E., and Wang, C. Y., 2005, Finite-element calculation of gravity anomaly for complex heterogeneous regions: Geophysics J. Int., **162**, 696 – 708.

Oldenburg, D. W., 1974, The inversion and interpretation of gravity anomalies: Geophysics, **39**, 526 – 536.

Parker, R. L., 1973, The rapid calculation of potential anomalies: Geophy J R Astr Soc, **31**, 447 – 455.

Pohanka, V., 2001, Application of the harmonic inversion method to the Kolarovo gravity anomaly: Contributions to Geophysics and Geodesy, **31**(4), 603 – 620.

Pohanka, V., 2003, The harmonic inversion method: calculation of the multi-domain density: Contributions to Geophysics and Geodesy, **33**(4), 247 – 266.

Shi, Y., 1992, Genetic algorithm and some of its geophysical applications: Acta Geophysica Sinica (in Chinese), **35**, 367 – 371.

Yao, C. L., Hao, T. Y., Guan, Z. N., and Zhang, Y. W., 2003, High-speed computation and efficient storage in 3-D gravity and magnetic inversion based on genetic algorithms: Chinese Journal of Geophysics (in Chinese), **46**(2), 252 – 258.

Zhang, J., Wang, C.Y., Shi, Y. L., Cai, Y. E., Chi, W. C., Dreger, D., Cheng, W. B., and Yuan, Y. H., 2004, Three-dimensional crustal structure in central Taiwan from gravity inversion with a parallel genetic algorithm: Geophysics, **69**(4), 917 – 924.

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