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Lamé parameters inversion based on elastic impedance and its application

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Abstract: The Connolly (1999) elastic impedance (EI) equation is a function of P-wave velocity, S-wave velocity, density, and incidence angle. Conventional inversion methods based on this equation can only extract P-velocity, S-velocity, and density data directly and the elastic impedance at different incidence angles are not at the same scale, which makes comparison difficult. We propose a new elastic impedance equation based on the Gray et al. (1999) Zoeppritz approximation using Lamé parameters to address the conventional inversion method's deficiencies. This equation has been normalized to unify the elastic impedance dimensions at different angles and used for inversion. Lamé parameters can be extracted directly from the elastic impedance data obtained from inversion using the linear relation between Lamé parameters and elastic impedance. The application example shows that the elastic parameters extracted using this new method are more stable and correct and can recover the reservoir information very well. The new method is an improvement on the conventional method based on Connolly's equation.

Key words: Gray approximation, elastic impedance inversion, normalization, and Lamé parameter.

Introduction

Fundamental physical properties of rock such as compressibility (bulk modulus) and shear rigidity (shear modulus) are easier to understand than acoustic velocities and impedances (Gray and Andersen, 2000). Lamé parameters are the Lamé constant (closely related to incompressibility) (λ) and shear rigidity (μ), in which the incompressibility (λ) is more sensitive to the pore fluids than to the matrix, while the rigidity is influenced by the matrix connectivity only (Dufour and Goodway, 1998). The Lamé parameters λ and μ don't contain unstable information without the ambiguity introduced by the density parameter ρ with errors obtained from $\lambda\rho$ and $\mu\rho$ inversion. Compared with $\lambda\rho$ and $\mu\rho$, the signal to noise ratio is improved by a factor of two for μ and four for λ (Gray, 2002). In order to describe the reservoir information, we hope to obtain rock physical parameter data volumes. The Connolly (1999) elastic impedance equation is a function of P-wave velocity, S-wave velocity, and density. Based on this equation, P-wave velocity, S-wave velocity, and density can be extracted from elastic impedance inversion directly but other rock physical parameters such as λ and μ can only be calculated using the extracted parameters indirectly (Connolly, 1999; Yin et al., 2004) which increases calculation errors. In order to obtain more accurate results and reduce the cumulative errors, it is hoped that these rock physical parameters can be extracted directly.

In this paper, we start from the fundamental elastic impedance equation expressed using the shear modulus μ, density ρ, and Lamé parameter λ. Using this new elastic impedance equation for inversion, the shear modulus, density, and Lamé parameter data can be extracted from elastic impedance data directly.

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Elastic impedance equation with Lamé parameters

Recently, the use of AVO technology to predict oil- and gas-bearing sand reservoirs has led to the discovery that other elastic parameters besides Poisson's ratio have a great effect on reflectivity. Cross-plots of elastic moduli can not only effectively extract lithology information but also better distinguish pore fluids. The Gray et al. (1999) approximation provides different approximations for the exact solution of reflectivity carried out using elastic moduli.

Gray approximation

Gray et al. (1999) re-expressed the Aki-Richards (1980) Zoeppritz equation approximation in terms of the parameters $\Delta\lambda/\lambda$, $\Delta\mu/\mu$, and $\Delta\rho/\rho$; that is, the Lamé constant, shear modulus, and density reflectivities, respectively:

$$R(\theta)=[\frac{1}{4}-\frac{1}{2}\left(\frac{\beta}{\alpha}\right)^2]\sec^2\theta\frac{\Delta\lambda}{\lambda}+\left(\frac{\beta}{\alpha}\right)^2\left(\frac{1}{2}\sec^2\theta-2\sin^2\theta\right)\frac{\Delta\mu}{\mu}+\frac{1}{4}\left(1-\tan^2\theta\right)\frac{\Delta\rho}{\rho}$$

where R is the reflectivity at incident angle θ, β is the S-wave velocity, α is the P-wave velocity, λ is the Lamé constant, μ is the shear modulus, and ρ is the density.

The main characteristic of this approximation is that the reflectivity is represented using the relative changes of elastic parameters which are sensitive to oil- and gas-bearing reservoirs directly and the relative changes of the elastic parameters can be extracted directly. At about 60°, the third term in equation (1) is approximately zero and the first two terms express the complete reflectivity. Within the range of incident angles less than 70°, the Gray approximation error is less than the Aki-Richards approximation.

Elastic impedance equation with Lamé parameters and its normalization

Based on equation (1) and using a similar method to the derivation of Connolly equation, we can obtain the new elastic impedance equation of Lamé parameters as

$$EI(\theta)=\lambda^{\left(\frac{1}{2}-K\right)\sec^2\theta}\mu^{K\left(\sec^2\theta-4\sin^2\theta\right)}\rho^{\left(1-\frac{1}{2}\sec^2\theta\right)}. \qquad (2)$$

Similar to Connolly equation, the equation is a function of angle θ, so it also has the problem that the dimension of the elastic impedance ($EI(\theta)$) varies with incident angle θ and its numerical values change significantly with θ. It is a disadvantage to compare $EI(\theta)$ at different angles and acoustic impedance (AI) (Whitcombe, 2002).

To remove the dimension as a function of θ, three reference constants λ_0, μ_0, and ρ_0 and constant factor A_0 are introduced to modify the EI function in Equation (2), which makes the dimension of $EI(\theta)$ the same as the AI and helps the comparison between them. $EI(\theta)$ predicts the correct value of acoustic impedance, $\alpha\rho$, at $\theta = 0$. The expression of factor A_0 can be derived (see Appendix) as $A_0 = (8\lambda_0\mu_0\rho_0^2)^{1/4}$.

The normalized form of the elastic impedance equation with Lamé parameters can be expressed as

$$EI(\theta) = A_0\left(\frac{\lambda}{\lambda_0}\right)^a\left(\frac{\mu}{\mu_0}\right)^b\left(\frac{\rho}{\rho_0}\right)^c, \qquad (3)$$

where $a=\left(\frac{1}{2}-K\right)\sec^2\theta$, $b = K(\sec^2\theta - 4\sin^2\theta)$, and $c=1-\frac{1}{2}\sec^2\theta$.

From this equation we can see that when $\lambda = \lambda_0$, $\mu = \mu_0$, and $\rho = \rho_0$, the elastic impedance is the constant $\alpha_0\rho_0$, i.e., the acoustic impedance. Thus, as θ changes, the dimension of normalized $EI(\theta)$ remains constant. The EI function will not vary greatly with θ, so these modifications allow for a direct comparison between different elastic impedance values $EI(\theta)$ across a range of angles in a manner that was not available with the previous formulation.

Inversion result analysis

Flow of the elastic impedance inversion

The elastic impedance inversion using equation (3) is similar to the inversion based on Connolly's elastic impedance equation. Both of them include four steps: (1)

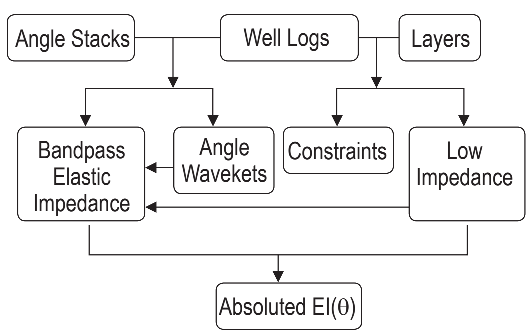

Fig.1 Flowchart of elastic impedance inversion

seismic data processing, (2) well log processing, (3) angle wavelet extraction, and (4) elastic impedance inversion (Wang et al., 2005). The detailed flowchart for the inversion is shown in Figure 1.

Seismic data processing: Before the elastic impedance inversion, angle gather stacks at different angles should be established to transform the seismic data volume from offset into angle gather stack volumes (partial stacks of incidence angle gathers).

Well log data processing: In order to constrain the inversion of angle gather stacks, pseudo well logs of elastic impedance at the location of wells have to be calculated by Equation (3) using the existing acoustic wave logs, shear wave logs, density logs, and angles that are provided by reservoir seismic data. When processing the elastic impedance inversion, besides their use to constrain the inversion, the pseudo elastic impedance well logs can also replace the frequency components lost during the propagation of seismic waves.

Angle wavelet extraction: When processing the elastic impedance inversion, wavelets at the different angle gather stacks must be extracted separately.

Elastic impedance inversion: Before elastic impedance inversion, the angle partial stacks and the elastic impedance logs of the corresponding incident angles at the well locations are extrapolated using the interpreted seismic horizons for control, to build low frequency models for the different angles. Using the elastic impedance logs and corresponding angle partial stack data and angle wavelets for constraint, seismic frequency band-limited elastic impedance can be obtained. In comparison with real absolute impedance, the frequency band-limited elastic impedance lacks a low frequency component, so the low frequency models should be incorporated into the frequency band-limited elastic impedance.

Extraction of lithology parameters

To extract the lithology parameters, we must solve Equation (3). Since equation (3) is nonlinear, it is necessary to make it linear. Take the log of both sides of equation (3) to get

$$\ln\frac{EI(\theta)}{A_0} = a(\theta)\ln\frac{\lambda}{\lambda_0} + b(\theta)\ln\frac{\mu}{\mu_0} + c(\theta)\ln\frac{\rho}{\rho_0}. \tag{4}$$

In the case of the same angle, the petrophysical parameters at different samples have equivalent $a(\theta)$, $b(\theta)$, and $c(\theta)$. So the previous equation can be written

$$\ln\frac{EI(t,\theta)}{A_0} = a(\theta)\ln\frac{\lambda(t)}{\lambda_0} + b(\theta)\ln\frac{\mu(t)}{\mu_0} + c(\theta)\ln\frac{\rho(t)}{\rho_0}. \tag{5}$$

Using elastic impedance and the λ, μ, and ρ logs at the wells, the $a(\theta)$, $b(\theta)$, $c(\theta)$ coefficients for every sample of a single angle can be determined. So, for the elastic impedance data of three different angles, nine constants $a(\theta_1)$, $b(\theta_1)$, $c(\theta_1)$, $a(\theta_2)$, $b(\theta_2)$, $c(\theta_2)$, $a(\theta_3)$, $b(\theta_3)$, and $c(\theta_3)$ can be obtained. If they are introduced into Equation (5) we have

$$\begin{cases} \ln\dfrac{EI(t,\theta_1)}{A_0} = a(\theta_1)\ln\dfrac{\lambda(t)}{\lambda_0} + b(\theta_1)\ln\dfrac{\mu(t)}{\mu_0} + c(\theta_1)\ln\dfrac{\rho(t)}{\rho_0} \\ \ln\dfrac{EI(t,\theta_2)}{A_0} = a(\theta_2)\ln\dfrac{\lambda(t)}{\lambda_0} + b(\theta_2)\ln\dfrac{\mu(t)}{\mu_0} + c(\theta_2)\ln\dfrac{\rho(t)}{\rho_0} \\ \ln\dfrac{EI(t,\theta_3)}{A_0} = a(\theta_3)\ln\dfrac{\lambda(t)}{\lambda_0} + b(\theta_3)\ln\dfrac{\mu(t)}{\mu_0} + c(\theta_3)\ln\dfrac{\rho(t)}{\rho_0} \end{cases}. \tag{6}$$

Using equation (6), λ, μ, and ρ for different samples can be obtained.

Case Study

To test the method, real seismic data from an area of Shengli Oilfield is inverted. The gas reservoirs of this area are the sand bodies of delta front facies, mainly the thick-bedded blocked sand bodies of estuarine bars. Connolly's elastic impedance equation is used to invert to elastic impedance data, from which P-wave velocity, S-wave velocity, and density data can be extracted directly. Then these P-wave velocity, S-wave velocity, and density data are used to get the elastic parameter (λ and μ) data indirectly. The new method in this paper is able to extract elastic parameter data from the inversion results directly.

Figure 2 shows comparisons of the λ logs extracted from the inversion using the conventional method and the new method proposed in this paper with the exact λ

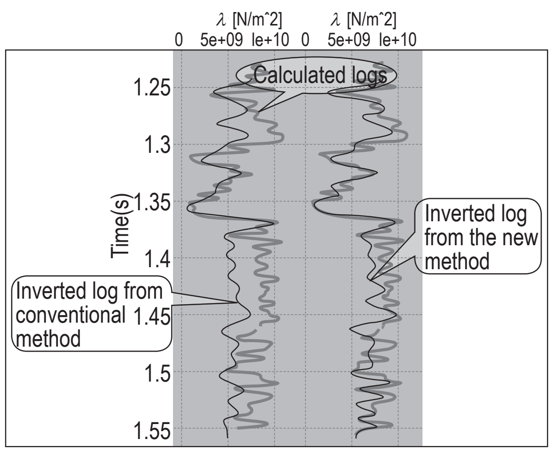

Fig.2 Thick log curves are the exact λ logs calculated from the P-wave velocity, S-wave velocity, and density curves of this well. Thin log curves are the λ logs extracted from inversion results. On the left, the thin curve is the λ log obtained from the conventional inversion method indirectly. On the right, the thin curve is the λ log obtained from the new method directly. The comparison of the two curves show that the λ curve obtained using the new method fits well with the calculated λ curve.

log obtained from estimation using the well log curve. We can see that the elastic log extracted from the new method is much closer to the estimated log than the elastic log extracted from the conventional method. From well log information it is known that the target gas bed near 1.35 s in this area has low λ and μ values. Figure 3 is a cross plot from the λ and μ well logs, from which analysis shows that the two logs can identify the target gas bed (red ellipse). Figures 4 and 5 are the elastic impedance λ profiles obtained from inversion using the new method. They show the target gas bed clearly and the inversion result at the well locations fit well with the exact λ log calculated from the P-wave velocity, S-wave velocity, and density curves for this well.

Incompressibility (λ)is also the "bulk modulus", which is the resistance to a change in volume caused by

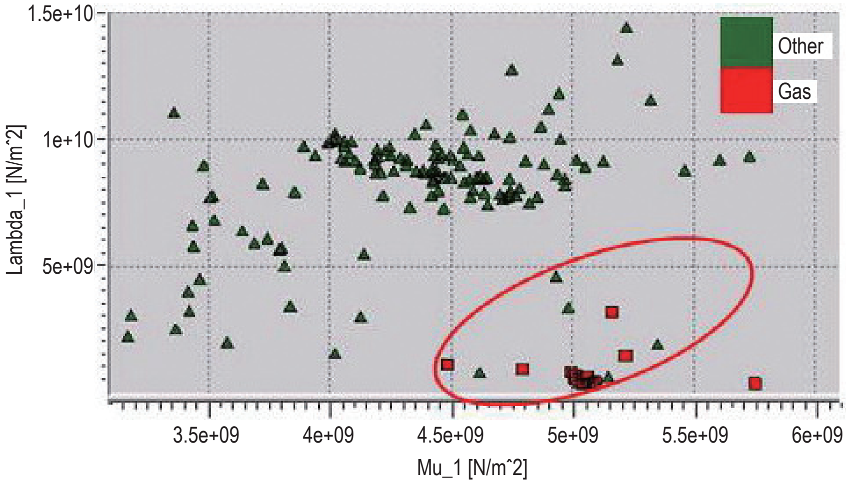

Fig.3 The crossplot of λ and μ. It shows that the target gas-bearing bed has low λ and μ values.

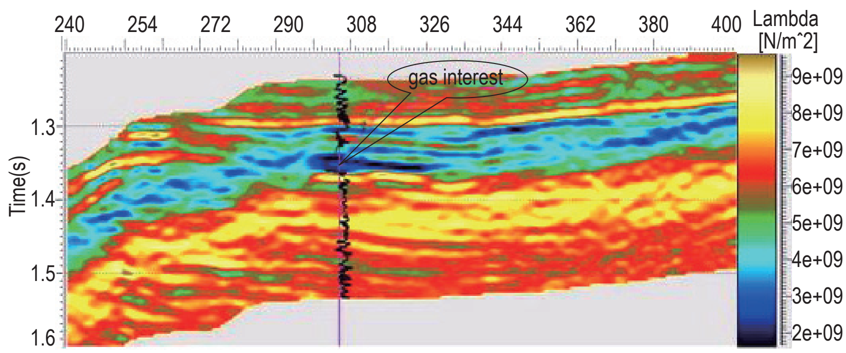

Fig. 4 A λ profile obtained from the new method intersecting a well. At the well location, an exact λ log calculated from the P-wave sonic, S-wave sonic, and density curves for this well is inserted. It shows that the inversion result agrees with the estimated λ log. The gas-bearing bed is identified on the inverted profile.

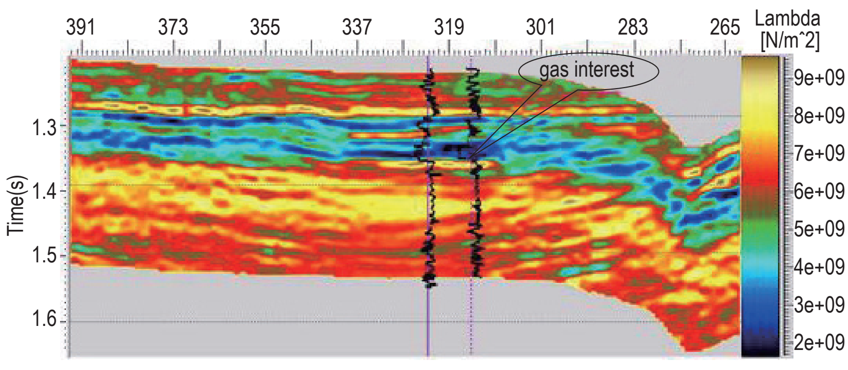

Fig. 5. An inverted λ profile through two wells. At the well locations, the exact λ log calculated from the P-wave sonic, S-wave sonic, and density curves for this well is inserted. Note that the inversion results at the well locations fit well with the exact curve.

a change in pressure. Compared to sand, gas is easier to compress, so it has low λ values. Figures 4 and 5 show that the gas bed has low values of λ, which is consistent with the theoretical result.

Conclusions

Based on the Gray et al. (1999) approximation, a new elastic impedance equation expressed in terms of Lamé's parameters is proposed. In order to unify the elastic impedance dimensions of different incident angles, this equation is normalized and then used for real seismic data inversion.

Since the λ and μ profiles can be inverted directly using this new method, compared to the conventional method that the λ and μ profiles estimated using P-wave velocity, S-wave velocity, and density profiles indirectly, the new method avoids a step which reduces the cumulative error effect and makes the Lamé parameter data more accurate. Using these data, we can obtain more accurate subsurface geologic information to reveal the distribution and potential of the oil- and gas bearing reservoirs more reliably.

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Appendix

The determination of factor :

$$EI(\theta) = A_0\left[\left(\frac{\lambda}{\lambda_0}\right)^a\left(\frac{\mu}{\mu_0}\right)^b\left(\frac{\rho}{\rho_0}\right)^c\right], \quad \text{(i)}$$

$$EI = \lambda^a \mu^b \rho^c$$

when θ= 0^0 and assuming K=α^2/β^2=1/4:

$$a = \left(\frac{1}{2} - K\right)\sec^2\theta = \frac{1}{4}$$

$$b = K\left(\sec^2\theta - 4\sin^2\theta\right) = 1/4$$

$$c = 1 - \frac{1}{2}\sec^2\theta = 1/2$$

$$\alpha\rho = \sqrt{\frac{\lambda+2\mu}{\rho}}\rho = \sqrt{(\lambda+2\mu)\rho}$$

$$= \rho^{\frac{1}{2}}(\lambda+2\mu)^{\frac{1}{2}}$$

$$= \rho^{\frac{1}{2}}(\lambda+2\mu)^{\frac{1}{4}}(\lambda+2\mu)^{\frac{1}{4}}$$

$$= \rho^{\frac{1}{2}}\lambda^{\frac{1}{4}}\mu^{\frac{1}{4}}\left(1+2\frac{\mu}{\lambda}\right)^{\frac{1}{4}}\left(\frac{\lambda}{\mu}+2\right)^{\frac{1}{4}}$$

$$= 8^{\frac{1}{4}}\rho^{\frac{1}{2}}\lambda^{\frac{1}{4}}\mu^{\frac{1}{4}}$$

So $A_0 =$ $A_0 = (8\lambda_0\mu_0\rho_0^2)^{\frac{1}{4}}$ and

$$EI(0) = A_0\left(\frac{\lambda}{\lambda_0}\right)^{\frac{1}{4}}\left(\frac{\mu}{\mu_0}\right)^{\frac{1}{4}}\left(\frac{\rho}{\rho_0}\right)^{\frac{1}{2}}$$

$$= (8\lambda_0\mu_0\rho_0^2)^{\frac{1}{4}}\left(\frac{\lambda}{\lambda_0}\right)^{\frac{1}{4}}\left(\frac{\mu}{\mu_0}\right)^{\frac{1}{4}}\left(\frac{\rho}{\rho_0}\right)^{\frac{1}{2}}. \quad \text{(ii)}$$

$$= (8\lambda\mu\rho^2)^{\frac{1}{4}} = \alpha\rho$$