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From flatness, GPI observers, GPI control and flat filters to observer-based ADRC

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Abstract

In this article, we establish the route taken by the author, and his research group, to bring differential flatness to the realm of active disturbance rejection control (ADRC). This avenue entitled: 1) generalized proportional integral observers (GPIO), as natural state and disturbance observers for flat systems, 2) generalized proportional integral (GPI) control, provided with extra integrations, to produce a modular controller known as flat filters (FF's) and, finally, 3) the establishing of an equivalence of observer based ADRC with FF's. The context is that of pure integration systems. The obtained controllers depend only on the order of the flat system and they are to be directly used on the basis of the available flat output signal in a universal, modular, fashion. The map is complemented with the relevant references where the intermediate techniques were illustrated and developed, over the years, in connection with laboratory experimental implementations.

Keywords: Flatness, GPI observers, GPI control, reduced order GPI observers, flat filters

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1 Introduction

Differential flatness is a system property establishing, in a natural manner, an input output description of the system, thus easing the controller design task in, both, SISO (single-input single-output) and MIMO (multiple inputs multiple outputs) nonlinear systems within a zero dynamics-free environment. All system states and the control inputs are differentially parameterizable in terms of the flat outputs and of a finite number of their time derivatives with a clear opportunity for identification of possible structural singularities. The flatness property substantially eases off-line trajectory planning issues, while trivializing feedback controller design. One of the advantages of the flatness property, in their input-to-flat-output representation of the dynamics, is that it allows to efficiently circumvent matching conditions, as it naturally leads to trivially matched input-to-flat-output models. Flatness allows for exact static feedback linearization, for SISO systems, and to clearly identify

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the need for static or dynamic feedback linearization, in MIMO nonlinear systems. Flatness easily leads to controller synthesis based on desirable input to flat outputs closed-loop dynamics, in total absence of unobservable zero dynamics. In the linear system case, flatness is completely equivalent to system's controllability, while flat outputs are trivially observable.

Nonlinear differentially flat systems are equivalent to perturbed chains of pure integration. Treating, via drastic model simplification, the effects of endogenous nonlinearities in combination with exogenous inputs into a single footing of total unknown, unstructured, disturbance (addressed as the *total disturbance*), immediately prompts for the need of, simultaneous, state and disturbance estimates. Generalized proportional integral (GPI) observers are extended state observers traditionally used for robust phase variable reconstruction and high-gain based disturbance estimation, including the estimation of a finite number of disturbance input time derivatives. Integral phase variables reconstruction, via iterated inputs and outputs integrations, with suitably added iterated output integral compensation is an alternative to GPI observer design, which is known as GPI control. The GPI control scheme establishes a means of circumventing nonlinear observers design in both SISO and MIMO nonlinear systems. The key point here is to carry out the GPI observer, or GPI controller design, for the unperturbed version of the system, and, then, assess the effect of the neglected disturbance on the closed-loop response of the system to the designed controller. The scheme results in attenuation or disturbance rejection properties and generates a desirable, robust, trajectory tracking, or output stabilization, performance.

The robustness consideration of GPI controller design, on flat systems simplified to perturbed chains of pure integrations, directly results in a high gain FF with desirable trajectory tracking qualities simultaneously achieved with low frequency input disturbance rejection and high frequency output measurement noise filtering or attenuation.

In this article, we establish the route, taken by the author and his colleagues, students and coworkers, to bring Differential Flatness to the realm of ADRC design. This entitled GPI observers, GPI control via integral reconstructors and, finally, the establishing of an important equivalence between Observer based ADRC and robust linear control based on FF's. This developments clarified, step by step, the way to establish a complete equivalence of ADRC via reduced order extended observers, and robust GPI control based on integral reconstructors also called FF's. The context, quite on purpose, is that of pure integration systems which is the fundamental paradigm of flat nonlinear systems. The obtained controllers can be directly used on the perturbed flat system in a universal controller fashion requiring only knowledge of the dimension of the nonlinear flat system. The map is complemented with the relevant references where the intermediate techniques were illustrated with the help of experimental results.

Section 2 introduces flatness and formulates the problem of controlling a flat output under unknown, time-varying, lumped input disturbances constituted by endogenous (state dependent) inputs and exogenous inputs. Section 3 illustrates the use of GPI observers and its relevance in the control of uncertain input-output models of flat systems and assesses the performance of the estimator in frequency domain terms. Section 4 explains GPI control in the context of an unperturbed chain of integrations. Section 5 places the uncertain control problem for flat systems in the context of robust FF's. Section 6 contains the equivalence of reduced order extended observers, for disturbance estimation in an ADRC scheme, with the FF approach.

To simplify the presentation, only the SISO nonlinear case will be treated throughout. Extension to MIMO flat systems, linearizable via static or dynamic feedback, is not particularly difficult. All technical assumptions are to be considered globally valid in the relevant state or phase space. For convenience, we only treat flat output stabilization problems. The results trivially extend to flat output reference trajectory tracking problems.

2 Flat systems

An n-dimensional, smooth, nonlinear system of the form: $\dot{x} = f(x,u), y = h(x)$, with $x \in \mathbb{R}^n$, $y \in \mathbb{R}$ and $u \in \mathbb{R}$, is said to be flat, with flat output $y \in \mathbb{R}$, if there exists a diffeomorphic map $\Phi : x \mapsto (y, \dot{y}, \dots, y^{(n-1)}) := \underline{y} \in \mathbb{R}^n$ and a smooth function $\psi : (\underline{y}, y^{(n)}) \in \mathbb{R}^n \times \mathbb{R} \mapsto u \in \mathbb{R}$, i.e.,

$$x = \Phi(\underline{y}), \quad u = \psi(\underline{y}, y^{(n)}). \tag{1}$$

Φ is said to differentially parameterize the n components of the state x. We assume that $\dfrac{\partial \psi}{\partial y^{(n)}} \neq 0$. The state dependent input coordinate transformation: $u = \psi(\underline{y}, v)$, confirms that the original system is equivalent to the pure integration, controllable, linear system, $y^{(n)} = v$. The relation $u = \psi(\underline{y}, y^{(n)})$ is addressed as the input-(flat) output description of the system, or simply, the input-output system.

The vast majority of examples of flat systems include the affine in the control input case: $\dot{x} = f(x) + g(x)u$, $y = h(x)$, with (f, g) a smooth pair of complete vector fields defined on the tangent space of $\mathbb{R}^n$. The input-output description of the nonlinear, affine in the control, flat system is readily determined by

$$y^{(n)} = L_f^n h(\Phi(\underline{y})) + [L_g L_f^{n-1} h(\Phi(\underline{y}))]u. \tag{2}$$

It will be assumed that the nonlinear input gain: $[L_g L_f^{n-1} h(\Phi(\underline{y}))]$, is nonzero in a sufficiently large open subset of $\mathbb{R}^n$. If nothing is specified about $L_g L_f^{n-1} h(\cdot)$, the previous regularity assumption will hold globally in $\mathbb{R}^n$.

Many engineering SISO nonlinear systems are flat (DC-to-DC converters such as the boost, the buck and the buck-boost converters; electric motors, such as DC motors, induction motors, variable reluctance motors, and permanent magnet synchronous motors; Airplane models, PVTOL systems, helicopters, some drones and marine vessels models). Many popular underactuated mechanical systems (ball and beam, inverted pendulum on a cart, the Furuta pendulum, the Kapitsa pendulum, etc.) are, generally speaking, *non flat*.

Suppose, for a moment, that the term $L_f^n h(\Phi(\underline{y}))$ is *not* precisely known, or difficult to "wire-up" in an experimental implementation of a certain output feedback control law strategy, implying its "exact cancellation". Contrary to this, assume also that the input gain, $b(\underline{y}) := L_g L_f^{n-1} h(\Phi(\underline{y}))$, is perfectly known. The term $L_f^n h(\Phi(\underline{y}))$, viewed as an unknown scalar time function: $\eta(t) = L_f^n h(\Phi(\underline{y}(t)))$, is then properly regarded as an *endogenous perturbation* input. Any external, unstructured, perturbation input, affecting the system's state model, acts as a *matched* perturbation input in the input-output model and it is addressed as the *exogenous perturbation*, denoted by $\vartheta(t)$. The total perturbation input is, hence, defined as $\xi(t) = \eta(t) + \vartheta(t)$. We consider then the *simplified model* as the perturbed integration system,

$$y^{(n)} = b(\underline{y})u + \xi(t). \tag{3}$$

Flatness clearly leads, in a natural manner, to the paradigmatic perturbed model customarily considered in ADRC. Endogenous and exogenous perturbation inputs are handled as the total additive perturbation input. Particularly simple regular cases include: constant input gain: $b(\underline{y}) = b$, output dependent input gain, $b(\underline{y}) = b(y)$. The general regular case, $b(\underline{y}) \neq 0$, is treated via a suitable homotopic equivalence of the closed-loop output solution trajectories with those pertaining unit input gain, trivially stable, closed-loop output trajectories. This uses a globally well defined, state-dependent, time coordinate transformation. In such cases, $b(\underline{y}, t)$, is largely unknown except, possibly, for its (unchanging) sign.

Let, without loss of generality, $b(\underline{y}(t)) > 0$ uniformly in time. Consider the implicit state-dependent time coordinate transformation,

$$\mathrm{d}\tau^n = b(\underline{y}(t))\mathrm{d}t^n, \ \left(\frac{\mathrm{d}\tau}{\mathrm{d}t}\right)^n = b(\underline{y}(t)) \tag{4}$$

with the corresponding differential equation exhibiting the trivial initial conditions: $\tau(0) = 0$. This defines a state-dependent time scaling transformation $t \mapsto \tau$. Since $b(\underline{y}(t)) = L_g L_f^{n-1} h(\Phi(\underline{y}(t)))$, is uniformly strictly positive, the solution trajectory, $\tau(t)$, is uniformly increasing, thus qualifying as a time-like variable. The transformation $\tau : t \mapsto \tau$, represented by the solution of the differential equation for τ, is, hence, globally invertible on the non-negative portion of $\mathbb{R}$, denoted by $\mathbb{R}^+$. The time transformed system is

$$\frac{d^n y}{\mathrm{d}\tau^n} = u + \tilde{\xi}(\tau), \ \ \tilde{\xi}(\tau) = \frac{\xi(\tau)}{b(\underline{y}(\tau))}. \tag{5}$$

For any static, or dynamic, feedback control law u (such as ADRC), and any given set of initial conditions for the phase variables, $\underline{y}(0)$, the trajectory, in the flat output phase space of the original system, is a smooth map, $\underline{y} : \mathbb{R}^+ \to \mathbb{R}^n$, defined by: $t \mapsto \underline{y}(t)$. The corresponding phase space trajectory, $\tau \mapsto \underline{y}(\tau)$, of the pure integration system, starting from the same initial condition, is a continuous deformation of that of the original system. In fact, the two maps: $t \mapsto \underline{y}(t)$ and $\tau \mapsto \underline{y}(\tau)$ belong to the same *homotopy* class.

Indeed, consider the map $z : [0, 1] \times \mathbb{R}^+ \to \mathbb{R}^n$ defined by

$$z(\rho, t) = \underline{y}((1 - \rho)t + \rho\phi(t)). \tag{6}$$

This map, continuously deforms phase space trajectories, $t \mapsto \underline{y}(t)$, of the nonlinear flat system into phase space trajectories, $t \mapsto \underline{y}(\tau(t))$, of the simplified system. The monotonicity of the function $\tau(t)$, with $\tau(0) = 0$, guarantee that $\lim_{t\to\infty} \underline{y}(t)$ and $\lim_{t\to\infty} \underline{y}(\tau(t))$ are the same. Since the phase trajectories for the original and the time-scaled pure integration system start at the same initial condition, and end at the same point at infinity, the time-scaling homotopy class existing between the trajectory maps is well defined. Trajectories homotopic, via time scale transformations, to stable trajectories are stable.

We, henceforth, consider, without loss of generality, pure integration systems of the form:

$$y^{(n)} = u + \xi(t). \tag{7}$$

3 Generalized proportional integral observers

The flat system equivalence to a perturbed pure integration system, with unit control input gain, still requires, for robust feedback purposes, of the following two items 1) the asymptotic estimation of the phase variables set $\{y, \dot{y}, \ldots, y^{(n-1)}\}$, simply denoted by $\underline{y}$, and 2) the accurate estimation of the disturbance input $\xi(t)$, as if it were an unstructured, purely time-varying, total disturbance input. It is clear that the prevailing linearity of the simplified system prompts, for the phase variables estimation purposes, the use of a linear Luenberger type of observer and the incorporation of a reasonable total disturbance model for the signal $\xi(t)$. Under the suitable disturbance smoothness assumption, an mth order Taylor time-polynomial approximation at time t, of the disturbance input, leads to the following self-updating linear approximation model:

$$\xi(t) = z_1(t),\ \ z_1^{(m)}(t) = 0. \tag{8}$$

Defining $z_i(t) = z^{(i-1)}$, $i = 1, 2, \ldots, m$, we immediately obtain

$$\begin{aligned} \dot{z}_1 &= z_2,\\ \dot{z}_2 &= z_3,\\ &\ \vdots\\ \dot{z}_{m-1} &= z_m,\\ \dot{z}_m &= 0. \end{aligned} \tag{9}$$

The disturbance approximation model corresponds with an $(m-1)$st order time polynomial which ultra-locally will be made to act as a self-updating polynomial spline approximating the actual value of the disturbance input. This self-updating character is bestowed through the disturbance estimation errors in the following manner.

Define the simplified plant phase variables: $y_i = y^{i-1}$, $i = 1, 2, \ldots, n$. Consider next the full order system model, including the disturbance model, and, also, its associated (perturbed) asymptotic extended observer:

$$\begin{aligned} &\dot{y}_1 = y_2, && \frac{\mathrm{d}}{\mathrm{d}t}\hat{y}_1 = \hat{y}_2 + \lambda_{m+n-1}(y - \hat{y}_1),\\ &\dot{y}_2 = y_3, && \frac{\mathrm{d}}{\mathrm{d}t}\hat{y}_2 = \hat{y}_3 + \lambda_{m+n-2}(y - \hat{y}_1),\\ &\ \vdots && \ \vdots\\ &\dot{y}_n = u + z_1, && \frac{\mathrm{d}}{\mathrm{d}t}\hat{y}_n = u + \hat{z}_1 + \lambda_m(y - \hat{y}_1),\\ &\dot{z}_1 = z_2, && \frac{\mathrm{d}}{\mathrm{d}t}\hat{z}_1 = \hat{z}_2 + \lambda_{m-1}(y - \hat{y}_1)\\ &\ \vdots && \ \vdots\\ &\dot{z}_m = 0, && \frac{\mathrm{d}}{\mathrm{d}t}\hat{z}_m = \lambda_0(y - \hat{y}_1). \end{aligned}$$

The (redundant) output estimation error, $e_y = y - \hat{y}_1$, is seen to satisfy the following perturbed linear dynamics,

$$e_y^{(m+n)} + \lambda_{m+n-1} e_y^{(m+n-1)} + \ldots + \lambda_1 \dot{e}_y + \lambda_0 e_y = \xi^{(m)}(t). \tag{10}$$

Clearly, the unperturbed version ($\xi(t) = 0$) of the output estimation error dynamics can be specified to become asymptotically exponentially stable through the choice of suitable Hurwitz design coefficients: $\{\lambda_0, \ldots, \lambda_{m+n-1}\}$. Let $\xi(s)$ denote the Laplace transform of the total disturbance signal $\xi(t)$. The injected estimation error dynamics is described by the perturbed band-pass stable filter,

$$e_y(s) = \frac{s^m \xi(s)}{s^{m+n} + \lambda_{m+n-1} s^{m+n-1} + \ldots + \lambda_0}, \tag{11}$$

which enjoys infinite attenuation at very low, and at very high, frequencies. At intermediate frequencies, where the minimum disturbance attenuation (or, actually, disturbance amplification) may be experienced, a high gain observer design, based on corresponding Hurwitz coefficients: κ_i, $i = 0, 1, \ldots, m+n$, defined by, $\lambda_i = \kappa_i/\epsilon^{m+n-i}$, $i = 0, 1, \ldots, m+n-1$ substantially attenuates to a desired level the mth derivative of the smooth total disturbance influence on the output estimation error and of its time derivatives. The parameter ϵ is a small positive real number. The frequency response of the output estimation error would be given, in normalized frequency terms, $\sigma = \epsilon s$, by

$$e_y(\sigma) = \frac{\epsilon^n \sigma^m \xi(\sigma)}{\sigma^{m+n} + \kappa_{m+n-1}\sigma^{m+n-1} + \dots + \kappa_1\sigma + \kappa_0}. \quad (12)$$

The attenuation effects of the high gain design parameter ϵ are clearly depicted.

As a result, as ϵ is made sufficiently small, an uniformly absolutely bounded total perturbation input, with uniformly absolutely bounded time derivatives, induces a phase variable estimation error which can be made as small in magnitude as desired. Clearly, for $j = 0, 1, \dots, n-1$,

$$\sigma^j e_y(\sigma) = \frac{\epsilon^{n-j} \sigma^{m+j} \xi(\sigma)}{\sigma^{m+n} + \kappa_{m+n-1}\sigma^{m+n-1} + \dots + \kappa_1\sigma + \kappa_0}, \quad (13)$$

which still enjoys infinite, low and high frequency, attenuation features and modest attenuation at intermediate frequencies as the order of the output estimation error time derivative, j, increases from 0 towards $n-1$.

The disturbance estimation error $e_\xi = \xi(t) - \hat{z}_1$ is seen to satisfy, in the time domain,

$$e_y^{(n)} + \lambda_{m+n-1} e_y^{(n-1)} + \dots + \lambda_{m+1}\dot{e}_y + \lambda_m e_y = \xi - \hat{z}_1. \quad (14)$$

As the estimation errors time derivatives, $e_y^{(j)}, j = 0, 1, \dots, n$, uniformly ultimately approach a neighborhood of the origin in the estimation error phase space, the disturbance estimation error, $\xi - \hat{z}_1$, approaches a small neighborhood of the origin of the real line, still conveniently determined by the small parameter ϵ.

It should be clear by now, that the approximation error $\xi(t) - z_1$, associated with the proposed Taylor polynomial model of the total disturbance, $\xi(t)$, exhibits an explicit linear dependance on the phase variable estimation errors. Forcing their contributions to be part of an $(m+n)$th order asymptotically exponentially stable linear dynamics perturbed by the disturbance model actual residual, the resulting disturbance estimate error automatically adapts to a small vicinity of zero, thus making the Taylor polynomial approximation truly self-adapting.

In the context of pure integration perturbed systems, GPI observers have been shown to be a generalization of Han's extended state observer, but one which is also capable of on-line estimating a finite number of time derivatives of the total disturbance input. High gain state estimation seems to be at the heart of observer based ADRC control.

It was, therefore, rather natural to combine GPI observers in an ADRC scheme for simplified models of totally perturbed flat nonlinear systems. The estimated phase variables completed a suitable linear feedback loop with rather accurate, though approximate, disturbance cancellation.

4 GPI control: dynamical output feedback control without observers

The stabilization of a perturbed pure integration system can be accomplished by the use of suitable classical compensation networks. The lack of universality of classical compensation networks is determined, primarily, by the nature of the disturbance function, by the order of the plant and by the knowledge of the control input gain. There exists a very close connection between GPI control, based on integral phase variables reconstructors, and classical output compensation networks. The presence of additive exogenous and endogenous (total) disturbances disrupts the established input error integration process aimed to obtain structural estimates of the output phase variables. To circumvent this inconvenience, one must first establish the structure of the GPI output compensator, regardless of the additive disturbances and, then, proceed to examine, and assess, the closed-loop performance in the presence of such unknown but bounded perturbation inputs.

Consider then the nth order pure integration system as simplified from the input-to-flat output dynamics,

$$y^{(n)} = u. \quad (15)$$

Iterated integrations of the input u yield structural estimates of the phase variables as follows:

$$\begin{aligned} y_i &:= \hat{y}^{(i)}(t) \\ &= \int_0^t \int_0^{\rho_1} \dots \int_0^{\rho_{n-i-1}} u(\rho_{n-i}) \mathrm{d}\rho_{n-i} \dots \mathrm{d}\rho_1 \\ &=: \int^{(n-i)} u(t), \quad i = 1, 2, \dots, n-1. \end{aligned} \quad (16)$$

Each estimate is off by an $(n-i-1)$th order time-polynomial, whose coefficients are exclusively dependent upon the unknown initial conditions associated with the estimated phase variable. Any linear feedback control scheme, based on the use of all these structural estimates of the phase variables (from the first, $y_1 = \hat{y} = \hat{y}^{(1)}$, to the $(n-1)$th, $y_{n-1} = \hat{y}^{(n-1)}$), must dully compensate, as classically done, for the resulting linear combination of the corresponding time-polynomial errors. This simply requires a suitable linear combination of iterated integrals of the output signal y, ranging from a first order output integration, for compensation of the constant errors, up to an iterated $(n-1)$th order output iterated integration for the compensation of the $(n-2)$th order time polynomial associated with the estimation y_1 of $\dot{y} = y^{(1)}$. The linear control scheme, thus requires only the measurements of the input signal u and of the output signal y for its implementation.

A compensator for the unperturbed chain of integrations is given by

$$\begin{aligned} u &= -\sum_{i=1}^{n-1} k_{2n-i-1} y_{n-i} - \sum_{j=0}^{n-1} k_{n-j-1} \Big(\int^{(j)} y\Big) \\ &= -\sum_{i=1}^{n-1} k_{2n-i-1} \Big(\int^{(i)} u\Big) - \sum_{j=0}^{n-1} k_{n-j-1} \Big(\int^{(j)} y\Big). \end{aligned} \tag{17}$$

In terms of the Laplace transforms, one obtains the implicit expression for the control input u

$$u(s) = -\Big[\sum_{i=1}^{n-1} \frac{k_{2n-i-1}}{s^i}\Big] u(s) - \Big[\sum_{j=0}^{n-1} \frac{k_{n-j-1}}{s^j}\Big] y(s). \tag{18}$$

Solving for $u(s)$ we have

$$u(s) = -\Big[\frac{k_{n-1}s^{n-1} + k_{n-2}s^{n-2} + \ldots + k_1 s + k_0}{s^{n-1} + k_{2n-2}s^{n-2} + \ldots + k_{n+1}s + k_n}\Big] y(s). \tag{19}$$

The closed-loop, unperturbed, system is readily obtained as

$$[s^{2n-1} + k_{2n-2}s^{2n-2} + \ldots + k_1 s + k_0] y(s) = 0. \tag{20}$$

The order of the dynamic output feedback compensator, for the unperturbed input output dynamics, is one less than the order of the plant. The output can be exponentially asymptotically stabilized via the suitable choice of the design parameters, $\{k_{2n-2}, \ldots, k_1, k_0\}$, as the coefficients of a $(2n-1)$ degree Hurwitz polynomial in the complex variable s. The above stabilizing classical compensation network is addressed as the *GPI controller*.

The above dynamic flat output feedback control scheme allows for flat output stabilization without explicitly using a linear observer exhibiting exponentially asymptotically stable (redundant) flat output estimation error.

5 Flat filters

The GPI controller is non robust with respect to additive disturbance inputs of the simplest kind (constant unknown disturbances, for example). As it was done in GPI observer based ADRC control of simplified, perturbed, flat systems, total additive disturbances may be modeled as finite order time polynomials (say, polynomials of order $m-1$). There are, however, at least, two equivalent manners of bestowing the self updating feature to such a finite order, linear, total perturbation model. One of them is through exogenous input extensions, coupled with imposition of closed-loop stability for the entire extended systems. This is dual to the GPI observer approach. We take, however, the alternative route of compensating, in the feedback control, the effects of the total perturbation input through a suitable finite linear combination of iterated output integrations.

Robustness of the previously exposed output feedback control scheme is obtained after adding m additional iterated output integrations and redefinition of the controller design parameters.

Consider the output feedback control law

$$\begin{aligned} u &= -\sum_{i=1}^{n-1} k_{2n+m-i-1} y_{n-i} - \sum_{j=0}^{n+m-1} k_{n+m-j-1} \Big(\int^{(j)} y\Big) \\ &= -\sum_{i=1}^{n-1} k_{2n+m-i-1} \Big(\int^{(i)} u\Big) - \sum_{j=0}^{n+m-1} k_{n+m-j-1} \Big(\int^{(j)} y\Big). \end{aligned} \tag{21}$$

Solving for $u(s)$, after using the Laplace transform operator on the implicit controller expression, we obtain

$$u(s) = -\Big[\frac{k_{n+m-1}s^{n+m-1} + \ldots + k_1 s + k_0}{s^m(s^{n-1} + k_{2n+m-2}s^{n-2} + \ldots + k_{n+m})}\Big] y(s). \tag{22}$$

The closed-loop system, in the absence of input disturbances, is given by

$$\begin{aligned} &[s^{2n+m-1} + k_{2n+m-2}s^{2n+m-2} + \ldots + k_{n+m}s^{n+m} \\ &+ k_{n+m-1}s^{n+m-1} + \ldots + k_2 s^2 + k_1 s + k_0] y(s) = 0, \end{aligned} \tag{23}$$

which can be made to asymptotically exponentially converge towards zero provided the appropriate (Hurwitz) gains are used.

For the perturbed pure integration system

$$y^{(n)} = u + \xi(t). \tag{24}$$

The previously derived dynamic output feedback controller is directly used on the perturbed system. One obtains the following closed-loop system, driven by the total perturbation input.

$$y(s) = \frac{s^m(s^{n-1} + k_{2n+m-2}s^{n-2} + \ldots + k_{n+m})\xi(s)}{s^{2n+m-1} + k_{2n+m-2}s^{2n+m-2} + \ldots + k_1 s + k_0}. \quad (25)$$

The frequency response of the output stabilization error (represented by y itself) exhibits significantly large attenuation at low and high frequencies, thus rejecting high frequency measurement noise and rejecting also typical low frequency disturbance inputs. For intermediate frequencies, a high gain parameter factor, in the form: $\kappa = k_i/\epsilon^{2n+m-1-i}$. This induces the normalized frequency relation,

$$y(\sigma) = \frac{\epsilon^{n-1}\sigma^m(\sigma^{n-1} + \kappa_{2n+m-2}\sigma^{n-2} + \ldots + \kappa_{n+m})\xi(\sigma)}{\sigma^{2n+m-1} + \kappa_{2n+m-2}\sigma^{2n+m-2} + \ldots + \kappa_1\sigma + \kappa_0}. \quad (26)$$

6 Flat filters and observer based ADRC: An equivalence

Consider the pure integration perturbed system

$$y^{(n)} = u + \xi(t). \quad (27)$$

We adopt, for the reduced order observer, the $n-1$ dimensional system state representation including an artificial velocity measurement ($y_2 = \dot{y} = \dot{y}_1$):

$$\begin{aligned}
\dot{y}_2 &= y_3,\\
&\vdots\\
\dot{y}_{n-1} &= y_n,\\
\dot{y}_n &= u + \xi,\\
y_2 &= \dot{y}_1.
\end{aligned} \quad (28)$$

A reduced order extended state observer (ROESO), which takes y_2 as the artificially measured output, including, also, m extra output integrations, is given by

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\hat{y}_2 &= \hat{y}_3 + \lambda_{m+n-2}(\dot{y} - \hat{y}_2),\\
\frac{\mathrm{d}}{\mathrm{d}t}\hat{y}_3 &= \hat{y}_4 + \lambda_{m+n-3}(\dot{y} - \hat{y}_2),\\
&\vdots\\
\frac{\mathrm{d}}{\mathrm{d}t}\hat{y}_{n-1} &= \hat{y}_n + \lambda_{m+1}(\dot{y} - \hat{y}_2),\\
\frac{\mathrm{d}}{\mathrm{d}t}\hat{y}_n &= u + \hat{\xi}_1 + \lambda_m(\dot{y} - \hat{y}_2),\\
\frac{\mathrm{d}}{\mathrm{d}t}\hat{\xi}_1 &= \hat{\xi}_2 + \lambda_{m-1}(\dot{y} - \hat{y}_2),\\
&\vdots\\
\frac{\mathrm{d}}{\mathrm{d}t}\hat{\xi}_{m-1} &= \hat{\xi}_m + \lambda_1(\dot{y} - \hat{y}_2),\\
\frac{\mathrm{d}}{\mathrm{d}t}\hat{\xi}_m &= \lambda_0(\dot{y} - \hat{y}_2).
\end{aligned} \quad (29)$$

Define

$$\begin{aligned}
\hat{\eta}_2 &= \hat{y}_2 - \lambda_{m+n-2}y,\\
\hat{\eta}_3 &= \hat{y}_3 - \lambda_{m+n-3}y,\\
&\vdots\\
\hat{\eta}_n &= \hat{y}_n - \lambda_m y,\\
\hat{\zeta}_1 &= \hat{\xi}_1 - \lambda_{m-1}y,\\
&\vdots\\
\hat{\zeta}_{m-1} &= \hat{\xi}_{m-1} - \lambda_1 y,\\
\hat{\zeta}_m &= \hat{\xi}_m - \lambda_0 y.
\end{aligned} \quad (30)$$

The ROESO is thus proposed to be

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\hat{\eta}_2 &= \hat{\eta}_3 - \lambda_{m+n-2}\hat{\eta}_2 + (\lambda_{m+n-3} - \lambda_{m+n-2}^2)y,\\
\frac{\mathrm{d}}{\mathrm{d}t}\hat{\eta}_3 &= \hat{\eta}_4 - \lambda_{m+n-3}\hat{\eta}_2\\
&\quad + (\lambda_{m+n-4} - \lambda_{n+m-3}\lambda_{m+n-2})y,\\
&\vdots\\
\frac{\mathrm{d}}{\mathrm{d}t}\hat{\eta}_{n-1} &= \hat{\eta}_n - \lambda_{m+1}\hat{\eta}_2 + (\lambda_m - \lambda_{m+1}\lambda_{m+n-2})y,\\
\frac{\mathrm{d}}{\mathrm{d}t}\hat{\eta}_n &= u + \hat{\zeta}_1 - \lambda_m\hat{\eta}_2 + (\lambda_{m-1} - \lambda_m\lambda_{m+n-2})y,\\
\frac{\mathrm{d}}{\mathrm{d}t}\hat{\zeta}_1 &= \hat{\zeta}_2 - \lambda_{m-1}\hat{\eta}_2 + (\lambda_{m-2} - \lambda_{m-1}\lambda_{m+n-2})y,\\
&\vdots\\
\frac{\mathrm{d}}{\mathrm{d}t}\hat{\zeta}_{m-1} &= \hat{\zeta}_m - \lambda_1\hat{\eta}_2 + (\lambda_0 - \lambda_1\lambda_{m+n-2})y,\\
\frac{\mathrm{d}}{\mathrm{d}t}\hat{\zeta}_m &= -\lambda_0\hat{\eta}_2 - \lambda_0\lambda_{m+n-2}y.
\end{aligned} \quad (31)$$

The estimates of the original phase variables may be computed from the following expressions:

$$\begin{aligned}
\hat{y}_1 &= y_1 = y,\\
\hat{y}_2 &= \hat{\eta}_2 + \lambda_{m+n-2}y,\\
\hat{y}_3 &= \hat{\eta}_3 + \lambda_{m+n-3}y,\\
&\vdots
\end{aligned}$$

$$\hat{y}_n = \hat{\eta}_n + \lambda_m y,$$
$$\hat{\xi}_1 = \hat{\xi} = \hat{\zeta}_1 + \lambda_{m-1} y,$$
$$\vdots$$
$$\hat{\xi}_{m-1} = \widehat{\xi^{(m-2)}} = \hat{\zeta}_{m-1} + \lambda_1 y,$$
$$\hat{\xi}_m = \widehat{\xi^{(m-1)}} = \hat{\zeta}_m + \lambda_0 y.$$

Defining also, for the original system,

$$\begin{aligned}
\eta_2 &= y_2 - \lambda_{m+n-2} y,\\
\eta_3 &= y_3 - \lambda_{m+n-3} y,\\
&\vdots\\
\eta_n &= y_n - \lambda_m,\\
\zeta_1 &= \xi_1 - \lambda_{m-1} y,\\
&\vdots\\
\zeta_{m-1} &= \xi_{m-1} - \lambda_1 y,\\
\zeta_m &= \xi_m - \lambda_0 y
\end{aligned} \tag{32}$$

with $\xi_1 = \xi, \xi_2 = \dot{\xi}, \dots, \xi_m = \xi^{(m-1)}$. One readily obtains

$$\begin{aligned}
\dot{\eta}_2 &= \eta_3 - \lambda_{m+n-2}\eta_2 + (\lambda_{m+n-3} - \lambda_{m+n-2}^2) y,\\
\dot{\eta}_3 &= \eta_4 - \lambda_{m+n-3}\eta_2 + (\lambda_{m+n-4} - \lambda_{n+m-3}\lambda_{m+n-2}) y,\\
&\vdots\\
\dot{\eta}_{n-1} &= \eta_n - \lambda_{m+1}\eta_2 + (\lambda_m - \lambda_{m+1}\lambda_{m+n-2}) y,\\
\dot{\eta}_n &= u + \zeta_1 - \lambda_m\eta_2 + (\lambda_{m-1} - \lambda_m\lambda_{m+n-2}) y,\\
\dot{\zeta}_1 &= \zeta_2 - \lambda_{m-1}\eta_2 + (\lambda_{m-2} - \lambda_{m-1}\lambda_{m+n-2}) y,\\
&\vdots\\
\dot{\zeta}_{m-1} &= \zeta_m - \lambda_1\eta_2 + (\lambda_0 - \lambda_1\lambda_{m+n-2}) y,\\
\dot{\zeta}_m &= \xi^{(m)} - \lambda_0\eta_2 - \lambda_0\lambda_{m+n-2} y.
\end{aligned} \tag{33}$$

The ROESO state, and disturbance, estimation errors are seen to satisfy:

$$\begin{aligned}
\dot{e}_2 &= e_3 - \lambda_{m+n-2} e_2,\\
\dot{e}_3 &= e_4 - \lambda_{m+n-1} e_2,\\
&\vdots\\
\dot{e}_{n-1} &= e_n - \lambda_{m+1} e_2,\\
\dot{e}_n &= (\xi - \hat{\xi}_1) - \lambda_m e_2,\\
\dot{e}_{\zeta_1} &= e_{\zeta_2} - \lambda_{m-1} e_2 = e_{\zeta_1} - \lambda_m e_2,\\
&\vdots\\
\dot{e}_{\zeta_{m-1}} &= e_{\zeta_m} - \lambda_1 e_2,\\
\dot{e}_{\zeta_m} &= \xi^{(m)} - \lambda_0 e_2.
\end{aligned} \tag{34}$$

In other words, the estimation error e_2 satisfies the linear perturbed differential equation:

$$e_2^{(n+m-1)} + \lambda_{m+n-2} e_2^{(n+m-2)} + \dots + \lambda_1 \dot{e}_2 + \lambda_0 e_2 = \xi^{(m)}. \tag{35}$$

From here, it easily follows that,

$$\begin{aligned}
&e_2(s)\\
&= \Big[\frac{s^m}{s^{n+m-1} + \lambda_{m+n-2} s^{m+n-2} + \dots + \lambda_1 s + \lambda_0}\Big]\xi(s),\\
&e_3(s)\\
&= \Big[\frac{s^m(s + \lambda_{m+n-2})}{s^{n+m-1} + \lambda_{m+n-2} s^{m+n-2} + \dots + \lambda_1 s + \lambda_0}\Big]\xi(s),\\
&\vdots\\
&e_n(s)\\
&= \Big[\frac{s^m(s^{n-2} + \lambda_{m+n-2} s^{n-3} + \dots + \lambda_{m+1})}{s^{n+m-1} + \lambda_{m+n-2} s^{m+n-2} + \dots + \lambda_1 s + \lambda_0}\Big]\xi(s),\\
&e_\xi(s)\\
&= \Big[\frac{s^m(s^{n-1} + \lambda_{m+n-2} s^{n-2} + \dots + \lambda_{m+1} s + \lambda_m)}{s^{n+m-1} + \lambda_{m+n-2} s^{m+n-2} + \dots + \lambda_1 s + \lambda_0}\Big]\xi(s),
\end{aligned} \tag{36}$$

which implies large attenuation of the low frequency input disturbance signal, $\xi(t)$, in the state and disturbance estimation errors, produced by the proposed ROESO.

Finally, let $\hat{y}_1 = y$. A ROESO-based ADRC controller, for output stabilization purposes, is proposed as

$$\begin{aligned}
u &= -\hat{\xi} - \sum_{j=1}^{n} \gamma_{j-1}\hat{y}_j\\
&= -\hat{\xi} - \sum_{j=1}^{n} \gamma_{j-1} y_j + \sum_{j=2}^{n} \gamma_{j-1}(y_j - \hat{y}_j)\\
&= -\hat{\xi} - \sum_{j=1}^{n} \gamma_{j-1} y_j + \sum_{j=2}^{n} \gamma_{j-1} e_j,
\end{aligned} \tag{37}$$

where y_j is the $(j-1)$th order time derivative of the output y.

The nth order time derivative of the output (itself the stabilization error) satisfies

$$\begin{aligned}
y_{n+1} = y^{(n)} &= (\xi - \hat{\xi}) - \sum_{j=1}^{n} \gamma_{j-1} y_j + \sum_{j=2}^{n} \gamma_{j-1} e_j\\
&= e_\xi - \gamma_0 y - \sum_{j=2}^{n} \gamma_{j-1} y_j + \gamma_{j-1} e_j.
\end{aligned} \tag{38}$$

Let $q(s)$ denote the following characteristic polynomial: $q(s) = s^{n+m-1} + \lambda_{n+m-2} s^{n+m-2} + \dots + \lambda_1 s + \lambda_0$ and let $p(s)$ denote the closed-loop control characteristic polynomial $p(s) = s^n + \gamma_{n-1} s^{n-1} + \dots + \gamma_1 s + \gamma_0$. Using the above expressions, one obtains, in the Laplace transform domain:

$$\begin{aligned} p(s)y(s) &= (s^n + \gamma_{n-1}s^{n-1} + \ldots + \gamma_0)y(s) \\ &= e_\xi(s) + \sum_{j=2}^{n} \gamma_{j-1}\Big[\frac{n_j(s)}{q(s)}\Big] \times \Big[\frac{n_{n+1}}{q(s)}\Big]\xi(s) \\ &\quad + \sum_{j=2}^{n} \gamma_{j-1}\Big[\frac{n_j(s)}{q(s)}\Big]\xi(s) \end{aligned} \tag{39}$$

with

$$\begin{aligned} n_j(s) &= s^m(s^{j-2} + \lambda_{n+m-2}s^{j-3} + \ldots + \lambda_{n+m-(j-1)}), \\ n_{n+1} &= s^m(s^{n-1} + \lambda_{m+n-2}s^{n-2} + \ldots + \lambda_{m+1}s + \lambda_m). \end{aligned} \tag{40}$$

The closed-loop system output, y_1, evolves, excited by the disturbance ξ, in accordance with the following dynamics:

$$y(s) = \Big[\frac{m(s)}{r(s)}\Big]\xi(s), \tag{41}$$

where

$$\begin{aligned} m(s) &= s^m[s^{n-1} + (\lambda_{m+n-2} + \gamma_{n-1})s^{n-2} + (\lambda_{m+n-3} \\ &\quad +\gamma_{n-1}\lambda_{m+n-2} + \gamma_{n-3})s^{n-3} + \ldots + (\lambda_m \\ &\quad +\gamma_{n-1}\lambda_{m+1} + \gamma_{n-2}\lambda_{m+2} + \ldots + \gamma_1\lambda_{m+n-1})], \\ r(s) &= p(s)q(s) \\ &= (s^n + \gamma_{n-1}s^{n-1} + \ldots + \gamma_1 s + \gamma_0)(s^{n+m-1} \\ &\quad +\lambda_{m+n-2}s^{m+n-2} + \ldots + \lambda_1 s + \lambda_0). \end{aligned} \tag{42}$$

Clearly the denominator $r(s)$ is of the form,

$$r(s) = s^{2n+m-1} + k_{2n+m-2}s^{2n+m-2} + \ldots + k_1 s + k_0, \tag{43}$$

while the numerator is of the form,

$$m(s) = s^{n+m-1} + k_{2n+m-2}s^{n+m-2} + \ldots + k_{m+n}s^m. \tag{44}$$

The characteristic polynomial of the closed-loop system factors into the product of the ROESO characteristic polynomial (i.e., $n + m - 1 = (n - 1) + m$) and the nth order characteristic polynomial of the closed-loop system, obtained by straightforward pole placement on the nth order pure integration plant system, as if all the phase variables had been available for feedback. All this is, evidently, in accordance with the observer controller design separation principle for state feedback through an observer in linear systems.

An output stabilization feedback controller for an nth order pure integration system, $y^{(n)} = u + \xi(t)$, was found to be characterized, in the frequency domain, by

$$u = -\Big[\frac{k_{m+n-1}s^{m+n-1} + \ldots + k_1 s + k_0}{s^m(s^{n-1} + k_{2n+m-2}s^{n-2} + \ldots + k_{m+n})}\Big]y(s) \tag{45}$$

with the filter gains chosen to guarantee a Hurwitz closed-loop characteristic polynomial. The closed-loop system is described by

$$y(s) = \Big[\frac{n(s)}{d(s)}\Big]\xi(s), \tag{46}$$

where

$$\begin{aligned} n(s) &= s^m(s^{n-1} + k_{2n+m-2}s^{n-2} + \ldots + k_{n+m}), \\ d(s) &= s^{2n+m-1} + k_{2n+m-2}s^{2n+m-2} \\ &\quad + \ldots + k_{n+m}s^{n+m} + \ldots + k_1 s + k_0, \end{aligned} \tag{47}$$

which is clearly identifiable with the ROESO-based ADRC closed-loop system, depicted in equation (41), thanks to the fact that the closed-loop characteristic polynomial coefficients uniquely determine all the coefficients in the flat filter controller. We therefore have

$$\begin{aligned} p(s) &= s^{2n+m-1} + k_{2n+m-2}s^{2n+m-2} + \ldots + k_{n+m}s^{n+m} \\ &\quad + \ldots + k_0 \\ &= (s^n + \gamma_{n-1}s^{n-1} + \gamma_1 s + \gamma_0)(s^{n+m-1} \\ &\quad +\lambda_{m+n-2}s^{m+n-2} + \ldots + \lambda_1 s + \lambda_0). \end{aligned}$$

Clearly, given a ROESO-based ADRC controller design, there exists a unique stable flat filter controller which has exactly the same set of fundamental transfer functions (sensitivity, complementary sensitivity and open loop transfer functions). On the other hand, given a FF controller design, there exists non-unique equivalent ROESO based ADRC controllers. This one-way equivalence is substantially helpful in synthesizing observer based ADRC control schemes in the form of a single linear controller, in the form of a stable proper transfer function with enhanced disturbance attenuation features and good low frequency trajectory tracking features. The equivalence has been tested, also, in several experimental settings.

7 Conclusions

In this article, a roadmap, intimately related to the author's gradual understanding, and linking, of ADRC with differential flatness and GPI control in the form of robust flat filters, has been provided through the basic developments leading from one end to the other. An exposition has been given of the inherent and natural relevance of differential flatness in the control of nonlinear uncertain systems for the SISO case. Also, the natural importance of classical compensation schemes in the control of uncertain nonlinear systems cannot be overemphasized. It is the author's belief that the key issue, and one which encounters serious difficulties and criticisms on ADRC in the automatic control community, is given by the unstructured nature of the simplified pure integration system in the realm of non-linear control, whose design methods have been traditionally dominated by meticulous consideration of the nonlinear state structure. When the cult to the nonlinearities is swept away, in the form of a total purely time varying disturbance, all the arsenal of robust linear control can be readily applied to great advantage. It is my personal belief that ADRC still has numerous development avenues, both, in theory and practise, in comparison with other nonlinear control and observer design methods.

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8 Brief description of the bibliography

References [1–4] represent, in the form of books, the contributions towards flatness, GPI control, GPI observers and ADRC of the author, his coworkers and students. Numerous application examples and laboratory experiments are described in detail in those four references. References [5–11] are book chapters containing early work on GPI observers, flatness and GPI control. References [12–41] constitute journal articles, published since 2010, containing diverse applications of GPI observers, GPI controllers and flatness based controllers. In many of these works striking coincidences and similarities may be found with ADRC schemes, at a time the author of this article was unaware of the vast potentials and appeal of the fascinating research field started by Prof. J. Han. Meeting Prof. Z. Gao in 2013, completely changed the perspectives and triggered the enthusiasm of the author for this area, while feeling the respectful need to start calling things by their proper name.

References

[1] H. Sira-Ramírez, A. Luviano-Juarez, M. Ramírez Neira, et al. *Active Disturbance Rejection Control of Dynamic Systems*. Oxford: Elsevier, 2017.

[2] H. Sira-Ramírez. *Sliding Mode Control: The Sigma-Delta Modulation Approach*. Control Engineering Series. Basel: Birkhäuser, 2015.

[3] H. Sira-Ramírez, C. García Rodríguez, J. Cortes-Romero, et al. *Algebraic Identification and Estimation Methods in Feedback Control Systems*. Hoboken: Wiley, 2014.

[4] H. Sira-Ramírez, S. Agrawal. *Differentially Flat Systems*. New York: Marcel Dekker, 2004.

[5] H. Sira-Ramírez, E. W. Zurita-Bustamante, M. A. Aguilar-Orduña, et al. Sliding mode control devoid of state measurements. *New Perspectives and Applications of Modern Control Theory*. J. B. Clempner, W. Yu (eds.). Cham: Springer, 2017: 73 – 102.

[6] H. Sira Ramírez, F. Gonzalez Montañez, J. Cortes Romero, et al. State observers for active disturbance rejection in induction motor control. *AC Motors Control – Advanced Design Techniques and Applications*. F. Giri (ed.). Hoboken: Wiley, 2014.

[7] E. Zurita-Bustamante, J. Linares-Flores, E. Guzman-Ramírez, et al. FPGA implementation of PID controller for the stabilization of a DC-DC "Buck" converter. *Frontiers in Advanced Control Systems*. G. L. de Oliveira-Serra (ed.). Chicago: INTECH Publishing, 2012: 215 – 230.

[8] J. Cortes-Romero, A. Luviano-Juarez, H. Sira-Ramírez. Sliding mode control design for induction motors: An input-output approach. *Sliding Mode Control*. A. Bartoszewicz (ed.). Chicago: INTECH Publishing, 2011; 135 – 154.

[9] H. Sira-Ramírez, J. Cortes-Romero, A. Luviano-Juarez. Robust linear control of nonlinear flat systems. *Robust Control, Theory and Applications*. A. Bartoszewicz (ed.). Chicago: INTECH Publishing, 2011: 455 – 476.

[10] H. Sira-Ramírez, A. Luviano-Juarez, J. Cortes-Romero. Sliding mode controller design: An input-output approach. *Sliding Modes after the First Decade of the 21st Century*. L. Fridman (ed.). London: Springer, 2011: 245 – 268.

[11] H. Sira-Ramírez, V. Feliu-Battle. A generalized PI sliding mode control of switched fractional systems. *Modern Sliding Mode Control Theory: New Perspectives and Applications*. G. Bartolini, L. Fridman, A. Pisano (eds.). Lecture Notes in Control and Information Sciences. Berlin: Springer, 2008: 201 – 221.

[12] H. Sira Ramírez, E. W. Zurita-Bustamante, E. Hernandez-Flores, et al. On a linear input-output approach for the control of nonlinear flat systems. *International Journal of Control*, 2018, 91(9): 2131 – 2146.

[13] C. Aguilar-Ibañez, H. Sira-Ramírez, J. A. Acosta. Stability of active disturbance rejection control for uncertain systems: A Lyapunov perspective. *International Journal of Robust and Nonlinear Control*, 2017, 27(18): 4541 – 4553.

[14] H. Sira-Ramírez, E. W. Zurita-Bustamante, A. Luviano-Juarez. Robust flat filtering control of a nonlinear manipulator-direct current motor system. *Journal of Dynamic Systems Measurment and Control*, 2017, 140(2): DOI 10.1115/1.4037386.

[15] A. Hernandez-Mendez, J. Linares-Flores, H. Sira-Ramírez, et al. A backstepping approach to decentralized active disturbance rejection control of interacting Boost converters. *IEEE Transactions on Industry Applications*, 2017, 53(4): 4063 – 4072.

[16] J. Cortes-Romero, A. Jimenez-Triana, H. Coral-Enríquez, et al. Algebraic estimation and active disturbance rejection in the control of flat systems. *Control Engineering Practice*, 2017, 61: 173 – 182.

[17] H. Sira-Ramírez, A. Hernandez-Mendez, J. Linares-Flores, et al. Robust flat filtering DSP based control of the boost converter. *Control Theory and Technology*, 2016, 14(3): 224 – 236.

[18] M. Ramírez-Neria, H. Sira-Ramírez, R. Garrido-Moctezuma, et al. On the linear control of underactuated nonlinear systems via tangent flatness and active disturbance rejection control: the case of the ball and beam system. *Journal of Dynamic Systems, Measurement and Control*, 2016, 138(10): DOI 10.1115/1.4033313.

[19] A. Gutierrez-Gilesa, M. A. Arteaga-Perez, H. Sira-Ramírez. Control de fuerza de robots manipuladores basado en observadores proporcionales integrales generalizados. *Revista Iberoamericana de Automatica e Informatica industrial*, 2016, 13(2): 238 – 246.

[20] E. Guerrero, H. Sira-Ramírez, A. Martínez, et al. On the robust control of parallel-cascade DC/DC buck converter. *IEEE Latin America Transactions*, 2016, 14(2): 595 – 601.

[21] C. Huang, H. Sira-Ramírez. Flatness-based active disturbance rejection control for linear systems with unknown time-varying coefficients. *International Journal of Control*, 2015, 88(12): 2578 – 2587.

[22] H. Sira-Ramírez, J. Linares-Flores, A. Luviano-Juarez, et al. Ultramodelos globales y el control por rechazo activo de perturbaciones en sistemas no lineales diferencialmente planos. *Revista Iberoamericana de Automatica e Informatica industrial*, 2015, 12(2): 133 – 144.

[23] M. Ramírez-Neria, H. Sira-Ramírez, A. Rodríguez-Angeles, et al. Active disturbance rejection control applied to a delta parallel robot in trajectory tracking tasks. *Asian Journal of Control*, 2015, 17(2): 636 – 647.

[24] R. Morales, H. Sira-Ramírez, J. A. Somolinos. Robust control of underactuated wheeled mobile manipulators using GPI disturbance observers. *Multibody Systems Dynamics*, 2014, 32(4): 511 – 533.

[25] H. Sira-Ramírez, J. Linares-Flores, C. García-Rodríguez, et al. On the control of the permanent magnet synchronous motor: An active disturbance rejection control approach. *IEEE Transactions on Control Systems Technology*, 2014, 22(5): 2056 – 2063.

[26] M. Ramírez-Neria, H. Sira-Ramírez, R. Garrido-Moctezuma, et al. Linear active disturbance rejection control of underactuated systems: The case of the Furuta pendulum. *ISA Transactions*, 2014, 53(4): 920 – 928.

[27] A. Rodríguez-Angeles, H. Sira-Ramírez, J. A. García-Antonio. A vehicle haptic steering by wire system based on high gain GPI observers. *Mathematical Problems in Engineering*, 2014: DOI 10.1155/2014/981276.

[28] H. Sira-Ramírez, R. Castro-Linares, G. Puriel-Gil. An active disturbance rejection approach to leader-follower controlled formation. *Asian Journal of Control*, 2014, 16(2): 382 – 395.

[29] J. A. Juarez-Abad, J. Linares-Flores, E. Guzman-Ramírez, et al. Generalized proportional integral tracking controller for a single phase multilevel cascade inverter: A FPGA implementation. *IEEE Transactions on Industrial Informatics*, 2014, 10(1): 256 – 266.

[30] M. Ramírez-Neria, J. L. García-Antonio, H. Sira-Ramírez, et al. An active disturbance rejection control of leader-follower Thomson's jumping rings. *Journal of Control Theory and Applications*, 2013, 30(12): 1563 – 1571.

[31] J. Cortes-Romero, H. D. Rojas-Cubides, H. Coral-Enríquez, et al. Active disturbance rejection approach for robust fault-tolerant control via observer assisted sliding mode control. *Mathematical Problems in Engineering*, 2013: DOI 10.1155/2013/609523.

[32] J. Cortes-Romero, A. Luviano-Juarez, H. Sira-Ramírez. A Delta operator approach for the discrete-time active disturbance rejection control on induction motors. *Mathematical Problems in Engineering*, 2013: DOI 10.1155/2013/572026.

[33] H. Sira-Ramírez , F. Gonzalez-Montañez, J. Cortes-Romero, et al. A robust linear field oriented voltage control for the induction motor: Experimental results. *IEEE Transactions on Industrial Electronics*, 2013, 60(8): 3025 – 3033.

[34] H. Sira-Ramírez, M. Oliver-Salazar. On the robust control of a buck-converter DC motor combination. *IEEE Transactions on Power Electronics*, 2013, 28(8): 3912 – 3922.

[35] H. Sira-Ramírez, A. Luviano-Juarez, J. Cortes-Romero. Robust input-output sliding mode control of the buck converter. *Control Engineering Practice*, 2013, 21(5): 671 – 678.

[36] H. Sira-Ramírez, C. Lopez-Uribe, M. Velasco-Villa. Linear observer-based active disturbance rejection control of the omnidirectional mobile robot. *Asian Journal of Control*, 2013, 15(1): 51 – 63.

[37] H. Sira-Ramírez, A. Luviano-Juarez, J. Cortes-Romero. Flatness-based linear output feedback control for disturbance rejection and tracking tasks on a Chua's circuit. *International Journal of Control*, 2012, 85(5): 594 – 602.

[38] E. Zurita-Bustamante, J. Linares-Flores, E. Guzman-Ramírez, et al. A comparison between the GPI and the PID controllers for the stabilization of a DC-DC buck converter: A field programmable gate array implementation. *IEEE Transactions on Industrial Electronics*, 2011, 58(11): 5251 – 5262.

[39] H. Sira-Ramírez, C. Nuñez, N. Visairo. Robust Sigma-Delta generalized proportional integral observer based control of a "buck" converter with uncertain loads. *International Journal of Control*, 2010, 83(8): 1631 – 1640.

[40] R. Morales-Herrera, H. Sira-Ramírez. Trajectory tracking for the magnetic ball levitation system via exact feedforward linearization and GPI control. *International Journal of Control*, 2010, 83(6): 1155 – 1166.

[41] A. Luviano, J. Cortes, H. Sira-Ramírez. Synchronization of chaotic oscillators by means of GPI observers. *International Journal of Bifurcations and Chaos in Applied Science and Engineering*, 2010, 20(5): 1509 – 1517.

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