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Orthogonal projection based subspace identification against colored noise

Jie HOU, Tao LIU†, Fengwei CHEN

School of Control Science and Engineering, Dalian University of Technology, Dalian Liaoning 116024, China

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Abstract

In this paper, a bias-eliminated subspace identification method is proposed for industrial applications subject to colored noise. Based on double orthogonal projections, an identification algorithm is developed to eliminate the influence of colored noise for consistent estimation of the extended observability matrix of the plant state-space model. A shift-invariant approach is then given to retrieve the system matrices from the estimated extended observability matrix. The persistent excitation condition for consistent estimation of the extended observability matrix is analyzed. Moreover, a numerical algorithm is given to compute the estimation error of the estimated extended observability matrix. Two illustrative examples are given to demonstrate the effectiveness and merit of the proposed method.

Keywords: Subspace identification, colored noise, orthogonal projection, extended observability matrix, consistent estimation

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1 Introduction

Owing to the convenience of using a state space model to describe a multivariable system, increasing attentions [1, 2] have been devoted to state space model identification. The state space identification methods (SIMs) have been increasingly explored in the past two decades owing to the robust properties and relatively low computational complexities [1, 3, 4]. A few subspace identification methods have been widely recognized for engineering applications with white noise, e.g., the canonical variate analysis (CVA) approach [5], the multiple-input-multiple-output error state space model identification (MOESP) method [6], the numerical subspace state space identification (N4SID) algorithm [7], and the instrumental variable method (IVM) [8]. It was pointed out [9] that the aforementioned SIMs differ from each other by using different weighting matrices to construct the instrumental variables (IVs) for consistent estimation of the extended observability matrix of the plant

†Corresponding author.

E-mail: liurouter@ieee.org. Tel.: +86-411-84706465.

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state-space model. The asymptotic properties of these identification algorithms were analyzed in [10, 11].

Since there are industrial systems likely subject to colored noise, e.g., harmonic signals are usually involved with industrial electric circuits and mechanical systems, identification of these systems with colored noise have therefore received increasing attentions [12–14] in the recent years. Although the existing SIMs can guarantee consistent estimation in the presence of white noise, biased estimation may be obtained when these SIMs are applied to these systems due to the autocorrelation between sampled output data arising from colored noise. A feasible approach to eliminate the estimation bias is the use of the IV technique. By taking the past input sequence as the IV to eliminate the influence of noise to the system output, an extended SIM named PI-MOESP was proposed in [15] to guarantee consistent estimation. By projecting the observed data onto the past input sequence, an orthogonal subspace identification method named ORT-CN was developed in [16] to eliminate the influence of colored noise. However, this method requires the input excitation to be a zero-mean uncorrelated stationary sequence to ensure identification accuracy.

In this paper, a subspace identification method based on double orthogonal projections is proposed to realize consistent estimation of the extended observability matrix in the presence of colored noise, by projecting the observed data onto the orthogonal complement of the future input sequence to eliminate the influence from the future input, and then projecting the data onto the past input sequence to eliminate the noise effect. Compared to the existing SIMs, e.g., PI-MOESP and ORT-CN, an important merit of the proposed method is that there is no limit on the input correlation as long as the persistent excitation condition is satisfied. Consistency analysis of the proposed algorithm is given with a proof. Moreover, an explicit formula of the estimation error of the extended observability matrix is derived, which can be easily used to evaluate the estimation errors of the system matrices. Two illustrative examples are given to demonstrate the effectiveness of the proposed method. The paper is organized as follows. The identification problem is introduced in Section 2. Section 3 gives a brief review of the IV-4SID algorithm and then presents the proposed method. Furthermore, the asymptotic properties of the proposed method are analyzed in Section 4. Two illustrative examples are given in Section 5. Finally, some conclusions are drawn in Section 6.

2 Problem description

Consider the following linear discrete-time invariant state-space model:

$$S: \begin{cases} x(t+1) = Ax(t) + Bu(t) + w(t), \\ y(t) = Cx(t) + Du(t) + v(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$,and $y(t) \in \mathbb{R}^{n_y}$ denote the system state, input, and output vectors, respectively. The process noise $w(t) \in \mathbb{R}^{n_u}$ and measurement noise $v(t) \in \mathbb{R}^{n_u}$ are assumed to be colored noise with unknown variance. The system matrices are denoted by (A, B, C, D) with appropriate dimensions. The following assumptions are considered in the paper.

A1) The system is asymptotically stable, i.e., all the eigenvalues of A lie inside the unit circle.

A2) The pair (A, C) is observable and the pair (A, B) is reachable.

A3) The noises $w(t)$, $v(t)$ and system input are statistically independent of each other, i.e.,

$$\mathrm{E}\left\{\begin{bmatrix} w(i) \\ v(i) \\ u(i) \end{bmatrix}\begin{bmatrix} w(i) \\ v(i) \\ u(i) \end{bmatrix}^{\mathrm{T}}\right\} = \begin{bmatrix} R_w & 0 & 0 \\ 0 & R_v & 0 \\ 0 & 0 & R_u \end{bmatrix}$$

where $E(\cdot)$ is the expectation operator, and where

$$R_u = \bar{E}(u(t)u^{\mathrm{T}}(t)) = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} \mathrm{E}[u(t)u(t)^{\mathrm{T}}]$$

denotes the autocorrelation matrix.

The objective of this paper is to propose a new SIM method to estimate the system matrices based on the measured input and output data.

Denote by p and f the past and future horizons, respectively. For convenience, we assume $p = f$ $(p > n_x)$. Denote the stacked future and past output vectors by $y_p(t) = [y(t-p)^{\mathrm{T}} \ \cdots \ y(t-2)^{\mathrm{T}} \ \ y(t-1)^{\mathrm{T}}]^{\mathrm{T}}$ and $y_f(t) = [y(t)^{\mathrm{T}} \ \cdots \ y(t+f-2)^{\mathrm{T}} \ \ y(t+f-1)^{\mathrm{T}}]^{\mathrm{T}}$, respectively. Similar definitions are given for $w_p(t)$, $w_f(t)$, $v_p(t)$, $v_f(t)$, $u_p(t)$ and $u_f(t)$. By iterating the state-space model in (1), we have

$$x(t-p) = A^p x(t-2p) + L_1 u_p(t-p) + L_2 w_p(t-p), \tag{2}$$

$$x(t) = A^p x(t-p) + L_1 u_p(t) + L_2 w_p(t), \tag{3}$$

$$y_p(t) = \Gamma x(t-p) + H u_p(t) + G w_p(t) + v_p(t), \tag{4}$$

$$y_f(t) = \Gamma x(t) + H u_f(t) + G w_f(t) + v_f(t), \tag{5}$$

where $\Gamma = [C^{\mathrm{T}}\ \cdots\ (CA^{f-1})^{\mathrm{T}}]^{\mathrm{T}}$ is the extended observability matrix. $L_1 = [A^{f-1}B\ \cdots\ AB\ \ B]$, $L_2 = [A^{f-1}\ \cdots\ A\ \ I]$ are the extended controllability matrices. The lower triangular Toeplitz matrices are, respectively,

$$H_f = \begin{bmatrix} D & & \mathbf{0} \\ CB & \ddots & \\ \vdots & \ddots & D \\ CA^{f-2} \cdots & CB & D \end{bmatrix} G_f = \begin{bmatrix} 0 & & \mathbf{0} \\ C & \ddots & \\ \vdots & \ddots & 0 \\ CA^{f-2} \cdots & C & 0 \end{bmatrix}.$$

Suppose that there are $N + p + f - 1$ sampled data and introduce the output block Hankel matrices $Y_p = [y_p(t)\ \cdots\ y_p(N)]$ and $Y_f = [y_f(t)\ \cdots\ y_f(N)]$. Similar definitions are given for W_p, W_f, V_p, V_f, U_p and U_f. Denote $X_p = [x(t-p)\ \cdots\ x(t-p+N-1)]$ and $X_f = [x(t)\ \cdots\ x(t+N-1)]$. It follows from (4) and (5) that

$$Y_p = \Gamma X_p + HU_p + GW_f + V_p, \tag{6}$$

$$Y_f = \Gamma X_f + HU_f + GW_f + V_f. \tag{7}$$

3 Orthogonal projection based identification method

First, a brief review of the well recognized IV-4SID method is presented to explain why most of existing SIMs are biased in the presence of colored noise. Then the proposed identification method is given accordingly.

3.1 Brief review of the IV-4SID algorithm

The key idea of IV-4SID is to estimate the range space of Γ. For this purpose, it eliminates both effects of the future input and future noise from Y_f. The first step is to annihilate the input term in (7) by projecting the data onto the orthogonal complement of U_f, i.e.,

$$Y_f\Pi_{U_f}^{\perp} = \Gamma X_f\Pi_{U_f}^{\perp} + GW_f\Pi_{U_f}^{\perp} + V_f\Pi_{U_f}^{\perp}, \tag{8}$$

where $\Pi_{U_f}^{\perp} = I_N - U_f^{\mathrm{T}}(U_fU_f^{\mathrm{T}})^{-1}U_f$, and $U_f\Pi_{U_f}^{\perp} = 0$.

Then, the following IV is used to annihilate the noise term in (8),

$$P = [Y_p^{\mathrm{T}}\ \ U_p^{\mathrm{T}}]^{\mathrm{T}}. \tag{9}$$

It follows that

$$Y_f\Pi_{U_f}^{\perp}P^{\mathrm{T}} = (\Gamma X_f\Pi_{U_f}^{\perp} + GW_f\Pi_{U_f}^{\perp} + V_f\Pi_{U_f}^{\perp})P^{\mathrm{T}}. \tag{10}$$

Accordingly, the range space of Γ is estimated by performing a singluar calue decomposition (SVD) on $Y_f\Pi_{U_f}^{\perp}P^{\mathrm{T}}$. The conditions for using the IV-4SID to guarantee consistent estimation for systems subject to white noise are that $\lim\limits_{N\to\infty}(GW_f + V_f)\Pi_{U_f}^{\perp}P^{\mathrm{T}} = 0$ (which means that $\lim\limits_{N\to\infty}(GW_f + V_f)\Pi_{U_f}^{\perp}P^{\mathrm{T}} \to 0$ with probability 1 (w.p.1) as $N \to \infty$) and $\mathrm{rank}(\lim\limits_{N\to\infty} X_f\Pi_{U_f}^{\perp}P^{\mathrm{T}}) = n_x$, as studied in the literature [9]. However, these conditions are not satisfied for systems with colored noise, due to the correlation between the IV and colored noise. The reason is analyzed as below.

The noise part in (10) can be asymptotically expressed as

$$\begin{aligned} &\lim_{N\to\infty}(GW_f + V_f)\Pi_{U_f}^{\perp}P^{\mathrm{T}} \\ &= G(r_{wp} - r_{wu_f}R_{u_f}^{-1}r_{u_fp}) + r_{vp} - r_{vu_f}R_{v_f}^{-1}r_{vp}, \end{aligned} \tag{11}$$

where $r_{vp} = \bar{E}(v_f(t)p^{\mathrm{T}}(t))$ denotes the correlation matrix between $p(t)$ and $v_f(t)$, where $p(t) = [y_p^{\mathrm{T}}\ \ u_p^{\mathrm{T}}]^{\mathrm{T}}$. Similar notations hold for r_{wp}, r_{wu_f}, r_{u_fp} and r_{vu_f}.

If the input is a persistent excitation of order f, which means that R_{u_f} is positive definite [10], we have

$$\lim_{N\to\infty}(GW_f + V_f)\Pi_{U_f}^{\perp}P^{\mathrm{T}} = Gr_{wp} - r_{vp}. \tag{12}$$

Substituting (2) into (4) yields

$$\begin{aligned} y_p(t) = {} & \Gamma[A^p x(t-2p) + L_1u_p(t-p) + L_2w_p(t-p)] \\ & + Hu_p(t) + Gw_p(t) + v_p(t). \end{aligned} \tag{13}$$

Then, substituting (13) into (12) yields

$$r_{wp} = \bar{E}\left\{w_f(t)\begin{bmatrix} x(t-2p) \\ u_p(t-p) \\ u_p(t) \\ w_p(t-p) \\ w_p(t) \\ v_p(t) \end{bmatrix}^{\mathrm{T}}\right\}\begin{bmatrix} [A^p]^{\mathrm{T}}\Gamma^{\mathrm{T}} \\ L_1^{\mathrm{T}}\Gamma^{\mathrm{T}} \\ H^{\mathrm{T}} \\ L_2^{\mathrm{T}}\Gamma^{\mathrm{T}} \\ G^{\mathrm{T}} \\ I \end{bmatrix}. \tag{14}$$

Due to the autocorrelation property of colored noise, there are $\bar{E}(w_f(t)w_p^{\mathrm{T}}(t)) \neq 0$, $\bar{E}(w_f(t)w_p^{\mathrm{T}}(t-p)) \neq 0$, $\bar{E}(v_f(t)v_p^{\mathrm{T}}(t)) \neq 0$, and $\bar{E}(v_f(t)v_p^{\mathrm{T}}(t-p)) \neq 0$, which indicate that $\lim\limits_{N\to\infty}(GW_f + V_f)\Pi_{U_f}^{\perp}P^{\mathrm{T}} \neq 0$. This is the reason why IV-4SID gives biased estimation in the presence of colored noise.

3.2 Proposed method

The noise effect can be removed from (10) to obtain consistent estimation by projecting $Y_f\Pi^{\perp}_{U_f}$ onto the column space of U_p, i.e.,

$$Y_f\Pi^{\perp}_{U_f}\Pi_{U_p} = (\Gamma X_f + GW_f + V_f)\Pi^{\perp}_{U_f}\Pi_{U_p}, \quad (15)$$

where $\Pi_{U_p} = U_p^{\mathrm{T}}(U_pU_p^{\mathrm{T}})^{-1}U_p$ denotes an orthogonal projection matrix of U_p.

Correspondingly, the noise part in (15) can be asymptotically expressed as

$$\begin{aligned}&\lim_{N\to\infty}(GW_f + V_f)\Pi^{\perp}_{U_f}\Pi_{U_p}\\ &= G(r_{wu_p} + r_{vu_p})R^{-1}_{u_p}u_p\\ &\quad -G(r_{wu_f} + r_{vu_f})R^{-1}_{u_f}r_{u_fu_p}R^{-1}_{u_p}u_p. \quad (16)\end{aligned}$$

If the input is a persistent excitation of order $\max(p, f)$, then $\lim_{N\to\infty}(GW_f + V_f)\Pi^{\perp}_{U_f}\Pi_{U_p} = 0$. We have

$$Y_f\Pi^{\perp}_{U_f}\Pi_{U_p} = \Gamma X_f\Pi^{\perp}_{U_f}\Pi_{U_p}. \quad (17)$$

Performing an SVD for the left-hand side of (17), we obtain

$$Y_f\Pi^{\perp}_{U_f}\Pi_{U_p} = [\hat{U}_1\ \ \hat{U}_1^{\perp}]\begin{bmatrix}\hat{\Sigma}_1 & 0\\ 0 & \hat{\Sigma}_2\end{bmatrix}\begin{bmatrix}\hat{V}_1^{\mathrm{T}}\\ (\hat{V}_1^{\perp})^{\mathrm{T}}\end{bmatrix}, \quad (18)$$

where $\hat{U}_1$ is the first n_x eigenvalues of (18).

The range space of extended observability matrix Γ is therefore obtained as

$$\hat{\Gamma} = \hat{U}_1. \quad (19)$$

With the estimated $\hat{\Gamma}$, the estimations of $\hat{A}$ and $\hat{C}$ can be extracted as

$$\hat{A} = (J_1\hat{\Gamma})^{\dagger}J_2\hat{\Gamma}, \quad (20)$$
$$\hat{C} = J_3\hat{\Gamma}, \quad (21)$$

where $J_1 = [I_{(f-1)n_y}\ 0_{(f-1)n_y\times n_y}]$, $J_2 = [0_{(f-1)\times n_y}\ I_{(f-1)n_y}]$ and $J_1 = [I_{n_y}\ 0_{n_y\times(f-1)n_y}]$.

The last step is to estimate B and D. By post-multiplying $(U_f)^{\dagger}$ and pre-multiplying $\hat{\Gamma}^{\perp}$ to both sides of (7) and using $\hat{\Gamma}^{\perp}\Gamma = 0$, we have

$$\hat{\Gamma}^{\perp}Y_fU_f^{\dagger} = \hat{\Gamma}^{\perp}H + \hat{\Gamma}^{\perp}(GW_f + V_f)U_f^{\dagger}. \quad (22)$$

If the input is a persistent excitation of order f, there is $\lim_{N\to\infty}\hat{\Gamma}^{\perp}(GW_f + V_f)U_f^{\dagger} = 0$.

For abbreviation, denote

$$M = \hat{\Gamma}^{\perp}Y_fU_f^{\dagger} = [M_1\ \cdots\ M_f], \quad (23)$$
$$L = \hat{\Gamma}^{\perp} = [L_1\ \cdots\ L_f], \quad (24)$$

where $M_k \in \mathbb{R}^{(fn_y-n_x)\times n_u}$, $L_k \in \mathbb{R}^{(fn_y-n_x)\times n_y}$ and $\hat{\Gamma}^{\perp} = \hat{U}_1^{\perp}$.

The estimation of B and D can be extracted by

$$\begin{bmatrix}\hat{D}\\ \hat{B}\end{bmatrix} = \begin{bmatrix}M_1\\ \vdots\\ M_f\end{bmatrix}\left[\begin{bmatrix}L_1 & \cdots & L_{f-1} & L_f\\ L_2 & \cdots & L_f & 0\\ \vdots & & \vdots & \vdots\\ L_f & \cdots & 0 & 0\end{bmatrix}\begin{bmatrix}I & 0\\ 0 & J_1\hat{\Gamma}\end{bmatrix}\right]^{\dagger}. \quad (25)$$

Hence, the proposed double orthogonal projections based subspace identification method, named as 2ORT-SIM, can be summarized as follows:

1) Eliminate the influence of the future input and colored noise to the future output by using (15).

2) Calculate the SVD of the projection matrix in (18).

3) Extract the estimates of system matrices $\hat{A}$ and $\hat{C}$ from (20) and (21).

4) Extract the estimates of system matrices $\hat{B}$ and $\hat{D}$ from (25).

Remark 1 The PI-MOESP method [15] extracted the range space of Γ by performing an SVD for $Y_f\Pi^{\perp}_{U_f}P^{\mathrm{T}}$. By reformulating $Y_f\Pi^{\perp}_{U_f}\Pi_{U_p}(Y_f\Pi^{\perp}_{U_f}\Pi_{U_p})^{\mathrm{T}} = Y_f\Pi^{\perp}_{U_f}P^{\mathrm{T}}W_1(Y_f\Pi^{\perp}_{U_f}P^{\mathrm{T}}W_1)^{\mathrm{T}}$ where $W_1 = R^{-1/2}_{u_p}$, the proposed 2ORT-SIM is equivalent to PI-MOESP with the column weighting matrix W_1. As discussed in the literature [17–19], using the column weighting matrix improves the estimation accuracy.

Remark 2 The ORT-SIM method [16] extracted the range space of Γ by performing an SVD for $Y_f\Pi_{U_p}$. When the input sequence is a zero-mean uncorrelated stationary sequence, the influence of the future input and colored noise can be eliminated, i.e., $\lim_{N\to\infty}(HU_f + GW_f + V_f)\Pi_{U_p} = 0$, such that the estimation of Γ is consistent. However, if the input sequence is an autocorrelated sequence, the influence of the future input cannot be eliminated, i.e., $\lim_{N\to\infty} HU_f\Pi_{U_p} \neq 0$, leading to biased estimation of Γ.

4 Asymptotic properties

The asymptotic properties including consistency and asymptotic error for estimating the extended observability matrix are studied below.

4.1 Consistent estimation

The following theorem is given for consistent estimation of the extended observability matrix by using the proposed method.

Theorem 1 Under the assumptions A1)–A3), the proposed 2ORT-SIM algorithm gives consistent estimation on $\hat{\Gamma}$ if the input is a persistent excitation of order $p+f$, i.e., there exists a non-singular matrix T with dimension $n_x \times n_x$ such that $\lim\limits_{N\to\infty} \hat{\Gamma} = \Gamma T$.

Proof It can be seen from (17) and (18) that a consistent estimate of $\hat{\Gamma}$ can be obtained if

$$\operatorname{rank}(\lim_{N\to\infty} X_f \Pi^{\perp}_{U_f} \Pi_{U_p}) = n_x. \tag{26}$$

It can be derived from (3) that

$$\begin{aligned}&\lim_{N\to\infty} X_f \Pi^{\perp}_{U_f} \Pi_{U_p}\\ &= \lim_{N\to\infty} (A^p X_p + L_1 U_p + L_2 W_p) \Pi^{\perp}_{U_f} \Pi_{U_p}.\end{aligned} \tag{27}$$

If p is sufficiently large and the input is a persistent excitation of order $\max(p, f)$, it follows

$$\lim_{N\to\infty} (A^p X_p + L_2 W_p) \Pi^{\perp}_{U_f} \Pi_{U_p} = 0. \tag{28}$$

Therefore, we have

$$\lim_{N\to\infty} X_f \Pi^{\perp}_{U_f} \Pi_{U_p} = \lim_{N\to\infty} L_1 U_p \Pi^{\perp}_{U_f} \Pi_{U_p}. \tag{29}$$

According to the assumptions of A1) and A2), we are sure that L_1 is a full row rank matrix. Hence, the rank condition in (26) is equivalent to

$$\lim_{N\to\infty} U_p \Pi^{\perp}_{U_f} \Pi_{U_p} > 0. \tag{30}$$

Note that

$$\begin{aligned}&U_p \Pi^{\perp}_{U_f} \Pi_{U_p}\\ &= [U_p U_p^{\mathrm{T}} - U_p U_p^{\mathrm{T}} (U_p U_p^{\mathrm{T}})^{-1} U_p U_p^{\mathrm{T}}] (U_p U_p^{\mathrm{T}})^{-1} U_p.\end{aligned} \tag{31}$$

If the input is a persistent excitation of order p, there stands $\lim\limits_{N\to\infty} (U_p U_p^{\mathrm{T}})^{-1} U_p > 0$, so the consistent estimation condition in (30) is equivalent to

$$\lim_{N\to\infty} [U_p U_p^{\mathrm{T}} - U_p U_p^{\mathrm{T}} (U_p U_p^{\mathrm{T}})^{-1} U_p U_p^{\mathrm{T}}] > 0, \tag{32}$$

which is equivalent to

$$\lim_{N\to\infty} \begin{bmatrix} U_p U_p^{\mathrm{T}} & U_p U_f^{\mathrm{T}} \\ U_f U_p^{\mathrm{T}} & U_f U_f^{\mathrm{T}} \end{bmatrix} > 0. \tag{33}$$

In fact, it holds that

$$\lim_{N\to\infty} \begin{bmatrix} U_p U_p^{\mathrm{T}} & U_p U_f^{\mathrm{T}} \\ U_f U_p^{\mathrm{T}} & U_f U_f^{\mathrm{T}} \end{bmatrix} = \bar{E}\left\{ \begin{bmatrix} u_p(t) \\ u_f(t) \end{bmatrix} \begin{bmatrix} u_p(t) \\ u_f(t) \end{bmatrix}^{\mathrm{T}} \right\}. \tag{34}$$

If the input is a persistent excitation of order $p+f$, the condition in (30) can surely be satisfied. This completes the proof. □

4.2 Estimation error

The true estimation of Γ can be computed from an SVD as following,

$$\Gamma X_f \Pi^{\perp}_{U_f} \Pi_{U_p} = [U_1 \ U_1^{\perp}] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^{\mathrm{T}} \\ (V_1^{\perp})^{\mathrm{T}} \end{bmatrix}. \tag{35}$$

Obviously, it can be simply taken as

$$\Gamma = U_1. \tag{36}$$

Since the estimated extended observability matrix $\hat{\Gamma}$ is computed from $Y_f \Pi^{\perp}_{U_f} \Pi_{U_p}$ as shown in (17), it follows from (18) that

$$Y_f \Pi^{\perp}_{U_f} \Pi_{U_p} = \hat{U}_1 \hat{\Sigma}_1 \hat{V}_1^{\mathrm{T}} + \hat{U}_1^{\perp} \hat{\Sigma}_2 (\hat{V}_1^{\perp})^{\mathrm{T}}. \tag{37}$$

Postmultiplying $\hat{V}_1$ to both sides of (37) and using $(\hat{V}_1^{\perp})^{\mathrm{T}} \hat{V}_1 = 0$ and $\hat{V}_1^{\mathrm{T}} \hat{V}_1 = I$, we obtain

$$Y_f \Pi^{\perp}_{U_f} \Pi_{U_p} \hat{V}_1 = \hat{U}_1 \hat{\Sigma}_1. \tag{38}$$

Premultiplying $(\hat{U}_1^{\perp})^{\mathrm{T}}$ to both sides of (38) and using (15) and $(\hat{U}_1^{\perp})^{\mathrm{T}} \Gamma X_f \Pi^{\perp}_{U_f} \Pi_{U_p} = 0$, we have

$$\begin{aligned}&(\hat{U}_1^{\perp})^{\mathrm{T}} Y_f \Pi^{\perp}_{U_f} \Pi_{U_p} \hat{V}_1\\ &= (\hat{U}_1^{\perp})^{\mathrm{T}} (G W_f + V_f) \Pi^{\perp}_{U_f} \Pi_{U_p} \hat{V}_1,\end{aligned} \tag{39}$$

$$\begin{aligned}&(\hat{U}_1^{\perp})^{\mathrm{T}} \hat{U}_1 \hat{\Sigma}_1\\ &= (\hat{U}_1^{\perp})^{\mathrm{T}} (U_1 + \delta U_1) \hat{\Sigma}_1 = (\hat{U}_1^{\perp})^{\mathrm{T}} \delta U_1 \hat{\Sigma}_1,\end{aligned} \tag{40}$$

where $\delta U_1 = \hat{U}_1 - U_1$.

It follows from (38)–(40) that

$$(\hat{U}_1^{\perp})^{\mathrm{T}} (G W_f + V_f) \Pi^{\perp}_{U_f} \Pi_{U_p} \hat{V}_1 = (\hat{U}_1^{\perp})^{\mathrm{T}} \delta U_1 \hat{\Sigma}_1. \tag{41}$$

Substituting $\hat{U}_1^{\mathrm{T}} = U_1^{\mathrm{T}} + \delta U_1^{\mathrm{T}}$ and $\hat{\Sigma}_1 = \Sigma_1 + \delta \Sigma_1$ into (41) yields

$$\begin{aligned}&(\hat{U}_1^{\perp})^{\mathrm{T}} (G W_f + V_f) \Pi^{\perp}_{U_f} \Pi_{U_p} [V_1 + \delta V_1]\\ &= (\hat{U}_1^{\perp})^{\mathrm{T}} \delta U_1 (\Sigma_1 + \delta \Sigma_1).\end{aligned} \tag{42}$$

Therefore,

$$\begin{aligned}&(\hat{U}_1^{\perp})^{\mathrm{T}}(GW_f+V_f)\Pi_{U_f}^{\perp}\Pi_{U_p}V_1\\&=(\hat{U}_1^{\perp})^{\mathrm{T}}\delta U_1\Sigma_1+(\hat{U}_1^{\perp})^{\mathrm{T}}\delta U_1\delta\Sigma_1\\&\quad-(\hat{U}_1^{\perp})^{\mathrm{T}}(GW_f+V_f)\Pi_{U_f}^{\perp}\Pi_{U_p}\delta V_1.\end{aligned}\tag{43}$$

By omitting the second term of $((GW_f+V_f)\Pi_{U_f}^{\perp}\Pi_{U_p})^2$ at the right-hand side of (43), we have

$$\begin{aligned}\delta U_1&=(GW_f+V_f)\Pi_{U_f}^{\perp}\Pi_{U_p}V_1\Sigma_1^{-1}\\&\quad+\mathrm{o}((GW_f+V_f)\Pi_{U_f}^{\perp}\Pi_{U_p})^2),\end{aligned}\tag{44}$$

where $\mathrm{o}((GW_f+V_f)\Pi_{U_f}^{\perp}\Pi_{U_p})^2)$ is infinitesimal when $N\to\infty$.

Since $\hat{\Gamma}=\hat{U}_1$ and $\Gamma=U_1$, the difference between $\hat{\Gamma}$ and its true value can be computed by

$$\delta\Gamma=\hat{\Gamma}-\Gamma=\delta U_1.\tag{45}$$

After the asymptotic error of the estimated extended observability matrix is computed, the asymptotic errors of the system matrices can be computed using the numerical methods given in the references [20,21]. Then the asymptotic error of the plant transfer function matrix can be easily computed [22].

5 Illustration

Two examples are used to demonstrate the effectiveness and merit of the proposed method. One is a benchmark example studied in the reference [23], and the other is an injection molding process in the reference [24].

Example 1 Consider a benchmark example studied in [23],

$$S:\begin{cases}x(t+1)=\begin{bmatrix}0.393&2.022\\-0.208&-0.685\end{bmatrix}x(t)+\begin{bmatrix}0.95\\1\end{bmatrix}u(t)+w(t),\\y(t)=\begin{bmatrix}0.326&-0.743\end{bmatrix}x(t)+0.95u(t)+v(t),\end{cases}$$

where $w(t)$ and $v(t)$ are colored noises, which are independently generated by $(1-0.75q^{-1}-2.5q^{-2})/(1-1.5q^{-1}+0.8q^{-2})e_1(t)$, where $e_1(t)$ is a white noise with variance of 0.05. For illustration, the input excitation is taken as $u(t)=((1-0.8q^{-1}+0.6q^{-2})e_2(t))$ where $e_2(t)$ is a white noise with variance of 5.

One thousand Monte Carlo (MC) tests are carried out with a data length of $N=3000$ and the past and further horizons $p=f=10$. For comparison, the proposed 2ORT-SIM together with PI-MOESP [15], ORT-SIM [16] and N4SID [7] are used to estimate the system matrices. Denote the true transfer function matrix (TFM) by $G(\mathrm{e}^{\mathrm{j}w})=C(zI-A)^{-1}B+D$ and the identified model transfer function matrix in the ith test of MC by $\hat{G}(\mathrm{e}^{\mathrm{j}w})=\hat{C}(zI-\hat{A})^{-1}\hat{B}+\hat{D}$. The averaged TFM magnitude plots are shown in Fig. 1. Fig. 2 shows the standard deviation (Std) of model errors (i.e., $G(\mathrm{e}^{\mathrm{j}w})-\hat{G}(\mathrm{e}^{\mathrm{j}w})$).

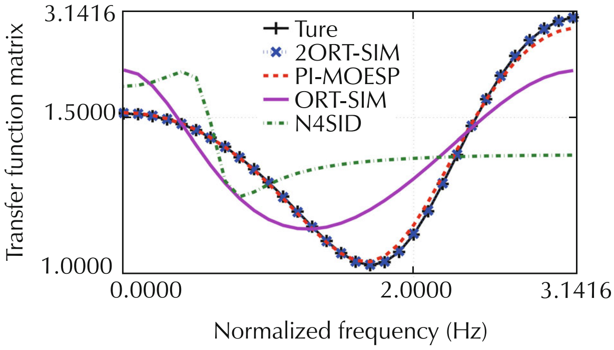

Fig. 1 Magnitude plot of the identified transfer function matrix for Example 1.

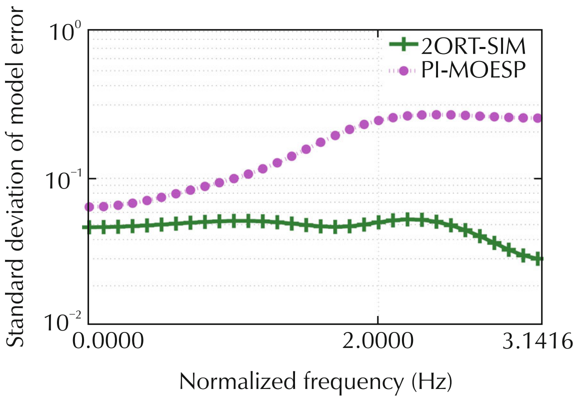

Fig. 2 Plot of the standard deviation of model error for Example 1.

It is seen that both the proposed 2ORT-SIM and PI-MOESP give consistent estimations, while the proposed method gives an improved accuracy compared to PI-MOESP. Note that the N4SID and ORT-SIM give biased estimation. Note that the ORT-SIM can only be used to obtain consistent estimation when the input excitation is a zero-mean uncorrelated stationary sequence.

Furthermore, to assess the accuracy of proposed method for estimating the asymptotic error of the extended observability matrix, the estimated errors of $(\delta A,\delta C)$ are computed directly from $\delta\Gamma$ through a linear operation as [25],

$$\delta A = -[(J_1\Gamma)^{\mathrm{T}}J_1\Gamma]^{-1}[(J_1\Gamma)^{\mathrm{T}}J_1\delta\Gamma + (J_1\delta\Gamma)^{\mathrm{T}}J_1\Gamma]A + [(J_1\delta\Gamma)^{\mathrm{T}}J_2\Gamma + (J_1\Gamma)^{\mathrm{T}}J_1\delta\Gamma],$$
$$\delta C = J_3\delta\Gamma.$$

Tables 1 shows the mean values along with the Std of $(\delta A, \delta C)$ by using the proposed asymptotic error estimation method. The true values obtained by using the true plant model to compute the estimation errors are also listed in Table 1, which demonstrates the effectiveness of the proposed estimation of asymptotic error.

Example 2 Consider an injection molding process studied in the reference [24],

$$S: \begin{cases} x(t+1) = \begin{bmatrix} 1.582 & -0.592 \\ 1 & -0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) + w(t), \\ y(t) = [1.69 \;\; 1.419]x(t) + 0.95u(t) + v(t). \end{cases}$$

Using the same input excitation as Example 1, one thousand MC tests are carried out for model identification. The above four methods are used again for comparison. The averaged TFM magnitude plots are shown in Fig. 3. It is seen that both the proposed 2ORT-SIM and PI-MOESP give consistent estimations, while the proposed method gives an improved accuracy compared to PI-MOESP. In contrast, the N4SID and ORT-SIM give biased estimation. Fig. 4 shows the Stds of model errors by using the proposed 2ORT-SIM and PI-MOESP. The mean values along with the Stds of $(\delta A, \delta C)$ are listed in Table 2, well demonstrating the effectiveness of the proposed estimation of asymptotic error.

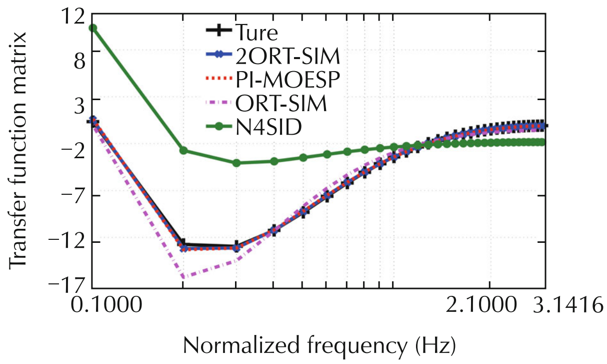

Fig. 3 Magnitude plot of the identified transfer function matrix for Example 2.

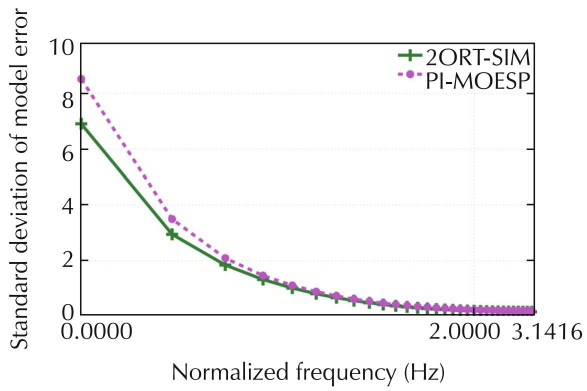

Fig. 4 Plot of the standard deviation of model error for Example 2.

Table 1 Estimation error of δA and δC by using the proposed 2ORT-SIM for Example 1.

Identification errors	δA	δC
Proposed method	$\begin{bmatrix} 0.0001 \pm 0.0086 & 0.0004 \pm 0.0265 \\ 0.0002 \pm 0.0121 & -0.0001 \pm 0.0457 \end{bmatrix}$	$[0.0000 \pm 0.0066 \;\; 0.0010 \pm 0.0311]$
True	$\begin{bmatrix} 0.0010 \pm 0.0041 & -0.0009 \pm 0.0210 \\ -0.0002 \pm 0.0100 & 0.0145 \pm 0.0382 \end{bmatrix}$	$[0.0000 \pm 0.0047 \;\; 0.0030 \pm 0.0068]$

Table 2 Estimation error of δA and δC by using the proposed 2ORT-SIM for Example 2.

Identification errors	δA	δC
Proposed method	$\begin{bmatrix} 0.0028 \pm 0.0015 & 0.0184 \pm 0.0107 \\ 0.0068 \pm 0.0016 & 0.0153 \pm 0.0171 \end{bmatrix}$	$[0.0004 \pm 0.0006 \;\; 0.0004 \pm 0.0094]$
True	$\begin{bmatrix} -0.0007 \pm 0.0010 & 0.0176 \pm 0.0068 \\ 0.0009 \pm 0.0012 & 0.0293 \pm 0.0134 \end{bmatrix}$	$[-0.0002 \pm 0.0001 \;\; 0.0060 \pm 0.0022]$

6 Conclusions

A bias-eliminated subspace identification method has been proposed for industrial applications subject to colored noise, to overcome the deficiency of existing SIMs that could not provide consistent estimation. An identification algorithm based on double orthogonal projections is developed by using the past input sequence rather than the output sequence to eliminate the influence of colored noise, such that consistent estimation of the extended observability matrix can be obtained. The persistent excitation condition for consistent estimation of the extended observability matrix is analyzed with a strict proof. Moreover, a numerical algorithm is given to compute the asymptotic error of the estimated extended observability matrix, which can be easily applied to compute the estimation errors of the system matrices. The applications to two illustrative examples have well demonstrated the effectiveness and good accuracy of the proposed identification method.

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Jie HOU received the B.Eng. degree in Automation from Beifang University of Nationalities, Yinchuan, China, in 2010, the M.Sc. degree in Control Science and Engineering from Chongqing University, Chongqing, China, in 2013. He is currently a Ph.D. candidate in the School of Control Science and Engineering, Dalian University of Technology. His research interest covers system identification. E-mail: jiehou.phd@hotmail.com.

Tao LIU received his Ph.D. degree in Control Science and Engineering from Shanghai Jiaotong University, Shanghai, China, in 2006. He is a professor in the Institute of Advanced Control Technology at Dalian University of Technology. His research interests include chemical and industrial process identification & modeling, robust process control, iterative learning control, batch process optimization. He is a member of the Technical Committee on Chemical Process Control of IFAC, Technical Committee on System Identification and Adaptive Control of the IEEE Control System Society, and Chinese Process Control Committee. E-mail: liurouter@ieee.org.

Fengwei CHEN received the B.Eng. and M.Eng. degrees from Wuhan University, Wuhan, China, in 2009 and 2011, respectively, and the Ph.D. degree from Université de Lorraine, Nancy, France, in 2014. He is currently working with Dalian University of Technology, Dalian, China. His research interests include system identification and signal processing. E-mail: fengwei.chen@dlut.edu.cn.