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Leader-following consensus for uncertain second-order nonlinear multi-agent systems

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Abstract

This paper studies the leader-following consensus problem for a class of second-order nonlinear multi-agent systems subject to linearly parameterized uncertainty and disturbance. The problem is solved by integrating the adaptive control technique and the adaptive distributed observer method. The design procedure is illustrated by an example with a group of Van der Pol oscillators as the followers and a harmonic system as the leader.

Keywords: Adaptive control, distributed control, multi-agent systems, nonlinear systems

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1 Introduction

In the past few years, the cooperative control problems for multi-agent systems have attracted extensive attention due to their applications in sensor networks, robotic teams, satellite clusters, unmanned air vehicle formations and so on. The consensus problem is one of the basic cooperative control problems, whose objective is to design a distributed control law for each agent such that the states (or outputs) of all agents synchronize to a common trajectory [1–4]. Depending on whether or not a multi-agent system has a leader, the consensus problem can be divided into two classes: leaderless and leader-following. The leaderless consensus problem does not specify the common trajectory [2, 3], while the leader-following consensus problem requires the states (or outputs) of all agents to track a desired trajectory generated by a so-called leader system [4–7].

An important class of multi-agent systems is the second-order nonlinear multi-agent systems. Recently, considerable efforts have been made to handle the leader-following consensus problem for various second-order nonlinear multi-agent systems [8–13]. For example, references [8–10] studied the leader-following consensus problem for some second-order nonlinear multi-agent systems under the assumption that the nonlinear functions satisfy the global Lipschitz condition or global Lipschitz-like condition. The system studied in reference [11] contains disturbance but no uncertainty. The systems considered in [12, 13] allow both disturbance

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and uncertainty, but the boundary of the uncertainty is known.

In this paper, we will further consider the leader-following consensus problem for a class of second-order nonlinear multi-agent systems subject to linearly parameterized uncertainty and disturbance. Compared with [8–10], we do not impose the global Lipschitz condition or the global Lipschitz-like condition on the nonlinear functions. Compared with [8–11], the nonlinear multi-agent system here contains both linearly parameterized uncertainty and disturbance. Finally, compared with [12, 13], our uncertainty can be arbitrarily large, and we do not assume the uncontrolled system has an equilibrium point at the origin.

Our distributed control law is based on a combination of the adaptive control technique and the adaptive distributed observer method developed in [14]. It turns out that such a control law is quite effective in dealing with the problem studied in this paper.

The rest of this paper is organized as follows. In Section 2, we will give our problem formulation and some preliminaries. In Sections 3, we will give our main result. In Section 4 we will provide an example to illustrate our design. Finally, in Section 5, we will finish the paper with some conclusions.

Notation For any column vectors a_i, $i = 1, \dots, s$, denote $\mathrm{col}(a_1, \dots, a_s) = [a_1^{\mathrm{T}} \ \cdots \ a_s^{\mathrm{T}}]^{\mathrm{T}}$. $\otimes$ denotes the Kronecker product of matrices. $\|x\|$ denotes the Euclidean norm of vector x. $\|A\|$ denotes the induced norm of matrix A by the Euclidean norm.

2 Problem formulation

Consider a class of second-order nonlinear multi-agent systems as follows:

$$\begin{cases} \dot{q}_i = p_i, \\ \dot{p}_i = f_i^{\mathrm{T}}(q_i, p_i)\theta_i + d_i(w) + u_i, \quad i = 1, \dots, N, \end{cases} \tag{1}$$

where $q_i, p_i \in \mathbb{R}^n$ are the states, $u_i \in \mathbb{R}^n$ is the input, $f_i(q_i, p_i) \in \mathbb{R}^{m \times n}$ is a matrix with every element being known continuous function, $\theta_i \in \mathbb{R}^m$ is an unknown constant parameter vector, $d_i(w) \in \mathbb{R}^n$ denotes the disturbance with $d_i(\cdot)$ being some C^1 function, and w is generated by the following linear exosystem

$$\dot{w} = S_{\mathrm{b}} w \tag{2}$$

with $w \in \mathbb{R}^{n_w}$ and $S_{\mathrm{b}} \in \mathbb{R}^{n_w \times n_w}$. It is assumed that the reference signal is generated by the following linear exosystem

$$\dot{x}_0 = S_{\mathrm{a}} x_0, \tag{3}$$

where $S_{\mathrm{a}} = \begin{bmatrix} 0_{n \times n} & I_n \\ S_{\mathrm{a}1} & S_{\mathrm{a}2} \end{bmatrix} \in \mathbb{R}^{2n \times 2n}$ and $x_0 = \mathrm{col}(q_0, p_0)$ with $q_0, p_0 \in \mathbb{R}^n$. Let $v = \mathrm{col}(x_0, w) \in \mathbb{R}^{n_v}$ and $S = \mathrm{diag}(S_{\mathrm{a}}, S_{\mathrm{b}}) \in \mathbb{R}^{n_v \times n_v}$ with $n_v = 2n + n_w$. Then we can put (2) and (3) together and get the following exosystem

$$\dot{v} = Sv. \tag{4}$$

System (1) and the exosystem (4) together can be viewed as a multi-agent system of $(N+1)$ agents with (4) as the leader and the N subsystems of (1) as N followers.

Next, we introduce some graph notation which can also be found in [15]. A digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a finite set of nodes $\mathcal{V} = \{1, \dots, N\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. An edge of $\mathcal{E}$ from node i to node j is denoted by (i, j), where node i and j are called the parent node and the child node of each other. Define $\mathcal{N}_i = \{j \,|\, (j, i) \in \mathcal{E}\}$, which is called the neighbor set of node i. The edge (i, j) is called undirected if $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$. The digraph $\mathcal{G}$ is called undirected if every edge in $\mathcal{E}$ is undirected. If the digraph $\mathcal{G}$ contains a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \dots, (i_k, i_{k+1})$, then the set $\{(i_1, i_2), (i_2, i_3), \dots, (i_k, i_{k+1})\}$ is called a path of $\mathcal{G}$ from node i_1 to node i_{k+1} and node i_{k+1} is said to be reachable from node i_1. A digraph is called connected if there exists a node i such that any other nodes are reachable from node i. The weighted adjacency matrix of the digraph $\mathcal{G}$ is a nonnegative matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ where $a_{ii} = 0$ and $a_{ij} > 0 \Leftrightarrow (j, i) \in \mathcal{E}$, $i, j = 1, \dots, N$. On the other hand, given a matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ satisfying $a_{ii} = 0$ and $a_{ij} \geqslant 0$ for $i, j = 0, 1, \dots, N$, we can always define a digraph $\mathcal{G}$ such that $\mathcal{A}$ is the weighted adjacency matrix of the digraph $\mathcal{G}$. We call $\mathcal{G}$ the digraph of $\mathcal{A}$.

With respect to the plant (1) and the exosystem (4), we can define a digraph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ with $\bar{\mathcal{V}} = \{0, 1, \dots, N\}$ and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$, where the node 0 is associated with the leader system (4) and the node i, $i = 1, \dots, N$, is associated with the ith subsystem of system (1). For $i = 1, \dots, N$, $j = 0, 1, \dots, N$ and $i \neq j$, $(j, i) \in \bar{\mathcal{E}}$ if and only if u_i can use the information of the jth subsystem for control. Let $\bar{\mathcal{A}} = [\bar{a}_{ij}] \in \mathbb{R}^{(N+1) \times (N+1)}$ be the weighted adjacency matrix of $\bar{\mathcal{G}}$. Let $\bar{\mathcal{N}}_i = \{j, (j, i) \in \bar{\mathcal{E}}\}$ denote the neighbor set of agent i.

We describe our control law as follows:

$$\begin{cases} u_i = h_i(q_i, p_i, \zeta_i, q_j, p_j, \zeta_j, j \in \bar{\mathcal{N}}_i), \\ \dot{\zeta}_i = l_i(q_i, p_i, \zeta_i, q_j, p_j, \zeta_j, j \in \bar{\mathcal{N}}_i), \quad i = 1, \dots, N, \end{cases} \tag{5}$$

where h_i and l_i are some nonlinear functions. A control law of the form (5) is called a distributed control law, since u_i only depends on the information of its neighbors and itself. Our problem is described as follows.

Problem 1 Given the multi-agent system (1), the exosystem (4) and a digraph $\bar{\mathcal{G}}$, design a control law of the form (5), such that, for any initial states $q_i(0)$, $p_i(0)$, $\zeta_i(0)$ and $v(0)$, $q_i(t)$ and $p_i(t)$ exist for all $t \geqslant 0$, and satisfy $\lim\limits_{t\to+\infty}(q_i(t) - q_0(t)) = 0$ and $\lim\limits_{t\to+\infty}(p_i(t) - p_0(t)) = 0$.

Remark 1 Note that, like in [12, 13], here we assume that the reference signal and the disturbance are generated by a linear exosystem (4) called the leader. Indeed, this formulation is more general than the case that the disturbance $d_i(w)$ is generated by an individual exosystem for each follower.

To solve our problem, we make two assumptions as follows.

Assumption 1 The exosystem (4) is neutrally stable, i.e., all the eigenvalues of S are semi-simple with zero real parts.

Assumption 2 Every node $i = 1, \dots, N$ is reachable from the node 0 in the diagraph $\bar{\mathcal{G}}$.

Remark 2 Assumption 1 is standard and has been used in [12]. Under Assumption 1, the exosystem (2) can generate arbitrarily large constant signals and some sinusoidal signals with arbitrary initial phases and amplitudes, and the exosystem (3) can generate sinusoidal signals with arbitrary initial phases and amplitudes. What's more, under Assumption 1, given any compact set $\mathcal{V}_0$, there exists a compact set $\mathcal{V}$ such that, for any $v(0) \in \mathcal{V}_0$, the trajectory $v(t)$ of the exosystem (4) remains in $\mathcal{V}$ for all $t \geqslant 0$.

Remark 3 Assumption 2 is also a standard assumption and has been used in many literatures on cooperative control problems of multi-agent systems [12–14, 16]. Note that Assumption 2 allows the network to be directed and thus is less restrictive than that in [11, 17].

3 Main result

In this section, we will consider the leader-following consensus problem for system (1) and exosystem (4).

We first recall the concept of the distributed observer for the leader system developed in [16] as follows:

$$\dot{\hat{v}}_i = S\hat{v}_i + \mu_0 \sum_{j=0}^{N} \bar{a}_{ij}(\hat{v}_j - \hat{v}_i), \quad i = 1, \dots, N, \tag{6}$$

where $\hat{v}_0 = v$, $\hat{v}_i \in \mathbb{R}^{n_v}$ for $i = 1, \dots, N$, μ_0 is any positive constant. By Theorem 1 and Remark 4 of [16], under Assumptions 1 and 2, we have $\lim\limits_{t\to+\infty}(\hat{v}_i - v) = 0$, $i = 1, \dots, N$. That is why we call (6) the distributed observer for (4).

However, a drawback of (6) is that the matrix S is used by every follower which may not be realistic in some applications. To overcome this drawback, an adaptive distributed observer was further proposed in [14] as follows:

$$\begin{cases} \dot{S}_i = \mu_1 \sum\limits_{j=0}^{N} \bar{a}_{ij}(S_j - S_i), \\ \dot{\hat{v}}_i = S_i\hat{v}_i + \mu_2 \sum\limits_{j=0}^{N} \bar{a}_{ij}(\hat{v}_j - \hat{v}_i), \quad i = 1, \dots, N, \end{cases} \tag{7}$$

where $S_0 = S$, $\hat{v}_0 = v$, $S_i \in \mathbb{R}^{n_v \times n_v}$, $\hat{v}_i \in \mathbb{R}^{n_v}$, $i = 1, \dots, N$, μ_1 and μ_2 are any positive constants. The adaptive distributed observer (7) is more realistic than the distributed observer (6), since here $\dot{S}_i$ depends on S at the time t iff the leader is the neighbor of the ith follower at time t, while the matrix S is used by every follower in (6).

Let $\tilde{v}_i = \hat{v}_i - v$ and $\tilde{S}_i = S_i - S$ for $i = 0, 1, \dots, N$. Then, for $i = 1, \dots, N$,

$$\begin{cases} \dot{\tilde{S}}_i = \mu_1 \sum\limits_{j=0}^{N} \bar{a}_{ij}(\tilde{S}_j - \tilde{S}_i), \\ \dot{\tilde{v}}_i = \tilde{S}_i\hat{v}_i + S\tilde{v}_i + \mu_2 \sum\limits_{j=0}^{N} \bar{a}_{ij}(\tilde{v}_j - \tilde{v}_i). \end{cases} \tag{8}$$

Let $\tilde{v} = \mathrm{col}(\tilde{v}_1, \dots, \tilde{v}_N)$, $\hat{v} = \mathrm{col}(\hat{v}_1, \dots, \hat{v}_N)$, $\tilde{S} = \mathrm{col}(\tilde{S}_1, \dots, \tilde{S}_N)$, and $\tilde{S}_{\mathrm{d}} = \mathrm{blockdiag}\{\tilde{S}_1, \dots, \tilde{S}_N\}$. Then (8) can be put into the following compact form

$$\begin{cases} \dot{\tilde{S}} = -\mu_1(H \otimes I_{n_v})\tilde{S}, \\ \dot{\tilde{v}} = (I_N \otimes S - \mu_2(H \otimes I_{n_v}))\tilde{v} + \tilde{S}_{\mathrm{d}}\hat{v}, \end{cases} \tag{9}$$

where $H = [h_{ij}]_{i,j=1}^{N}$ with $h_{ij} = -\bar{a}_{ij}$ for $i \neq j$ and $h_{ii} = \sum\limits_{j=0}^{N} \bar{a}_{ij}$. Then we introduce the following lemma.

Lemma 1 (Lemma 2 of [14]) Under Assumptions 1 and 2, we have

$$\lim_{t\to+\infty}\tilde{S}(t)=0 \tag{10}$$

exponentially and

$$\lim_{t\to+\infty}\tilde{v}(t)=0 \tag{11}$$

exponentially.

Remark 4 Let $\xi_i = D\hat{v}_i$ with $D=[I_n \ \ 0_{n\times(n+n_w)}]$ and $\tilde{v}_{\mathrm{d}i} \triangleq \mu_2 \sum\limits_{j=0}^{N} \bar{a}_{ij}(\hat{v}_j-\hat{v}_i)$. Then, by Lemma 1, we have

$$\lim_{t\to+\infty}\tilde{v}_{\mathrm{d}i}(t)=0, \tag{12}$$

$$\lim_{t\to+\infty}(\xi_i(t)-q_0(t))=\lim_{t\to+\infty}D\tilde{v}_i(t)=0, \tag{13}$$

$$\lim_{t\to+\infty}(\dot{\xi}_i(t)-\dot{q}_0(t))=\lim_{t\to+\infty}D\dot{\tilde{v}}_i(t)=0. \tag{14}$$

To synthesize our control law, let

$$p_{\mathrm{r}i}=DS_i\hat{v}_i-\alpha(q_i-\xi_i), \tag{15}$$

where α is a positive constant, and

$$s_i=p_i-p_{\mathrm{r}i}. \tag{16}$$

Then, our control law is as follows:

$$\begin{cases} u_i=-f_i^{\mathrm{T}}(q_i,p_i)\hat{\theta}_i-d_i(\hat{w}_i)-k_is_i+\dot{p}_{\mathrm{r}i},\\ \dot{\hat{\theta}}_i=f_i(q_i,p_i)s_i,\\ \dot{S}_i=\mu_1\sum\limits_{j=0}^{N}\bar{a}_{ij}(S_j-S_i),\\ \dot{\hat{v}}_i=S_i\hat{v}_i+\mu_2\sum\limits_{j=0}^{N}\bar{a}_{ij}(\hat{v}_j-\hat{v}_i),\ \ i=1,\dots,N, \end{cases} \tag{17}$$

where k_i is some positive constant , $\hat{w}_i=[0_{n_w\times 2n} \ \ I_{n_w}]\hat{v}_i$, and

$$\dot{p}_{\mathrm{r}i}=D(\dot{S}_i\hat{v}_i+S_i\dot{\hat{v}}_i)-\alpha(\dot{q}_i-\dot{\xi}_i). \tag{18}$$

The closed-loop system composed of (1) and (17) is as follows:

$$\begin{cases} \dot{q}_i=p_i,\ \ i=1,\dots,N,\\ \dot{p}_i=-f_i^{\mathrm{T}}(q_i,p_i)\tilde{\theta}_i+\tilde{d}_i(\tilde{w}_i,w)-k_is_i+\dot{p}_{\mathrm{r}i},\\ \dot{\hat{\theta}}_i=f_i(q_i,p_i)s_i,\\ \dot{\tilde{S}}_i=\mu_1\sum\limits_{j=0}^{N}\bar{a}_{ij}(\tilde{S}_j-\tilde{S}_i),\\ \dot{\tilde{v}}_i=\tilde{S}_i\hat{v}_i+S\tilde{v}_i+\mu_2\sum\limits_{j=0}^{N}\bar{a}_{ij}(\tilde{v}_j-\tilde{v}_i), \end{cases} \tag{19}$$

where $\tilde{\theta}_i=\hat{\theta}_i-\theta_i$, $\tilde{w}_i=\hat{w}_i-w$ and $\tilde{d}_i(\tilde{w}_i,w)=d_i(w)-d_i(\hat{w}_i)$. It is easy to see that $\tilde{d}_i(0,w)=0$ for all $w\in\mathbb{R}^{n_w}$. Under Assumption 1, by Remark 2, we know that $w\in\mathcal{W}$ for all $t\geqslant 0$ with $\mathcal{W}$ being some compact subset of $\mathbb{R}^{n_w}$. Then, by Lemma 7.8 of [18], there exists some smooth function $\bar{d}_i(\tilde{w}_i)\geqslant 1$ such that, for all $w\in\mathcal{W}$,

$$\|\tilde{d}_i(\tilde{w}_i,w)\|^2\leqslant\bar{d}_i(\tilde{w}_i)\|\tilde{w}_i\|^2. \tag{20}$$

Now we give our result as follows.

Theorem 1 Under Assumptions 1 and 2, the leader-following consensus problem for the system composed of (1) and (4) is solvable by the distributed control law (17).

Proof Let

$$V=\frac{1}{2}\sum_{i=1}^{N}(s_i^{\mathrm{T}}s_i+\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i). \tag{21}$$

Then the time derivative of V along the trajectory of the closed-loop system (19) is given by

$$\begin{aligned}\dot{V}&=\sum_{i=1}^{N}(s_i^{\mathrm{T}}\dot{s}_i+\tilde{\theta}_i^{\mathrm{T}}\dot{\tilde{\theta}}_i)\\ &=\sum_{i=1}^{N}(s_i^{\mathrm{T}}(\dot{p}_i-\dot{p}_{\mathrm{r}i})+\tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i)\\ &=\sum_{i=1}^{N}(s_i^{\mathrm{T}}(-f_i^{\mathrm{T}}(q_i,p_i)\tilde{\theta}_i+\tilde{d}_i(\tilde{w}_i,w)-k_is_i)\\ &\quad+\tilde{\theta}_i^{\mathrm{T}}f_i(q_i,p_i)s_i)\\ &=\sum_{i=1}^{N}(s_i^{\mathrm{T}}\tilde{d}_i(\tilde{w}_i,w)-k_is_i^{\mathrm{T}}s_i)\\ &\leqslant\sum_{i=1}^{N}(\frac{1}{4}\|s_i\|^2+\|\tilde{d}_i(\tilde{w}_i,w)\|^2-k_i\|s_i\|^2)\\ &\leqslant\sum_{i=1}^{N}(-(k_i-\frac{1}{4})\|s_i\|^2+\bar{d}_i(\tilde{w}_i)\|\tilde{w}_i\|^2).\end{aligned} \tag{22}$$

Choosing $k_i\geqslant\dfrac{5}{4}$ gives

$$\dot{V}\leqslant\sum_{i=1}^{N}(-\|s_i\|^2+\bar{d}_i(\tilde{w}_i)\|\tilde{w}_i\|^2) \tag{23}$$

and thus

$$\begin{aligned}V(t)&=\int_0^t\dot{V}(\tau)\mathrm{d}\tau+c_0\\ &\leqslant\int_0^t\sum_{i=1}^{N}\bar{d}_i(\tilde{w}_i)\|\tilde{w}_i\|^2\mathrm{d}\tau+c_0,\end{aligned} \tag{24}$$

where c_0 is some constant. Since, by Lemma 1, under Assumptions 1 and 2, $\lim\limits_{t\to+\infty} \tilde{v}_i(t) = 0$ exponentially which implies $\lim\limits_{t\to+\infty} \tilde{w}_i(t) = 0$ exponentially. Thus $\lim\limits_{t\to+\infty} \int_0^t \sum\limits_{i=1}^N \bar{d}_i(\tilde{w}_i)\|\tilde{w}_i\|^2 \mathrm{d}\tau$ exists and hence $\lim\limits_{t\to+\infty} V(t)$ exists and is finite since $\bar{d}_i(\cdot)$ is smooth and $\lim\limits_{t\to+\infty} \tilde{w}_i(t) = 0$ exponentially. Thus, $V(t)$ as well as s_i and $\tilde{\theta}_i$, $i = 1, \dots, N$, is bounded for all $t \geqslant 0$. By (16) and (18), $s_i(t)$ is differentiable for all $t \geqslant 0$ and so is $\dot{V}(t)$. By (15) and (16), we have

$$\dot{q}_i + \alpha q_i = s_i + DS_i\hat{v}_i + \alpha\xi_i. \tag{25}$$

(25) can be viewed as a stable first order linear system in q_i with a bounded input since s_i, S_i, $\hat{v}_i$ and ξ_i are all bounded, both q_i and $\dot{q}_i$ are bounded. Therefore, from (15) and (18), $p_{\mathrm{r}i}$ and $\dot{p}_{\mathrm{r}i}$ are both bounded. From the second equation of (19), $\dot{p}_i$ is bounded. Thus $\dot{s}_i = \dot{p}_i - \dot{p}_{\mathrm{r}i}$ is also bounded. Note that

$$\begin{aligned}\ddot{V} = \sum_{i=1}^N (s_i^{\mathrm{T}}(&\frac{\partial\tilde{d}_i(\tilde{w}_i, w)}{\partial\tilde{w}_i}\dot{\tilde{w}}_i + \frac{\partial\tilde{d}_i(\tilde{w}_i, w)}{\partial w}\dot{w}) \\ &+ \dot{s}_i^{\mathrm{T}}\tilde{d}_i(\tilde{w}_i, w) - 2k_i s_i^{\mathrm{T}}\dot{s}_i).\end{aligned} \tag{26}$$

Since s_i, $\dot{s}_i$, w, $\dot{w}$, $\tilde{w}_i$ and $\dot{\tilde{w}}_i$ are all bounded, we can conclude that $\ddot{V}(t)$ is bounded for all $t \geqslant 0$. Then, by Barbalat's Lemma, $\lim\limits_{t\to+\infty} \dot{V}(t) = 0$ and thus, from (23), we have $\lim\limits_{t\to+\infty} s_i(t) = 0$ for $i = 1, \dots, N$. Next, by (7), (15) and (16), we have

$$\begin{aligned}\dot{q}_i - \dot{\xi}_i + \alpha(q_i - \xi_i) &= p_i - D\dot{\hat{v}}_i + \alpha(q_i - \xi_i) \\ &= s_i + DS_i\hat{v}_i - D\dot{\hat{v}}_i \\ &= s_i - D\tilde{v}_{\mathrm{d}i}.\end{aligned} \tag{27}$$

From Remark 4, under Assumptions 1 and 2, by Lemma 1, we know that $\lim\limits_{t\to+\infty} \tilde{v}_{\mathrm{d}i}(t) = 0$. Note that equation (27) can be viewed as a stable first order differential equation in $q_i - \xi_i$ with $s_i - D\tilde{v}_{\mathrm{d}i}$ as the input, and this input is bounded for all $t \geqslant 0$ and tends to zero as $t \to +\infty$, then we conclude that both $q_i - \xi_i$ and $\dot{q}_i - \dot{\xi}_i$ are bounded for all $t \geqslant 0$ and tend to zero as $t \to +\infty$. Together with (13), (14) and the following equations

$$\begin{cases} q_i - q_0 = (q_i - \xi_i) + (\xi_i - q_0), \\ p_i - p_0 = (\dot{q}_i - \dot{\xi}_i) + (\dot{\xi}_i - \dot{q}_0), \end{cases} \tag{28}$$

our proof is thus completed. □

4 An example

Consider the leader-following consensus problem for a group of Vol del Pol systems as follows:

$$\begin{cases} \dot{q}_i = p_i, \quad i = 1, 2, 3, 4, \\ \dot{p}_i = -\theta_{1i}q_i + \theta_{2i}p_i(1 - q_i^2) + d_i(w) + u_i, \end{cases} \tag{29}$$

where $q_i, p_i \in \mathbb{R}$, $w = [w_1 \;\; w_2]^{\mathrm{T}}$, $d_1(w) = w_1^2$, $d_2(w) = w_2^2$, $d_3(w) = w_1 w_2$ and $d_4(w) = w_1^2 + w_2^2$. Clearly, system (29) is in the form of (1) with $f_i^{\mathrm{T}}(q_i, p_i) = [-q_i \;\; p_i(1 - q_i^2)]$ and $\theta_i = [\theta_{1i} \;\; \theta_{2i}]^{\mathrm{T}}$.

The exosystem is in the form of (4) with $S_{\mathrm{a}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $S_{\mathrm{b}} = \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix}$. Clearly, Assumption 1 is satisfied.

The communication graph is described by Fig. 1 where the node 0 is associated with the leader and the other nodes are associated with the followers. Clearly, every node $i = 1, 2, 3, 4$ is reachable from the node 0 in the diagraph $\bar{\mathcal{G}}$ and thus Assumption 2 is satisfied. From Fig. 1, we obtain that the adjacency matrix of $\bar{\mathcal{G}}$ is

$$\bar{\mathcal{A}} = [\bar{a}_{ij}]_{i,j=0}^4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}. \tag{30}$$

Then, by Theorem 1, we can design a distributed control law as follows:

$$\begin{cases} u_i = -f_i^{\mathrm{T}}(q_i, p_i)\hat{\theta}_i - d_i(\hat{w}_i) - 4s_i + \dot{p}_{\mathrm{r}i}, \\ \dot{\hat{\theta}}_i = f_i(q_i, p_i)s_i, \\ \dot{S}_i = \sum\limits_{j=0}^N \bar{a}_{ij}(S_j - S_i), \\ \dot{\hat{v}}_i = S_i\hat{v}_i + 10\sum\limits_{j=0}^N \bar{a}_{ij}(\hat{v}_j - \hat{v}_i), \quad i = 1, 2, 3, 4, \end{cases} \tag{31}$$

where

$$\begin{cases} f_i^{\mathrm{T}}(q_i, p_i) = [-q_i \;\; p_i(1 - q_i^2)], \\ \hat{w}_i = \mathrm{col}(\hat{w}_{1i}, \hat{w}_{2i}) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\hat{v}_i, \\ d_1(\hat{w}_1) = \hat{w}_{11}^2, \quad d_2(\hat{w}_2) = \hat{w}_{22}^2, \\ d_3(\hat{w}_3) = \hat{w}_{13}\hat{w}_{23}, \quad d_4(\hat{w}_4) = \hat{w}_{14}^2 + \hat{w}_{24}^2. \end{cases} \tag{32}$$

s_i and $\dot{p}_{ri}$ are defined as in (16) and (18) with $D = [1\ \ 0\ \ 0\ \ 0]$ and $\alpha = 1$.

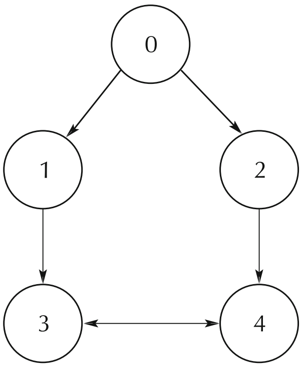

Fig. 1 Communication graph $\bar{\mathcal{G}}$.

Simulation is performed with

$$\begin{cases} \theta_1 = [3\ \ 4]^{\mathrm{T}},\ \ \theta_2 = [1\ \ 3]^{\mathrm{T}}, \\ \theta_3 = [2\ \ 5]^{\mathrm{T}},\ \ \theta_4 = [4\ \ 2]^{\mathrm{T}} \end{cases} \tag{33}$$

and the following initial conditions:

$$\begin{aligned}
&[q_1(0)\ \ p_1(0)] = [4\ \ -3],\ \ [q_2(0)\ \ p_2(0)] = [3\ \ 4],\\
&[q_3(0)\ \ p_3(0)] = [-2\ \ 1],\ \ [q_4(0)\ \ p_4(0)] = [1\ \ -2],\\
&v(0) = [1\ \ -1\ \ 2\ \ 0]^{\mathrm{T}},\\
&\hat{v}_1(0) = [-1\ \ 2\ \ 1\ \ -3]^{\mathrm{T}},\ \ \hat{v}_2(0) = [5\ \ 1\ \ -1\ \ 4]^{\mathrm{T}},\\
&\hat{v}_3(0) = [4\ \ -2\ \ 5\ \ 3]^{\mathrm{T}},\ \ \hat{v}_4(0) = [-3\ \ -4\ \ 3\ \ -1]^{\mathrm{T}},\\
&S_1(0) = \begin{bmatrix} 1 & 2 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -3 \\ 0 & 0 & 4 & 1 \end{bmatrix},\ \ \hat{\theta}_1(0) = [-1\ \ 2]^{\mathrm{T}},\\
&S_2(0) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 5 & 1 \end{bmatrix},\ \ \hat{\theta}_2(0) = [1\ \ -1]^{\mathrm{T}},\\
&S_3(0) = \begin{bmatrix} 0 & -2 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2 \end{bmatrix},\ \ \hat{\theta}_3(0) = [-2\ \ 1]^{\mathrm{T}},\\
&S_4(0) = \begin{bmatrix} -2 & 1 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 3 & 1 \end{bmatrix},\ \ \hat{\theta}_4(0) = [2\ \ -1]^{\mathrm{T}}.
\end{aligned}$$

Fig. 2 shows the states of the leader system which are bounded for all time $t \geqslant 0$. Figs. 3–6 show the estimation errors of the observer for each follower. It can be seen that all four estimations of leader's states converge to the leader's states as $t \to +\infty$.

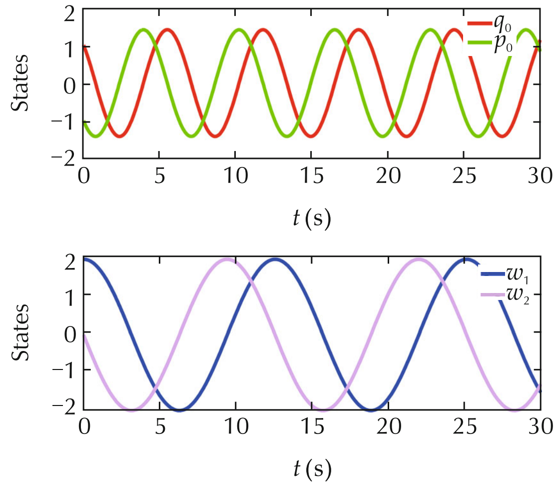

Fig. 2 States of leader system: $v = \mathrm{col}(q_0, p_0, w_1, w_2)$.

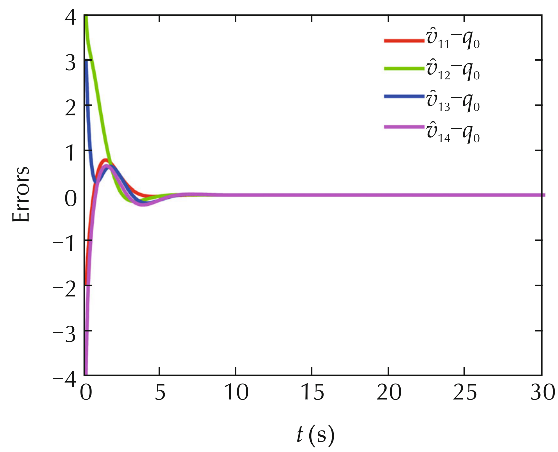

Fig. 3 Estimation errors: $\hat{v}_{1i} - q_0$.

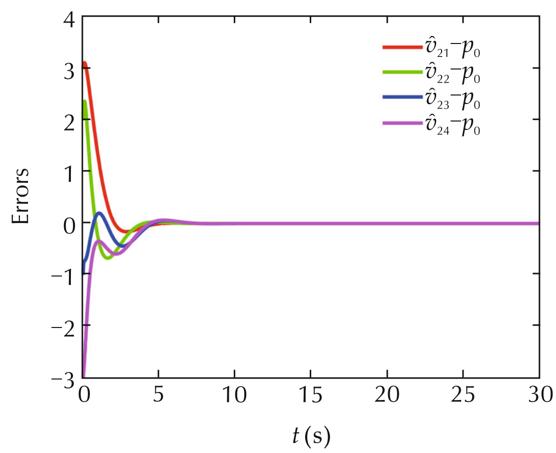

Fig. 4 Estimation errors: $\hat{v}_{2i} - p_0$.

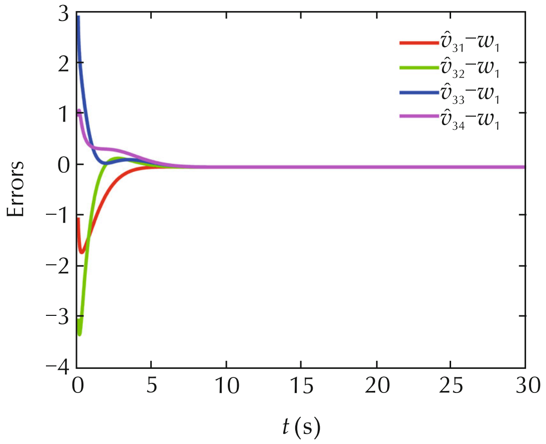

Fig. 5 Estimation errors: $\hat{v}_{3i} - w_1$.

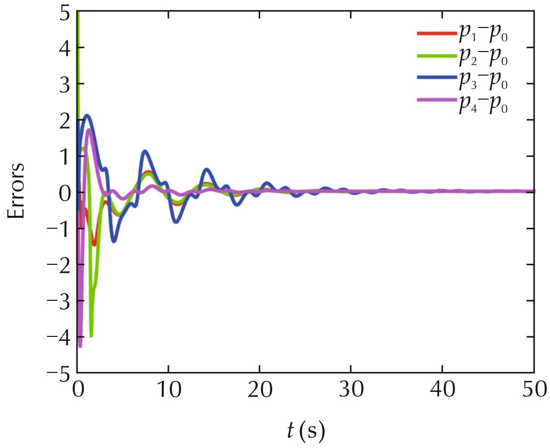

Fig. 8 Tracking errors: $p_i - p_0$.

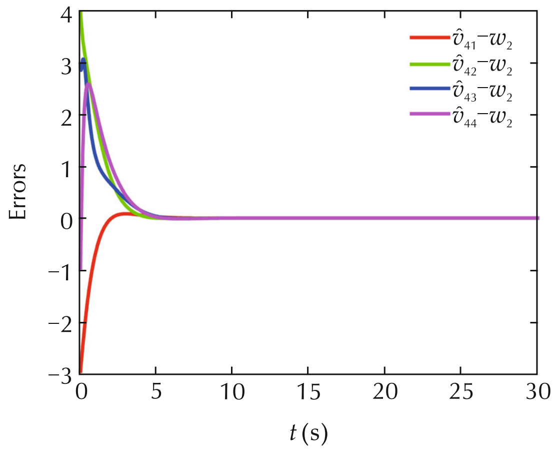

Fig. 6 Estimation errors: $\hat{v}_{4i} - w_2$.

Figs. 7 and 8 further show the tracking performance of q_i and p_i. As expected, the states of all followers approach the states of the leader asymptotically.

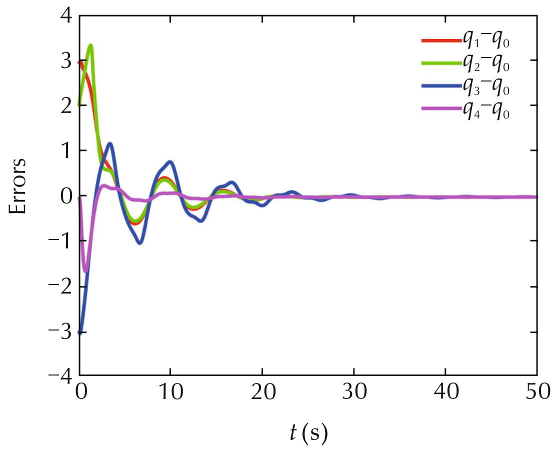

Fig. 7 Tracking errors: $q_i - q_0$.

5 Conclusions

In this paper, we have studied the leader-following consensus problem for a class of second-order nonlinear multi-agent systems subject to linearly parameterized uncertainty and disturbance. We have solved the problem by integrating the adaptive control technique and the adaptive distributed observer method. It is interesting to further consider the case where the network topology is switching and satisfies the jointly connected condition.

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