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Quasi-Newton-type optimized iterative learning control for discrete linear time invariant systems

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Abstract

In this paper, a quasi-Newton-type optimized iterative learning control (ILC) algorithm is investigated for a class of discrete linear time-invariant systems. The proposed learning algorithm is to update the learning gain matrix by a quasi-Newton-type matrix instead of the inversion of the plant. By means of the mathematical inductive method, the monotone convergence of the proposed algorithm is analyzed, which shows that the tracking error monotonously converges to zero after a finite number of iterations. Compared with the existing optimized ILC algorithms, due to the superlinear convergence of quasi-Newton method, the proposed learning law operates with a faster convergent rate and is robust to the ill-condition of the system model, and thus owns a wide range of applications. Numerical simulations demonstrate the validity and effectiveness.

Keywords: Iterative learning control, optimization, quasi-Newton method, inverse plant

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1 Introduction

Iterative learning control (ILC) has been acknowledged as one of the effective techniques that achieves perfect trajectory tracking while the system implements the tracking task repetitively over a fixed time-interval, see, e.g., [1–6]. Arimoto et al. [1] firstly introduced ILC as an intelligent teaching mechanism called "betterment process" for robot manipulators. Numerous ILC contributions have come forth over the past three decades scoping from theoretical investigations to practical applications. As one of the important theoretical issues, the convergence analysis has been discussed by Amann et al. [2], Xu [3], and Meng et al. (2-D analysis approach) [5] and Ruan et al. [6, 7]. In addition, the robustness has been discussed from many aspects, such as stochastic noise in [8], iteration-varying disturbances in [9], model uncertainty in [10] and time-delay uncertainty in [11], etc., stability and initial state learning have been researched in [12] and [13], respectively. Another significant contribution to ILC theory is optimal ILC algorithms in articles [14–19]. In terms of the applications of ILC, the categories mainly include robotics in [20], rotary systems in [21], chemical batch processing in [22], etc.

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Ahn et al. [23] provided a summary and review of the recent trends in ILC research from both the application and the theoretical aspects.

In optimization field, it is known that the gradient method, the conjugate direction method, the Newton method as well as the quasi-Newton method have been acknowledged as effective optimization techniques for their widely applications in the areas of industry, agriculture, military and medical treatment, see, e.g., [24–26]. Guided by the practicability and the validity of the optimization techniques, a number of investigations have been made which focus on embedding some of the above-mentioned optimization methods into the ILC algorithms [14–19]. Referring to the scheme of optimization technique, the mode of such an optimized ILC updating law is to generate a sequence of optimized control inputs by minimizing performance index function.

In this aspect, Amann et al. [2] firstly introduced the concept of optimal ILC algorithm for linear systems based on optimization theory and made a comprehensive analysis of norm-optimal ILC (NOILC). After that, Owens and Feng [15] proposed a parameter optimization ILC (POILC) for discrete linear time-invariant systems and derived its monotone convergence under the assumption that the system satisfies a positivity condition. Besides, Owens et al. [16] offered a gradient-type ILC algorithm and analyzed its convergence in a rigorous manner. The analogous work was to establish an inverse model ILC scheme named as a Newton-type ILC algorithm, and made a comprehensive analysis in term of the convergence and the robustness as shown in [17]. It is noticed that the optimized ILC strategies in [15–17] are model-based, of which both the necessity and the sufficiency of the monotone convergence are involved.

Theoretically, it is thus no doubt that the inverse model ILC scheme owns a one-step terminative performance for the case when the inversion of the model of the plant is precisely identified in prior and well-conditioned. In reality, on the one hand, the inverse model algorithm is quite sensitive to the perturbation incurred by some measurement noise or slow changing of the system parameters. On the other hand, the inverse model technique may not work for the model imprecision. This implies that the inverse model ILC is hardly realizable in practical executions. In order to avoid the complexities of matrix inversion, Owens et al. [18] developed a polynomial approximation ILC (PA-ILC) algorithm which replaces the inverse model of the plant by a polynomial. However, the ILC algorithm requires plenty of computation to capture the inversions of the system matrices and thus is just implementable to a lower dimensional plant or a less operational processing. Besides, as the searching directory of the gradient-type ILC mechanism, article [16] prevailed to a saw path with a very small learning step when the iteration-wise approximate optimum is close to the desired one, the convergent rate of the gradient-type ILC is to some extent not satisfactory, especially when the system is ill-conditioned. In 2013, Yang and Ruan [19] developed a type of conjugate direction ILC scheme for linear discrete time-invariant systems to speed up the convergent rate.

In spite of the above-mentioned executable limitation, the inverse mode ILC mechanism remains referable to develop an efficient learning law. As such, a quasi-Newton-type ILC updating law is a candidate, which adopts an approximate matrix to replace the inverse model of the plant. This motivates the paper to develop a quasi-Newton-type ILC strategy for discrete linear time-invariant systems and derives its convergence as well.

The paper is organized as follows. Section 2 presents two types of quasi-Newton ILC schemes abbreviated as SR1-ILC algorithm and SR2-ILC algorithm, respectively. Section 3 exhibits the monotonic convergence. Numerical simulations are illustrated in Section 4 and the conclusions are drawn in Section 5.

2 Quasi-Newton-type optimized ILC algorithm

Consider a class of repetitive discrete linear time-invariant single-input-single-output (SISO) systems described as follows:

$$\begin{cases} x(t+1) = Ax(t) + Bu(t), \\ y(t) = Cx(t), \\ x(0) = 0, \end{cases} \tag{1}$$

where $t \in I$ represents the sampling time with $I = \{0, 1, \dots, N\}$ denoting the set of the sampling times and N standing for the total number of the sampling times, respectively. The variables $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are respectively an n-dimensional state vector, a scalar input and a scalar output at the sampling time t. A, B and C are matrices with appropriate dimensions, respectively, satisfying $CB \neq 0$. The model system (1) is reformulated in a lifted input-output form as follows:

$$y = Gu, \tag{2}$$

where

$$u = [u(0)\ \ u(1)\ \cdots\ u(N-1)]^{\mathrm{T}},$$
$$y = [y(1)\ \ y(2)\ \cdots\ y(N)]^{\mathrm{T}},$$
$$G = \begin{bmatrix} CB & 0 & 0 & \cdots & 0 \\ CAB & CB & 0 & \cdots & 0 \\ CA^2B & CAB & CB & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ CA^{N-1}B & CA^{N-2}B & CA^{N-3}B & \cdots & CB \end{bmatrix}.$$

It is obvious that G is an invertible Markov parameters-based transfer matrix.

Let $y_{\mathrm{d}} = [y_{\mathrm{d}}(1)\ \ y_{\mathrm{d}}(2)\ \cdots\ y_{\mathrm{d}}(N)]^{\mathrm{T}}$ be a given desired trajectory of system (2) and $e = y_{\mathrm{d}} - y = [y_{\mathrm{d}}(1) - y(1)\ \ y_{\mathrm{d}}(2) - y(2)\ \cdots\ y_{\mathrm{d}}(N) - y(N)]^{\mathrm{T}} = y_{\mathrm{d}} - Gu$ denotes the tracking error of system (2). The ILC algorithm is to design a sequence of inputs $\{u_{k+1}\}$ so that it may drive system (2) to track the desired trajectory y_{d} as precisely as possible as the iteration index approaches infinity. That is to say,

$$\lim_{k\to\infty} \|e_{k+1}\|_2 = 0,$$

where

$$e_{k+1} = y_{\mathrm{d}} - y_{k+1},$$
$$y_{k+1} = [y_{k+1}(1)\ \ y_{k+1}(2)\ \cdots\ y_{k+1}(N)]^{\mathrm{T}}$$

refers to the output of system (2) driven by $u_{k+1} = [u_{k+1}(0)\ \ u_{k+1}(1)\ \cdots\ u_{k+1}(N-1)]^{\mathrm{T}}$. Such an ILC sequence u_{k+1} can be produced by solving an optimization problem for system (2) as follows:

$$\min_u F(u) = \frac{1}{2}\|e\|^2 = \frac{1}{2}u^{\mathrm{T}}G^{\mathrm{T}}Gu - (G^{\mathrm{T}}y_{\mathrm{d}})^{\mathrm{T}}u + \frac{1}{2}y_{\mathrm{d}}^{\mathrm{T}}y_{\mathrm{d}}. \tag{3}$$

Recall that the Newton-type ILC updating law of the optimization (3) developed in article [17] takes a form of

$$u_{k+1} = u_k + \beta G^{-1}e_k, \tag{4}$$

where $k = 1, 2, \ldots$ is the iterative number. $e_k = y_{\mathrm{d}} - y_k$ is the tracking error between the desired trajectory y_{d} and the output y_k, and β is termed as a learning step length.

Obviously, the above Newton-type ILC is an inversion-model scheme, which requires amounts of computation for inversion and is usually fit for a well-conditioned system.

For the purpose of avoiding the computational complexity of matrix inversion and enriching the fitness of the system mode, a feasible manner is to replace the learning gain matrix G^{-1} of the scheme (4) by a matrix with less computation or generally conditioned. As such, an iteration-relevant matrix H_kG^{T} is adopted for the substitution and the corresponding ILC scheme named as the quasi-Newton-type optimized ILC algorithm is as follows:

$$u_{k+1} = u_k + \beta_k H_k G^{\mathrm{T}} e_k. \tag{5}$$

Here, H_k is the kth approximation of $(G^{\mathrm{T}}G)^{-1}$ and is updated in accordance with the quasi-Newton condition. β_k is termed as the kth learning step length that obtained by exact linear search method.

The quasi-Newton ILC algorithm (5) is specified as follows.

u_1 and H_1 are given arbitrarily,

$$y_1 = Gu_1,$$
$$\bar{e}_1 = G^{\mathrm{T}}(y_{\mathrm{d}} - y_1),$$
$$\beta_1 = \frac{\bar{e}_1^{\mathrm{T}}(H_1\bar{e}_1)}{(H_1\bar{e}_1)^{\mathrm{T}}G^{\mathrm{T}}G(H_1\bar{e}_1)};$$

when $k \geqslant 1$,

$$u_{k+1} = u_k + \beta_k H_k \bar{e}_k, \tag{6}$$
$$y_{k+1} = Gu_{k+1}, \tag{7}$$
$$e_{k+1} = y_{\mathrm{d}} - y_{k+1}, \tag{8}$$
$$\bar{e}_{k+1} = G^{\mathrm{T}}e_{k+1}, \tag{9}$$
$$H_{k+1} = H_k + \Delta H_k, \tag{10}$$
$$\beta_{k+1} = \frac{\bar{e}_{k+1}^{\mathrm{T}}(H_{k+1}\bar{e}_{k+1})}{(H_{k+1}\bar{e}_{k+1})^{\mathrm{T}}G^{\mathrm{T}}G(H_{k+1}\bar{e}_{k+1})}, \tag{11}$$

where ΔH_k is a correction term which can be constructed in various forms so that the matrix H_{k+1} satisfies the quasi-Newton condition

$$H_{k+1}\tilde{e}_k = \Delta u_k,\ \ k = 1, 2, \ldots, \tag{12}$$

where

$$\Delta u_k = u_{k+1} - u_k = \beta_k H_k \bar{e}_k \tag{13}$$

is assigned as the searching direction and

$$\tilde{e}_k = \nabla F(u_{k+1}) - \nabla F(u_k) = G^{\mathrm{T}}G\Delta u_k \tag{14}$$

is termed as the gradient difference vector and the expression (12) is called as a secant equation.

Two typical correction forms are symmetrical-rank-1 and symmetrical-rank-2 expressed as follows:

I) Symmetrical-Rank-1 (SR1) correction formula is

$$\Delta H_k = \frac{(\Delta u_k - H_k\tilde{e}_k)(\Delta u_k - H_k\tilde{e}_k)^{\mathrm{T}}}{(\Delta u_k - H_k\tilde{e}_k)^{\mathrm{T}}\tilde{e}_k}. \tag{15}$$

The above (6)–(11) together with the SR1 correction form (15) compose an SR1-ILC algorithm.

II) Symmetrical-Rank-2 (SR2) correction formula is

$$\Delta H_k = \frac{\Delta u_k \Delta u_k^{\mathrm{T}}}{\Delta u_k^{\mathrm{T}}\tilde{e}_k} - \frac{H_k\tilde{e}_k\tilde{e}_k^{\mathrm{T}}H_k}{\tilde{e}_k^{\mathrm{T}}H_k\tilde{e}_k}. \tag{16}$$

The symmetric-rank-2 form is induced by DFP correction in [27]. The above (6)–(11) plus the correction form (16) is called an SR2-ILC algorithm.

3 Convergence analysis

The monotonicity of the tracking error and the termination at the finite iteration of the quasi-Newton-type optimized ILC algorithm can be concluded in the following theorem. In order to derive convergence property, some properties of the searching directions are exhibited as Lemmas 1–3 as follows.

Lemma 1 Suppose that the sequence of the gradient difference vectors $\{\tilde{e}_j\}_{j=1}^{k+1}$ and the searching direction vectors $\{\Delta u_j\}_{j=1}^{k+1}$ are generated by the SR1-ILC algorithm (6)–(11) and (15) satisfying $(\Delta u_{k+1} - H_k\tilde{e}_{k+1})^{\mathrm{T}}\tilde{e}_{k+1} \neq 0$. Then, the following secant equations are true:

$$H_{k+1}\tilde{e}_j = \Delta u_j,\quad j = 1,2,\ldots,k. \tag{17}$$

Proof Since the assumption that $(\Delta u_{k+1} - H_k\tilde{e}_{k+1})^{\mathrm{T}} \times\tilde{e}_{k+1} \neq 0$, the SR1-ILC updating law is well-defined. The secant equations are derived by mathematical inductive method as follows.

Step 1 (For the case when $k = 1$) From the quasi-Newton condition (12), the SR1-ILC updating law satisfies the secant equation, that is, $H_2\tilde{e}_1 = \Delta u_1$ is true.

Assume that the secant equations (17) are true for the the case when $k = m$ and $m = 1,2,\ldots$, that is,

$$H_{m+1}\tilde{e}_j = \Delta u_j,\quad j = 1,2,\ldots,m. \tag{18}$$

Step 2 Induce that for the case when $k = m+1$ the conclusion is true.

From (10) and (15), we have

$$\begin{aligned} &H_{(m+1)+1}\tilde{e}_j \\ &= H_{m+1}\tilde{e}_j \\ &\quad + \frac{(\Delta u_{m+1} - H_{m+1}\tilde{e}_{m+1})(\Delta u_{m+1} - H_{m+1}\tilde{e}_{m+1})^{\mathrm{T}}\tilde{e}_j}{(\Delta u_{m+1} - H_{m+1}\tilde{e}_{m+1})\tilde{e}_{m+1}}. \end{aligned} \tag{19}$$

From the assumption (18), we have

$$\begin{aligned} &(\Delta u_{m+1} - H_{m+1}\tilde{e}_{m+1})^{\mathrm{T}}\tilde{e}_j \\ &= \Delta u_{m+1}^{\mathrm{T}}\tilde{e}_j - \tilde{e}_{m+1}^{\mathrm{T}}(H_{m+1}\tilde{e}_j) \\ &= \Delta u_{m+1}^{\mathrm{T}}\tilde{e}_j - \tilde{e}_{m+1}^{\mathrm{T}}\Delta u_j \\ &= \Delta u_{m+1}^{\mathrm{T}}(G^{\mathrm{T}}G\Delta u_j) - (G^{\mathrm{T}}G\Delta u_{m+1})^{\mathrm{T}}\Delta u_j \\ &= 0. \end{aligned} \tag{20}$$

Substituting (18) and (20) into (19), we have

$$\begin{aligned} &H_{(m+1)+1}\tilde{e}_j \\ &= H_{m+1}\tilde{e}_j \\ &= \Delta u_j,\quad j = 1,2,\ldots,m+1. \end{aligned} \tag{21}$$

This means that (17) holds when $k = m+1$. □

Remark 1 From Lemma 1, it is observed that the secant equation is guaranteed to depend on not only the current tracking error but also all previous tracking errors. The SR1-ILC algorithm has the hereditary property $H_{k+1}\tilde{e}_j = \Delta u_j$ for $j = 1,2,\ldots,k$, where $H_{k+1}\tilde{e}_j = \Delta u_j$ is named as the secant equation for $\tilde{e}_j = \nabla F(u_{j+1}) - \nabla F(u_j)$ and $\Delta u_j = u_{j+1} - u_j$.

Lemma 2 Suppose that the searching direction vectors $\Delta u_1, \Delta u_2, \ldots, \Delta u_k$ are $G^{\mathrm{T}}G$-conjugate and $k \leqslant N$. Then, the searching direction vectors $\Delta u_1, \Delta u_2, \ldots, \Delta u_k$ are linearly independent.

Proof Suppose that there exists such a set of numbers $\alpha_1, \alpha_2, \ldots, \alpha_k$ that the equality

$$\alpha_1\Delta u_1 + \alpha_2\Delta u_2 + \cdots + \alpha_k\Delta u_k = 0 \tag{22}$$

holds. Then, (22) yields

$$\begin{aligned} 0 &= \Delta u_i^{\mathrm{T}}G^{\mathrm{T}}G(\alpha_1\Delta u_1 + \alpha_2\Delta u_2 + \ldots + \alpha_k\Delta u_k) \\ &= \alpha_i\Delta u_i^{\mathrm{T}}G^{\mathrm{T}}G\Delta u_i,\quad i = 1,2,\ldots,k. \end{aligned}$$

Because $\Delta u_i^{\mathrm{T}}G^{\mathrm{T}}G\Delta u_i \neq 0$, it is no other than $\alpha_i = 0$ for all $i = 1,2,\ldots,k$. □

Remark 2 A set of nonzero searching direction vectors $\{\Delta u_1, \ldots, \Delta u_k\}$ is said to be conjugate with respect to the symmetric positive definite matrix $G^{\mathrm{T}}G$ if

$$\Delta u_i^{\mathrm{T}}G^{\mathrm{T}}G\Delta u_j = 0,\quad i \neq j.$$

Lemma 3 Suppose that the sequence of the gradient difference vectors $\{\tilde{e}_j\}$ and the searching direction vectors $\{\Delta u_k\}$ are generated by SR2-ILC algorithm (6)–(11) and (16). Then, the searching direction vectors $\Delta u_1, \Delta u_2, \dots, \Delta u_k$ are conjugate for all $k \leqslant N$, and the following secant equations are true

$$H_{k+1}\tilde{e}_j = \Delta u_j, \quad j = 1, 2, \dots, k, \tag{23}$$
$$\Delta u_k^{\mathrm{T}} G^{\mathrm{T}} G \Delta u_j = 0, j = 1, 2, \dots, k-1. \tag{24}$$

Proof By using the mathematical inductive method, we can prove (23) and (24) to be true.

For the case when $k = 1$, from the quasi-Newton condition (12), we have $H_2\tilde{e}_1 = \Delta u_1$, it shows that (23) is true.

For the case when $k = 2$, from the property of exact line search $\bar{e}_{k+1}^{\mathrm{T}}\Delta u_k = 0$, we obtain

$$\Delta u_2^{\mathrm{T}} G^{\mathrm{T}} G \Delta u_1 = \beta_2 \bar{e}_2^{\mathrm{T}} H_2 \tilde{e}_1 = \beta_2 \bar{e}_2^{\mathrm{T}} \Delta u_1 = 0. \tag{25}$$

From (14) and (25), we have

$$\Delta u_2^{\mathrm{T}} \tilde{e}_1 = \Delta u_2^{\mathrm{T}} G^{\mathrm{T}} G \Delta u_1 = 0, \tag{26}$$
$$\begin{aligned} \tilde{e}_2^{\mathrm{T}} H_2 \tilde{e}_1 &= \tilde{e}_2^{\mathrm{T}} \Delta u_1 = (G^{\mathrm{T}} G \Delta u_2)^{\mathrm{T}} \Delta u_1 \\ &= \Delta u_2 G^{\mathrm{T}} G \Delta u_1 = 0. \end{aligned} \tag{27}$$

Thus,

$$\begin{aligned} H_3\tilde{e}_1 &= H_2\tilde{e}_1 + \frac{\Delta u_2 \Delta u_2^{\mathrm{T}} \tilde{e}_1}{\Delta u_2^{\mathrm{T}} \tilde{e}_2} - \frac{H_2 \tilde{e}_2 \tilde{e}_2^{\mathrm{T}} H_2 \tilde{e}_1}{\tilde{e}_2^{\mathrm{T}} H_2 \tilde{e}_2} \\ &= H_2 \tilde{e}_1 = \Delta u_1. \end{aligned} \tag{28}$$

In addition, from the quasi-Newton condition (12), we have

$$H_3 \tilde{e}_2 = \Delta u_2. \tag{29}$$

Equalities (29) and (28) show that (23) is true and (26) shows that (24) is true for the case when $k = 2$.

Assume that the secant equations (23) and (24) are true for the index $k = m$ and $m = 1, 2, \dots$, that is

$$H_{m+1}\tilde{e}_j = \Delta u_j, \quad j = 1, 2, \dots, m, \tag{30}$$
$$\Delta u_m^{\mathrm{T}} G^{\mathrm{T}} G \Delta u_j = 0, \quad j = 1, 2, \dots, m-1. \tag{31}$$

For the case $k = m+1$, since $\bar{e}_{m+1} \neq 0$, the property of exact line search $\bar{e}_{m+1}^{\mathrm{T}} \Delta u_m = 0$ and inductive assumption (31) for all $j \leqslant m$, we have

$$\begin{aligned} &\bar{e}_{m+1}^{\mathrm{T}} \Delta u_j \\ &= \bar{e}_{j+1}^{\mathrm{T}} \Delta u_j + \sum_{i=j+1}^{m} (\bar{e}_{i+1} - \bar{e}_i)^{\mathrm{T}} \Delta u_j \\ &= \bar{e}_{j+1}^{\mathrm{T}} \Delta u_j - \sum_{i=j+1}^{m} \tilde{e}_i^{\mathrm{T}} \Delta u_j \\ &= 0 - \sum_{i=j+1}^{m} \Delta u_i^{\mathrm{T}} G^{\mathrm{T}} G \Delta u_j \\ &= 0. \end{aligned} \tag{32}$$

By (13), (14), (30) and (32), we have

$$\begin{aligned} &\Delta u_{m+1}^{\mathrm{T}} G^{\mathrm{T}} G \Delta u_j \\ &= \beta_{m+1} \bar{e}_{m+1}^{\mathrm{T}} H_{m+1} \tilde{e}_j \\ &= \beta_{m+1} \bar{e}_{m+1}^{\mathrm{T}} \Delta u_j = 0, \quad j \leqslant m. \end{aligned} \tag{33}$$

It shows that (24) holds for the case when $k = m+1$.

By the quasi-Newton condition, when $j = m+1$, we have

$$H_{(m+1)+1} \tilde{e}_{m+1} = \Delta u_{m+1}. \tag{34}$$

For all $j \leqslant m$, from (14) and (33), we have

$$\Delta u_{m+1}^{\mathrm{T}} \tilde{e}_j = \Delta u_{m+1}^{\mathrm{T}} G^{\mathrm{T}} G \Delta u_j = 0, \tag{35}$$
$$\begin{aligned} \tilde{e}_{m+1}^{\mathrm{T}} H_{m+1} \tilde{e}_j &= \tilde{e}_{m+1}^{\mathrm{T}} \Delta u_j \\ &= (G^{\mathrm{T}} G \Delta u_{m+1})^{\mathrm{T}} \Delta u_j = 0. \end{aligned} \tag{36}$$

By (35) and (36), we have

$$\begin{aligned} H_{(m+1)+1} \tilde{e}_j &= H_{m+1} \tilde{e}_j + \frac{\Delta u_{m+1} \Delta u_{m+1}^{\mathrm{T}} \tilde{e}_j}{\Delta u_m^{\mathrm{T}} \tilde{e}_{m+1}} \\ &\quad - \frac{H_{m+1} \tilde{e}_{m+1} \tilde{e}_{m+1}^{\mathrm{T}} H_{m+1} \tilde{e}_j}{\tilde{e}_{m+1}^{\mathrm{T}} H_{m+1} \tilde{e}_{m+1}} \\ &= H_{m+1} \tilde{e}_j. \end{aligned} \tag{37}$$

Expressions (34) and (37) indicate that $H_{(m+1)+1}\tilde{e}_j = \Delta u_j$ holds, for all $j = 1, 2, \dots, m+1$. □

Theorem 1 Assume that the tracking errors $\{e_k\}$ are generated by SR1-ILC algorithm (6)–(11) and (15), then the following conclusions hold.

i) If the searching direction vectors $\{\Delta u_j\}_{j=1}^{N}$ are linearly independent, then the tracking error e_k converges to zero at the $(N+1)$th iteration, which means that $e_{N+2} = 0$ if $(N+1)$th iteration is performed.

ii) If the searching direction vectors $\Delta u_1, \Delta u_2, \dots, \Delta u_k$ are linearly dependent, $\Delta u_1, \Delta u_2, \dots, \Delta u_{k-1}$ are linearly independent, $k \leqslant N$ and (11) is replaced by $\beta_k = 1$, then $e_{k+1} = -\nabla F(u_{k+1}) = 0$, which means that the input signal u_{k+1} is the solution and the tracking error $e_{k+1} = 0$.

Proof From Lemma 1 we have

$$\Delta u_j = H_{N+1} \tilde{e}_j = H_N G^{\mathrm{T}} G \Delta u_j, \quad j = 1, \dots, N.$$

If the search direction vectors $\{\Delta u_j\}_{j=1}^{N}$ are linearly independent, then $H_{N+1}G^{\mathrm{T}}G = I$, which implies that $H_{N+1} = (G^{\mathrm{T}}G)^{-1}$. Thus,

$$\begin{aligned}
e_{N+2} &= y_{\mathrm{d}} - Gu_{N+2} \\
&= y_{\mathrm{d}} - Gu_{N+1} - G\beta_N H_N \bar{e}_{N+1} \\
&= e_{N+1} - G\frac{\bar{e}_{N+1}^{\mathrm{T}}(H_{N+1}\bar{e}_{N+1})H_{N+1}\bar{e}_{N+1}}{(H_{N+1}\bar{e}_{N+1})^{\mathrm{T}}G^{\mathrm{T}}G(H_{N+1}\bar{e}_{N+1})} \\
&= e_{N+1} - \frac{G\cdot\bar{e}_{N+1}^{\mathrm{T}}(G^{\mathrm{T}}G)^{-1}\bar{e}_{N+1}\cdot(G^{\mathrm{T}}G)^{-1}\bar{e}_{N+1}}{\bar{e}_{N+1}^{\mathrm{T}}((G^{\mathrm{T}}G)^{-1})^{\mathrm{T}}(G^{\mathrm{T}}G)(G^{\mathrm{T}}G)^{-1}\bar{e}_{N+1}} \\
&= e_{N+1} - G\frac{\bar{e}_{N+1}^{\mathrm{T}}(G^{\mathrm{T}}G)^{-1}\bar{e}_{N+1}}{\bar{e}_{N+1}^{\mathrm{T}}((G^{\mathrm{T}}G)^{-1})^{\mathrm{T}}\bar{e}_{N+1}}(G^{\mathrm{T}}G)^{-1}\bar{e}_{N+1} \\
&= e_{N+1} - G(G^{\mathrm{T}}G)^{-1}\bar{e}_{N+1} \\
&= e_{N+1} - G(G^{\mathrm{T}}G)^{-1}G^{\mathrm{T}}e_{N+1} \\
&= 0.
\end{aligned}$$

Consider the case when the searching direction vectors $\Delta u_1, \Delta u_2, \ldots, \Delta u_k$ become linearly dependent. Let Δu_k be a linear combination of the previous iterations, we have

$$\Delta u_k = \xi_1\Delta u_1 + \ldots + \xi_{k-1}\Delta u_{k-1}. \tag{38}$$

For some scalar ξ_i, from (14), (17) and (38) we obtain that

$$\begin{aligned}
H_k\tilde{e}_k &= H_k G^{\mathrm{T}}G\Delta u_k \\
&= \xi_1 H_k G^{\mathrm{T}}G\Delta u_1 + \ldots + \xi_{k-1}H_k G^{\mathrm{T}}G\Delta u_{k-1} \\
&= \xi_1 H_k\tilde{e}_1 + \xi_{k-1}H_k\tilde{e}_{k-1} \\
&= \xi_1\Delta u_1 + \ldots + \xi_k\Delta u_{k-1} \\
&= \Delta u_k.
\end{aligned}$$

Since $\tilde{e}_k = \nabla F(u_{k+1}) - \nabla F(u_k) = \bar{e}_k - \bar{e}_{k+1} = H_k\bar{e}_k$ and $\Delta u_k = u_{k+1} - u_k = H_k\bar{e}_k$, we obtain that

$$H_k(\bar{e}_k - \bar{e}_{k+1}) = H_k\bar{e}_k,$$

which, by the non-singularity of H_k, implies that $\bar{e}_{k+1} = 0$, that is $\nabla F(u_{k+1}) = 0$. Thus, the input signal u_{k+1} is the solution. Since $\bar{e}_{k+1} = G^{\mathrm{T}}e_{k+1} = 0$ and G is nonsingular, then we have $e_{k+1} = 0$. □

Theorem 2 Assume that the tracking errors $\{e_k\}$ are generated by the SR2-ILC algorithm (6)–(11) and (16). Then, the tracking error e_k is reduced to zero at the most $(N+1)$th iteration. This implies that $e_{N+2} = 0$ if the $(N+1)$th iteration is performed.

Proof If the $(N+1)$th iteration is performed, it follows from (24) in Lemma 3 that the vectors of search directions $\Delta u_1, \Delta u_2, \ldots, \Delta u_N$ are conjugate with respect to the matrix $G^{\mathrm{T}}G$. Thus, the searching direction vectors $\Delta u_1, \Delta u_2, \ldots, \Delta u_N$ are linearly independent by Lemma 2. From (23) in Lemma 3, it yields

$$H_{N+1}\tilde{e}_j = \Delta u_j, \quad j = 1, 2, \ldots, N,$$

that is,

$$H_{N+1}G^{\mathrm{T}}G\Delta u_j = \Delta u_j, \quad j = 1, 2, \ldots, N.$$

Therefore, $H_{N+1} = (G^{\mathrm{T}}G)^{-1}$.

Additionally,

$$\begin{aligned}
e_{N+2} &= y_{\mathrm{d}} - Gu_{N+2} \\
&= y_{\mathrm{d}} - Gu_{N+1} - G\beta_N H_N\bar{e}_{N+1} \\
&= e_{N+1} - G\frac{\bar{e}_{N+1}^{\mathrm{T}}(H_{N+1}\bar{e}_{N+1})H_{N+1}\bar{e}_{N+1}}{(H_{N+1}\bar{e}_{N+1})^{\mathrm{T}}G^{\mathrm{T}}G(H_{N+1}\bar{e}_{N+1})} \\
&= e_{N+1} - \frac{G\cdot\bar{e}_{N+1}^{\mathrm{T}}\left(G^{\mathrm{T}}G\right)^{-1}\bar{e}_{N+1}\cdot\left(G^{\mathrm{T}}G\right)^{-1}\bar{e}_{N+1}}{\bar{e}_{N+1}^{\mathrm{T}}\left((G^{\mathrm{T}}G)^{-1}\right)^{\mathrm{T}}(G^{\mathrm{T}}G)\left(G^{\mathrm{T}}G\right)^{-1}\bar{e}_{N+1}} \\
&= e_{N+1} - G\frac{\bar{e}_{N+1}^{\mathrm{T}}\left(G^{\mathrm{T}}G\right)^{-1}\bar{e}_{N+1}}{\bar{e}_{N+1}^{\mathrm{T}}\left((G^{\mathrm{T}}G)^{-1}\right)^{\mathrm{T}}\bar{e}_{N+1}}\left(G^{\mathrm{T}}G\right)^{-1}\bar{e}_{N+1} \\
&= e_{N+1} - G(G^{\mathrm{T}}G)^{-1}\bar{e}_{N+1} \\
&= e_{N+1} - G(G^{\mathrm{T}}G)^{-1}G^{\mathrm{T}}e_{N+1} \\
&= 0.
\end{aligned}$$

□

Theorem 3 Assume that the tracking errors $\{e_k\}$ are generated by quasi-Newton-type optimized ILC algorithm (6)–(11) for both the correction forms (15) and (16) for all k, then the norm of tracking error is monotone decreasing, that is, $\|e_{k+1}\|_2 \leqslant \|e_k\|_2$.

Proof

$$\begin{aligned}
&\|e_{k+1}\|_2^2 - \|e_k\|_2^2 \\
&= \|y_{\mathrm{d}} - Gu_{k+1}\|_2^2 - \|e_k\|_2^2 \\
&= \|y_{\mathrm{d}} - G(u_k + \beta_k H_k\bar{e}_k)\|_2^2 - \|e_k\|_2^2 \\
&= \|y_{\mathrm{d}} - Gu_k - \beta_k GH_k\bar{e}_k\|_2^2 - \|e_k\|_2^2 \\
&= \|e_k - \beta_k GH_k\bar{e}_k\|_2^2 - \|e_k\|_2^2 \\
&= (e_k - \beta_k GH_k\bar{e}_k)^{\mathrm{T}}(e_k - \beta_k GH_k\bar{e}_k) - e_k^{\mathrm{T}}e_k \\
&= -2\beta_k\bar{e}_k^{\mathrm{T}}H_k\bar{e}_k + \beta_k^2(H_k\bar{e}_k)^{\mathrm{T}}(G^{\mathrm{T}}G)H_k\bar{e}_k \\
&= -2\frac{\bar{e}_k^{\mathrm{T}}(H_k\bar{e}_k)}{(H_k\bar{e}_k)^{\mathrm{T}}(G^{\mathrm{T}}G)(H_k\bar{e}_k)}\bar{e}_k^{\mathrm{T}}(H_k\bar{e}_k) \\
&\quad + \Big(\frac{\bar{e}_k^{\mathrm{T}}(H_k\bar{e}_k)}{(H_k\bar{e}_k)^{\mathrm{T}}(G^{\mathrm{T}}G)(H_k\bar{e}_k)}\Big)^2(H_k\bar{e}_k)^{\mathrm{T}}(G^{\mathrm{T}}G)(H_k\bar{e}_k) \\
&= -\frac{(\bar{e}_k^{\mathrm{T}}(H_k\bar{e}_k))^2}{(H_k\bar{e}_k)^{\mathrm{T}}(G^{\mathrm{T}}G)(H_k\bar{e}_k)} \\
&\leqslant 0.
\end{aligned}$$

Hence, the result $\|e_{k+1}\|_2 \leqslant \|e_k\|_2$ is proven. □

Remark 3 From Theorems 1–3, it is clarified that the SR1(SR2) ILC has the property of quadratic termination, that is, the tracking error e_k converges monotonically to zero at the most $(N+1)$th iteration. While as the NOILC in [2], Gradient-ILC in [16] and PA-ILC in [18] are convergent with nonzero quotient convergence factors. This implies that the proposed quasi-Newton ILC may guarantee zero-tracking error at a finite iteration whilst the NOILC in [2], Gradient-ILC in [16] and the PA-ILC in [18] only guarantee the tracking error is at most very small.

4 Numerical simulations

In order to manifest the feasibility and practicality of the proposed SR1-ILC (SR2-ILC) scheme to a wide range of systems, this section gives simulation results for three examples, of which Example 1 is a well-conditioned system whilst Example 2 is an ill-conditioned system and Example 3 is a real system. For the same initial input u_1, SR1-ILC and SR2-ILC can generate the same sequence of inputs u_k, the details can be referred to the articles [27] and [28].

Example 1 Consider the following discrete time-invariant SISO system, which was adopted in [29].

$$\begin{cases} x(t+1) = \begin{bmatrix} -0.1 & -0.1 \\ 0.1 & 0.78 \end{bmatrix} x(t) + \begin{bmatrix} -0.1 \\ -0.1 \end{bmatrix} u(t), \\ y(t) = \begin{bmatrix} 0.5 & 0 \end{bmatrix} x(t). \end{cases} \tag{39}$$

The desired trajectory is predetermined as

$$y_{\mathrm{d}}(t) = 0.5\exp(\frac{t}{100})\sin(\frac{6t}{50}),$$
$$t \in I = \{0, 1, \dots, 100\}.$$

The initial state is set $x_k(1) = [0 \ \ 0]^{\mathrm{T}}$ and the beginning input $u_1(t) = 0$. It is calculated that the condition number of the matrix $G^{\mathrm{T}}G$ is 1.6145 and the largest singular value of $G^{\mathrm{T}}G$ is 0.3524. This means that the exampled system (39) is well-conditioned.

The comparative tracking errors in 2-norm of the SR1-ILC (SR2-ILC) algorithm with that of the norm optimal ILC (NOILC) [2] and gradient-based ILC (Gradient-ILC) [16] are presented in Fig. 1. The tracking outputs of the SR1-ILC (SR2-ILC) at the 5th iteration and that of the NOILC and Gradient-ILC are exhibited in Fig. 2. It is seen from Figs. 1 and 2 that the tracking error of the SR1-ILC (SR2-ILC) algorithm converges faster than the others.

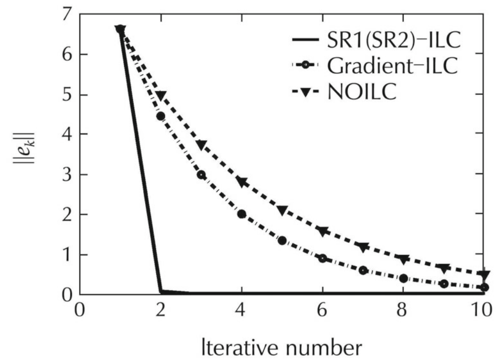

Fig. 1 Tracking errors of SR1(SR2)-ILC, Gradient-ILC and NOILC.

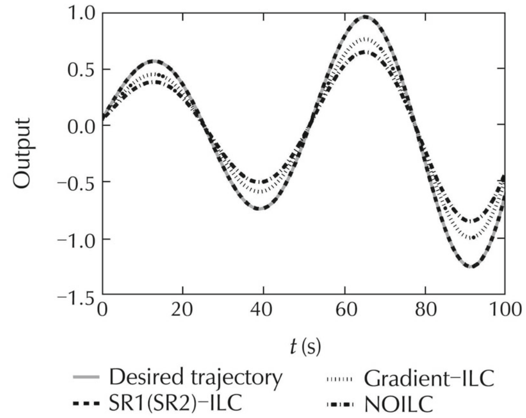

Fig. 2 Tracking outputs of SR1(SR2)-ILC, Gradient-ILC and NOILC at the 5th iteration.

Example 2 Consider the following discrete time-invariant SISO systems that was used in [30].

$$\begin{cases} x(t+1) = \begin{bmatrix} 0.8187 & 0 & 0 \\ 0.4526 & 0.9915 & 0 \\ 0.0023 & 0.0100 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.0197 \\ 0.0211 \end{bmatrix} u(t), \\ y(t) = [0 \ \ 0 \ \ 1]x(t). \end{cases} \tag{40}$$

The discrete time interval $t \in I = \{1, 2, \dots, 1200\}$. Set the initial state $x_k(1) = [0 \ 0 \ 0]^{\mathrm{T}}$, the beginning input $u_1(t) = 0$. The desired trajectory is given as $y_{\mathrm{d}}(t) = 10 \times (1 + \sin(\frac{\pi}{600}(t-1) - \frac{\pi}{2}))$. It is computed that the largest singular value of $G^{\mathrm{T}}G$ is 31.9026 and the condition number is 9.2306×10^6. This implies that system (40) is ill-conditioned. Fig. 3 displays the tracking errors in 2-norm of SR1-ILC (SR2-ILC), whilst Fig. 4 exhibits the tracking errors of SR1-ILC (SR2-ILC), NOILC in natural logarithm of 2-norm.

From Fig. 3, it is found that the tracking error of the SR1-ILC (SR2-ILC) converges to zero at the 10th iteration and Fig. 4 illustrates that the tracking error of the SR1-ILC (SR2-ILC) converges faster than that of NOILC algorithm. Additionally, Fig. 5 gives a comparable tracking errors of natural logarithm form of 2-norm of SR1-ILC (SR2-ILC) with those produced by the PA-ILC [18] with polynomial degree being $L = 1, 2, \ldots, 7$, respectively. It shows that the convergence of tracking error of SR1-ILC (SR2-ILC) is faster than the PA-ILC after the 15th iteration. As system (40) is extremely ill-conditioned, its convergent assumption with respect to the Gradient-ILC algorithm is not guaranteed.

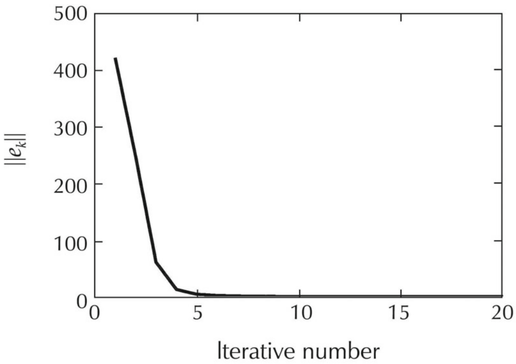

Fig. 3 Tracking error of SR1(SR2)-ILC.

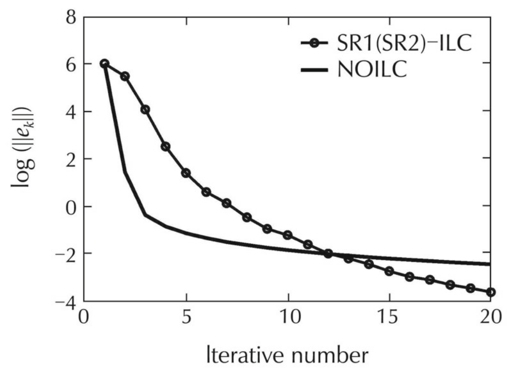

Fig. 4 Tracking errors of SR1(SR2)-ILC and NOILC.

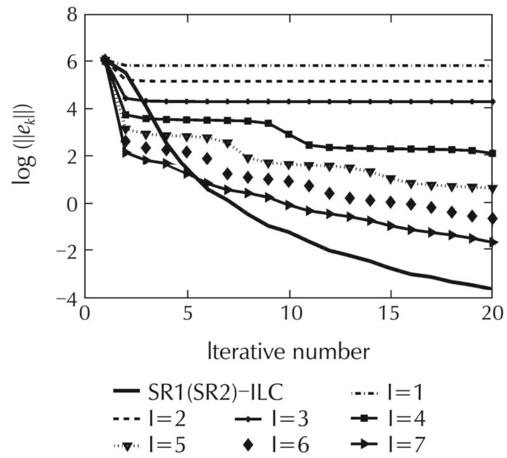

Fig. 5 Tracking errors of SR1(SR2)-ILC and PA-ILC.

Example 3 In microelectronics manufacturing, in order to guarantee the rapid thermal processing to work at the designed set-point, the temperature of the monocrystal reactor must be tuned to follow an operating trajectory [6]. As the rapid thermal processing is usually scheduled as a repetitive batch process, the ILC scheme is adequately to be utilized so that transient temperature of the reactor to follow a desired trajectory. Suppose that the transfer function of the reactor is identified as $G_{\mathrm{P}}(s) = \dfrac{K}{(\tau_{\mathrm{W}}s+1)(\tau_{\mathrm{L}}s+1)}$, where K is the process gain, τ_{W} denotes the heating time constant of the crystal and τ_{L} the heating time constant of the crystal light. Conventionally, the power ratio of the crystal light is tuned by a proportional-derivative-integral (PID) controller. Given that the transfer function of the PID controller is $G_{\mathrm{C}}(s) = \dfrac{K_{\mathrm{C}}}{1 + 1/\tau_{\mathrm{I}}s + \tau_{\mathrm{D}}s}$, where K_{C}, τ_{I} and τ_{D} are proportional, integral and derivative gains, respectively [6]. By converting the dynamics of frequency domain into that of time domain and then discretizing the PID-controller-tuned closed-loop control system with the 0.05s sampling step, the discrete time system is described as follows:

$$\begin{cases} x(t+1) = \begin{bmatrix} 1 & 0.05 & 0 \\ 0 & 1 & 0.05 \\ -0.05a_0 & -0.05a_1 & 1-0.05a_2 \end{bmatrix} x(t) \\ \qquad\qquad + \begin{bmatrix} 0 \\ 0 \\ 0.05 \end{bmatrix} u(t), \\ y(t) = [b_0 \ \ b_1 \ \ b_2] x(t), \end{cases} \tag{41}$$

where

$$a_0 = \frac{KK_{\mathrm{C}}}{\tau_{\mathrm{I}}\tau_{\mathrm{W}}\tau_{\mathrm{L}}}, \ a_1 = \frac{1+KK_{\mathrm{C}}}{\tau_{\mathrm{W}}\tau_{\mathrm{L}}},$$
$$a_2 = \frac{(\tau_{\mathrm{W}}+\tau_{\mathrm{L}}) + KK_{\mathrm{C}}\tau_{\mathrm{D}}}{\tau_{\mathrm{W}}\tau_{\mathrm{L}}},$$
$$b_0 = \frac{KK_{\mathrm{C}}}{\tau_{\mathrm{I}}\tau_{\mathrm{W}}\tau_{\mathrm{L}}}, \ b_1 = \frac{KK_{\mathrm{C}}}{\tau_{\mathrm{W}}\tau_{\mathrm{L}}}, \ b_2 = \frac{KK_{\mathrm{C}}\tau_{\mathrm{D}}}{\tau_{\mathrm{W}}\tau_{\mathrm{L}}}.$$

By setting a group of parameters as $K = 0.9$, $\tau_{\mathrm{W}} = 5$, $\tau_{\mathrm{L}} = 1$, $K_{\mathrm{C}} = 1.51$, $\tau_{\mathrm{I}} = 15$ and $\tau_{\mathrm{D}} = 3.33$. Set the initial state $x_k(0) = [0 \ \ 0 \ \ 0]^{\mathrm{T}}$ and the initial $u_1(t) = 0$, where $t \in [0, 100]$. The desired trajectory is defined as $y_{\mathrm{d}}(t) = 1 - \exp(-0.4t)$. The comparative tracking error of the proposed quasi-Newton ILC scheme with that of the Gradient-type ILC and the NOILC is exhibited in Fig. 6, whilst the outputs at the 12th iteration of the the proposed quasi-Newton ILC scheme, the Gradient-type ILC and the NOILC are displayed in Fig. 7.

It is seen from Figs. 6 and 7 that the proposed quasi-Newton SR1 (SR2)-ILC owns better tracking performance.

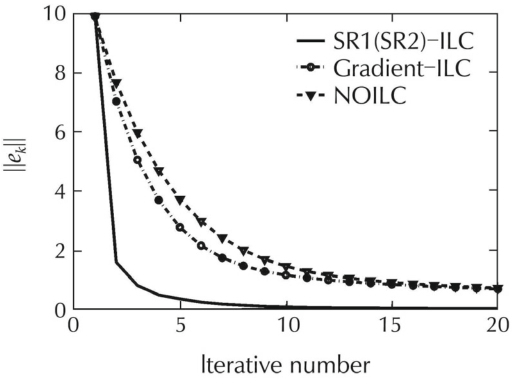

Fig. 6 Comparative tracking errors.

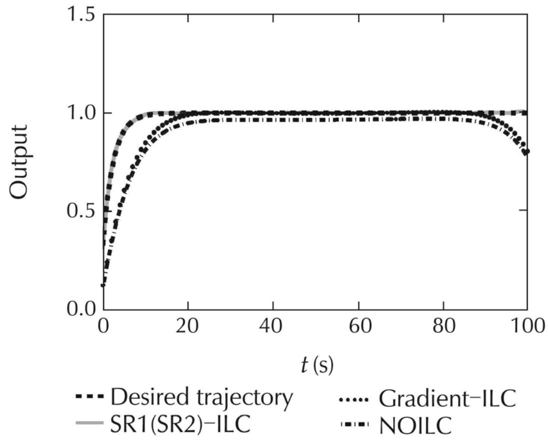

Fig. 7 Comparative tracking performance at the 12th iteration.

5 Conclusions

In this paper, a quasi-Newton-type optimized ILC is proposed for a class of discrete linear time-invariant SISO systems. The idea is to use an approximation matrix to replace the inverse model of the plant. The convergence analysis indicates that the proposed ILC algorithm performs well with the tracking error vanishing within finite iterations. Numerical simulations testify that the proposed quasi-Newton-type optimized ILC scheme is effective though the system is ill-conditioned. However, the proposed scheme requires a precise knowledge of the system. In reality, the system is unavoidably perturbed by noise and sometimes the system is nonlinear. How to solve the perturbation and the nonlinearity is challenging in the future.

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