



# Electrocardiogram (ECG) pattern modeling and recognition via deterministic learning

Xunde DONG<sup>1</sup>, Cong WANG<sup>1†</sup>, Junmin HU<sup>1</sup>, Shanxing OU<sup>2</sup>

*1.School of Automation Science and Engineering, South China University of Technology, Guangzhou Guangdong 510640, China;*

*2.Department of Radiology, General Hospital of Guangzhou Military Command, Guangzhou Guangdong 510010, China*

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## Abstract:

A method for electrocardiogram (ECG) pattern modeling and recognition via deterministic learning theory is presented in this paper. Instead of recognizing ECG signals beat-to-beat, each ECG signal which contains a number of heartbeats is recognized. The method is based entirely on the temporal features (i.e., the dynamics) of ECG patterns, which contains complete information of ECG patterns. A dynamical model is employed to demonstrate the method, which is capable of generating synthetic ECG signals. Based on the dynamical model, the method is shown in the following two phases: the identification (training) phase and the recognition (test) phase. In the identification phase, the dynamics of ECG patterns is accurately modeled and expressed as constant RBF neural weights through the deterministic learning. In the recognition phase, the modeling results are used for ECG pattern recognition. The main feature of the proposed method is that the dynamics of ECG patterns is accurately modeled and is used for ECG pattern recognition. Experimental studies using the Physikalisch-Technische Bundesanstalt (PTB) database are included to demonstrate the effectiveness of the approach.

**Keywords:** ECG; Pattern recognition; Deterministic learning; Dynamics; Temporal features

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## 1 Introduction

As a valuable tool for patient monitoring or diagnosis in clinical practice, an electrocardiogram (ECG) is widely used for the detection of a broad range of cardiac conditions, e.g., heart rate variability (HRV), myocardial ischemia and myocardial infarction. Due to the enormous volume of the non-stationary ECG data available,

ECG analysis is very time consuming. Thus, automatic ECG recognition and analysis is very important in detecting cardiac disease. The process of automatic ECG recognition generally consists of two steps: i) the extraction and selection of ECG pattern features, and ii) the design of classification systems. The performance of ECG pattern classification strongly depends on the characterization power of the features extracted from

<sup>†</sup>Corresponding author.

E-mail: wangcong@scut.edu.cn. Tel.: +86-20-87114612.

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the ECG data and the design of the classification model [1]. ECG features can be extracted in time domain, in frequency domain, or represented as statistical measures. Lots of schemes using different techniques have been proposed for feature extraction, such as wavelet transform (WT) [1–10], principal component analysis (PCA) [11–13], KL transforms [14], Hermite function [15], and Hilbert transform [16]. For the design of ECG classification systems, many schemes have also been presented, including morphology method [17], fuzzy inference engines [18], particle swarm optimization [1, 19], support vector machines [20–22], hidden Markov model [23], independent component analysis [24, 25], nearest neighbor method [9], linear discriminant analysis [26, 27], and artificial neural network (ANN). In particular, ANN has often been proposed as tools for realizing classifiers that are able to deal even with nonlinear discrimination between classes and to accept incomplete or ambiguous input patterns [28]. Many promising ANN-based techniques have also been applied to the ECG pattern recognition and classification, including multilayer perceptrons (MLPs) [29–34], fuzzy neural networks [35–39], modular neural network [40, 41], radial basis function network [42–44], etc.

Although much progress has been achieved, so far fully automatic classification of ECG patterns is still a challenging problem. The main difficulty lies in the significant variations in the morphologies of ECG waveforms for different patients and under different temporal and physical conditions [1, 34]. For example, in some cases patients with identical defects may not have completely similar ECG waveforms, while in other cases two various diseases may have nearly the same ECG signals [45]. Moreover, in healthy individuals as well as within different patient categories, a large interindividual variability in the ECG waveforms is presented [46]. The existence of large variations in the morphologies of ECG waveforms is mainly due to the fact that ECG waveforms are temporal or dynamical patterns describing electrical activities of the heart. Thus, automatic classification and diagnosis of ECG signals virtually belongs to the problem of temporal pattern recognition. As a matter of fact, recognition of temporal patterns is among the most difficult tasks in the pattern recognition area [47]. One challenging issue is how to appropriately represent the time-varying patterns in a time-independent manner. Further, similarity definition and recognition of temporal patterns are also difficult problems. It has been pointed

out that the methods for temporal pattern processing should be fundamentally different from those for static pattern processing [48].

Recently, a deterministic learning theory [49–51] was proposed for representation, similarity definition and rapid recognition of temporal patterns. This theory was mainly developed using concepts and theories of system identification, adaptive control, and radial basis function (RBF) networks. By using the deterministic learning theory, the dynamics of a temporal pattern is accurately modeled and then the temporal pattern can be effectively represented in a time-invariant and spatially distributed manner. That is, a temporal pattern is represented as a set of constant weights of an RBF network. Moreover, this representation contains complete information of its state trajectory and its underlying system dynamics along the state trajectory. A similarity definition of temporal patterns was given based on system dynamics. With the time-invariant representation and the similarity definition of temporal patterns, a mechanism for rapid recognition of temporal patterns was proposed in [50, 51]. In the recognition mechanism, the learned knowledge can be quickly recalled and used for recognition. A bank of estimators are constructed using the time-invariant representation. By comparing the set of estimators with a test pattern, a set of recognition errors are generated, and are taken as the similarity measure between the training patterns and the test pattern. The recognition of a test dynamical pattern is achieved rapidly because the recognition process takes place from the beginning of measuring the state of the test pattern, without identifying the system dynamics of the test pattern and so without comparing system dynamics of corresponding dynamical patterns via numerical computation. Furthermore, as complete information of temporal patterns is used in the recognition mechanism, it will be more suitable for accurate recognition of complex temporal patterns.

In this paper, we present a method for ECG pattern modeling and recognition via deterministic learning theory, aiming at classify ECG signals into different types corresponding to different heart diseases. The proposed method is completely based on the temporal features (i.e., the dynamics) of ECG patterns rather than statics features. The underlying dynamics within an ECG pattern is accurately modeled by using RBF neural networks (NNs). On this basis, an ECG pattern is represented by using complete information of its state trajectory and its underlying system dynamics along the

state trajectory. Based on the representation, a mechanism of recognition of ECG patterns with low computational effort is presented. Since complete information of ECG patterns is used for automatic classification, the presented method is appropriate to solve the problems caused by the large variations of ECG. To test the proposed method, two types of ECG recordings, healthy recordings and myocardial infarction (MI) recordings, taken from Physikalisch-Technische Bundesanstalt (PTB) database [52] are used in the paper. Experimental results demonstrate the effectiveness of the proposed method.

Compared with existing results on ECG pattern modeling and recognition the proposed method has the following features: 1) In existing studies, each heartbeat was recognized and classified as one type of beats (e.g., normal beat, premature ventricular contraction, atrial premature beat). However, in this paper, each ECG signal contains a number of heartbeats, and is recognized and classified as one type of heart diseases. The objectives of the two kinds of methods are different. 2) In most existing ECG modeling studies, it is the states of ECG signals that were modeled. However, in this paper, it is the underlying system dynamics rather than the states of ECG signals that are accurately modeled and then used for ECG pattern recognition. Especially, it is accurately modeled by using the deterministic learning theory. 3) The recognition process of a test ECG pattern takes place from the beginning of measuring the state of test ECG pattern, without numerical computation associated with identifying the test ECG pattern dynamics and comparison of system dynamics of the two ECG patterns. Thus, the recognition can be achieved with low computational effort.

The remainder of this paper is organized as follows. A brief introduction of the deterministic learning is presented in Section 2. Section 3 is devoted to the dynamical model used for showing the proposed method. In Section 4, on the basis of the dynamical model mentioned in Section 3, we present the mechanism of ECG pattern recognition of via deterministic learning. In Section 5, two types of ECG recordings taken from PTB database are used to demonstrate the effectiveness of the proposed method. Section 6 is a discussion of the proposed method. Section 7 concludes the paper.

## 2 Deterministic learning

Deterministic learning theory was proposed for NN approximation of nonlinear dynamical systems [49].

Among various types of NN architectures, a dynamical version of the localized RBF neural networks is used in deterministic learning theory. The RBF networks can be considered as two-layer networks in which the hidden layer performs a fixed nonlinear transformation with no adjustable parameters; that is, the input space is mapped into a new space. The output layer then combines the outputs in the latter space linearly. Therefore, they belong to a class of linearly parameterized networks. It has been proven that an RBF network can approximate any continuous function (i.e., universal approximation) to arbitrary accuracy. Usually, it can be described in the following form:

$$f(x) = \sum_{i=1}^N w_i s_i(x) = W^T S(x), \tag{1}$$

where  $x$  is the input vector,  $W = [w_1 \cdots w_N]^T \in \mathbb{R}^N$  is the weight vector,  $N > 1$  is the NN node number, and  $S(x) = [s_1(\|x - \xi_1\|) \cdots s_N(\|x - \xi_N\|)]$ , is the regressor vector, with  $s_i(\cdot)$  being a radial basis function, and  $\xi_i$  ( $i = 1, \dots, N$ ) being distinct neurons in state space. The radial basis function used in deterministic learning is Gaussian function, which has the following form:  $\phi(\|x - \xi_i\|) = \exp[-\frac{(x - \xi_i)^T(x - \xi_i)}{\eta^2}]$ , where  $\eta$  is the bandwidth of the each neuron.

Consider a general nonlinear dynamical system in the following form:

$$\dot{x} = f(x;p), \quad x(t_0) = x_0, \tag{2}$$

where  $x$  is the state of the system which is measurable,  $p$  is a constant vector of system parameters, and  $f(x;p)$  is a continuous but unknown nonlinear function.

To achieve identification of the unknown system dynamics  $f(x;p)$ , the following dynamical model using the RBF network is employed:

$$\dot{\hat{x}} = -a(\hat{x} - x) + \hat{W}^T S(x), \tag{3}$$

where  $\hat{x}$  is the state of the dynamical model,  $x$  is the state of system (2),  $a > 0$  is a design constant, RBF network  $\hat{W}^T S(x)$  are used to approximate the unknown  $f(x;p)$  in equation (2) with  $\hat{W} = [w_1 \cdots w_N]^T \in \mathbb{R}^N$  and  $S(x) = [s_1(\|x - \xi_1\|) \cdots s_N(\|x - \xi_N\|)]^T$ ,  $s_i(\cdot)$  being Gaussian function, and  $\xi_i$  ( $i = 1, \dots, N$ ) being distinct points in state space. The weight estimates  $\hat{W}$  are updated by the following law:

$$\dot{\hat{W}} = \hat{W} = -\Gamma S(x)\tilde{x} - \sigma\Gamma\hat{W}, \tag{4}$$

where  $\Gamma = \Gamma^T > 0$ , and  $\sigma > 0$  is a small value. Based on the dynamical model (3) and the weight update law (4), the approximation for the unknown  $f(x;p)$  can be obtained:

$$f(x;p) = \bar{W}^T S(x) + \epsilon,$$

where  $\bar{W} = \text{mean}_{t \in [t_a, t_b]} \hat{W}$ , "mean" is the arithmetic mean,  $0 < t_a < t_b$  represents a piece of time segment after the transient process, and  $\epsilon$  is the approximation error which can be made arbitrarily small. Thus, the identification of  $f(x;p)$  using only the information of system state  $x$  is achieved, and is expressed as  $\bar{W}^T S(x)$ , a time-invariant manner (For details, see [51]).

**Remark 1** A temporal (dynamical) pattern is defined as a recurrent system trajectory generated from the dynamical system as system (2). The class of recurrent trajectories includes periodic, quasi-periodic, almost-periodic, and even chaotic trajectories. This definition of a dynamical pattern covers a wide class of temporal patterns studied in the literature.

### 3 Dynamical model

A dynamical model which is capable of replicating many of the important features of the human ECG had been introduced in [53]. It has a variable number of free parameters that make it adaptable to many normal and abnormal ECG signals [54]. By changing parameters of the dynamical model, different morphologies for the PQRST-complex can be generated. The effectiveness of different techniques for ECG analysis could be assessed by using the synthetic ECG. For example, in [55], the data generated by mixing the synthetic ECG signals with random realizations of ECG noise are used to evaluate performance of various of ECG enhancers. In [56], the dynamical model is used to quantify the errors in spectral estimates of HRV due to resampling and beat replacement. With the simplicity and flexibility, this model can be easily used as a base for ECG processing [57], some modified nonlinear dynamic models have developed and are used for ECG denoising and baseline wandering [54, 57].

To illustrate the proposed method, a dynamical model which is a little different from the dynamical model proposed in [53] is used in this paper. The difference is that the baseline wander, which was modeled in the dynamical model proposed in [53], is not considered in our study, since the purpose of this paper is to propose a new method for ECG pattern recognition rather than for

ECG preprocessing. It is given by a set of three ordinary equations as follows:

$$\begin{cases} \dot{x}_1 = \alpha x_1 - \omega x_2, \\ \dot{x}_2 = \alpha x_2 + \omega x_1, \\ \dot{x}_3 = f(x;p), \end{cases} \quad (5)$$

where  $\alpha = 1 - \sqrt{x_1^2 + x_2^2}$ ,  $\Delta\theta_l = \theta - \theta_l$ ,  $\theta = \text{atan2}(x_2, x_1)$ ,  $\omega$  is the angular velocity of the trajectory as it moves around the limit cycle in the  $(x_1, x_2)$  plane,

$$f(x;p) = - \sum_{l \in \{P, Q, R, S, T\}} A_l \Delta\theta_l \exp\left(-\frac{\Delta\theta_l^2}{2B_l^2}\right)$$

with  $x = [x_1 \ x_2 \ x_3]$ ,  $p = [A_l \ B_l \ \theta_l]$  being the constant vector of system parameters,  $A_l, l \in \{P, Q, R, S, T\}$  represent the amplitudes of the  $P, Q, R, S, T$  waves;  $B_l, l \in \{P, Q, R, S, T\}$  represent the width (i.e., time duration) of  $P, Q, R, S, T$  waves;  $\theta_l, l \in \{P, Q, R, S, T\}$  represent the locations on the circle where  $P, Q, R, S, T$  waves occur [58]. A three-dimensional trajectory generated by (5) corresponding to  $(x_1, x_2, x_3)$  is illustrated in Fig. 1 and is denoted as  $\varphi_\zeta$ . The  $x_3$  variable from the dynamical model (5) yields a synthetic ECG with realistic PQRST morphology (Fig. 2). In the following section, the dynamical model and the trajectory will be used to illustrate the proposed method.

**Remark 2** In the dynamical model proposed in [53], the baseline wander is modeled with the  $x_{30}$  which is assumed to be a relatively low amplitude sinusoidal component coupled with the respiratory frequency [54], where

$$f(x;p) = - \sum_{l \in \{P, Q, R, S, T\}} A_l \Delta\theta_l \exp\left(-\frac{\Delta\theta_l^2}{2B_l^2}\right) - (x_3 - x_{30}).$$

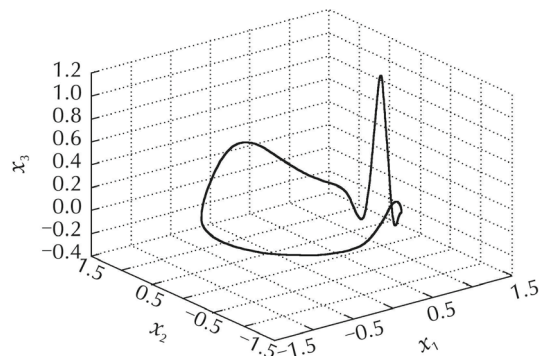


Fig. 1 Trajectory generated by the dynamical model (5) with certain parameters.

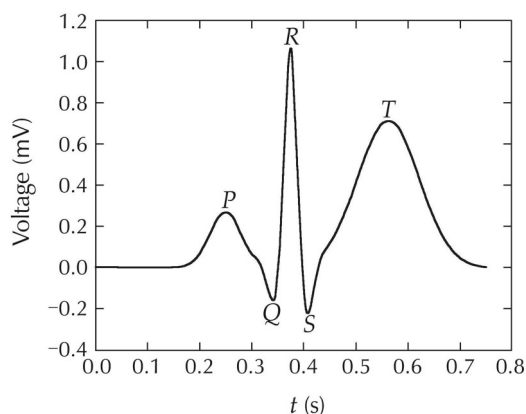


Fig. 2 Morphology of one PQRST-complex of the ECG with big T-wave generated by dynamical model (5), corresponding to the  $x_3$  variable.

### 4 Methods

In this section, based on the dynamical model mentioned in Section 3, we present an approach for recognition of ECG patterns by using the deterministic learning theory. The approach will be shown in two phases: the training phase and the test phase. In the training phase, the dynamics of the ECG signals is accurately modeled and is expressed as constant RBF networks. In the test phase, a similarity definition of ECG patterns is given first. Based on the constant RBF networks and the similarity definition, we propose a mechanism for recognition of ECG patterns. For a test ECG pattern, a bank of estimators are constructed using the constant RBF networks. By comparing the set of estimators with the test ECG pattern, a set of recognition errors are generated and are taken as the similarity measure between the training ECG patterns and the test ECG pattern. According to the smallest error principle, the test ECG pattern can be recognized.

#### 4.1 Identification phase

In this section, we will discuss the identification of the ECG system dynamics  $f(x;p)$ . Based on the deterministic learning, the following dynamical model using the RBF networks is employed to identify the dynamics  $f(x;p)$  of system (5).

$$\dot{\hat{x}}_3 = -a(\hat{x}_3 - x_3) + \hat{W}^T S(X), \tag{6}$$

where  $X = [x_1 \ x_2]$ ,  $\hat{x}_3$  is the estimation of  $x_3$  in system (5),  $a > 0$  being design constants, RBF networks  $\hat{W}^T S(X)$  are used to approximate  $f(x;p)$ . The weight estimates  $\hat{W}$  are updated by the following law:

$$\dot{\hat{W}} = \dot{W} = -\Gamma(S(X)\tilde{x}_3 + \sigma\hat{W}), \tag{7}$$

where  $\Gamma = \Gamma^T > 0$ , and  $\sigma > 0$  is a small value.

Consider the adaptive system consisting of the non-linear dynamical system (5), the dynamical model (6), and the NN weight updating law (7). For the trajectory  $\varphi_\zeta$  shown in Fig. 1, with initial values  $\hat{W}(0) = 0$ , according to Theorem 1 in [59], we have: i) all signals in the adaptive system remain uniformly bounded; ii) the state estimation error  $\tilde{x}_3 = \hat{x}_3 - x_3$  converges to zero; iii)  $f(x;p) = \hat{W}^T S(X) + \epsilon$ , where  $\epsilon$  is the approximation error which can be made arbitrarily small. Thus, accurate identification of ECG system dynamics is achieved along the trajectory  $\varphi_\zeta$ . ECG patterns are effectively represented by the accurate NN approximations of system dynamics. The representation is time-invariant in the sense that it is independent of the time attribute. The representation is also spatially distributed, since fundamental information is stored in a large number of neurons distributed along the trajectory of the ECG pattern. Thus, complete information of both the ECG pattern state and the underlying ECG dynamics is utilized for appropriate representation of the ECG pattern. In other words, a complete representation of ECG patterns is achieved.

In order to show the identification effect of the proposed method, synthetic ECG signal shown in Fig. 2 is accurately identified. A dynamical model (such as (6)) corresponding to the synthetic ECG signal is first constructed using RBF networks. The weight estimates  $\hat{W}$  of the RBF network are updated by equation (7). According to Theorem 1 in [59], the weights of the RBF networks  $\hat{W}$  will be converge to their optimal values  $W^*$ . The convergence of the neural weights is shown in Fig. 3. It can be seen from this figure that the neural weights indeed converge to their optimal values. From Fig. 4, we can see that good NN approximations of  $f(x;p)$  is obtained using the proposed method. In other words, the accurate identification of  $f(x;p)$  is actually achieved.

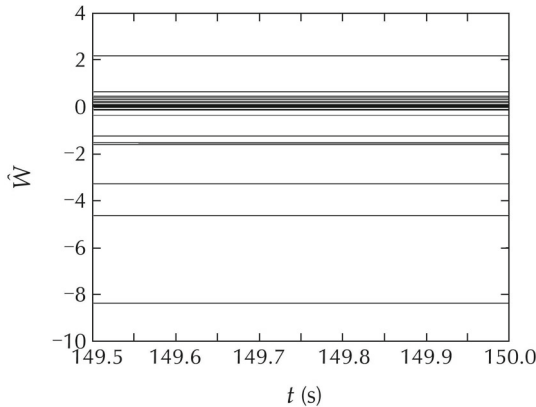


Fig. 3 Partial parameter convergence  $\hat{W}$  after the transient process. Each line corresponds to one neuron of the RBF network.

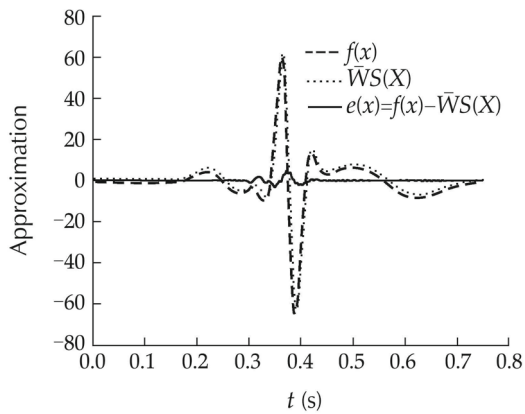


Fig. 4 Function approximation, where  $f(x)$  is known,  $\bar{W}^T S(X)$  is the approximation of  $f(x)$ , and  $e(X)$  is the approximating error.

### 4.2 Recognition phase

Based on the time-invariant representation of the system dynamics of ECG patterns (i.e., constant RBF neural weights), we propose a mechanism for recognition of ECG patterns. Using the constant RBF networks obtained in the training phase, we construct a dynamical model for each training ECG pattern [50]. For the  $k$ th ( $k = 1, \dots, M$ ) training ECG pattern  $\varphi_c^k$ , a dynamical model is constructed by using the time-invariant representation  $\bar{W}^{kT}$  as

$$\dot{\tilde{x}}_3^k = -b(\tilde{x}_3^k - x_3) + \bar{W}^{kT} S(x), \tag{8}$$

where  $\tilde{x}_3^k$  is the state of the dynamical (template) model,  $x = [x_1 \ x_2 \ x_3]$  is the state of the input test ECG pattern  $\varphi_c$  generated from equation (5), and  $b > 0$  is a design parameter that is kept the same for all training ECG patterns and normally smaller than  $a$  ( $a$  is given in equation

(6)). Then, corresponding to the test ECG pattern  $\varphi_c$  and the dynamical model (8) (for training pattern  $\varphi_c^k$ ), we obtain the following recognition error system:

$$\dot{\tilde{x}}_3^k = -b\tilde{x}_3^k + \bar{W}^{kT} S(x) - f(x, p), \tag{9}$$

where  $\tilde{x}_3^k = \tilde{x}_3^k - x_3$  is the state tracking (or synchronization) error.

According to Theorem 2 in [50], the state estimation errors  $\tilde{x}_3^k$  are approximately proportional to the differences between the system dynamics of test ECG pattern  $\varphi_c$  and the identified system dynamics of training ECG pattern  $\varphi_c^k$ . Thus, the synchronization errors can be taken as similarity measures between the test and the training ECG patterns. Accordingly, in this paper, we propose the following definition of similarity for ECG patterns.

**Definition 1** The test ECG pattern  $\varphi_c$  is recognized as similar to the training ECG pattern  $\varphi_c^k$  if the average  $L_1$  norm of the state estimation error  $\tilde{x}^k$  is the smallest.

$$\|\tilde{x}^k(t)\|_1 = \frac{1}{T} \int_{t-T}^t |\tilde{x}^k(\tau)| d\tau, \quad t \geq T, \tag{10}$$

where  $T$  is the period of the heart beat.

From the above analysis, we take the following method to recognize a test ECG pattern from a set of training ECG patterns:

- Identify the system dynamics of a set of training ECG patterns  $\varphi_c^k, k = 1, \dots, M$ .
- Construct a set of dynamical models (8) for the training dynamical patterns  $\varphi_c^k$ .
- Take the state  $x(t)$  of a test ECG pattern  $\varphi_c$  as the RBFN input to the dynamical models (8), and compute the average  $L_1$  norm of the state estimation error  $\tilde{x}^k$ .
- Take the training dynamical pattern whose corresponding dynamical model yields the smallest  $\|\tilde{x}^k\|_1$  as the one most similar to the test dynamical pattern  $\varphi_c$  in the sense of similarity definition given above.

**Remark 3** The recognition of a test ECG pattern can be achieved with low computational effort because the recognition process takes place from the beginning of measuring the state of the test ECG pattern, without feature extraction from test ECG pattern.

## 5 Results

A set of ECG records taken from PTB diagnostic ECG database [52] is used to evaluate the proposed method. PTB database is an ECG collection that was provided by

the National Metrology Institute of Germany for teaching and research purposes, and for algorithm evaluation [60]. It contains more than 27000 ECG records, and each record includes 15 simultaneously measured signals: the conventional 12 leads together with the three Frank leads ECG ( $V_X, V_Y, V_Z$ ). Especially, each record of PTB database has a detailed clinical summary, including age, gender, diagnosis and so on. In the database, 52 subject’s records diagnosed as healthy and 148 subject’s records diagnosed as myocardial infarction (MI) by physicians. A set containing 52 healthy ECG signals taken from the 52 healthy subjects and 148 MI ECG signals taken from the 148 MI subjects (one recording per patient) is used in our experiment. The data length is 10 seconds. Since ECG signal recorded in each lead is the projection of the heart vector onto the lead vector, so we only use the three Frank leads ECG to recognize and classify the ECG patterns. It is worth noting that, no matter identify or recognize a ECG pattern, ECG pre-processing have to be done to remove various noises, especially the baseline wandering since it will seriously affect the periodicity of the ECG signal.

Based on the ECG recordings taken from PTB database, four experiments are conducted in the paper. The performance is quantified by the following three indices: accuracy, specificity (SPE) and sensitivity (SEN), which are defined as follows:

$$\text{Accuracy} \stackrel{\text{def}}{=} \frac{TP + TN}{TP + TN + FP + FN} (\%), \quad (11)$$

$$\text{Specificity} \stackrel{\text{def}}{=} \frac{TN}{TN + FP} (\%), \quad (12)$$

$$\text{Sensitivity} \stackrel{\text{def}}{=} \frac{TP}{TP + FN} (\%), \quad (13)$$

where  $TP$  (true positives) is the number of correctly recognized MI ECG signals,  $TN$  (true negatives) is the number of correctly recognized healthy ECG signals,  $FP$  (false positives) is the number of erroneously recognized MI ECG signals, and  $FN$  (false negatives) is the number of erroneously recognized healthy ECG signals.

In the first experiment, 10 healthy ECG signals and 25 MI ECG signals are randomly selected as training patterns. The remaining ECG signals, 42 healthy ECG signals and 123 MI ECG signals, are used as test patterns. In the second experiment, 25 healthy ECG patterns and 50 MI ECG patterns are selected as training patterns. Correspondingly, the remaining 25 healthy ECG signals and 98 MI ECG signals are used as test patterns. The obtained test recognition accuracies of experiment 1 and experiment 2 are 81% and 90%. Detailed results of the two experiments are given in Table 1, where H and MI denote healthy and MI ECG signal, N denotes the number of mis-recognized ECG signals.

To further estimate the performance of the proposed method,  $k$ -fold cross-validation method is used in the following two experiments. It is a standard technique in machine learning and is popular for estimating generalization ability of a classifier. The data set is divided into  $k$  subsets, and the proposed method is repeated  $k$  times. Each time, one of the  $k$  subsets is used as the test set and the other  $k - 1$  subsets are put together to form a training set. Then, the average error across all  $k$  trials is computed. In the paper, 3- and 5-fold cross-validation are used to further estimate the proposed method. The obtained test recognition accuracies of 3- and 5-fold cross-validation are 89.5% and 85.5%, respectively. Detailed results are given in Tables 2 and 3, where H, MI and N are the same as in Table 1.

Table 1 Recognition results of Experiments 1 and 2.

Experiment	Training number			Test number			Accuracy (%)	SPE (%)	SEN (%)	N
	H	MI	Total	H	MI	Total				
1	10	25	35	42	123	165	81	69	85	31
2	25	50	75	27	98	125	90	85	92	12

Table 2 Recognition results of Experiment 3.

Fold	Training number			Test number			Accuracy (%)	SPE (%)	SEN (%)	N
	H	MI	Total	H	MI	Total				
1	35	99	134	17	49	66	87.8	70.6	91.8	9
2	35	99	134	17	49	66	90.9	88.2	91.8	6
3	34	98	132	18	50	68	89.7	77.8	94.0	7
Average results							89.5	78.9	92.5	7.3

Table 3 Recognition results of Experiment 4.

Fold	Training number			Test number			Accuracy (%)	SPE (%)	SEN (%)	N
	H	MI	Total	H	MI	Total				
1	42	118	160	10	30	40	87.5	70.0	93.3	5
2	42	118	160	10	30	40	85.0	70.0	90.0	6
3	42	118	160	10	30	40	87.5	80.0	90.0	5
4	41	119	160	11	29	40	87.5	81.8	89.6	5
5	42	118	160	11	29	40	80.0	63.4	85.8	8
Average results							85.5	73.0	89.9	5.8

As an example, identification results of s0302lrem (patient 116) are given as follows. After denoising, space vector ( $V_X, V_Y, V_Z$ ) of s0302lrem is shown in Fig. 5. As mentioned in Section 4.1, three dynamical models using the RBF networks are employed to identify the dynamics of  $V_X, V_Y$  and  $V_Z$ . The neural weights estimates  $\hat{W}_X, \hat{W}_Y$  and  $\hat{W}_Z$  are updated along the trajectory shown in Fig. 5. As described for Fig. 3 in the last paragraph of Section 4.1, the neural weights of the three RBF networks are converge to their optimal values. The convergence of the neural weights is shown in Fig. 6. It can be seen from Fig. 6 that the neural weights are essentially unchanged during the last two seconds, in other words, the weights estimates indeed converge to their optimal values. The underlying system dynamics along the state trajectory shown in Fig. 5 is identified and is shown in Fig. 7. The dynamics of the other training ECG patterns can also be identified by the same procedure. Then, a pattern library can be obtained.

With a pattern library, a test ECG pattern can be recognized according to the similarity definition of ECG patterns. As an example, a healthy ECG data s0336lrem (patient 185) is recognized by using the proposed method, with the pattern library of the first experiment. A set of dynamical models for the training ECG patterns are constructed first, and the state of s0336lrem is used as the RBF network input to the dynamical models. Then, comparing the test ECG pattern s0336lrem with the dynamical models, we obtain a set of state tracking errors  $\tilde{x}^k, k = 1, 2, \dots, 35$ , where  $\tilde{x} = (\tilde{V}_X, \tilde{V}_Y, \tilde{V}_Z)$ . The average  $L_1$  norms of  $\tilde{x}^k$  is computed as follows:

$$\|\tilde{x}^k\|_1 = \frac{1}{3}(\|\tilde{V}_X^k\|_1 + \|\tilde{V}_Y^k\|_1 + \|\tilde{V}_Z^k\|_1). \quad (14)$$

The average  $L_1$  norms of  $\tilde{x}^k$  for the 35 training patterns are shown in Fig. 8. According to the smallest error principle, test ECG pattern s0036lrem is recognized as similar to training pattern s0302lrem. That is, ECG pattern

s0036lrem is classified into healthy group.

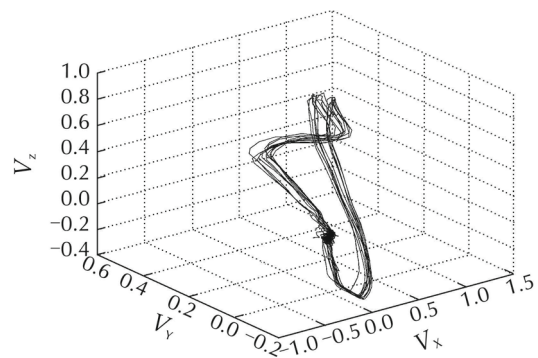
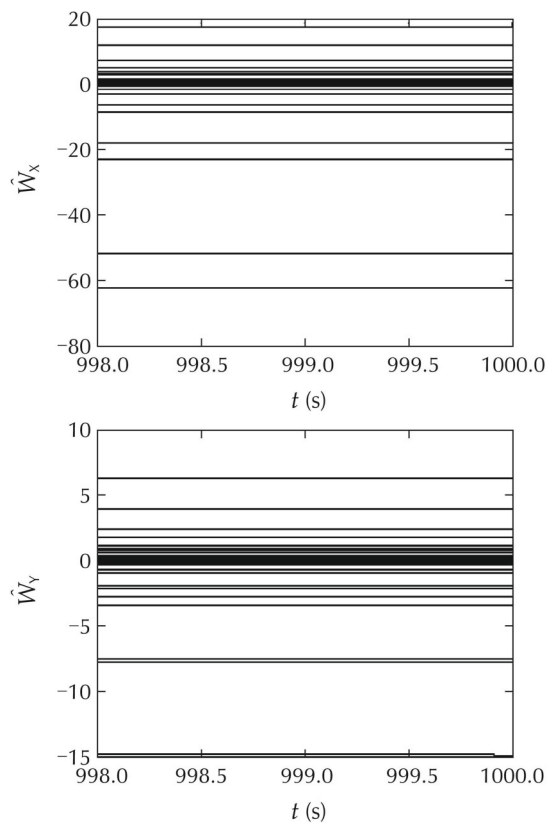


Fig. 5 The space vector of s0302lrem with 10 seconds length (patient 116).





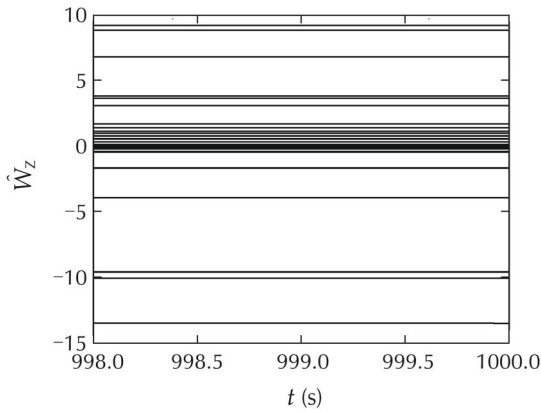


Fig. 6 Partial parameter convergence  $\hat{W}_X$ ,  $\hat{W}_Y$  and  $\hat{W}_Z$  after the transient process, which are corresponding to the three Frank lead ECGs  $V_X, V_Y, V_Z$ . Each line corresponds to one neuron of the RBF network.

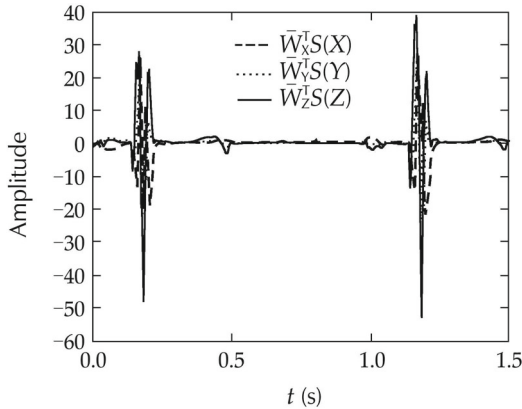


Fig. 7 The dynamics of s0302lrem (the first 1.5 seconds) we identified,  $\hat{W}_X^T S(X)$ ,  $\hat{W}_Y^T S(Y)$  and  $\hat{W}_Z^T S(Z)$ , corresponding to the three Frank leads ECG.

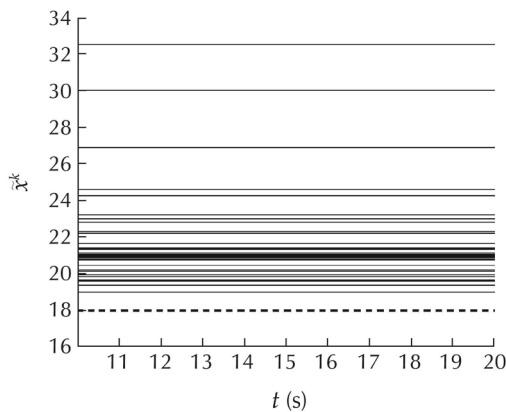


Fig. 8 Average  $L_1$  norms of  $\hat{x}^k$  for the 35 training patterns, where  $\hat{x}^k, k = 1, 2, \dots, 35$  are the state tracking errors obtained by comparing the test ECG pattern s0036lrem with the 35 training ECG patterns. The dash line corresponds to the training ECG pattern s0302lrem, a healthy ECG data (patient 116).

## 6 Discussion

As shown in Section 4.1, a synthetic ECG signal shown in Fig. 2 is generated by the dynamical model (5), and the dynamics  $f(x;p)$  of the synthetic ECG signals is accurately modeled and represented as constant RBF networks. Likewise, the dynamics of practical ECG signals can also be accurately modeled, an example is given in Section 5. On this basis, four experiments with different number of training ECG patterns are given. The number of the training ECG patterns of the second experiment is more than 2 times of the first experiment. From the results of the two experiments, we can see that the accuracy of the second experiment is higher than the first experiment. Especially, the accuracy of the second experiment for healthy and MI ECG patterns are 16 percentage points and 7 percentage points higher than the first experiment. This shows that the accuracy can be improved by increasing the number of the training ECG patterns.

From Tables 2 and 3, we can see the accuracies of 3- and 5-fold cross-validation are 89.5% and 85.5%, respectively. There is a little difference between the accuracies of the two experiments. Additionally, the accuracy of experiment 2 (90%) is very close to experiment 3 (89.5%). These show the proposed method has good generations. On the another hand, these also show the accuracy based on the set used in the paper is difficult to further improve, since the number of the records is very limited and the records (52 healthy ECG signals and 148 MI ECG signals) are far from representative of all of the healthy ECG patterns and MI ECG patterns.

As the existence of individual differences in human heart, the recognition accuracy based on a small pattern library is more dependent on the selection of training ECG patterns. That is the main reason for the lower accuracy of the first experiment (which can be seen in Table 1). Also for the reason, there is a large variation of specificity of different folds (e.g., the difference between the specificity of Folds 1 and 4 of 5-fold cross-validation experiment is 11.8 percentage point). Moreover, the training ECG patterns used in the experiments may not be universally representative, since the impact of various factors (e.g., ages, gender, physical conditions) for ECG patterns is not taken into account. However, in addition to the individual differences, there also exist similarities in the mechanism and physiological among patients with the same disease. Thus, if there is a large pattern library containing adequate ECG patterns with different

ages and different physical conditions (e.g., healthy conditions, various diseases), most of test ECG pattern will be accurately recognized. With further increase of the training ECG pattern, the accuracy will be satisfactory for the clinical application.

Thus, the method may be used in automatic detection of heart diseases in the near future. A large number of ECG patterns can be obtained conveniently in clinical. ECG recordings of various heart diseases diagnosed by physicians can be added in a ECG pattern library. Based on a very large scale pattern library which contains ECG patterns with various heart diseases, a more accurate detection of heart diseases will be achieved. As the rapid development of computer technology, the establishment and processing of a very large scale pattern library is not a problem. As an estimated  $10^6$  ECGs performed per day [61], the proposed method will be very helpful to improve diagnosis efficiency and to reduce workload of physicians.

## 7 Conclusions

In this paper, we have proposed an approach for recognition of ECG signals. Each ECG signal is recognized rather than beat-to-beat, which is completely different from existing studies. It includes the following elements: a time-invariant representation for ECG patterns, a similarity measure based on ECG dynamics, a mechanism for ECG pattern recognition with low computational effort. It has been shown, that the dynamics of ECG is effectively expressed as constant RBF networks. With the time-invariant manner, a set of estimators are constructed. For a test ECG pattern, a set of errors are obtained by comparing it with the set of dynamical estimators. Based on the similarity definition of ECG patterns, the test ECG pattern is recognized according to the smallest residual error principle. Compared with the existing ECG recognition approaches, the main advantage of the proposed method is that the dynamics which contains complete information of ECG patterns is used for recognition. Another advantage is that it does not need to extract various static features from test ECG pattern. Moreover, numerical computation associated with identifying test ECG pattern dynamics and comparison of dynamics of two ECG patterns is also not required. Accordingly, various complex algorithms for features extraction are avoided and the recognition can be achieved with low computational effort.

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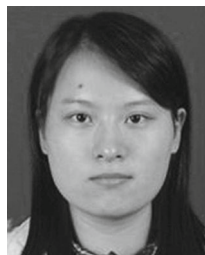
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**Xunde DONG** is a Ph.D. candidate at the Center for Control and Optimization, School of Automation, South China University of Technology. His research interest covers adaptive NN control/identification, deterministic learning theory and distributed parameter system. E-mail: dong.xd@mail.scut.edu.cn.



**Cong WANG** received the B.E. and M.E. degrees from Beijing University of Aeronautic & Astronautics in 1989 and 1997, respectively, and the Ph.D. degree from the Department of Electrical & Computer Engineering, The National University of Singapore in 2002. From 2001 to 2004, he did his postdoctoral research at the Department of Electronic Engineering, City University of Hong Kong. He has been with the School of Automation, South China University of Technology, Guangzhou, China, since 2004, where he is currently a professor. He has authored and co-authored over 60 international journal and conference papers and the book “Deterministic Learning Theory for Identification, Recognition and Control”. He serves as an associate editor of the *IEEE Transactions on Neural Networks and Learning Systems* since 2012, and as an Associate Editor for *Journal of Control Theory & Applications*, and *ACTA AUTOMATI-CA SINICA* (two best journals in systems and control area in China) since 2008 and 2011, respectively. He is a member of the Technical Committee on Intelligent Control of the IEEE CSS. His research interest includes intelligent control, neural networks, nonlinear systems and control, dynamical pattern recognition, pattern-based control, dynamical systems, and oscillation fault diagnosis. E-mail: wangcong@scut.edu.cn.



**Junmin HU** is a Ph.D. candidate at the Center for Control and Optimization, School of Automation, South China University of Technology. Her research interest covers adaptive NN control/identification, deterministic learning theory and oscillation fault diagnosis. E-mail: hujunmin852@163.com.



**Shanxing OU** is a doctor of Medicine, and is currently director of the Department of Radiology, General Hospital of Guangzhou Military Command, Guangzhou, China. He is a member of the standing committee of the Chinese People’s Liberation Army society of radiology. He serves as the chairman of committee of Guangzhou Military Command society of radiology. His research interest includes cardiothoracic surgery disease, the imaging diagnosis of heart diseases, coronary heart disease, and myocardial ischemia. E-mail: oushanxing@sina.com.