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Second-order terminal sliding mode control for hypersonic vehicle in cruising flight with sliding mode disturbance observer

Ruimin ZHANG [1,2], Changyin SUN [1†], Jingmei ZHANG [1], Yingjiang ZHOU [1]
1.School of Automation, Southeast University, Nanjing Jiangsu 210096, China;
2.School of Mathematics & Statistics, Henan University of Science & Technology, Luoyang Henan 471003, China

Abstract: This paper focuses on the design of nonlinear robust controller and disturbance observer for the longitudinal dynamics of a hypersonic vehicle (HSV) in the presence of parameter uncertainties and external disturbances. First, by combining terminal sliding mode control (TSMC) and second-order sliding mode control (SOSMC) approach, the second-order terminal sliding control (2TSMC) is proposed for the velocity and altitude tracking control of the HSV. The 2TSMC possesses the merits of both TSMC and SOSMC, which can provide fast convergence, continuous control law and high-tracking precision. Then, in order to increase the robustness of the control system and improve the control performance, the sliding mode disturbance observer (SMDO) is presented. The closed-loop stability is analyzed using the Lyapunov technique. Finally, simulation results illustrate the effectiveness of the proposed method, as well as the improved overall performance over the conventional sliding mode control (SMC).

Keywords: Hypersonic vehicle; Second-order sliding mode control; Terminal sliding mode control; Sliding mode disturbance observer

1 Introduction

The flight control design of the hypersonic vehicle (HSV) is a significant challenge due to the particular characteristics of its dynamics. Airframe/engine integrated design leads to strong coupling between the propulsion system, the structural dynamics and the airframe. High Mach numbers and altitude make the flight control extremely sensitive to changes in atmospheric conditions as well as physical and aerodynamic parameters. As a result, HSV is a highly nonlinear, strongly coupling, uncertain system. Several nonlinear controllers have been presented in the literatures for the tracking control of the HSV. A backstepping control approach was used by [1–2] to design nonlinear controllers to achieve good-tracking performance. In [3–4], neural network technique was used to develop nonlinear adaptive controllers. The proposed neural adaptive controllers can guarantee the stability of the closed-loop system under parameter uncertainties. Durmaz [5] designed a sliding mode controller with adaptive sliding surfaces which was robust to external disturbances. In addition, some nonlinear control strategies based on input-output linearization technique have been proposed to design tracking control systems, such as sliding mode control [6–9], dynamic inversion control [10].

Among these control approaches, SMC attracts extensive attention due to its simplicity and robustness to parameter uncertainties and external disturbances. However, there are also some disadvantages in SMC. The first is the well-known chattering phenomenon which can be harmful to the actuators. Especially, when large parameter uncertainties or external disturbances appear, chattering will be aggravated. In order to alleviate chattering, a boundary layer approach is usually adopted [6–8]. However, this implies a small deterioration in accuracy and robustness. The second is that the convergence rate of conventional SMC with a linear sliding surface is relatively slow. By contrast, TSMC has some superior properties such as fast, finite time convergence and higher-tracking precision [11]. Therefore, a terminal sliding mode controller in [9] was proposed to achieve fast-tracking control for the HSV. Moreover, chattering still exists in TSMC.

In order to eliminate chattering and improve convergence rate, motivated by Feng's work [11–12], we combine TSMC with SOSMC to design the 2TSMC for the tracking control of the HSV longitudinal dynamic model. 2TSMC owns the common advantages of TSMC and SOSMC, which not only eliminates chattering, but also speeds up system convergence and improves tracking precision.

Anyway, there is still the reaching phase problem in SMC. That is to say, SMC's robustness is invalid during the reaching phase, which also exists in 2TSMC. To solve this problem, the SMDO is presented to compensate for parameter uncertainties and external disturbances and increase the robustness of the control system. Thus, the whole phase including reaching phase and sliding phase possesses robustness.

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†Corresponding author. E-mail: cysun@seu.edu.cn. Tel.: +86-13913036082; fax: + 86-25-83794974.
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Moreover, most of studies for the longitudinal model of the HSV only consider parameter uncertainties, such as [6–10]. Few consider external disturbances except [5], which actually exist due to rigorous atmospheric conditions. However, the study in [5] does not involve the parameter uncertainties. Therefore, it is significative that both parameter uncertainties and external disturbances are considered to design a control system with satisfied performance.

For the above analysis, we propose a second-order terminal sliding mode controller with SMDO for the longitudinal model of the HSV under parameter uncertainties and external disturbances. The proposed method provides a continuous control input and achieves fast convergence and robust tracking performance. Compared with conventional SMC, our method has a better control performance.

2 Problem formulation

2.1 Hypersonic vehicle model

Consider the longitudinal dynamic model of a hypersonic vehicle developed by NASA Langley Research Center [6]. The differential equations are given as follows:

$$\begin{cases} \dot{V} = \dfrac{T\cos\alpha - D}{m} - \dfrac{\mu\sin\gamma}{r^2}, \\ \dot{\gamma} = \dfrac{L + T\sin\alpha}{mV} - \dfrac{(\mu - V^2 r)\cos\gamma}{Vr^2}, \\ \dot{h} = V\sin\gamma, \\ \dot{\alpha} = q - \dot{\gamma}, \\ \dot{q} = \dfrac{M_{yy}}{I_{yy}}, \end{cases} \tag{1}$$

where V, γ, h, α, q represent the vehicle's velocity, flight path angle, altitude, angle of attack and pitch rate, respectively. m, I_{yy}, r represent the mass of vehicle, moment of inertia and the radial distance from center of the earth, respectively. T, D, L, M_{yy} represent thrust, drag, lift and pitching moment, respectively.

$$\begin{cases} L = 0.5\rho V^2 S C_L, \\ D = 0.5\rho V^2 S C_D, \\ T = 0.5\rho V^2 S C_T, \\ M_{yy} = 0.5\rho V^2 S\bar{c}[C_{\mathrm{M}}(\alpha) + C_{\mathrm{M}}(\delta_{\mathrm{e}}) + C_{\mathrm{M}}(q)], \\ r = h + R_{\mathrm{E}}, \end{cases} \tag{2}$$

where ρ is the density; S is the reference area, δ_{e} is elevator deflection; and $C_T, C_L, C_D, C_{\mathrm{M}}$ are the corresponding force and moment coefficients, respectively.

The engine dynamics are modeled by the following second-order system:

$$\ddot{\beta} = -2\xi w_n\dot{\beta} - w_n^2\beta + w_n^2\beta_{\mathrm{c}}.$$

The control inputs are throttle settings β_{c} and the elevator deflection δ_{e}; and the outputs are the velocity V and the altitude h, i.e., $u = [\beta_{\mathrm{c}}\ \ \delta_{\mathrm{e}}]^{\mathrm{T}}$, $y = [V\ \ h]^{\mathrm{T}}$.

2.2 Parameter uncertainties and external disturbances

Because the variability of the vehicle characteristics with flight conditions (such as fuel consumption, uncertain atmosphere), significant parameter uncertainties and external disturbances affect the vehicle model.

Parameter uncertainties are taken into account the same as [6], which are adopted as

$$\begin{cases} m = m_0(1 + \Delta m),\ \ I_{yy} = I_0(1 + \Delta I_{yy}), \\ S = S_0(1 + \Delta S),\ \ \bar{c} = \bar{c}_0(1 + \Delta\bar{c}), \\ c_{\mathrm{e}} = c_{\mathrm{e}0}(1 + \Delta c_{\mathrm{e}}),\ \ \rho = \rho_0(1 + \Delta\rho), \\ |\Delta m| \leqslant 0.03,\ \ |\Delta I_{yy}| \leqslant 0.02,\ \ |\Delta S| \leqslant 0.03, \\ |\Delta\bar{c}| \leqslant 0.02,\ \ |\Delta\rho| \leqslant 0.03,\ \ |\Delta c_{\mathrm{e}}| \leqslant 0.02, \end{cases} \tag{3}$$

where $m_0, I_0, S_0, \bar{c}_0, c_{\mathrm{e}}, \rho_0$ are nominal values.

For external disturbances, we only consider wind disturbance similar as [5]. Wind disturbance affects the related aerodynamic coefficients by changing the angle of attack of the vehicle relative to the wind. It is assumed that the disturbance effect is additive and affects only the aerodynamic coefficients about the angle of attack. The additional disturbance terms are given below:

$$\begin{aligned} &C_{Lw} = 0.6203\Delta\alpha_w, \\ &C_{Dw} = 0.6405\Delta\alpha_w^2 + 0.0043378\Delta\alpha_w + 0.003772, \\ &C_{Mw} = -0.035\Delta\alpha_w^2 + 0.036617\Delta\alpha_w + 5.3261\times 10^{-6}. \end{aligned}$$

Here, $\Delta\alpha_w$ is the additional angle of attack and equal to the difference between the vehicle angle of attack α and the angle of attack relative to the wind α_w.The additional angle of attack is assumed to be a fluctuating variable around its non zero mean value, which is expressed as follows:

$$\Delta\alpha_w = -0.25^\circ + 0.025^\circ(\sin t) + \sin(\sqrt{3}t) + \sin(\sqrt{5}t).$$

The lift and drag forces and the pitching moment, represented by equation (2), are affected by wind disturbance through the additional angle of attack. That is to say, additional lift, drag forces and pitching moment are expressed as follows:

$$\begin{cases} L_w = 0.5\rho V^2 S C_{Lw}, \\ D_w = 0.5\rho V^2 S C_{Dw}, \\ M_{yyw} = 0.5\rho V^2 S\bar{c}C_{Mw}. \end{cases} \tag{4}$$

The aim of control design is to track the velocity and altitude commands $y_{\mathrm{d}} = [V_{\mathrm{d}}\ \ h_{\mathrm{d}}]^{\mathrm{T}}$ and achieve good performance under the above-mentioned parameter uncertainties and external disturbances.

3 Input-output linearization

To facilitate the control design, the input-output linearization technique is applied to transform the longitudinal model of the HSV into the affine form [6], as follows:

$$\begin{cases} V^{(3)} = f_{\mathrm{v}} + \Delta f_{\mathrm{v}} + (b_{11} + \Delta b_{11})\beta_{\mathrm{c}} + (b_{12} + \Delta b_{12})\delta_{\mathrm{e}}, \\ h^{(4)} = f_{\mathrm{h}} + \Delta f_{\mathrm{h}} + (b_{21} + \Delta b_{21})\beta_{\mathrm{c}} + (b_{22} + \Delta b_{22})\delta_{\mathrm{e}}, \end{cases} \tag{5}$$

where $f_{\mathrm{v}}, f_{\mathrm{h}}, b_{11}, b_{12}, b_{21}, b_{22}$ are defined as

$$\begin{aligned} f_{\mathrm{v}} &= \frac{w_1\ddot{x}_0 + \dot{x}\varOmega_2\dot{x}}{m}, \\ f_{\mathrm{h}} &= 3\ddot{V}\dot{\gamma}\cos\gamma - 3\dot{V}\dot{\gamma}^2\sin\gamma + 3\dot{V}\ddot{\gamma}\cos\gamma - 3V\dot{\gamma}\ddot{\gamma}\sin\gamma \\ &\quad -V\dot{\gamma}^3\cos\gamma + (w_1\ddot{x}_0 + x\varOmega_2\dot{x})\frac{\sin\gamma}{m} \\ &\quad +V\cos\gamma(\pi_1\ddot{X}_0 + \dot{X}^{\mathrm{T}}\varPi_2\dot{X}), \\ b_{11} &= [\rho V^2 S c_\beta \frac{w_n^2}{2} m]\cos\alpha, \\ b_{12} &= -(c_{\mathrm{e}}\rho V^2 S\frac{\bar{c}}{2} m I_{yy})(T\sin\alpha + D_{\mathrm{a}}), \end{aligned}$$

$$b_{21} = (\rho V^2 S c_\beta \frac{w_n^2}{2} m) \sin(a + \gamma),$$
$$b_{22} = (c_{\rm e} \rho V^2 S \frac{\bar{c}}{2} m I_{yy})[T \cos(\alpha + \gamma) + L_{\rm a} \cos\gamma - D_{\rm a} \sin\gamma],$$

where $\Delta f_{\rm v}, \Delta f_{\rm h}, \Delta b_{11}, \Delta b_{12}, \Delta b_{21}, \Delta b_{22}$ are bounded uncertain terms induced by parameter uncertainties and external disturbances. For convenience, we define $\varDelta_1, \varDelta_2$ as the lumped uncertainties $\varDelta_1 = \Delta f_{\rm v} + \Delta b_{11}\beta_{\rm c} + \Delta b_{12}\delta_{\rm e}$, $\varDelta_2 = \Delta f_{\rm h} + \Delta b_{21}\beta_{\rm c} + \Delta b_{22}\delta_{\rm e}$.The unknown lumped uncertainties are assumed bounded, i.e., $|\varDelta_i| \leqslant L_i$, L_i is a positive constant $(i = 1, 2)$.

4 Control design

In this section, we develop a robust control approach based on the combination of 2TSMC and SMDO. The control design consists of the following steps.

The first step is to develop the 2TSMC to achieve robust tracking of the velocity and altitude commands. The second is to design the SMDO to increase the robustness of the control system and improve control performance.

4.1 Second-order terminal sliding mode controller

First, two linear sliding surfaces are defined respectively as

$$\begin{cases} s_1 = \dot{e}_1 + \lambda_1 e_1, \ \ e_1 = V - V_{\rm d}, \\ s_2 = (\dfrac{\rm d}{{\rm d}t} + \lambda_2)^2 e_2, \ \ e_2 = h - h_{\rm d}, \end{cases} \tag{6}$$

where $\lambda_1, \lambda_2 > 0$, $V_{\rm d}$, $h_{\rm d}$ are the velocity and altitude commands.

Next, two terminal sliding surfaces are chosen respectively as [13] to guarantee the sliding surfaces s_1, s_2 converge to zero in finite time and achieve the second-order sliding mode control.

$$\sigma_i = \dot{s}_i + \beta_i {\rm sg}(s_i)^{\gamma_i}, \ \ i = 1, 2, \tag{7}$$

where $\beta_i > 0$, $0.5 < \gamma_i < 1$, ${\rm sg}(s_i)^{\gamma_i} = |s_i|^{\gamma_i} \operatorname{sgn} s_i$.

Furthermore, select the following continuous terminal sliding mode type reaching law [13] to enhance finite-time reaching phase and achieve continuous control:

$$\dot{\sigma}_i = -k_{i1}\sigma_i - k_{i2}{\rm sg}(\sigma_i)^{p_i}, \ \ i = 1, 2, \tag{8}$$

where $k_{i1}, k_{i2} > 0$, $0 < p_i < 1$.

By designing an appropriate controller, the reaching law (8) can drive terminal sliding surfaces (7) to zero in finite time. Assume that $t_{{\rm r}i}$ is the time when σ_i reaches zero from $\sigma_i(0) \neq 0$. Solving the differential equation (8) analytically, we can obtain the finite time $t_{{\rm r}i}$.

$$t_{{\rm r}i} = \frac{\ln(1 + \frac{k_{i1}}{k_{i2}} |\sigma_i(0)|^{1-p_i})}{k_1(1 - p_i)}, \ \ i = 1, 2.$$

Then, $\sigma_i = 0, t \geqslant t_{{\rm r}i}$ and the system dynamics is determined by $\dot{s}_i + \beta_i {\rm sg}(s_i)^{\gamma_i} = 0, \ \ i = 1, 2$. Similarly, we also can get the finite time $t_{{\rm s}i}$ which is taken to travel from $s(t_{{\rm r}i}) \neq 0$ to $s(t_{{\rm r}i} + t_{{\rm s}i}) = 0$.

$$t_{{\rm s}i} = \frac{|s_i(t_{{\rm r}i})|^{1-\gamma_i}}{\beta_i(1 - \gamma_i)}, \ \ i = 1, 2.$$

As a result, the second-order sliding mode control is realized for s_i. Both s_i and $\dot{s}_i$ reach zero in finite time and the system will stay on the sliding mode $s_i = \dot{s}_i = 0$. After sliding surface s_i reaches zero, the errors e_i will converge to zero asymptotically according to (6).

According to the above analysis, we consider the second time derivatives of s_1, s_2.

$$\begin{cases} \ddot{s}_1 = e_1^{(3)} + \lambda_1 \ddot{e}_1 \\ \quad = -V_{\rm d}^{(3)} + f_{\rm v} + \lambda_1 \ddot{e}_1 + \varDelta_1 + b_{11}\beta_{\rm c} + b_{12}\delta_{\rm e}, \\ \ddot{s}_2 = e_2^{(4)} + 2\lambda_2 e_2^{(3)} + \lambda_2^2 \ddot{e}_2 \\ \quad = -h_{\rm d}^{(4)} + f_{\rm h} + 2\lambda_2 e_2^{(3)} + \lambda_2^2 \ddot{e}_2 + \varDelta_2 + b_{21}\beta_{\rm c} \\ \qquad + b_{22}\delta_{\rm e}. \end{cases} \tag{9}$$

By differentiating the sliding variable σ_1 and σ_2 with respect to time, we have

$$\begin{cases} \dot{\sigma}_1 = \ddot{s}_1 + \beta_1\gamma_1 |s_1|^{\gamma_1 - 1} \dot{s}_1, \\ \dot{\sigma}_2 = \ddot{s}_2 + \beta_2\gamma_2 |s_2|^{\gamma_2 - 1} \dot{s}_2. \end{cases} \tag{10}$$

Substituting (9) into (10), we obtain

$$\begin{bmatrix} \dot{\sigma}_1 \\ \dot{\sigma}_2 \end{bmatrix} = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} + \begin{bmatrix} \varDelta_1 \\ \varDelta_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \beta_{\rm c} \\ \delta_{\rm e} \end{bmatrix}, \tag{11}$$

$$\varphi_1 = -V_{\rm d}^{(3)} + f_{\rm v} + \lambda_1 \ddot{e}_1 + \beta_1\gamma_1 |s_1|^{\gamma_1 - 1} \dot{s}_1,$$
$$\varphi_2 = -h_{\rm d}^{(4)} + f_{\rm h} + 2\lambda_2 e_2^{(3)} + \lambda_2^2 \ddot{e}_2 + \beta_2\gamma_2 |s_2|^{\gamma_2 - 1} \dot{s}_2.$$

From (8) and (11), the second-order terminal sliding mode controller can be designed as

$$u = B^{-1} \begin{bmatrix} -\varphi_1 - k_{11}\sigma_1 - k_{12}{\rm sg}(\sigma_1)^{p_1} \\ -\varphi_2 - k_{21}\sigma_2 - k_{22}{\rm sg}(\sigma_2)^{p_2} \end{bmatrix} - B^{-1} \begin{bmatrix} \varDelta_1 \\ \varDelta_2 \end{bmatrix} = u_0 - B^{-1}\varDelta, \tag{12}$$

where $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, $\varDelta = [\varDelta_1 \ \ \varDelta_2]^{\rm T}$, and

$$u_0 = B^{-1} \begin{bmatrix} -\varphi_1 - k_{11}\sigma_1 - k_{12}{\rm sg}(\sigma_1)^{p_1} \\ -\varphi_2 - k_{21}\sigma_2 - k_{22}{\rm sg}(\sigma_2)^{p_2} \end{bmatrix}$$

denotes the control law for the nominal system of the longitudinal model. $\varDelta$ is unknown and not available in general. In order to increase the robustness of the controller and improve control performance, the SMDO is proposed to estimate the uncertain terms.

It can be seen from (11) that φ_i contains the term $|s_i|^{\gamma_i - 1} \dot{s}_i$ which has negative fractional power $\gamma_i - 1$, and so the singularity may occur if $s_i = 0$ and $\dot{s}_i \neq 0$. However, once the system enters the sliding mode, this situation will never occur because $\sigma_i = 0$ leads to $\dot{s}_i = -\beta_i |s_i|^{\gamma_i} \operatorname{sgn} s_i$ and then $|s_i|^{\gamma_i - 1} \dot{s}_i = -\beta_i |s_i|^{2\gamma_i - 1} \operatorname{sgn} s_i$; and it is easy to be concluded that if γ_i is chosen as $0.5 < \gamma_i < 1$, the term $-\beta_i |s_i|^{2r_i - 1} \operatorname{sgn} s_i$ will be nonsingular. Therefore, the singularity may only occur during the reaching phase $(i = 1, 2)$.

To solve the singularity problem, the approach proposed in [14] is used.

Define a new variable $\bar{s}_i$ as

$$\bar{s}_i = \begin{cases} |s_i|^{\gamma_i - 1} \dot{s}_i, & \text{if } \ s_i \neq 0 \ \text{ and } \ \dot{s}_i \neq 0, \\ |\delta|^{\gamma_i - 1} \dot{s}_i, & \text{if } \ s_i = 0 \ \text{ and } \ \dot{s}_i \neq 0, \\ 0, & \text{if } \ s_i = 0 \ \text{ and } \ \dot{s}_i = 0, \end{cases} \quad i = 1, 2,$$

where δ is a small positive constant.

Thus, φ_1 and φ_2 can be rewritten as

$$\varphi_1 = -V_{\mathrm{d}}^{(3)} + f_{\mathrm{v}} + \lambda_1 \ddot{e}_1 + \beta_1\gamma_1\bar{s}_1,$$
$$\varphi_2 = -h_{\mathrm{d}}^{(4)} + f_{\mathrm{h}} + 2\lambda_2 e_2^{(3)} + \lambda_2^2\ddot{e}_2 + \beta_2\gamma_2\bar{s}_2.$$

As a result, the singularity is avoided in control design.

4.2 Sliding mode disturbance observer

The SMDO is an effective method to compensate for parameter uncertainties and external disturbances which has some advantages such as finite time estimation, low computation effort and simple structure [15–16].

Because control inputs $\beta_{\mathrm{c}}, \delta_{\mathrm{e}}$ on the right hand side of (11) are cross coupling to the sliding variables σ_1 and σ_2, so a new control variable is introduced to eliminate the coupling: $\bar{u} = Bu$. Then, (11) can be rewritten as

$$\dot{\sigma}_i = \varphi_i + \varDelta_i + \bar{u}_i, \quad i = 1, 2. \tag{13}$$

In order to design the SMDO to estimate uncertain term $\varDelta_i$ The auxiliary sliding variables are introduced:

$$\begin{cases} l_i = \sigma_i + z_i, \\ \dot{z}_i = -\varphi_i - \bar{u}_i - v_i, \end{cases} \quad i = 1, 2, \tag{14}$$

where l_i and z_i are auxiliary sliding variable and intermediate variable, respectively. v_i is auxiliary sliding mode control.

By differentiating l_i with respect to time, we have

$$\dot{l}_i = \dot{\sigma}_i + \dot{z}_i = \varDelta_i - v_i, \quad i = 1, 2. \tag{15}$$

Then, we design auxiliary sliding mode control v_i to stabilize the sliding variable l_i to zero

$$v_i = w_i \mathrm{sgn}\, l_i, \quad w_i > L_i + \eta_i, \quad \eta_i > 0. \tag{16}$$

It is easy to verify that sliding variable l_i is driven to zero in finite time based on the Lyapunov approach.

Introducing a Laypunov function $V_i = \dfrac{1}{2} l_i^2$ and differentiating it, we obtain

$$\dot{V}_i = l_i\dot{l}_i = l_i(\varDelta_i - v_i) \leqslant |l_i|(L_i - w_i) \leqslant -\eta_i |l_i|. \tag{17}$$

By analyzing (17), it can be inferred that l_i converges to zero in finite time $t_{\mathrm{a}i}$ ($t_{\mathrm{a}i} \leqslant \dfrac{|l_i(0)|}{\eta_i}$).

Once sliding variable l_i reaches zero, the system dynamics is governed by equivalent control $v_{\mathrm{eq}i}$ which can be obtained by filtering the control input v_i using a low pass filter (LPF).

$$v_{\mathrm{eq}i} = LPF(v_i) = \frac{1}{\tau s + 1} v_i. \tag{18}$$

Thus, (15) becomes

$$\varDelta_i - v_{\mathrm{eq}i} = 0, \quad i = 1, 2. \tag{19}$$

Therefore, uncertain term $\varDelta_i$ is exactly estimated by $\hat{\varDelta}_i = v_{\mathrm{eq}i}$ in finite time $t_{\mathrm{a}i}$ $(i = 1, 2)$.

Then, the final second-order terminal sliding mode controller with SMDO is designed as

$$u = B^{-1}\begin{bmatrix} -\varphi_1 - k_{11}\sigma_1 - k_{12}\mathrm{sg}(\sigma_1)^{p_1} - \hat{\varDelta}_1 \\ -\varphi_2 - k_{21}\sigma_2 - k_{22}\mathrm{sg}(\sigma_2)^{p_2} - \hat{\varDelta}_2 \end{bmatrix}. \tag{20}$$

Remark 1 The parameters of controller k_{ij}, p_i, w_i $(i, j = 1, 2)$ must be selected to ensure that the convergence of auxiliary sliding variable l_i is faster than that of σ_i $(i = 1, 2)$. In other words, sliding variable σ_i $(i = 1, 2)$ is stabilized to zero only after the uncertain term is estimated.

5 Stability analysis

In this section, the stability of the proposed approach is analyzed. First, we introduce the following lemma.

Lemma 1 [17] Assume that a continuous, the positive definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leqslant -aV^{\eta}(t), \ \forall t \geqslant t_0, \ V(t_0) \geqslant 0,$$

where $a > 0, 0 < \eta < 1$ are constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \leqslant V^{1-\eta}(t_0) - a(1-\eta)(t - t_0), \ t_0 \leqslant t \leqslant t_1,$$

and $V(t) = 0$, $\forall t \geqslant t_1$ with t_1 given by $t_1 = t_0 + \dfrac{V^{1-\eta}(t_0)}{a(1-\eta)}$.

Then, Theorem 1 is given below to prove the stability and robustness of the proposed approach.

Theorem 1 For HSV's dynamic model in (1), if the control law (20) based on the combination of 2TSMC and SMDO is applied, the robust stability of the closed-loop system in the presence of the parameter uncertainties and external disturbances is guaranteed.

Proof Rewrite equations (11) and (20) in a compact form:

$$\dot{\sigma} = \varphi + \Delta + Bu, \ \ u = B^{-1}(-\varphi - \phi - \hat{\Delta}),$$

where $\sigma = [\sigma_1 \ \ \sigma_2]^{\mathrm{T}}$, $\varphi = [\varphi_1 \ \ \varphi_2]^{\mathrm{T}}$, $\hat{\varDelta} = [\hat{\varDelta}_1 \ \ \hat{\varDelta}_2]^{\mathrm{T}}$, and

$$\phi = \begin{bmatrix} k_{11}\sigma_1 + k_{12}\mathrm{sg}(\sigma_1)^{p_1} \\ k_{21}\sigma_2 + k_{22}\mathrm{sg}(\sigma_2)^{p_2} \end{bmatrix}.$$

Then, consider the following Lyapunov function:

$$V = \frac{1}{2}\sigma^{\mathrm{T}}\sigma.$$

Differentiating V with respect to time, we have

$$\begin{aligned} \dot{V} &= \frac{1}{2}\sigma^{\mathrm{T}}\sigma = \sigma^{\mathrm{T}}\dot{\sigma} = \sigma^{\mathrm{T}}(\varphi + \varDelta + Bu) \\ &= \sigma^{\mathrm{T}}\{\varphi + \varDelta + B(B^{-1}(-\varphi - \phi - \hat{\varDelta}))\} \\ &= \sigma^{\mathrm{T}}(-\phi + \varDelta - \hat{\varDelta}) = -\sigma^{\mathrm{T}}\phi + \sigma^{\mathrm{T}}\tilde{\varDelta}, \end{aligned}$$

where $\tilde{\varDelta} = \varDelta - \hat{\varDelta}$, because sliding variable σ_i converges to zero only after the uncertain term $\varDelta_i$ is estimated in finite time $t_{\mathrm{a}i}$. Therefore, $\tilde{\varDelta}_i = \varDelta_i - \hat{\varDelta}_i \to 0, t > \bar{t}_{\mathrm{a}}, \bar{t}_{\mathrm{a}} = \max(t_{\mathrm{a}1}, t_{\mathrm{a}2})$.

Therefore, for $t > \bar{t}_{\mathrm{a}}$, we have

$$\begin{aligned} \dot{V} &= -\sigma^{\mathrm{T}}\phi \\ &= -(k_{11}\sigma_1^2 + k_{12}|\sigma_1|^{p_1+1} + k_{21}\sigma_2^2 + k_{22}|\sigma_2|^{p_2+1}) \\ &\leqslant -(k_{12}|\sigma_1|^{p_1+1} + k_{22}|\sigma_2|^{p_2+1}) \\ &\leqslant -k(|\sigma_1|^{p_1+1} + |\sigma_2|^{p_2+1})|_{k=\min(k_{12},k_{22})} \\ &= -k\{(\sigma_1^2)^{\frac{p_1+1}{2}} + (\sigma_2^2)^{\frac{p_2+1}{2}}\} \\ &\leqslant -k(\sigma_1^2 + \sigma_2^2)^{\frac{p+1}{2}}|_{p=\max(p_1,p_2)} \\ &= -k(\sigma^{\mathrm{T}}\sigma)^{\frac{p+1}{2}} = -k2^{\frac{p+1}{2}}V^{\frac{p+1}{2}} \end{aligned}$$

By Lemma 1, the sliding variables σ_1, σ_2 will be driven to zero in finite time with the controller (20). Then, according to (7), s_1, s_2 reach zero within finite time. Thus, the proof is completed.

6 Simulations

This section presents the simulation results to demonstrate the performance of the proposed design. The simulations are carried out for the HSV at trimmed cruise conditions with $V = 15060\,\mathrm{ft\cdot s^{-1}}$ and $h = 110000\,\mathrm{ft}$. The velocity and altitude commands are chosen to be $V_\mathrm{d} = 15160\,\mathrm{ft\cdot s^{-1}}$, $h_\mathrm{d} = 110500\,\mathrm{ft}$. The model parameters are set in accordance with the ones in [6].

The conventional SMC approach with integral sliding surface is also simulated for the comparative study. To design conventional sliding mode controller, integral sliding surfaces are chosen as in [6–7].

$$\begin{cases} s_1 = (\dfrac{\mathrm{d}}{\mathrm{d}t} + \mu_1)^3 \displaystyle\int_0^t e_1(\tau)\mathrm{d}\tau, \;\; e_1 = V - V_\mathrm{d}, \\ s_2 = (\dfrac{\mathrm{d}}{\mathrm{d}t} + \mu_2)^4 \displaystyle\int_0^t e_2(\tau)\mathrm{d}\tau, \;\; e_2 = h - h_\mathrm{d}. \end{cases} \tag{21}$$

The constant rate reaching law is chosen as [6–8].

$$\dot{s}_i = -k_i \mathrm{sgn}\, s_i, \;\; i = 1, 2. \tag{22}$$

Combining (21) with (22), we obtain the conventional sliding mode controller as follows:

$$u_{SMC} = B^{-1}\begin{bmatrix} v_1 - k_1 \mathrm{sgn}\, s_1 \\ v_2 - k_2 \mathrm{sgn}\, s_2 \end{bmatrix}, \tag{23}$$

$$v_1 = \dddot{V}_\mathrm{d} - 3\mu_1 \ddot{e}_1 - 3\mu_1^2 \dot{e} - \mu_1^3 e_1,$$

$$v_2 = h_\mathrm{d}^{(4)} - 4\mu_2 e_2^{(3)} - 6\mu_2^2 \ddot{e} - 4\mu_2^3 \dot{e}_2 - \mu_2^4 e_2.$$

First, we consider the tracking control problem for the nominal model, i.e., no parameter uncertainties and external disturbances. The 2TSMC without SMDO is used. The parameters of the 2TSMC in (20) are selected as $\lambda_1 = 1.5, \lambda_2 = 1.1, \beta_1 = \beta_2 = 2, \gamma_1 = \gamma_2 = 0.85, k_{11} = k_{21} = 1.8, k_{12} = k_{22} = 0.01, p_1 = p_2 = 0.5, \delta = 0.01$.

The parameters of conventional sliding mode controller (23) are set as $\mu_1 = \mu_2 = 0.5$, $k_1 = k_2 = 10$.

Fig. 1 shows the closed-loop tracking performance using both control approaches.

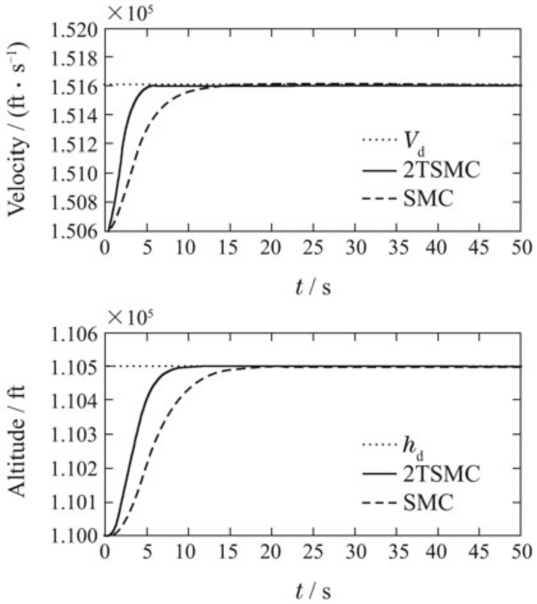


Fig. 1 Tracking performance of 2TSMC and SMC for the nominal model.

Two controllers show satisfied performances in the tracking of the velocity and altitude commands. However, the performance of the 2TSMC is better than that of the conventional SMC due to the 2TSMC possesses fast and higher-precision tracking characteristics.

In Fig. 2, the simulation result shows that control inputs produced by 2TSMC are continuous and the chattering has been eliminated. Apparently control chattering appears, when using conventional SMC.

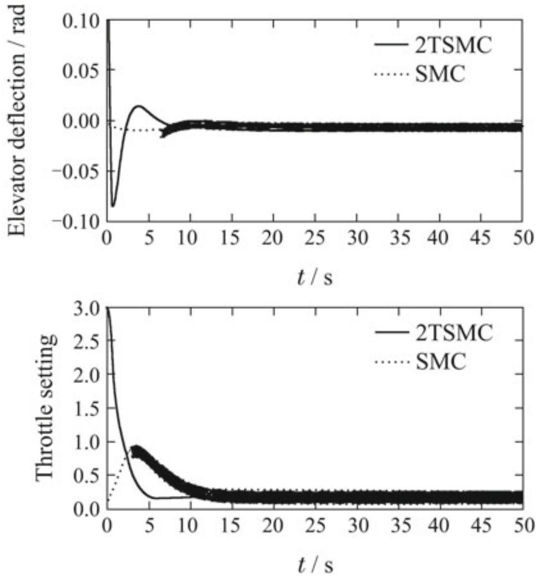


Fig. 2 Control inputs of 2TSMC and SMC for the nominal model.

Second, the tracking control problem under the parameter uncertainties and external disturbances is considered.

The parameters of 2TSMC are set the same as the ones for nominal model.

The parameters of conventional sliding mode controller in (23) are chosen as $\mu_1 = \mu_2 = 0.5, k_1 = k_2 = 30$.

Fig. 3 shows the tracking performance of the 2TSMC without SMDO under $\Delta \neq 0$ and $\Delta = 0$, respectively. From Fig. 3, it can be concluded that 2TSMC reveals its robustness, and but it is evident that the performance is lowered under $\Delta \neq 0$. Therefore, SMDO is applied to improve the control performance. The parameters of SMDO are chosen to be $w_1 = w_2 = 5, \tau = 0.01$.

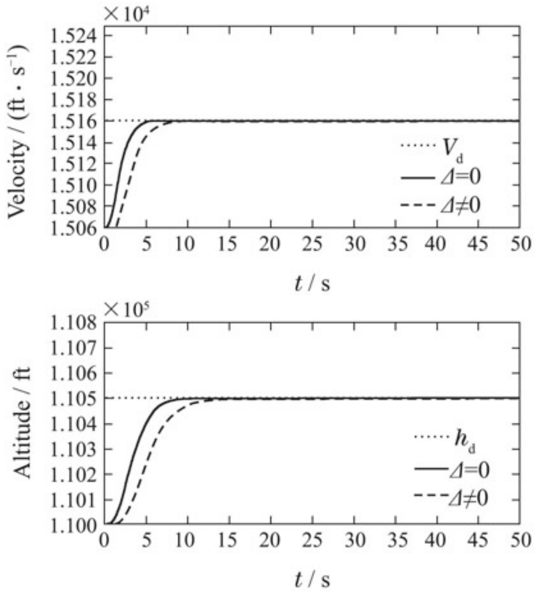


Fig. 3 Control performance of 2TSMC under $\Delta = 0$ and $\Delta \neq 0$.

Fig. 4 shows that 2STMC with SMDO exhibits excellent tracking perfomance. The velocity and altitude converge to the desired commands in a short time in spite of parameter uncertainties and external disturbances. From Fig. 4, we also can see that the control performance of conventional SMC is deteriorated. Moreover, Fig. 5 demonstrates that the chattering becomes more severe in conventional SMC. However, our approach still produces continuous control input and avoids chattering phenomenon.

Fig. 6 shows the responses of the flight-path angle, angle of attack and pitch rate, respectively, which all converge in a short time. Fig. 7 shows that the sliding variables and their derivatives converge to zero quickly and it can be concluded that SOSMC is realized.

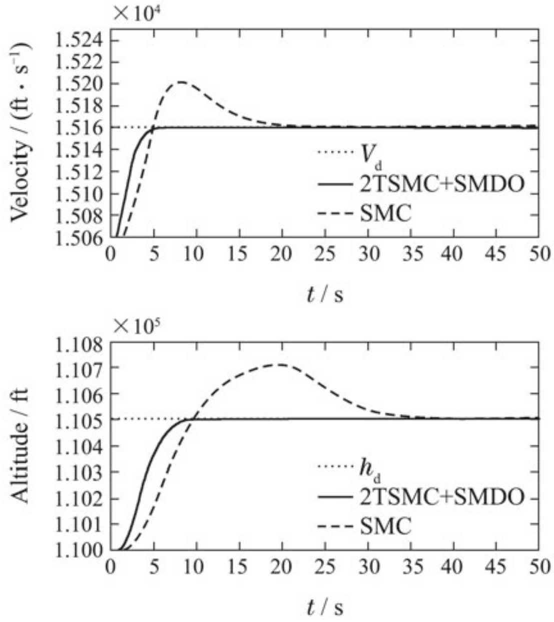


Fig. 4 Control performance of 2TSMC with SMDO and SMC.

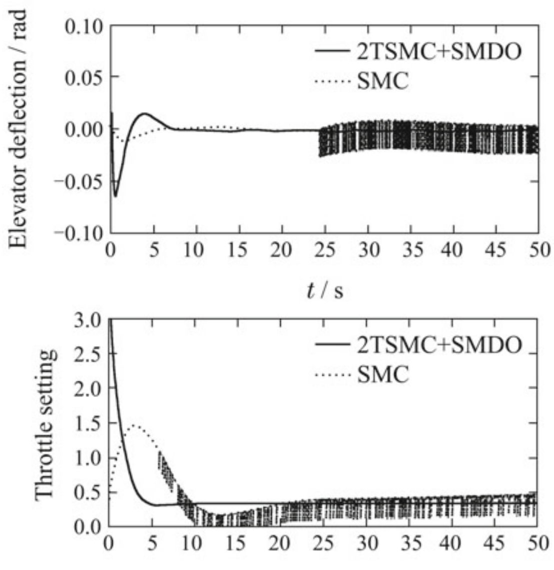


Fig. 5 Control inputs of 2TSMC with SMDO and SMC.

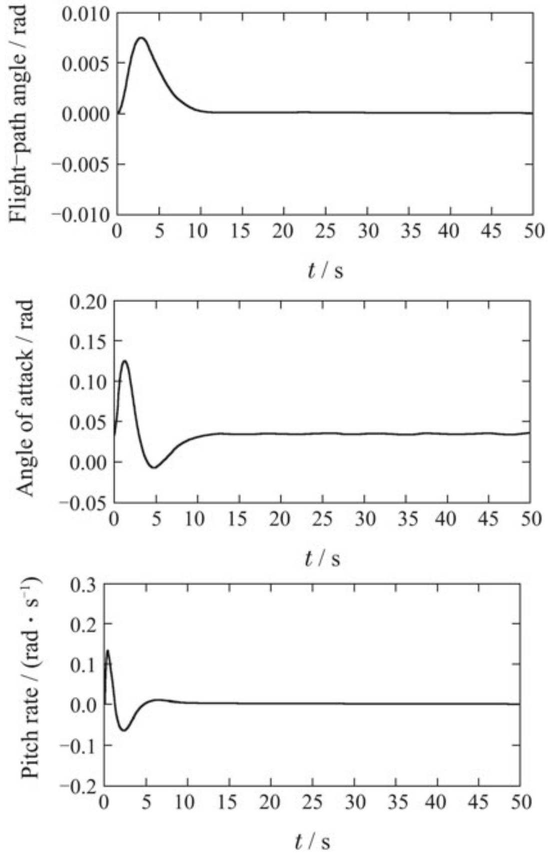


Fig. 6 Flight-path angle, angle of attack and pitch rate versus time.

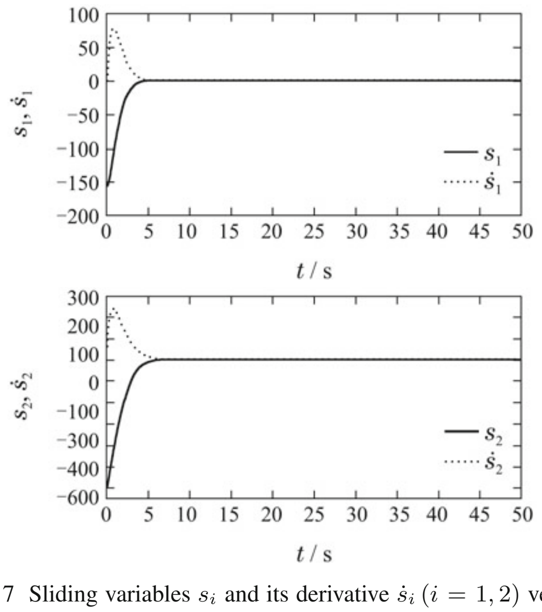


Fig. 7 Sliding variables s_i and its derivative $\dot{s}_i$ $(i = 1, 2)$ versus time.

7 Conclusions

In this paper, a robust control approach based on 2TSMC and SMDO is developed to solve the tracking control problem for the longitudinal model of the HSV. First, input-output linearization is used to transform the no-affine nonlinear model into an affine nonlinear model. Second, second-order terminal sliding mode controller is designed to provide fast convergence and high tracking precision. Then, SMDO is employed to increase the robustness of the control system and improve the control performance. The combination of 2TSMC and SMDO enables to obtain high control performance in spite of parameter uncertainties and external disturbances. Simulation results demonstrate the effectiveness and superiority of the proposed method compared with the conventional SMC.

References

[1] D. Gao, Z. Sun, X. Luo, et al. Fuzzy adaptive control for hypersonic vehicle via backstepping method. *Control Theory & Application*, 2008, 25(5): 805 – 810 (in Chinese).

[2] J. Chen, C. Pan. Dynamic surface backstepping control design for one hypersonic vehicle. *Proceedings of International Conference on Mechatronics and Automation*. Changchun: IEEE, 2009: 4770 – 4774.

[3] H. Xu, M. Mirmirani, P. A. Ioannou. Robust neural adaptive control of a hypersonic aircraft. *AIAA Guidance, Navigation and Control Conference and Exhibit*. Austin, Texas: AIAA, 2003: 1 – 8.

[4] B. Xu, D. Gao, S. Wang. Adaptive neural control based on HGO for hypersonic flight vehicles. *Science China Information Sciences*, 2011, 54(3): 511 – 520.

[5] B. Durmaz, M. K. Ozgoren, M. U. Salamcik. Sliding mode control for nonlinear systems with adaptive sliding surface. *Transactions of the Institute of Measurement and Control*, 2012, 34(1): 1 – 35.

[6] H. Xu, M. Mirmirani. Adaptive sliding mode control design for a hypersonic flight vehicle. *Journal of Guidance,Control and Dynamic*, 2004, 27(5): 829 – 838.

[7] H. Li, W. Jian, W. Sun. GA based design of sliding mode control law for hypersonic vehicle. *Journal of Beijing Universith of Aeronautics and Astronautices*, 2009, 35(9): 1144 – 1147.

[8] Y. Liu, Y. Lu. Longitudinal inversion flight control based on variable structure theory for hypersonic vehicle. *Jouranl of Information and Control*, 2006, 35(3): 388 – 392.

[9] X. Hu, C. Ding, L. Wu, et al. Terminal sliding mode control design for air-breathing hypersonic vehicles. *3rd International Symposium on Systems and Control in Aeronautics and Astronautics*. Harbin: IEEE, 2010: 18 – 22.

[10] Q. Wang, R. F. Stengel. Robust nonlinear control of a hypersonic aircraft. *Journal of Guidance, Control and Dynamics*, 2000, 23(4): 577 – 585.

[11] Y. Feng, X. Han, Y. Wang, et al. Second-order terminal sliding mode control of uncertain multivariable systems. *International Journal of Control*, 2007, 80(6): 856 – 862.

[12] Y. Feng, X. Zheng, X. Yu. Second order nonsingular terminal sliding mode decomposed control of uncertain multivariable systems. *Asian Journal of Control*, 2003, 5(4): 505 – 512.

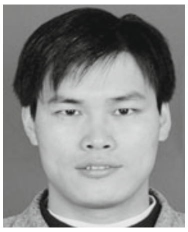
[13] S. Yu, X. Yu, B. Shirinzadeh, et al. Continuous finite time control for robotic manipulators with terminal sliding mode. *Automatica*, 2005, 41(11): 1957 – 1964.

[14] D. Zhao, S. Li, F. Gao. A new sliding mode control for robotic manipulators. *International Jouranl of Control*, 2009, 82(10): 1804 – 1813.

[15] C. Hall, Y. Shtessel. Sliding mode disturbance observer based control for a reusable launch vehicle. *Journel of Guidance, Control and Dynamics*, 2006, 29(6): 1315 – 1329.

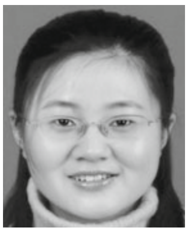
[16] L. Besnard, Y. B. Shtessel. Control of a quadrotor vehicle using sliding mode disturbance observer. *Proceedings of American Control Conference*. New York: IEEE, 2007: 5230 – 5325.

[17] S. Bhat, D. Bernstein. Finite time stability of continuous autonomous system. *SIAM Jouunal of Control and Optimization*, 2000, 38(3): 751 – 766.



Ruimin ZHANG received his M.S. degree from the Department of Mathematics, East China Normal University, in 2004. He is currently pursuing his Ph.D. degree at the School of Automation, Southeast University. His research interests include nonlinear control and flight control. E-mail: zhangrim@163.com.

Changyin SUN is a professor at the School of Automation at Southeast University, China. He received his M.S. and Ph.D. degrees in Electrical Engineering from Southeast University, Nanjing, China, in 2001 and in 2003, respectively. His research interests include intelligent control, neural networks, SVM, data-driven control, theory and design of intelligent control systems, optimization algorithms, and pattern recognition. He is an associate editor of IEEE Transactions on Neural Networks, Neural Processing Letters, and International Journal of Swarm Intelligence Research and Recent Patents on Computer Science. E-mail: cysun@seu.edu.cn.



Jingmei ZHANG received her M.S. degree from the School of Mathematical Science, Shanxi University, in 2006. She is currently pursuing her Ph.D. degree at the School of Automation, Southeast University. Her research interests include nonlinear control and flight control. E-mail: meizijingjing@163.com.

Yingjiang ZHOU received his M.S. degree from the College of Electrical Engineering, Hohai University, in 2010. He is currently pursuing his Ph.D. degree at the School of Automation, Southeast University. His research interests include nonlinear control and flight control. E-mail: zhou_4714@163.com.