J Control Theory Appl 2012 10 (4) 533–538
DOI 10.1007/s11768-012-0216-6

Global output feedback stabilization of upper-triangular nonlinear time-delay systems

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Abstract: In this paper, the problem of global output feedback stabilization for a class of upper-triangular nonlinear systems with time-varying time-delay in the state is considered. The uncertain nonlinearities are assumed to be higher-order in the unmeasurable states. Based on the extended homogeneous domination approach, using a low gain observer in combination with controller, the delay-independent output feedback controller makes closed-loop system globally asymptotically stable under a homogeneous growth condition.

Keywords: Upper-triangular nonlinear time-delay system; Lyapunov-Krasovskii functionals; Globally output feedback stabilization

1 Introduction

Since time-delay which exists in many control systems such as chemical progress, manufacture progress and high-speed networks often has great influence on the stability and performance of controlled systems and can't be eliminated due to its inherent nature, there have been considerable efforts in stability analysis of time-delay systems (see [1–7] and references therein). The well-known techniques that deals with stability analysis for linear time-delay systems is Lyapunov-Razumikhin method (see [8]). The results are often obtained in the form of linear matrix inequalities (LMI). The paper [9] presented a new delay-dependant and parameter-dependant robust stability criterion for linear continuous time systems using LMI. However the useful tools such as LMI are hard to apply to nonlinear systems with time-delay. In recent years, a class of Lyapunov-Krasovskii functionals have been used as checking criteria for time-delay nonlinear systems stability. The uncertainties of unknown time-delay were compensated for using appropriate Lyapunov-Krasovskii functionals for a class of parametric-strict-feedback nonlinear systems in [10].

Over the past few years, the global stabilization problem of triangular structural nonlinear systems by output feedback control has been addressed in numerous studies (see [11–12]). As investigated in [11], by using high gain observer and observer backstepping globally asymptotically stabilizing output feedback was designed for lower triangular systems. The problem of robust feedback control is considered for systems in lower triangular form under the global Lipschitz-like condition on the unmeasurable states with output dependent incremental rate in [12]. However little attention has been focused on nonlinear time-delay upper-triangular structure systems.

In general, from both practical and theoretical points of view, it is somewhat difficult to design controllers for upper-triangular systems with nonlinearities satisfying a linear growth. Recently, a homogeneous nonlinear observer design is introduced in [13]. It has been shown under the linear growth assumption that the problem of global output feedback stabilization of a class of upper-triangular nonlinear systems is solved by homogeneous domination approach. However the problem of how to globally stabilize the upper-triangular system with time-delay is still quite open. For example, the global output feedback stabilization problem of the following system:

$$\begin{cases} \dot{x}_1 = x_2 + x_3(t-d(t)) \cdot \sin x_3, \\ \dot{x}_2 = x_3 + u^3, \\ \dot{x}_3 = u, \\ y = x_1, \end{cases} \tag{1}$$

remains unsolved due to the presence of time-delay. Therefore, the problem of how to globally stabilize the upper-triangular system is of practical and theoretical importance. The purpose of this paper is to tackle this problem and provide a systematic design method.

In order to solve this problem, we extend the homogeneous domination method for output feedback stabilization to the case with time-delay. The novelty of the homogeneous domination approach is that no precise information of the nonlinearities and time-delay is needed. Therefore this is different from most of conventional approaches. Then we use the extended homogeneous domination approach and low gain observer and controller to deal with the upper-triangular systems even when the nonlinearities are inherently of higher-order.

The paper will generalize the results for the systems considered in [13] to systems with delays in the state. The global output stabilization problem for uncertain strict feedback upper-triangular nonlinear systems with output dependant incremental rate is solved in this paper. While in [11], it was only considered the global output stabilization prob-

Received 20 September 2010; revised 30 June 2011.
This work was supported by the National Natural Science Foundation of China (No. 60736022).

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lem for uncertain strict feedback nonlinear systems with constant incremental rate. Particularly we extend properties of homogeneous systems with time-delay and give the extended homogeneous domination approach.

2 Problem formulation

This paper considers a class of upper-triangular nonlinear systems with time-delay of the form:

$$\begin{cases} \dot{x}_1 = x_2 + \phi_1(t, x_3, \ldots, x_n, x_{3d}, \ldots, x_{nd}, u), \\ \dot{x}_2 = x_3 + \phi_2(t, x_4, \ldots, x_n, x_{4d}, \ldots, x_{nd}, u), \\ \quad \vdots \\ \dot{x}_{n-1} = x_n + \phi_{n-1}(t, u), \\ \dot{x}_n = u, \ \ y = x_1, \end{cases} \tag{2}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the system state, input and output, respectively. The continuous functions $\phi_i(\cdot), 1 \leqslant i \leqslant n-1$ represent nonlinear perturbation, which are supposed possibly unknown.

The time-varying delay function $d(t)$ is assumed to satisfy the condition: $0 \leqslant d(t) \leqslant r, \dot{d}(t) \leqslant \eta < 1$, where r and η are given non-negative constants. The assumption shows that the time delay may change from time to time and because of the upper bound the delay cannot increase as fast as the time itself, but the rate of changing is bounded. In this paper, for simplicity of notation, we write the delayed state variables $x_{id} = x_i(t - d(t)), i = 3, \ldots, n$. $x(t) = \varphi(t) \in C, t \in [-r, 0]$, where $C = C([-r, 0], \mathbb{R}^n)$ denotes the space of continuous functions that map the interval $[-r, 0]$ into $\mathbb{R}^n$, $\varphi(t)$ is continuous vector valued initial function with the norm $\|\varphi\| = \sup\limits_{s \in [-r,0]} \|\varphi(s)\|$.

The general notion of homogeneous differential equation, review of the corresponding theoretical results and numerous references can be found in [14–15]. Throughout this paper, for fixed coordinates $(x, x_d) = (x_1, \ldots, x_n, x_{1d}, \ldots, x_{nd}) \in \mathbb{R}^{2n}$ and real number $r_i > 0, i = 1, \ldots, n$, we propose the following notions based on properties of homogeneous systems.

Definition 1 The dilation $\Delta_{\epsilon d}(x, x_d)$ is defined by $\Delta_{\epsilon d}(x, x_d) = (\epsilon^{r_1} x_1, \ldots, \epsilon^{r_n} x_n, \epsilon^{r_1} x_{1d}, \ldots, \epsilon^{r_n} x_{nd}), \forall \epsilon > 0$ with r_i being called as the weights of the coordinates (we definite dilation weight with time-delay $\Delta_d = (r_1, \ldots, r_n, r_1, \ldots, r_n)$).

Definition 2 A function $V \in C(\mathbb{R}^{2n}, \mathbb{R})$ is said to be homogeneous of degree τ if there is a real number $\tau \in \mathbb{R}$ such that $\forall (x, x_d) \in \mathbb{R}^{2n} \backslash \{0\}, \epsilon > 0, V(\Delta_{\epsilon d}(x, x_d)) = \epsilon^\tau V(x, x_d)$.

Definition 3 A vector field $f \in C(\mathbb{R}^{2n}, \mathbb{R})$ is said to be homogeneous of degree τ if there is a real number $\tau \in \mathbb{R}$ such that for $i = 1, \ldots, 2n, \ \ \forall (x, x_d) \in \mathbb{R}^{2n} \backslash \{0\}, \epsilon > 0, f_i(\Delta_{\epsilon d}(x, x_d)) = \epsilon^{\tau + r_i} f_i(x, x_d)$.

Definition 4 A homogeneous $p-\mathrm{norm}$ is defined as $\|(x, x_d)\|_{\Delta_d, p} = (\sum\limits_{i=1}^n |x_i|^{p/r_i} + \sum\limits_{i=1}^n |x_{id}|^{p/r_i})^{1/p}$ for a constant $p \geqslant 1$, here we choose $p = 2$ for simplicity and write $\|(x, x_d)\|_{\Delta_d}$.

Next, we give the following lemmas based on the homogeneous properties before giving the extended homogeneous theorem.

Lemma 1 Suppose $V: \mathbb{R}^N \to \mathbb{R}$ is a homogeneous function of degree τ with respect to the dilation Δ, then $\dfrac{\partial V}{\partial x_i}$ is homogeneous of degree $\tau - r_i$ with r_i being the homogeneous weight of x_i.

Lemma 2 Given a dilation weight $\Delta = (r_1, \ldots, r_n)$, suppose $V_1(x)$ and $V_2(x)$ are homogeneous functions of degree τ_1 and τ_2, respectively, then $V_1(x)V_2(x)$ is also homogeneous with respect to the same dilation weight Δ. Moreover, the homogeneous degree of $V_1 V_2$ is $\tau_1 + \tau_2$.

With the help of aforementioned lemmas, we are ready to prove the following homogeneous domination theorem with time-delay.

Theorem 1 (Homogeneous domination theorem with time-delay) Suppose that the following conditions hold:

H1) $\dot{\mathcal{L}} = F(\mathcal{L}), F(0) = 0$, is globally asymptotically stable where $F: \mathbb{R}^N \to \mathbb{R}^N$ is a homogeneous vector field of degree τ with respect to dilation weight $\Delta = (r_1, r_2, \ldots, r_N)$.

H2) For $i = 1, \ldots, N$, there are positive constants K_i and C_i such that

$$\begin{aligned} &|G_i(t, \epsilon, \mathcal{L}, \mathcal{L}(t - d(t)))| \\ &\leqslant C_i \varepsilon^{1+K_i} (\|\mathcal{L}\|_\Delta^{\tau + r_i} + \|\mathcal{L}(t - d(t))\|_{\Delta_d}^{\tau + r_i}), \end{aligned}$$

then there is a small enough constant $\varepsilon > 0$ such that the following system $\dot{\mathcal{L}} = \varepsilon F(\mathcal{L}) + G(t, \epsilon, \mathcal{L}, \mathcal{L}(t - d(t)))$ is uniformly globally asymptotically stable.

Proof From H1), we know $\dot{\mathcal{L}} = F(\mathcal{L})$ is globally asymptotically stable. By Lemmas 1 and 2, there is a positive definite Lyapunov function $V_1(\mathcal{L})$, which is homogeneous with a degree of $\mu(\tau + \mu > 0)$ for the same dilation weight $\Delta = (r_1, r_2, \ldots, r_N)$ and $\dfrac{\partial V_1}{\partial \mathcal{L}} F(\mathcal{L})$ is homogeneous of degree of $\tau + \mu$, such that

$$\frac{\partial V_1}{\partial \mathcal{L}} F(\mathcal{L}) = \sum_{i=1}^n \frac{\partial V_1}{\partial \mathcal{L}_i} F_i(\mathcal{L}) \leqslant -\hat{C} \|\mathcal{L}\|_\Delta^{\tau + \mu},$$

for a constant $\hat{C} > 0$. By condition H2) and Young's inequality, we have

$$\begin{aligned} &\frac{\partial V_1}{\partial \mathcal{L}} G(t, \epsilon, \mathcal{L}, \mathcal{L}(t - d(t))) \\ &= \sum_{i=1}^N \frac{\partial V_1}{\partial \mathcal{L}_i} G_i(t, \epsilon, \mathcal{L}, \mathcal{L}(t - d(t))) \\ &\leqslant \sum_{i=1}^N (\|\mathcal{L}\|_\Delta^{\tau - r_i})(C_i \varepsilon^{1+K_i} (\|\mathcal{L}\|_\Delta^{\tau + r_i} + \|\mathcal{L}(t - d(t))\|_{\Delta_d}^{\tau + r_i})) \\ &\leqslant \epsilon^{1+K} \bar{C} (\|\mathcal{L}\|_\Delta^{\tau + \mu} + \|\mathcal{L}(t - d(t))\|_{\Delta d}^{\tau + \mu}), \end{aligned}$$

where $K = \min\{K_i\} > 0, \bar{C} > 0$.

Consider the following Lyapunov-Krasovskii functional:

$$V_2(t) = \epsilon \int_{t-d(t)}^t \|\mathcal{L}(s)\|_\Delta^{\tau + \mu} \mathrm{d}s, \ \ \epsilon > 0$$

then, we have

$$\dot{V}_2 = \epsilon \|\mathcal{L}\|_\Delta^{\tau + \mu} - \epsilon (1 - \dot{d}(t) \|\mathcal{L}(t - d(t))\|_{\Delta_d}^{\tau + \mu}, \epsilon > 0.$$

Taking $V = V_1 + V_2$, and we have

$$\begin{aligned} \dot{V} &= \dot{V}_1 + \dot{V}_2 \\ &= \frac{\partial V_1}{\partial \mathcal{L}} (\epsilon F(\mathcal{L}) + G(t, \epsilon, \mathcal{L}, \mathcal{L}(t - d(t)))) + \dot{V}_2 \\ &\leqslant -\epsilon \hat{C} \|\mathcal{L}\|_\Delta^{\tau + \mu} + \epsilon^{1+K} \bar{C} (\|\mathcal{L}\|_\Delta^{\tau + \mu} + \|\mathcal{L}(t - d)\|_{\Delta_d}^{\tau + \mu}) \end{aligned}$$

$$+\epsilon\|\mathcal{L}\|_{\Delta}^{\tau+\mu}-\epsilon(1-\eta)\|\mathcal{L}(t-d(t))\|_{\Delta_d}^{\tau+\mu}$$
$$\leqslant -(\epsilon\hat{C}-\epsilon^{1+K}\bar{C}-\epsilon)\|\mathcal{L}\|_{\Delta}^{\tau+\mu}-((1-\eta)\epsilon-\epsilon^{1+K}\bar{C})\cdot\|\mathcal{L}(t-d(t))\|_{\Delta_d}^{\tau+\mu}.$$

Obviously, if the constant ϵ is small enough, i.e. $0<\epsilon<1$, then we can make $(1-\eta)\epsilon-\epsilon^{1+K}\bar{C}>0, \epsilon\hat{C}-\epsilon^{1+K}\bar{C}-\epsilon=\epsilon(\hat{C}-1)-\epsilon^{1+K}\bar{C}>0$. As a result, the system $\dot{\mathcal{L}}=\epsilon F(\mathcal{L})+G(t,\epsilon,\mathcal{L},\mathcal{L}(t-d(t)))$ is uniformly globally asymptotically stable.

The homogeneous domination approach for output feedback stabilization was used in [13] without time-delay. It has been proven to be useful to deal with upper-triangular systems with linear growth condition. In this paper, to deal with time-delay, Theorem 1 employs Lyapunov-Krasovskii functional to dominate time-delay term.

3 Preliminary results

In this section, we briefly review a new output feedback stabilization result presented in [13]. Based on the homogeneous theory, the result provides a systematic design tool for the construction of dynamic compensation.

Consider a linear system

$$\dot{z}_i=z_{i+1},\ i=1,\dots,n-1,\ \dot{z}_n=v,\ y=z_1, \quad (3)$$

where v is input, y is output. For system (3), one can construct a reduced homogeneous observer

$$\begin{cases}\dot{\hat{\eta}}_2=f_{n+1}(z_1,\hat{\eta}_2)=-\lambda_1\hat{z}_2,\\ \hat{z}_2=\mathrm{sgn}(\hat{\eta}_2+\lambda_1 z_1)|\hat{\eta}_2+\lambda_1 z_1|^{r_2/r_1},\\ \quad\vdots\\ \dot{\hat{\eta}}_k=f_{n+k-1}(z_1,\hat{\eta}_2,\dots,\hat{\eta}_k)=-\lambda_{k-1}\hat{z}_k,\\ \hat{z}_k=\mathrm{sgn}(\hat{\eta}_k+\lambda_{k-1}\hat{z}_{k-1})|\hat{\eta}_k+\lambda_{k-1}\hat{z}_{k-1}|^{r_k/r_{k-1}}\end{cases} \quad (4)$$

where $k=3,\dots,n$, and $r_1=1, r_i=r_{i-1}+m, m=-\dfrac{q}{p}$, q is a positive even integer, p is a positive odd integer. r_i are the homogeneous dilations and $\lambda_i>0, i=1,\dots,n-1$ are observer gains. The sign function is defined by

$$\mathrm{sgn}\, s=\begin{cases}1, & \text{if } s>0,\\ -1, & \text{if } s<0.\end{cases}$$

The controller can be constructed as

$$v=-b_n(\hat{z}_n^{1/r_n}+b_{n-1}(\hat{z}_{n-1}^{1/r_{n-1}}+\dots+b_2(\hat{z}_2^{1/r_2}+b_1 z_1)))^{r_n+m}, \quad (5)$$

with $\hat{z}_1=z_1, b_i=\beta_i^{1/r_{i-1}}, i=1,\dots,n-1, b_n=\beta_n$ and

$$\hat{z}_1^*=0,\ \hat{\xi}_1=\hat{z}_1-\hat{z}_1^*$$
$$\vdots$$
$$\hat{z}_k^*=-\hat{\xi}_{k-1}^{r_k}\beta_{k-1},\ \hat{\xi}_k=\hat{z}_k^{1/r_k}-\hat{z}_k^{*1/r_k},$$

for appropriate controller constants $\beta_k>0, k=1,\dots,n$. Denote

$$\mathcal{L}=(z_1\ \cdots\ z_n\ \hat{\eta}_2\ \cdots\ \hat{\eta}_n)^{\mathrm{T}}, \quad (6)$$
$$F(\mathcal{L})=(z_2\ \cdots\ z_n\ v\ f_{n+1}\ \cdots\ f_{2n-1})^{\mathrm{T}}. \quad (7)$$

The closed-loop system (3)–(5) can be rewritten in a compact form $\dot{\mathcal{L}}=F(\mathcal{L})$. Moreover, $F(\mathcal{L})$ is homogeneous of degree m with dilation weight

$$\varDelta=(r_2,r_2,\dots,r_{2n-1})=(1,m+1,\dots,(n-1)m+1,1,m+1,\dots,(n-2)m+1). \quad (8)$$

Lemma 3 For linear system (3), for any constant $m\in(-\dfrac{1}{n},0)$, there is a homogeneous output feedback controller (5) of degree m rendering system (3) uniformly asymptotically stable.

4 Main results

The objective of this paper is to design a state feedback controller u which globally stabilizes system (2) in the presence of the commonly required higher-order or linear condition. To solve the problem, we make the following assumption on system (2).

Assumption 1 There is a constant $m(-\dfrac{1}{n}<m\leqslant 0)$ such that for $i=1,\dots,n-1$,

$$\begin{aligned}&\phi_i(t,x_{i+2},\dots,x_n,x_{i+2}(t-d(t)),\dots,x_n(t-d(t)),u)\\ &\leqslant c(|x_{i+2}|^{(im+1)/((i+1)m+1)}+\dots\\ &\quad+|x_n|^{(im+1)/((n-1)m+1)}\\ &\quad+|x_{i+2}(t-d(t))|^{(im+1)/((i+1)m+1)}+\dots\\ &\quad+|x_n(t-d(t))|^{(im+1)/((n-1)m+1)}+|u|^{(im+1)/(nm+1)}).\end{aligned}$$

In this section, we show that under Assumption 1, the problem of global stabilization for system (2) is solvable. We apply the extended homogeneous domination approach (Theorem 1) to design a scaled output feedback controller that globally stabilizes the time-varying time-delay upper-triangular system (2).

Theorem 2 Under Assumption 1, there is an output feedback controller globally stabilizing the upper triangular system (2).

Proof Supposed r_i is the same as the notation, $r_i\in(0,1]$ is a ratio of two positive odd integers. Defining new coordinates

$$z_i=\frac{x_i}{\epsilon^{i-1}},\ z_i(t-d(t))=\frac{x_i(t-d(t))}{\epsilon^{i-1}},\ v=\frac{u}{\epsilon^n},$$

with $i=1,\dots,n$, $0<\epsilon<1$, here v is as same as (5). Under the new coordinates, system (2) can be rewritten as the following system:

$$\dot{z}_i=\epsilon z_{i+1}+\frac{\phi_i}{\epsilon^{i-1}},\ i=1,\dots,n,$$
$$\dot{z}_n=\frac{u}{\epsilon^{n-1}}=\epsilon v.$$

By Assumption 1, we have

$$\begin{aligned}&\phi_i(t,x_{i+2},\dots,x_n,x_{i+2,d},\dots,x_{nd},u)\\ &\leqslant c(|\epsilon^{i+1}z_{i+2}|^{(im+1)/((i+1)m+1)}+\dots+\\ &\quad|\epsilon^{n-1}z_n|^{(im+1)/((n-1)m+1)}+\\ &\quad|\epsilon^{i+1}z_{i+2}(t-d)|^{(im+1)/((i+1)m+1)}+\dots+\\ &\quad|\epsilon^{n-1}z_n(t-d)|^{(im+1)/((n-1)m+1)}+|\epsilon^n v|^{(im+1)/(nm+1)}).\end{aligned}$$

Because $\epsilon<1$, thus

$$\begin{aligned}&|\frac{\phi_i}{\epsilon^{i-1}}|\\ &\leqslant c\epsilon^2[(|z_{i+2}^{(im+1)/(i+1)m+1}+\dots+|z_n|^{(im+1)/(i+1)m+1}\\ &\quad+|v|^{(im+1)/(nm+1)})+(|z_{i+2}(t-d)|^{(im+1)/((i+1)m+1)}\\ &\quad+\dots+|z_n(t-d)|^{(im+1)/((n-1)m+1)}\\ &\quad+|v|^{(im+1)/(nm+1)})]\end{aligned}$$

$$\leqslant \tilde{c}\epsilon^2(\|\mathcal{L}\|_{\Delta}^{im+1} + \|\mathcal{L}(t-d(t))\|_{\Delta_d}^{im+1}),\ i=1,\dots,n.$$

Utilizing the observer with a scaling gain ϵ :

$$\dot{\hat{\eta}}_2 = -\epsilon\lambda_1\hat{z}_2,\ \ \dot{\hat{\eta}}_3 = -\epsilon\lambda_2\hat{z}_3,\ \dots,\ \dot{\hat{\eta}}_n = -\epsilon\lambda_{n-1}\hat{z}_n,$$
$$\hat{z}_2 = \mathrm{sgn}(\hat{\eta}_2 + \lambda_1 z_1)^{r_2/r_1}|\hat{\eta}_2 + \lambda_1 z_1|^{r_2/r_1},$$
$$\hat{z}_3 = \mathrm{sgn}(\hat{\eta}_3 + \lambda_2 \hat{z}_2)^{r_3/r_2}|\hat{\eta}_3 + \lambda_2 \hat{z}_2|^{r_3/r_2},$$
$$\hat{z}_n = \mathrm{sgn}(\hat{\eta}_n + \lambda_{n-1} \hat{z}_{n-1})^{r_n/r_{n-1}}|\hat{\eta}_n + \lambda_{n-1} \hat{z}_{n-1}|^{r_n/r_{n-1}},$$

where the observer gains $\lambda_i, i=1,\dots,n-1$ are exactly chosen as [13]. Thus the closed-loop system can be described as

$$\dot{\mathcal{L}} = \epsilon F(\mathcal{L}) + (\phi_1(\cdot), \frac{\phi_2}{\epsilon}, \dots, \frac{\phi_{n-1}}{\epsilon^{n-2}}, 0, \dots, 0)^{\mathrm{T}}.$$

Choose the same construction $V_1 = U(\mathcal{L})$ in [13], we have

$$\frac{\partial U(\mathcal{L})}{\partial \mathcal{L}} F(\mathcal{L}) \leqslant -c_1\|\mathcal{L}\|_{\Delta}^{2+m},$$

which implies the system $\dot{\mathcal{L}} = F(\mathcal{L})$ with homogeneous of degree m asymptotically stable.

By Theorem 1, we can choose a small enough gain ϵ such that the scaled homogeneous output feedback controller renders systems (2) uniformly globally asymptotically stable.

Now we extend the result obtained in the preceding theorem(Theorem 2) to a more general nonlinear systems that are not necessarily in the triangular form. Consider the following non-triangular nonlinear system:

$$\begin{cases} \dot{x}_i = x_{i+1} + \phi_i(t, x, x(t-d(t)), u),\ i=1,\dots,n-1, \\ \dot{x}_n = u,\ \ y = x_1. \end{cases} \tag{9}$$

Since ϕ_i is not in the upper-triangular form, Theorem 2 is not applicable to system (9). However, based on Theorem 1,Theorem 2 can be extended to systems under the following assumption.

Assumption 2 There are a negative constant $m \in (-\frac{1}{n}, 0]$ and positive constants $\mu_i > 0, i=1,\dots,n-1$ such that $\forall\epsilon \in (0,1), \forall(t,z,v) \in \mathbb{R}^{n+2}$

$$|\phi_i(t, z_1, \epsilon z_2, \dots, \epsilon^{n-1}z_n, z_1(t-d), \epsilon z_2(t-d), \dots, \epsilon^{n-1}z_n(t-d), \epsilon^n v)|$$
$$\leqslant c\epsilon^{i+\mu_i}(\sum_{k=1}^{n}|z_k|^{im+1/(k-1)m+1}$$
$$+\sum_{k=1}^{n}|z_k(t-d)|^{im+1/(k-1)m+1} + |v|^{im+1/nm+1}).$$

Theorem 3 Under Assumption 2, the problem of global output feedback stabilization of non-triangular system can be solved.

Proof Using the same argument in the proof of Theorem 2, for system (9) we use the exactly same observer and controller, system (9) can be rewritten as

$$\dot{\mathcal{L}} = \epsilon F(\mathcal{L}) + (\phi_1(\cdot)\ \ \frac{\phi_2}{\epsilon}\ \ \cdots\ \ \frac{\phi_{n-1}}{\epsilon^{n-2}}\ \ 0\ \ \cdots\ \ 0)^{\mathrm{T}}.$$

Assumption 2 leads to

$$|G_i(t, \epsilon, \mathcal{L}, \mathcal{L}(t-d))|$$
$$= |\frac{\phi_i(t, z_1, \dots, \epsilon^{n-1}z_n, z_{1d}, \dots, \epsilon^{n-1}z_{nd}, \epsilon^n v)}{\epsilon^{i-1}}|$$
$$\leqslant c\epsilon^{1+\mu_i}(\sum_{k=1}^{n}|z_k|^{(im+1)/((k-1)m+1)}$$
$$+\sum_{k=1}^{n}|z_k(t-d)|^{(im+1)/((k-1)m+1)} + |v|^{(im+1)/(nm+1)},$$

for $\forall\epsilon \in (0,1), z_{id} = z_i(t-d), (t, z, z(t-d), v) \in \mathbb{R}^{2n+2}$. As a direct consequence of Theorem 1, the global output feedback stabilization of (9) can be achieved.

5 Examples

We now present two examples to illustrate the effectiveness of the proposed method.

Example 1 Consider the upper-triangular system (1), here we choose $d(t) = 0.5(1+\sin t)$ and $\dot{d}(t) \leqslant 0.5$. By taking

$$m = -\frac{2}{7} \in (-\frac{1}{3}, 0),$$

system (1) satisfies Assumption 3.1. As a matter of fact, by Young's inequality, we have

$$|x_3(t-d(t))\sin x_3|$$
$$\leqslant \frac{3}{5}|x_3(t-d)|^{3/5} + \frac{2}{5}|\sin x_3|^{5/2}$$
$$\leqslant \frac{3}{5}|x_3(t-d)|^{3/5} + \frac{2}{5}|\sin x_3|^{5/3}$$
$$\leqslant |x_3(t-d)|^{3/5} + |x_3|^{5/3}$$
$$= |x_3(t-d)|^{(m+1)/(2m+1)} + |x_3|^{(m+1)/(2m+1)},$$
$$|u^3| \leqslant |u|^3 = |u|^{(2m+1)/(3m+1)}.$$

Construct the nonsmooth observer

$$\begin{cases} \dot{\hat{\eta}}_2 = -\epsilon\lambda_1\hat{z}_2, \\ \dot{\hat{\eta}}_3 = -\epsilon\lambda_2\hat{z}_3, \\ \hat{z}_2 = \mathrm{sgn}(\hat{\eta}_2 + \lambda_1 z_1)^{5/7}|\hat{\eta}_2 + \lambda_1 z_1|^{5/7}, \\ \hat{z}_3 = \mathrm{sgn}(\hat{\eta}_3 + \lambda_2 \hat{z}_2)^{3/5}|\hat{\eta}_3 + \lambda_2 \hat{z}_2|^{3/5}, \end{cases} \tag{10}$$

$$u = -\epsilon^3 b_3(\hat{z}_3^{7/3} + b_2(\hat{z}_2^{7/5} + b_1 z_1))^{1/7}, \tag{11}$$

where $\lambda_1, \lambda_2, b_1, b_2$ and b_3 are appropriate positive constants.

Here we choose $\lambda_1 = 3, \lambda_2 = 3.1, b_1 = 0.5, b_2 = 1, b_3 = 3$. By Theorem 2, there is a small gain $0 < \epsilon = 0.8 < 1$ such that the output feedback controller (11) renders system (1) globally asymptotically stable. Figs. 1–4 show the response of the closed-loop system (1), (10) and (11) with initial conditions $[x_1(t)\ \ x_2(t)\ \ x_3(t)] = [1\ \ 2\ \ 3], [\hat{\eta}_2(t)\ \ \hat{\eta}_3(t)] = [5\ \ 4]$, for $t \in [-0.5, 0]$.

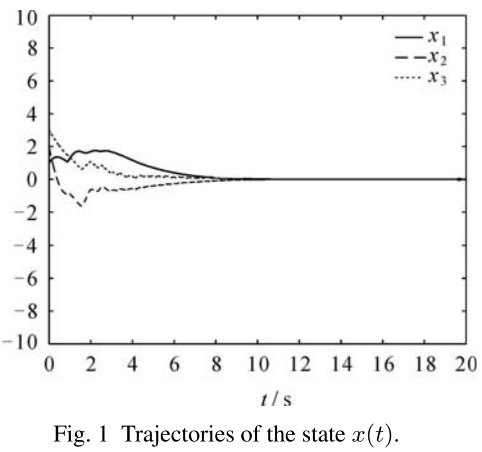

Fig. 1 Trajectories of the state $x(t)$.

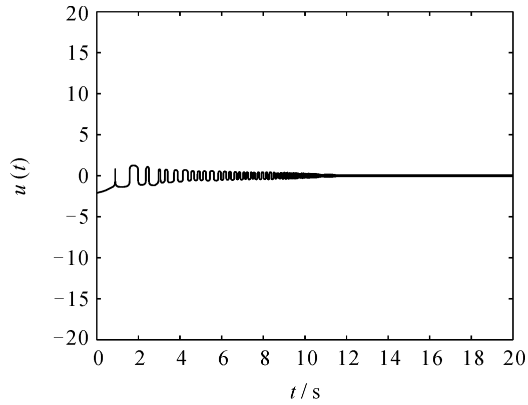

Fig. 2 Trajectory of control input $u(t)$.

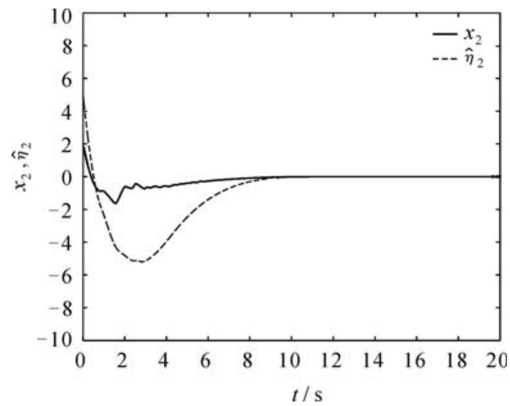

Fig. 3 Trajectories of x_2 and $\hat{\eta}_2$.

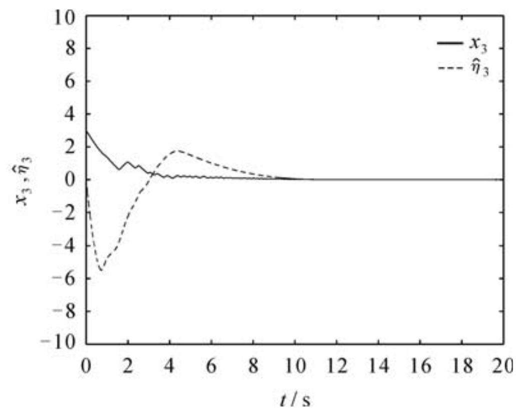

Fig. 4 Trajectories of x_3 and $\hat{\eta}_3$.

Remark 1 The observer (10) and the controller (11) are constructed only based on the nominal system (3). No precise information of the nonlinearities is needed. It means that the same dynamic controller (10)–(11) can be applied to different nonlinear systems as long as Assumption 3.1 is satisfied. This advantage can greatly reduce the design complexity normally associated with dynamic output feedback design.

Note that system (2) is a special case of system (9), so Theorem 3 can deal with general cases.

Example 2 Consider a non-triangular system with time-delay

$$\begin{cases} \dot{x}_1 = x_2 + x_3^{5/3}(t - d(t)), \\ \dot{x}_2 = x_3 + \ln(1 + x_1^2)u, \\ \dot{x}_3 = u, \\ y = x_1, \end{cases} \tag{12}$$

here we choose $d(t) = 0.3(1 + \cos t)$ and $\dot{d}(t) \leqslant 0.3$. Obviously, by taking $m = -\dfrac{2}{7} \in (-\dfrac{1}{3}, 0)$, system satisfies Assumption 3.2. When $\lambda_1 = 4, \lambda_2 = 4.1, b_1 = 0.5, b_2 = 1$ and $b_3 = 3, \epsilon = 0.7$, system (12) with initial condition $[x_1(t) \ \ x_2(t) \ \ x_3(t)] = [5 \ \ 8 \ \ 3], [\hat{\eta}_2(t) \ \ \hat{\eta}_3(t)] = [5 \ \ 4]$, for $t \in [-0.3, 0]$, by Theorem 3 we can explicitly construct output feedback controller (10)–(11), it guarantees that the system is globally asymptotically stable. Figs. 5–6 show the effectiveness of the proposed method.

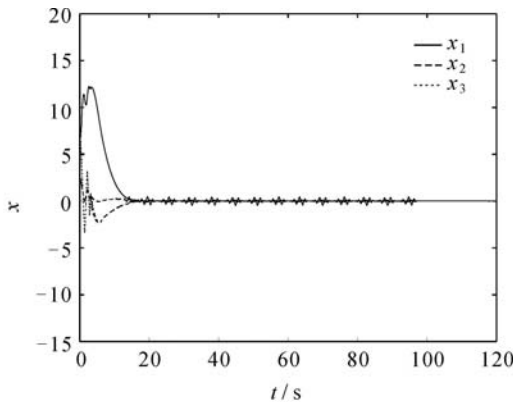

Fig. 5 Trajectories of the state $x(t)$.

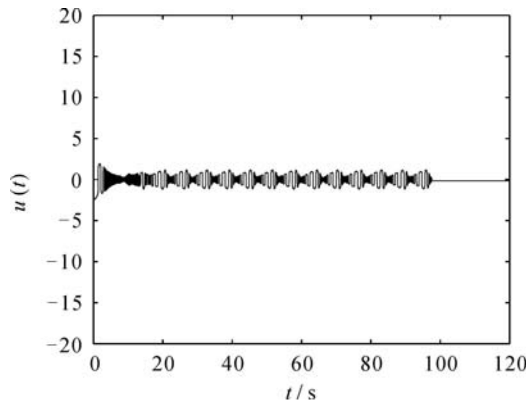

Fig. 6 Trajectory of the control input $u(t)$.

6 Conclusions

In this paper, the output feedback stabilization problem of a class of upper-triangular nonlinear systems with time-varying time-delay has been addressed. Several linear growth conditions that guarantee the existence of state feedback controller have been given. By utilizing the extended homogeneous domination theorem, the stabilization of a nonlinear system output feedback controller was constructed. Illustrative examples and simulations are given. It is also shown that design approach is applicable to more general cases.

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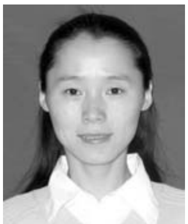

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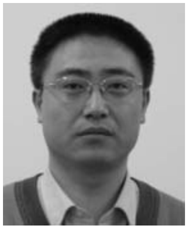

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