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Robust output-feedback control for stochastic nonlinear systems with modeling errors

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Abstract: In this paper, the stabilization problem of a stochastic nonlinear system with modeling errors is considered. An augmented observer is first presented to counteract the unmeasurable states as well as modeling errors. An adaptive output feedback controller is designed such that all signals in the closed-loop system are bounded in probability and the output is regulated to the origin almost surely.

Keywords: Stochastic systems; Nonlinear control; Modeling error; Backstepping

1 Introduction

Robust controller designs for the nonlinear systems in the presence of disturbances and modeling errors have received considerable attention over the last three decades, and several useful methodologies have been proposed. The robust control problem with only additive actuator disturbances was addressed and completely solved by Sontag in [1–3] by introducing the concept of input-to-state stability (ISS). In [4–6], these results were extended to the case of dynamic uncertainties by using a nonlinear small-gain theorem. In [7], for a nonlinear system in the presence of additive actuator disturbance, a high gain controller was proposed such that the output can be regulated to the origin under some strict assumptions suit for linear controller design. The situation becomes even more complex when a nonlinear system is in the presence of random disturbance. It can be easily seen that so many standard results in deterministic nonlinear control can only hold in a probability less than 1 in the stochastic setting (please refer to [8–16] for more detail).

In this paper, we consider the robust control problem for some classes of nonlinear systems with three types of uncertainties: 1) modeling errors and additive actuator disturbances; 2) internal nonlinear uncertainties; 3) external stochastic disturbance. The novelties can be founded by comparisons with the previous references.

- In [17] (the deterministic case) and [14–15] (the stochastic case), the modeling errors are constants and the output can only be regulated to a neighborhood of origin. In this paper, the modeling errors are unknown functions, and the output can be regulated to zero.
- In [7] (the deterministic case), only constant r_n was considered, i.e., $r_i = 0, i = 1, \ldots, n-1$. In this paper, all these errors are unknown functions and strict liner growth conditions are removed.
- Compared with [15, 18–19], a novel observer is constructed which is augmented by an identifier to signal caused by disturbances. The observer can serve multipurpose: the unmeasurable state is estimated, the unknown parameters are considered and a signal caused by modeling errors is identified (see (9) and (10)).
- An adaptive output-feedback controller is designed such that all signals of the closed-loop system are bounded in probability and the output can be regulated to the origin.

The following notations are used throughout the paper: $\mathbb{R}^n$ denotes the real n-dimensional space; $\mathbb{R}_+$ denotes the set of all nonnegative real numbers; $\mathbb{R}^{m\times n}$ denotes the real $m \times n$ matrix space. C^i denotes the set of all functions with continuous ith partial derivative; $\mathcal{K}$ denotes the set of all functions: $\mathbb{R}_+ \to \mathbb{R}_+$, which are continuous, strictly increasing and vanish at zero; $\mathcal{K}_\infty$ denotes the set of all functions which are of class $\mathcal{K}$ and unbounded. For a vector x, $|x|$ denotes its usual Euclidean norm, x^{T} denotes its transpose and $\bar{x}_i = (x_1, \ldots, x_i)^{\mathrm{T}}$; $\|X\|_{\mathrm{F}}$ denotes the Frobenius norm of a matrix X defined by $\|X\|_{\mathrm{F}} = (\mathrm{tr}\{XX^{\mathrm{T}}\})^{\frac{1}{2}}$, where $\mathrm{tr}(\,\cdot\,)$ denotes the matrix trace.

2 Mathematical preliminaries and problem formulation

Consider the nonlinear stochastic system

$$\mathrm{d}x = f(x,t)\mathrm{d}t + g(x,t)\mathrm{d}W,\ \ x(0) = x_0 \in \mathbb{R}^n, \quad (1)$$

where $x \in \mathbb{R}^n$ is the state, $W(t)$ is an m-dimensional independent standard Wiener process (or Brownian motion), the underlying complete filtration space is taken to be the quartet $(\varPsi, \mathcal{F}, \mathcal{F}_{\mathrm{t}}, P)$ with $\mathcal{F}_{\mathrm{t}}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all P-null sets), and functions f: $\mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ and g: $\mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^{n\times m}$ are locally Lipschitz and bounded in $x \in \mathbb{R}^n$.

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For $V(x) \in C^2(\mathbb{R}^n;\mathbb{R}_+)$, introduce the infinitesimal generator by

$$\mathcal{L}V(x) = V_x(x,t)f(x,t) + \frac{1}{2}\mathrm{tr}[g^{\mathrm{T}}(x,t)V_{xx}(x,t)g(x,t)], \tag{2}$$

where

$$V_x(x) = (\frac{\partial V(x)}{\partial x_1},\dots,\frac{\partial V(x)}{\partial x_n}),$$
$$V_{xx}(x) = (\frac{\partial^2 V(x)}{\partial x_p \partial x_q})_{n\times n}.$$

For stability analysis, the definition of boundedness in probability and the corresponding criterion are first presented. The former comes from [20], and the later is similar to that of [11] with slight difference that the origin is not necessarily the equilibrium of (1).

Definition 1 A stochastic process $x(t)$ is said to be bounded in probability if the random variables $|x(t)|$ are bounded in probability uniformly in t, i.e.,

$$\lim_{R\to\infty}\sup_{t>t_0} P\{|x(t)|>R\}=0. \tag{3}$$

Lemma 1 Consider system (1) and suppose there exists a C^2 function $V(x)$ and class $\mathcal{K}_\infty$ function α_1 and α_2, such that

$$\begin{cases}\alpha_1(|x|)\leqslant V(x)\leqslant \alpha_2(|x|),\\ \mathcal{L}V(x)\leqslant -W(x),\end{cases} \tag{4}$$

where $W(x)$ is a continuous and nonnegative function. Then for each $x_0 = x(0)\in\mathbb{R}^n$, there exists a unique strong solution $x(t) := x(x_0,t,\omega)$ of (1), which is bounded in probability, and moreover,

$$P\{\lim_{t\to\infty} W(x(t)) = 0\} = 1. \tag{5}$$

In the rest of this paper, we will consider the stochastic nonlinear system as follows:

$$\begin{cases}\mathrm{d}x_i(t) = (x_{i+1}(t)+\varPhi_i(\bar{x}_i(t))+r_i(t))\mathrm{d}t\\ \qquad +\varPsi_i(\bar{x}_i(t))\mathrm{d}W_i(t),\\ \mathrm{d}x_n(t) = (u(t)+\varPhi_n(x(t))+r_n(t))\mathrm{d}t\\ \qquad +\varPsi_n(x(t))\mathrm{d}W_n(t),\\ y(t)=x_1(t),\ \ i=1,2,\dots,n-1,\end{cases} \tag{6}$$

where $u\in\mathbb{R}$, $y\in\mathbb{R}$ and $x\in\mathbb{R}^n$ are input, output and state, respectively, $\varPhi_1,\dots,\varPhi_n\in\mathbb{R}$ and $\varPsi_1,\dots,\varPsi_n\in\mathbb{R}$ are locally Lipschitz functions, $W_1,\dots,W_n\in\mathbb{R}$ are independent standard Wiener processes, unknown function r_i denotes the unknown modeling error, and only the output y is available for feedback.

The control objective is to design an adaptive output-feedback controller robust against modeling error such that all signals in the closed-loop system are bounded in probability and the output can be regulated to the origin in almost surely sense. Throughout this paper, the following hypothesis is imposed.

A1 For every $1\leqslant i\leqslant n$, there exists an unknown constant θ_i satisfying

$$|\varPhi_i(\bar{x}_i)|^2\leqslant\theta_i\phi_i(y^2),\ \ |\varPsi_i(\bar{x}_i)|^2\leqslant\theta_i\psi_i(y^2), \tag{7}$$

where ϕ_i and ψ_i are nonnegative smooth functions with $\phi_i(0)=0$ and $\psi_i(0)=0$.

Remark 1 As in [7, 18], a more complicated case in which only output y is measurable is considered. When all the states can be obtained, the nonlinearities $\varPhi_i(\,\cdot\,)$ and $\varPsi_i(\,\cdot\,)$ can be assumed to depend on the other states.

According to assumption A1, there exist smooth functions $\bar{\phi}_i(y)$ and $\bar{\psi}_i(y)$ such that

$$\phi_i(y)=y\bar{\phi}_i(y),\ \ \psi_i(y)=y\bar{\psi}_i(y). \tag{8}$$

3 Main result

3.1 Observer design

Since $x_i\ (i=2,\dots,n)$ is unavailable and there exists the modeling error r_i, an augmented observer is introduced as follows:

$$\begin{cases}\mathrm{d}\hat{x}_i = (\hat{x}_{i+1}+k_{i+1}y-k_i(\hat{x}_1+k_1y))\mathrm{d}t,\\ \qquad\qquad 1\leqslant i\leqslant n-2,\\ \mathrm{d}\hat{x}_{n-1} = (\hat{r}+u+k_ny-k_{n-1}(\hat{x}_1+k_1y))\mathrm{d}t,\\ \mathrm{d}\hat{r} = -k_n(\hat{x}_1+k_1x_1)\mathrm{d}t,\end{cases} \tag{9}$$

where $\hat{r}$ is the estimation of $r := \sum\limits_{j=1}^{n} r_j^{(n-j)}$ and $k = (k_1,\dots,k_n)^{\mathrm{T}}$ is chosen such that

$$A_0 = \left[\begin{array}{c:c} -k & \begin{array}{c} I_{n-1}\\ 0\cdots0\end{array}\end{array}\right]$$

is asymptotically stable. From (6) and (9), the augmented observer error e can be defined by

$$\begin{cases}e=(e_1,\dots,e_n)^{\mathrm{T}},\\ e_i=\dfrac{1}{\theta^*}(x_{i+1}-\hat{x}_i+\sum\limits_{j=1}^{i}r_j^{(i-j)}-k_iy),\\ \qquad\qquad i=1,\dots,n-1,\\ e_n=-\hat{r}+r-k_ny,\end{cases} \tag{10}$$

which satisfies

$$\mathrm{d}e = A_0e\mathrm{d}t+\varDelta_1\mathrm{d}t+A_0\varDelta_2\mathrm{d}W \tag{11}$$

with

$$\begin{cases}\theta^*=\max\{1,\theta_1,\dots,\theta_n\}\\ \varDelta_1=\dfrac{1}{\theta^*}(\varPhi_2-k_1\varPhi_1,\dots,\varPhi_n-k_{n-1}\varPhi_1,-k_n\varPhi_1)^{\mathrm{T}},\\ \varDelta_2=\dfrac{1}{\theta^*}\mathrm{diag}\{\varPsi_1,\dots,\varPsi_n\},\\ W=(W_1,\dots,W_n)^{\mathrm{T}}.\end{cases} \tag{12}$$

Combining (7), (8) and (12) gives that

$$\begin{cases}|\varDelta_1|^4\leqslant 2(\sum\limits_{i=2}^{n}(\bar{\phi}_i+k_{i-1}^2\bar{\phi}_i))^2y^4,\\ \|\varDelta_2\|_{\mathrm{F}}^4\leqslant\sum\limits_{i=1}^{n}\bar{\psi}_i^2y^4.\end{cases} \tag{13}$$

For the backstepping controller design, some preliminaries should be first given. Consider the Lyapunov function candidate

$$V_e=\frac{1}{2}(e^{\mathrm{T}}Pe)^2, \tag{14}$$

where P satisfies $PA_0+A_0^{\mathrm{T}}P=-d_0I$, and d_0 is a design parameter. The infinitesimal generator of (13) satisfies

$$\begin{aligned}\mathcal{L}V_e =\ & 2(e^{\mathrm{T}}Pe)(A_0e+\varDelta_1)^{\mathrm{T}}Pe\\ &+\mathrm{tr}[\varDelta_2^{\mathrm{T}}A_0^{\mathrm{T}}(2Pee^{\mathrm{T}}P+e^{\mathrm{T}}PeP)A_0\varDelta_2]\\ \leqslant\ & -d_0\lambda_{\mathrm{m}}e^4+2(e^{\mathrm{T}}Pe)e^{\mathrm{T}}P\varDelta_1\\ &+\mathrm{tr}[\varDelta_2^{\mathrm{T}}A_0^{\mathrm{T}}(2Pee^{\mathrm{T}}P+e^{\mathrm{T}}PeP)A_0\varDelta_2],\end{aligned} \tag{15}$$

where $\lambda_{\rm m}$ is the smallest eigenvalue of P. By the aid of Young's inequality, for any design parameters $\delta_1, \delta_2, \delta_3 > 0$, it comes from (13) that

$$\begin{aligned}
&2(e^{\rm T}Pe)e^{\rm T}P\varDelta_1\\
&\leqslant 3\delta_1|e|^4 + 2\delta_1^{-3}\|P\|_{\rm F}^8(\sum_{i=2}^{n}(\bar{\phi}_i + k_{i-1}^2\bar{\phi}_i))^2y^4,\\
&2{\rm tr}[\varDelta_2^{\rm T}Pee^{\rm T}P\varDelta_2]\\
&\leqslant \delta_2|e|^4 + \delta_2^{-3}\|P\|_{\rm F}^4\|A_0\|_{\rm F}^4\sum_{i=1}^{n}\bar{\psi}_i^2y^4,\\
&{\rm tr}[\varDelta_2^{\rm T}e^{\rm T}PeP\varDelta_2]\\
&\leqslant \delta_3|e|^4 + \delta_3^{-1}\|P\|_{\rm F}^4\|A_0\|_{\rm F}^4\sum_{i=1}^{n}\bar{\psi}_i^2y^4. \qquad (16)
\end{aligned}$$

Substituting (16) into (15) yields

$$\mathcal{L}V_e \leqslant \varpi y^3 - (d_0\lambda_{\rm m} - b_0)|e|^4, \qquad (17)$$

where

$$\begin{aligned}
\varpi &= 2\delta_1^{-3}\|P\|_{\rm F}^8(\sum_{i=2}^{n}(\bar{\phi}_i + k_{i-1}^2\bar{\phi}_i))^2y\\
&\quad +(\delta_2^{-3} + \delta_3^{-1})\|P\|_{\rm F}^4\|A_0\|_{\rm F}^4\sum_{i=1}^{n}\bar{\psi}_i^2y,\\
b_0 &= 3\delta_1 + \delta_2 + \delta_3.
\end{aligned}$$

3.2 Controller design

Introducing the transformation

$$z_1 = y,\ \ z_{i+1} = \hat{x}_i - \alpha_i(y, \hat{x}_1, \dots, \hat{x}_{i-1}, \hat{\theta}), \quad i = 2, \dots, n \qquad (18)$$

with $z_{n+1} = 0$ and $\hat{x}_n = u + \hat{r}$, where α_i is stabilizing function to be designed later. It follows from (9) and (18) that

$$\begin{aligned}
{\rm d}z_i = &\big(z_{i+1} + \alpha_i + \eta_i - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\\
&-\frac{\partial\alpha_{i-1}}{\partial y}(e_1\theta^* + \varPhi_1) - \frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial y^2}\varPsi_1^2\big){\rm d}t\\
&-\frac{\partial\alpha_{i-1}}{\partial y}\varPsi_1{\rm d}W_1, \qquad (19)
\end{aligned}$$

where

$$\begin{aligned}
\eta_i = &\ k_iy - k_{i-1}(\hat{x}_1 + k_1y) - \frac{\partial\alpha_{i-1}}{\partial y}(\hat{x}_1 + k_1y)\\
&-\sum_{j=1}^{i-2}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} + k_{j+1}y - k_j(\hat{x}_1 + k_1y)).
\end{aligned}$$

Step 1 Choose the Lyapunov function candidate

$$V_1 = \frac{1}{4}y^4 + V_e + \frac{\gamma}{2}\tilde{\theta}^2, \qquad (20)$$

where $\tilde{\theta} = \theta - \hat{\theta}$, $\theta = \max\{\theta^*, \theta^{*4}\}$, $\theta^* = \max\{\theta_1, \dots, \theta_n\}$, $\hat{\theta}$ is the estimation of θ and $\gamma \geqslant 0$ is a design parameter. In view of (10) and (17)–(19), from (20), one has

$$\begin{aligned}
\mathcal{L}V_1 \leqslant &\ y^3(\hat{x}_1 + e_1\theta^* + k_1y + \varPhi_1) + \frac{3}{2}y^2\varPsi_1^2\\
&+\varpi y^3 - (d_0\lambda_{\rm m} - b_0)|e|^4 - \gamma\tilde{\theta}\dot{\hat{\theta}}. \qquad (21)
\end{aligned}$$

Combining(7) and (8) gives that

$$\frac{3}{2}y^2\varPsi_1^2 \leqslant \frac{3}{2}y^2\theta_1^2y^2\bar{\psi}_1 \leqslant \frac{3}{2}\theta\bar{\psi}_1y^4. \qquad (22)$$

According to Young's inequality, for any design parameters $d_{11}, d_{12}, d_{13} > 0$, one has

$$\begin{cases}
z_1^3z_2 \leqslant \frac{3}{4}d_{11}|z_1|^4 + \frac{1}{4}d_{11}^{-3}|z_2|^4,\\
y^3e_1\theta^* \leqslant \frac{3}{4}d_{12}^{-\frac{1}{3}}y^4\theta + \frac{1}{4}d_{12}|e|^4,\\
y^3\varPhi_1 \leqslant \frac{3}{4}d_{13}y^4 + \frac{1}{4}d_{13}^{-3}\theta y^4\bar{\phi}_1^2.
\end{cases} \qquad (23)$$

Substituting (22) and (23) into (21) yields that

$$\begin{aligned}
\mathcal{L}V_1 \leqslant &\ \frac{1}{4}d_{11}^{-3}z_2^4 + y^3(\frac{3}{4}d_{11}z_1\\
&+\alpha_1 + k_1y + \frac{3}{4}d_{13}y + \varpi + \omega_1\hat{\theta})\\
&+(\omega_1y^3 - \gamma\dot{\hat{\theta}})\tilde{\theta} - (d_0\lambda_{\rm m} - b_1)|e|^4, \qquad (24)
\end{aligned}$$

where

$$\begin{aligned}
b_1 &= b_0 + \frac{d_{12}}{4},\\
\omega_1 &= \frac{1}{4}d_{13}^{-3}y\bar{\phi}_1^2(y) + \frac{3}{2}\bar{\psi}_1(y)y + \frac{3}{4}d_{12}^{-\frac{1}{3}}y.
\end{aligned}$$

By choosing tuning τ_1 and stabilizing function $\alpha_1(y, \hat{x}_1, \hat{\theta})$ as

$$\begin{aligned}
\tau_1 &= \omega_1y^3, \qquad (25)\\
\alpha_1 &= -c_1z_1 - k_1z_1 - \frac{3}{4}d_{11}z_1 - \frac{3}{4}d_{13}z_1\\
&\quad -\varpi - \omega_1\hat{\theta} \qquad (26)
\end{aligned}$$

from (24), one has

$$\begin{aligned}
\mathcal{L}V_1 \leqslant &\ \frac{1}{4}d_{11}^{-3}z_2^4 - c_1z_1^4 - (d_0\lambda_{\rm m} - b_1)|e|^4\\
&+(\tau_1 - \gamma\dot{\hat{\theta}})\tilde{\theta}. \qquad (27)
\end{aligned}$$

Step i $(i = 2, \dots, n)$ Assume that one has designed smooth function $\tau_j, \alpha_j\ (2 \leqslant j \leqslant i-1)$ such that the infinitesimal generator of $V_{i-1} = V_{i-2} + \frac{1}{4}z_{i-1}^4$ satisfies

$$\begin{aligned}
\mathcal{L}V_{i-1} \leqslant &\ \frac{1}{4}d_{i-1,2}^{-3}z_i^4 - \sum_{j=1}^{i-1}c_jz_j^4\\
&-(d_0\lambda_{\rm m} - b_{i-1})|e|^4 + \sum_{j=2}^{i-1}\frac{1}{4}d_{j1}y^4\\
&+\sum_{j=2}^{i-1}\frac{1}{2}d_{j4}y^4 + \tilde{\theta}(\tau_{i-1} - \gamma\dot{\hat{\theta}})\\
&+\gamma^{-1}\sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}z_j^3(\tau_{i-1} - \gamma\dot{\hat{\theta}}), \qquad (28)
\end{aligned}$$

where $c_j, d_{jk} > 0\ (j = 2, \dots, i-1, k = 1, \dots, 4)$ are design parameters. In the sequel, we will prove that (28) holds for the ith Lyapunov function candidate $V_i = V_{i-1} + \frac{1}{4}z_i^4$. From (9) and (18), the infinitesimal generator of V_i satisfies

$$\begin{aligned}
\mathcal{L}V_i \leqslant &\ \mathcal{L}V_{i-1} + z_i^3(z_{i+1} + \alpha_i + \eta_i\\
&-\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} - \frac{\partial\alpha_{i-1}}{\partial y}(e_1\theta^* + \varPhi_1))\\
&-\frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial y^2}\varPsi_1^2z_i^3 + \frac{3}{2}z_i^2(\frac{\partial\alpha_{i-1}}{\partial y})^2\varPsi_1^2. \qquad (29)
\end{aligned}$$

According to Young's inequality, for any positive real numbers d_{i1}, d_{i2}, d_{i3} and d_{i4}, it is easy to verify that

$$\begin{cases} z_i^3 z_{i+1} \leqslant \frac{3}{4} d_{i1}|z_i|^4 + \frac{1}{4} d_{i1}^{-3}|z_{i+1}|^4, \\ -z_i^3 \frac{\partial \alpha_{i-1}}{\partial y} e_1 \theta^* \\ \leqslant \frac{1}{4} z_i^4 + d_{i2}^{-1} z_i^4 (\frac{\partial \alpha_{i-1}}{\partial y})^4 \theta + \frac{1}{4} d_{i2}|e|^4, \\ -z_i^3 \frac{\partial \alpha_{i-1}}{\partial y} \varPhi_1 \\ \leqslant \frac{1}{4} z_i^4 + d_{i3}^{-1} z_i^4 (\frac{\partial \alpha_{i-1}}{\partial y})^4 \bar{\phi}_1^2 \theta + \frac{1}{4} d_{i3} y^4, \\ \frac{3}{2} z_i^2 (\frac{\partial \alpha_{i-1}}{\partial y})^2 \varPsi_1^2 \leqslant \frac{9}{4} d_{i4}^{-1} (\frac{\partial \alpha_{i-1}}{\partial y})^4 \bar{\psi}_1^2 z_i^4 \theta + \frac{1}{4} d_{i4} y^4, \\ -\frac{1}{2} z_i^3 \frac{\partial^2 \alpha_{i-1}}{\partial y^2} \varPsi_1^2 \\ \leqslant \frac{1}{4} d_{i4}^{-1} z_i^2 (\frac{\partial^2 \alpha_{i-1}}{\partial y^2})^2 \bar{\psi}_1^2 z_i^4 \theta + \frac{1}{4} d_{i4} y^4. \end{cases} \tag{30}$$

Substituting (30) into (29) leads to

$$\begin{aligned} \mathcal{L}V_i \leqslant\ & \frac{1}{4} d_{i1}^{-3} z_{i+1}^4 + z_i^3 (\frac{1}{2} z_i + \frac{3}{4} d_{i1} z_i \\ & + \alpha_i + \eta_i - \gamma^{-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \tau_i + \omega_i \hat{\theta}) \\ & + \omega_i z_i^3 \tilde{\theta} - (d_0 \lambda_{\rm m} - b_i)|e|^4 - \sum_{j=1}^{i-1} c_j z_j^4 \\ & + \sum_{j=2}^{i} \frac{1}{4} d_{j3} y^4 + \sum_{j=2}^{i} \frac{1}{2} d_{j4} y^4 + \tilde{\theta}(\tau_{i-1} \\ & - \gamma \dot{\hat{\theta}}) + \sum_{j=2}^{i-1} \gamma^{-1} \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} z_j^3 (\tau_{i-1} - \gamma \dot{\hat{\theta}}) \\ & + \gamma^{-1} z_i^3 \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} (\tau_i - \gamma \dot{\hat{\theta}}), \end{aligned} \tag{31}$$

where

$$\begin{aligned} \omega_i &= d_{i2}^{-1} (\frac{\partial \alpha_{i-1}}{\partial y})^4 z_i + d_{i3}^{-1} (\frac{\partial \alpha_{i-1}}{\partial y})^4 \bar{\phi}_1^2 z_i \\ &\quad + \frac{1}{4} d_{i4}^{-1} (9 (\frac{\partial \alpha_{i-1}}{\partial y})^4 + z_i^2 (\frac{\partial^2 \alpha_{i-1}}{\partial y^2})^2) \bar{\psi}_1^2 z_i, \\ b_i &= b_{i-1} + \frac{1}{4} d_{i2}. \end{aligned}$$

By selecting tuning function τ_i and stabilizing function α_i as

$$\begin{cases} \tau_i = \tau_{i-1} + \omega_i z_i^3, \\ \alpha_i = -c_i z_i - \frac{1}{2} z_i - \frac{1}{4} d_{i-1}^{-3} z_i - \frac{3}{4} d_{i1} z_i - \eta_i \\ \qquad - \gamma^{-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \tau_i - \omega_i \hat{\theta} - \omega_i \sum_{j=2}^{i-1} \gamma^{-1} \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} z_j^3, \end{cases} \tag{32}$$

(30) can be turned to

$$\begin{aligned} \mathcal{L}V_i \leqslant\ & \frac{1}{4} d_{i1}^{-3} z_{i+1}^4 - \sum_{j=1}^{i} c_j z_j^4 - (d_0 \lambda_{\rm m} - b_i)|e|^4 \\ & + (\tau_i - \gamma \dot{\hat{\theta}}) \tilde{\theta} + \sum_{j=2}^{i} \frac{1}{4} d_{j3} y^4 + \sum_{j=2}^{i} \frac{1}{2} d_{j4} y^4. \end{aligned} \tag{33}$$

At the final step, we get control law and update law

$$u = \alpha_n - \hat{r}, \ \ \dot{\hat{\theta}} = \frac{1}{\gamma} \tau_n. \tag{34}$$

Since d_0 is independent of $\lambda_{\rm m}, \delta_1, \delta_2, \delta_3, d_{j2}$, and c_1 is independent of d_{j3}, d_{j4}, they can be selected to satisfy

$$\begin{cases} \frac{d_0 \lambda_{\rm m}}{2} \geqslant 3\delta_1 + \delta_2 + \delta_3 + \frac{1}{4} \sum_{j=1}^{i} d_{j2}, \\ \frac{1}{2} c_1 \geqslant \sum_{j=2}^{n} \frac{1}{4} d_{j3} + \sum_{j=2}^{n} \frac{1}{2} d_{j4}. \end{cases} \tag{35}$$

Finally, one has

$$\mathcal{L}V_n \leqslant -\frac{1}{2} c_1 z_1^4 - \sum_{j=2}^{n} c_j z_j^4 - \frac{d_0 \lambda_{\rm m}}{2} |e|^4. \tag{36}$$

3.3 Stability analysis

Now, the main result in this paper can be presented as follows.

Theorem 1 By choosing the design parameters appropriately, the closed-loop system consist of (6) and (34) has a unique solution, which is bounded in probability for any $x_0 \in \mathbb{R}^n$, and moreover,

$$P\{\lim_{t\to\infty} y(t) = 0\} = 1, \ \ P\{\lim_{t\to\infty} \hat{r}(t) = r\} = 1. \tag{37}$$

Proof From the continuity and nonnegativity of

$$W(z, \tilde{\theta}, e) = \frac{1}{2} c_1 z_1^4 + \sum_{j=2}^{n} c_j z_j^4 + \frac{d_0 \lambda_{\rm m}}{2} |e|^4,$$

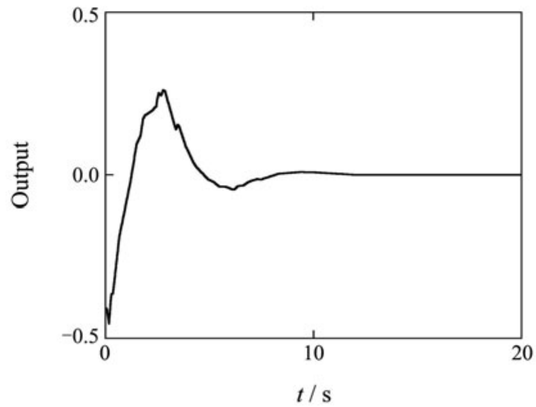
according to Lemma 1, for each $x_0 = x(0) \in \mathbb{R}^n$, there exists a unique strong solution to the closed-loop system, which is bounded in probability, and moreover,

$$P\{\lim_{t\to\infty} W(z, \tilde{\theta}, e) = 0\} = 1, \tag{38}$$

which means that (37) holds.

4 A simulation example

Consider system (6) ($n = 2$). Choose $\varPhi_1(\bar{x}_1) = \theta_1 y^2$, $\varPhi_2(\bar{x}_2) = \theta_2 y^3$, $\varPsi_1(\bar{x}_1) = \theta_1 y$, and $\varOmega_2(\bar{x}_2) = \theta_2 y^2$, with $\theta_1 = \theta_2 = 1$, where W being a scaler Wiener process. The observer is given by (9). The update and control law are given by (34). Choose the initial values $x_1(0) = -0.4$, $x_2(0) = -0.4$, $r(0) = 0.1$, $r_1 = 0.1 \cos t$, $r_2 = 0.3 \sin t$, the design parameters $k_1 = 1$, $k_2 = 1$, which satisfy the matrix A is Hurwitz, $d_0 = 0.1$, $\delta_1 = 0.1$, $\delta_2 = 1$, $\delta_3 = 1$, $d_{11} = 0.2$, $d_{12} = 1$, $d_{13} = 0.3$, $d_{21} = 1$, $d_{22} = 1$, $d_{23} = 0.5$, $d_{24} = 1$, $\hat{\theta}(0) = 0.5$, $\hat{x}_1(0) = 1.8$, $c_1 = 1.25$ (satisfying $c_1 \geqslant \frac{1}{2} d_{23} + d_{24}$), $c_2 = 0.1$ and $\gamma = 1$. Fig. 1 demonstrates that the output of stochastic nonlinear system with uncertainties can be regulated to the origin asymptotically.



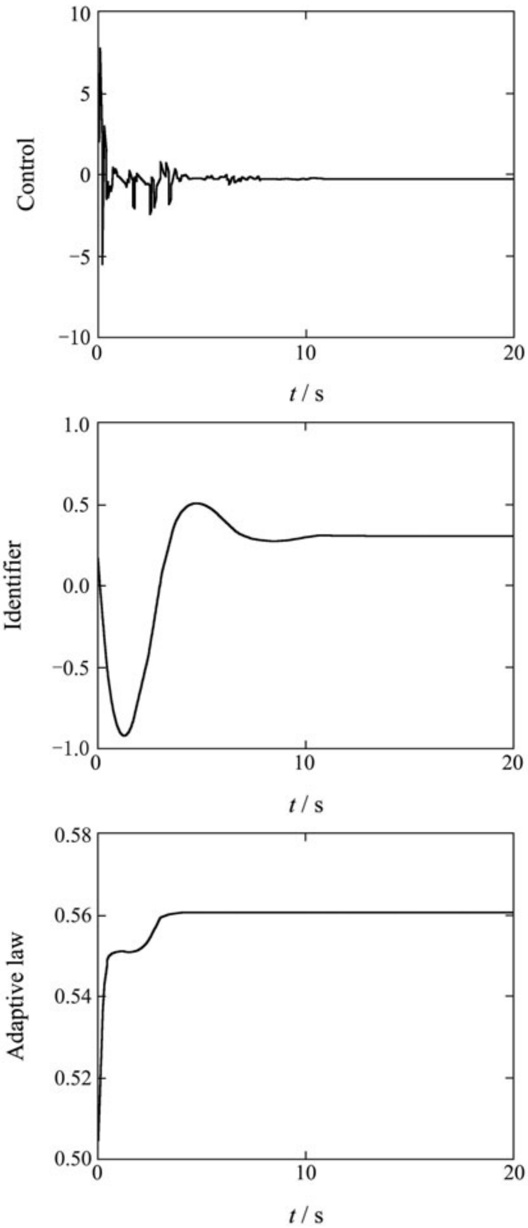


Fig. 1 The responses of stochastic nonlinear system.

5 Conclusions

In this paper, for a class of stochastic nonlinear systems in the presence of modeling errors, an adaptive output feedback backstepping controller is designed, which is robust against uncertainties not necessarily being constants. It is proved that all signals in the closed-loop system are bounded in probability and the output can be regulated to the origin almost surely. The efficiency of proposed method are verified by a simulation example.

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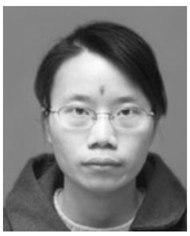

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