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Iterative learning control for nonlinear systems with uncertain state delay and arbitrary initial error

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Abstract: Most of the existing iterative learning control algorithms proposed for time-delay systems are based on the condition that the time-delay is precisely available, and the initial state is reset to the desired one or a fixed value at the start of each operation, which makes great limitation on the practical application of corresponding results. In this paper, a new iterative learning control algorithm is studied for a class of nonlinear system with uncertain state delay and arbitrary initial error. This algorithm needs to know only the boundary estimation of the state delay, and the initial state is updated, while the convergence of the system is guaranteed. Without state disturbance and output measurement noise, the system output will strictly track the desired trajectory after successive iteration. Furthermore, in the presence of state disturbance and measurement noise, the tracking error will be bounded uniformly. The convergence is strictly proved mathematically, and sufficient conditions are obtained. A numerical example is shown to demonstrate the effectiveness of the proposed approach.

Keywords: Iterative learning control; State-delay system; Initial condition; Uncertain disturbances

1 Introduction

The iterative learning control (ILC) method is a branch of intelligent control with strict mathematical description. When the same tasks are performed by a controlled system through repetitive high-speed operations, a controller in an iterative form, which generates control inputs progressively improving tracking performance and eventually leading to the perfect tracking along the entire span of the specified trajectory, can be designed with iterative learning control methodology. It just requires less a priori knowledge about the controlled systems and relies on less online-calculation burden. In the recent two decades, ILC has been extensively studied by experts and scholars with significant progress in both theory and application [1–4]. It is well known that time delays including control delays and state delays, are unavoidable in most industrial processes. As time delay is frequently encountered in actual systems and is often a source of instability, iterative learning control for systems with time delay has received increasing attention from researchers in recent years [5–7]. For example, Xu [8] discussed the problem of iterative control of batch processes with modeling errors and time delays, and the processes were represented by a transfer function plus dead time. Ji and Luo [9] gave the sufficient condition for the convergence of the linear system with control delay. Some monotone iterative schemes for computing the solution of a system of nonlinear difference equations that arise from a class of nonlinear reaction-diffusion equations with time delays were presented by Feng [10], while Hou and Ruan [11] studied the weighted leading open-loop PD-type iterative learning control algorithm for large-scale linear industrial processes with time delays. Yan [12] discussed the robust stability of interval dynamical systems with time delay by constructing suitable control matrix and iterative function. Despite the fact that nonlinear systems with state delays are pervasive in practical application, they have barely been investigated.

During the iterative process, a common assumption is that the initial state should be consistent with the desired value or set to a fixed point (which can be different from the desired initial state) in each iteration. Since the disturbance of initial state will directly affect the precision of trajectory tracking, this assumption is very important to the stability analysis of the system [13, 14]. However, in practice, zero initial deviation is difficult to be guaranteed; therefore, the research on the robustness of the initial state deviation is of great important significance. Sun [15] offered a novel initial state learning law, which allowed initial repositioning errors and initial states not to be in specified positions, but it was peculiar to the LTI systems. A discrete-time adaptive iterative learning control (AILC) scheme was presented in [16] to deal with systems with time-varying parametric uncertainties. When the initial states were random and the reference trajectory was iteration varying, the new ALIC could achieve the pointwise convergence over a finite time interval asymptotically along the iterative learning axis. A constructive discrete-time adaptive ILC approach was designed, which could perform well when the initial state value and the target trajectory were varying along the iteration axis [17, 18].

However, it can be found that most of the existing iterative learning control algorithms specific to time-delay systems only are restricted to those whose delay is precisely given, and initial resetting error is not allowed [19, 20].

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Moreover, under these disturbed conditions, the iterative algorithm uniform convergence still fails to be analyzed.

In this paper, a simple form of iterative learning control algorithm is proposed for a class of nonlinear systems with uncertain state delay and arbitrary initial error, and the system is of great generality and representative. This algorithm does not require the precise state delay but only its upper and lower bounds. When the learning convergence condition is satisfied, the control error will ultimately enter into some neighborhood of the desired value in the presence of the disturbances in the running process of the system. The neighbor size is decided by the bounds of the disturbances. The smaller the disturbances, the closer the learning control process gets to the desired one. Besides, the learning schemes under consideration are of robust convergence, which allows initial repositioning errors, and the initial states need not be specified positions. Sufficient conditions guaranteeing the convergence of the tracking error are stated and proven. A numerical example is presented to demonstrate the effectiveness of the proposed algorithm.

2 Problem formulation

The nonlinear system with uncertain state-delay and disturbances can be expressed in the following form:

$$\begin{cases} \dot{x}_k(t) = f\left(x_k(t), x_k(t-\tau), t\right) + B(t)u_k(t) \\ \qquad\quad + w_k\left(x_k(t), t\right), \\ y_k(t) = g\left(x_k(t), t\right) + v_k(t), \end{cases} \tag{1}$$

where $t \in [0, T]$; $k = 0, 1, 2, \dots$ is the iterative number; $x_k(t) \in \mathbb{R}^n$, $u_k(t) \in \mathbb{R}^m$, and $y_k(t) \in \mathbb{R}^r$ are the state, control input and output of the system, respectively. τ is an uncertain time delay, which satisfies $0 \leqslant \tau_1 \leqslant \tau \leqslant \tau_2 < T$, wherein τ_1, τ_2 are the lower and upper bounds of τ, respectively. $w_k(x_k(t), t) \in \mathbb{R}^n$ and $v_k(t) \in \mathbb{R}^r$ are the state disturbance and the output disturbance of the system, respectively. The functions $f : \mathbb{R}^n \times \mathbb{R}^n \times [0, T] \to \mathbb{R}^n$ and $B \in \mathbb{R}^{n\times m}$ are both piecewise continuous on $[0, T]$. $g : \mathbb{R}^n \times [0, T] \to \mathbb{R}^r$ is differentiable for all x and t. Besides, f, B, and g are not precisely known.

Relevant conditions of the nonlinear system are as follows.

Assumption 1 For any realizable bounded output trajectory $y_{\rm d}(t)$, there exists a unique control input $u_{\rm d}(t)$ to make the following equations hold:

$$\begin{cases} \dot{x}_{\rm d}(t) = f\left(x_{\rm d}(t), x_{\rm d}(t-\tau), t\right) + B(t)u_{\rm d}(t), \\ y_{\rm d}(t) = g\left(x_{\rm d}(t), t\right), \end{cases} \tag{2}$$

where $x_{\rm d}(t)$ is the desired state.

Assumption 2 Functions f, $B(t)$ and partial derivative g_x are all uniformly bounded, and the bounds are b_f, b_B, b_{gx}, respectively. f, g , g_x, and g_t are uniformly globally Lipschitz in x, $\forall t \in [0, T]$. That is,

$$\begin{aligned} &\|f\left(x_1(t), x_1\left(t-\tau\right), t\right) - f\left(x_2(t), x_2\left(t-\tau\right), t\right)\| \\ &\leqslant k_{f1}\|x_1(t)-x_2(t)\| + k_{f2}\left\|x_1\left(t-\tau\right)-x_2\left(t-\tau\right)\right\|, \end{aligned} \tag{3}$$

$$\|h\left(x_1(t), t\right) - h\left(x_2(t), t\right)\| \leqslant k_h \left\|x_1(t)-x_2(t)\right\|, \quad h \in \{g,\ g_x,\ g_t\}, \tag{4}$$

where k_{f1}, k_{f2}, and k_h are all Lipschitz constants.

Assumption 3 $B(t)$ is a full column rank matrix that satisfies $B(t)(B^{\rm T}(t)B(t))^{-1}B^{\rm T}(t) = \tilde{I}_n$, $\forall t \in [0, T]$. In particular, $\tilde{I}_n$ is a constant matrix that satisfies $\|\tilde{I}_n\| \leqslant 1$.

Assumption 4 Functions w and $\dfrac{{\rm d}v}{{\rm d}t}$ are bounded, and the upper boundaries of which are b_w and b_v, respectively. For example, $b_w = \sup\limits_{t\in[0,T],\forall x(t)\in\mathbb{R}^n} \|w\left(x(t), t\right)\|$.

Remark 1 In Assumption 3, $B(t)$ is a full column rank matrix to ensure the nonsingularity of $B^{\rm T}(t)B(t)$ on $[0, T]$.

The objective of the control can be described as follows: For any given desired trajectory $y_{\rm d}(t)$, the initial error of each iteration is not zero, and we find a control input $u_k(t)$ that is obtained through iteration to make the output $y_k(t)$ approach the desired output $y_{\rm d}(t)$ as closely as possible along the iterative axis as $k \to \infty$.

Thus, the explicit control law of nonlinear process with uncertain state delay and arbitrary initial error is finally given by

$$u_{k+1}(t) = u_k(t) + B_L(t)(\dot{e}_k(t+\tau_1) + \dot{e}_k(t+\tau_2)), \tag{5}$$

$$x_{k+1}(0) = x_k(0) + \tilde{I}_n L e_k(0), \tag{6}$$

where $B_L(t) = (B^T(t)B(t))^{-1}B^T(t)L$; $\tilde{I}_n$ is defined by Assumption 3; the initial learning state $x_1(0)$ can be selected discretionarily on $\mathbb{R}^n$; the first iterative input $u_1(t)$ can be chosen as any continuous admissible control; the kth iterative output error is $e_k(t) = y_{\rm d}(t) - y_k(t)$; L is adjustable constant gain matrix whose order is $n \times m$; and τ_1, τ_2 are the estimated lower and upper bounds of τ, respectively.

3 Main results

Lemma 1 [21] If $h(t) = \int_0^t {\rm e}^{a(t-\tau)} f(\tau){\rm d}\tau$, there is $\|h\|_\lambda \leqslant \dfrac{1-{\rm e}^{-\lambda T}}{\lambda}\|f\|_\lambda$ under the condition $a = 0$, where $t \in [0, T]$.

Lemma 2 [21] Real data sequence $\{a_k\}$ is supposed to satisfy

$$pa_k + qa_{k-1} \leqslant d_k, \quad k = 1, 2, \dots$$

wherein $\{d_k\}$ is the given real disturbance sequence. If $p > -q \geqslant 0$, then

1) $d_k \leqslant \bar{d}$ $(\forall k)$ implies

$$a_k \leqslant a_0 + \frac{\bar{d}}{p+q},\ \forall k,$$

$$\lim_{k\to\infty} \sup a_k \leqslant \frac{\bar{d}}{p+q}.$$

2) $\lim\limits_{k\to\infty} d_k = d_\infty$ implies

$$\lim_{k\to\infty} \sup a_k \leqslant \frac{d_\infty}{p+q}.$$

Theorem 1 When Assumptions 1–3 and the condition

$$\begin{aligned} &\|I - B_L(t)[g_{x{\rm d}}(t+\tau_1)B(t+\tau_1) \\ &+ g_{x{\rm d}}(t+\tau_2)B(t+\tau_2)]\| \leqslant \rho < 1 \end{aligned} \tag{7}$$

are satisfied, the control error $\|\Delta u_k(t)\|$, the state error $\|\Delta x_k(t)\|$, and the output error $\|\Delta e_k(t)\|$ of the uncertain state-delay nonlinear system (1) with arbitrary initial error (regardless of the state disturbances and output disturbances) will converge to zero as $k \to \infty$, where $g_{x{\rm d}}(\cdot)$ is the derivative of $g(\cdot,\cdot)$ with respect to x_{d}. When the system's external disturbances having boundary exists, the tracking error is uniformly bounded, and the convergent value is determined by the bounds of the disturbances.

To make rigorous analysis, the following notations are introduced:

$$f(x,t)=f(x(t),x(t-\tau),t),\ \ g(x,t)=g(x(t),t),$$
$$g_x(x,t)=g_x(x(t),t),\ \ \Delta u_k(t)=u_{\mathrm{d}}(t)-u_k(t),$$
$$\Delta x_k(t)=x_{\mathrm{d}}(t)-x_k(t),\ \ \Delta g_t(x_k)=g_t(x_{\mathrm{d}})-g_t(x_k),$$
$$\Delta g_{xk}(t)=g_{x\mathrm{d}}(t)-g_{xk}(t),\ \ w(x,t)=w(x(t),t),$$
$$\Delta f(x_k,t)=f(x_{\mathrm{d}},t)-f(x_k,t).$$

In addition, let

$$\begin{cases}\mathit{\Gamma}_{g\Delta f}^{d1}=g_{x\mathrm{d}}(t+\tau_1)\Delta f(x_k,t+\tau_1),\\ \mathit{\Gamma}_{g\Delta f}^{d2}=g_{x\mathrm{d}}(t+\tau_2)\Delta f(x_k,t+\tau_2),\\ \mathit{\Gamma}_{\Delta gf}^{k1}=\Delta g_{xk}(t+\tau_1)f(x_k,t+\tau_1),\\ \mathit{\Gamma}_{\Delta gf}^{k2}=\Delta g_{xk}(t+\tau_2)f(x_k,t+\tau_2),\\ \mathit{\Gamma}_{gB}^{d1}=g_{x\mathrm{d}}(t+\tau_1)B(t+\tau_1),\\ \mathit{\Gamma}_{gB}^{d2}=g_{x\mathrm{d}}(t+\tau_2)B(t+\tau_2),\\ \mathit{\Gamma}_{\Delta gB}^{k1}=\Delta g_{xk}(t+\tau_1)B(t+\tau_1),\\ \mathit{\Gamma}_{\Delta gB}^{k2}=\Delta g_{xk}(t+\tau_2)B(t+\tau_2),\\ \mathit{\Gamma}_{gw}^{d1}=g_{x\mathrm{d}}(t+\tau_1)w_k(x_k,t+\tau_1),\\ \mathit{\Gamma}_{gw}^{d2}=g_{x\mathrm{d}}(t+\tau_2)w_k(x_k,t+\tau_2),\\ \mathit{\Gamma}_{\Delta gw}^{k1}=\Delta g_{xk}(t+\tau_1)w_k(x_k,t+\tau_1),\\ \mathit{\Gamma}_{\Delta gw}^{k2}=\Delta g_{xk}(t+\tau_2)w_k(x_k,t+\tau_2).\end{cases}\tag{8}$$

Proof The $(k+1)$th control error can be derived from (5)

$$\begin{aligned}\Delta u_{k+1}(t)&=u_{\mathrm{d}}(t)-u_{k+1}(t)\\&=\Delta u_k(t)-B_L(t)\left(\dot e_k(t+\tau_1)+\dot e_k(t+\tau_2)\right).\end{aligned}\tag{9}$$

Considering $\dot e_k(t+\tau_1)$, according to (1), there is

$$\begin{aligned}&\dot e_k(t+\tau_1)\\&=\dot g(x_{\mathrm{d}},t+\tau_1)-\dot g(x_k,t+\tau_1)-\dot v_k(t+\tau_1)\\&=g_{x\mathrm{d}}(t+\tau_1)\dot x_{\mathrm{d}}(t+\tau_1)+g_{t+\tau_1}(x_{\mathrm{d}})\\&\quad-g_{xk}(t+\tau_1)\dot x_k(t+\tau_1)-g_{x\mathrm{d}}(t+\tau_1)\dot x_k(t+\tau_1)\\&\quad-g_{t+\tau_1}(x_k)+g_{x\mathrm{d}}(t+\tau_1)\dot x_k(t+\tau_1)-\dot v_k(t+\tau_1)\\&=\mathit{\Gamma}_{g\Delta f}^{d1}+\mathit{\Gamma}_{\Delta gf}^{k1}+\mathit{\Gamma}_{\Delta gB}^{k1}u_k(t+\tau_1)+\mathit{\Gamma}_{gB}^{d1}\Delta u_k(t+\tau_1)\\&\quad+\Delta g_{t+\tau_1}(x_k)-\dot v_k(t+\tau_1)-\mathit{\Gamma}_{gw}^{d1}+\mathit{\Gamma}_{\Delta gw}^{k1},\end{aligned}\tag{10}$$

where $\mathit{\Gamma}_{g\Delta f}^{d1}$, $\mathit{\Gamma}_{\Delta gf}^{k1}$, $\mathit{\Gamma}_{gB}^{d1}$, $\mathit{\Gamma}_{\Delta gB}^{k1}$, $\mathit{\Gamma}_{gw}^{d1}$, and $\mathit{\Gamma}_{\Delta gw}^{k1}$ are defined in (8).

Using Taylor's formula to expand $\Delta u_k(t+\tau_i)$ at t, and omitting the higher order terms, there is

$$\Delta u_k(t+\tau_i)=\Delta u_k(t)+\Delta\dot u_k\tau_i,\tag{11}$$

where $i\in\{1,2\}$.

Taking the norms on both sides of (10), and according to (11), Assumptions 1, 2, and 4, there is

$$\begin{aligned}&\|\dot e_k(t+\tau_1)\|\\&\leqslant m_1\|\Delta x_k(t+\tau_1)\|+m_2\|\Delta x_k(t+\tau_1-\tau)\|\\&\quad+m_3+m_4\tau_1+b_{\dot v},\end{aligned}\tag{12}$$

where

$$\begin{aligned}m_1&=b_{gx}k_{f1}+(b_f+b_w+b_Bb_u)k_{gx}+k_{gt},\\m_2&=b_{gx}k_{f2},\\m_3&=b_{gx}b_Bb_{\Delta u}+b_{gx}b_w,\\m_4&=b_{gx}b_Bb_{\Delta\dot u}.\end{aligned}$$

Similarly, the following results can be derived:

$$\begin{aligned}&\dot e_k(t+\tau_2)\\&=\dot g(x_d,t+\tau_2)-\dot g(x_k,t+\tau_2)-\dot v_k(t+\tau_2)\\&=g_{x\mathrm{d}}(t+\tau_2)\dot x_d(t+\tau_2)+g_{t+\tau_2}(x_d)-g_{t+\tau_2}(x_k)\\&\quad-g_{xk}(t+\tau_2)\dot x_k(t+\tau_2)-g_{x\mathrm{d}}(t+\tau_2)\dot x_k(t+\tau_2)\\&\quad+g_{x\mathrm{d}}(t+\tau_2)\dot x_k(t+\tau_2)-\dot v_k(t+\tau_2)\\&=\mathit{\Gamma}_{g\Delta f}^{d2}+\mathit{\Gamma}_{\Delta gf}^{k2}+\mathit{\Gamma}_{\Delta gB}^{k2}u_k(t+\tau_2)+\mathit{\Gamma}_{gB}^{d2}\Delta u_k(t+\tau_2)\\&\quad+\Delta g_{t+\tau_2}(x_k)-\dot v_k(t+\tau_2)-\mathit{\Gamma}_{gw}^{d2}+\mathit{\Gamma}_{\Delta gw}^{k2},\end{aligned}\tag{13}$$

where $\mathit{\Gamma}_{g\Delta f}^{d2}$, $\mathit{\Gamma}_{\Delta gf}^{k2}$, $\mathit{\Gamma}_{gB}^{d2}$, $\mathit{\Gamma}_{\Delta gB}^{k2}$, $\mathit{\Gamma}_{gw}^{d2}$, and $\mathit{\Gamma}_{\Delta gw}^{k2}$ are defined in (8). Moreover,

$$\begin{aligned}&\|\dot e_k(t+\tau_2)\|+m_2\|\Delta x_k(t+\tau_2-\tau)\|\\&\leqslant m_1\|\Delta x_k(t+\tau_2)\|+m_3+m_4\tau_2+b_{\dot v}.\end{aligned}\tag{14}$$

Substituting (10), (11), and (13) into (9), there is

$$\begin{aligned}&\Delta u_{k+1}(t)\\&=\left[1-B_L(t)\left(\mathit{\Gamma}_{gB}^{d1}+\mathit{\Gamma}_{gB}^{d2}\right)\right]\Delta u_k(t)\\&\quad-B_L(t)\left[\mathit{\Gamma}_{g\Delta f}^{d1}+\mathit{\Gamma}_{\Delta gf}^{k1}+\mathit{\Gamma}_{gB}^{d1}\Delta\dot u_k(t)\tau_1+\Delta g_{t+\tau_1}(x_k)\right.\\&\quad\left.+\mathit{\Gamma}_{\Delta gB}^{k1}u_k(t+\tau_1)-\dot v_k(t+\tau_1)-\mathit{\Gamma}_{gw}^{d1}+\mathit{\Gamma}_{\Delta gw}^{k1}\right]\\&\quad-B_L(t)\left[\mathit{\Gamma}_{g\Delta f}^{d2}+\mathit{\Gamma}_{\Delta gf}^{k2}+\mathit{\Gamma}_{\Delta gB}^{k2}\Delta\dot u_k(t)\tau_2+\Delta g_{t+\tau_2}(x_k)\right.\\&\quad\left.+\mathit{\Gamma}_{\Delta gB}^{k2}u_k(t+\tau_2)-\dot v_k(t+\tau_2)-\mathit{\Gamma}_{gw}^{d2}+\mathit{\Gamma}_{\Delta gw}^{k2}\right].\end{aligned}\tag{15}$$

Taking norms on the left and right sides of (15) and then according to Assumption 2, there is

$$\begin{aligned}&\|\Delta u_{k+1}(t)\|\\&\leqslant\rho\|\Delta u_k(t)\|+k_1\|\Delta x_k(t+\tau_1)\|\\&\quad+k_2\|\Delta x_k(t+\tau_1-\tau)\|+k_1\|\Delta x_k(t+\tau_2)\|\\&\quad+k_2\|\Delta x_k(t+\tau_2-\tau)\|+k_0+k_3,\end{aligned}\tag{16}$$

where

$$\begin{aligned}\rho&=\sup_{t\in[0,T]}\left\|1-B_L(t)\left(\mathit{\Gamma}_{gB}^{d1}+\mathit{\Gamma}_{gB}^{d2}\right)\right\|,\\k_1&=\sup_{t\in[0,T]}\|B_L(t)\left(b_{gx}k_{f1}+(b_f+b_w)k_{gx}\right.\\&\qquad\left.+k_{gx}b_Bb_u+k_{gt}\right)\|,\\k_2&=\sup_{t\in[0,T]}\|B_L(t)b_{gx}k_{f2}\|,\\k_3&=\sup_{t\in[0,T]}\|2B_L(t)\left(b_{\dot v}+b_{gx}b_w\right)\|,\\k_0&=\sup_{t\in[0,T]}\|B_L(t)b_{gx}b_Bb_{\Delta\dot u}(\tau_1+\tau_2)\|.\end{aligned}$$

Considering $\|\Delta x_{k+1}(t+\tau_*)\|$, $\tau_*\in\{\tau_1,\tau_1-\tau,\tau_2,\tau_2-\tau\}$ and using (5), (6), (12), and (14), we have

$$\begin{aligned}&\|\Delta x_{k+1}(t+\tau_*)\|\\&=\|\Delta x_k(0)+\tilde I_nL\Delta e_k(0)+\int_0^{t+\tau_*}(\dot x_{\mathrm{d}}(\sigma)-\dot x_{k+1}(\sigma))\mathrm{d}\sigma\|\\&\leqslant\|\Delta x_k(0)\|+\|L\|\|\Delta e_k(0)\|\\&\quad+\int_0^{t+\tau_*}\|f(x_{\mathrm{d}},\sigma)-f(x_{k+1},\sigma)+B(\sigma)[u_{\mathrm{d}}(\sigma)-u_k(\sigma)]\\&\quad-\tilde I_nL[\dot e_k(\sigma+\tau_1)+\dot e_k(\sigma+\tau_2)]-w_{k+1}(x_{k+1},\sigma)\|\mathrm{d}\sigma\\&\leqslant\|\Delta x_k(0)\|+\|L\|\|\Delta e_k(0)\|\end{aligned}$$

$$+\int_0^{t+\tau_*}\{k_{f1}\|\Delta x_{k+1}(\sigma)\|+k_{f2}\|\Delta x_{k+1}(\sigma-\tau)\| +b_B\|\Delta u_k(\sigma)\|+\|L\|[m_1\|\Delta x_k(\sigma+\tau_1)\| +m_2\|\Delta x_k(\sigma+\tau_1-\tau)\|+m_0+m_1\|\Delta x_k(\sigma+\tau_2)\| +m_2\|\Delta x_k(\sigma+\tau_2-\tau)\|]\}\mathrm{d}\sigma+n_1, \tag{17}$$

where

$$m_0=2m_3+m_4(\tau_1+\tau_2),$$
$$n_1=2(T+\tau_*)\|L\|b_{\dot v}+(T+\tau_*)b_w.$$

Taking λ-norms on both sides of the comparison sign of (17), using Lemma 1, there is

$$\begin{aligned}&\|\Delta x_{k+1}(t+\tau_*)\|_\lambda\\ &\leqslant\|\Delta x_k(0)\|+\|L\|\,\|\Delta e_k(0)\|\\ &\quad+k_{f1}a_1\|\Delta x_{k+1}(t+\tau_*)\|_\lambda\\ &\quad+k_{f2}a_1\|\Delta x_{k+1}(t+\tau_*)\|_\lambda\\ &\quad+b_Ba_1\|\Delta u_k(t+\tau_*)\|_\lambda\\ &\quad+\|L\|\{2m_1a_1\|\Delta x_k(t+\tau_*)\|_\lambda\\ &\quad+2m_2a_1\|\Delta x_k(t+\tau_*)\|_\lambda+a_0m_0\}+n_1,\end{aligned} \tag{18}$$

where $a_0=\dfrac{1-\mathrm{e}^{-\lambda(T+\tau_*)}}{\lambda}$, $a_1=\dfrac{1-\mathrm{e}^{-\lambda(t+\tau_*)}}{\lambda}$.

Thus, from (18), we have

$$\begin{aligned}&(1-k_{f1}a_1-k_{f2}a_1)\|\Delta x_{k+1}(t+\tau_*)\|_\lambda\\ &\leqslant\|\Delta x_k(0)\|+\|L\|\,\|\Delta e_k(0)\|+\|L\|\,a_0m_0\\ &\quad+2(m_1+m_2)a_1\|L\|\,\|\Delta x_k(t+\tau_*)\|_\lambda\\ &\quad+b_Ba_1\|\Delta u_k(t+\tau_*)\|_\lambda+n_1.\end{aligned} \tag{19}$$

According to system (1), as well as the iterative learning algorithms (5) and (6), there is

$$\begin{aligned}x_{k+1}(t)&=x_{k+1}(0)+\int_0^t[f(x_{k+1},\sigma)+B(\sigma)u_{k+1}(\sigma)\\ &\qquad+w_{k+1}(x_{k+1},\sigma)]\mathrm{d}\sigma\\ &=x_k(0)+\tilde I_nLe_k(0)+\int_0^t\{f(x_{k+1},\sigma)\\ &\qquad+B(\sigma)u_k(\sigma)+\tilde I_nL[\dot e_k(\sigma+\tau_1)\\ &\qquad+\dot e_k(\sigma+\tau_2)]+w_{k+1}(x_{k+1},\sigma)\}\mathrm{d}\sigma\\ &=x_k(t)-\int_0^tf(x_k,\sigma)\mathrm{d}\sigma+\int_0^tf(x_{k+1},\sigma)\mathrm{d}\sigma\\ &\qquad+\tilde I_nLe_k(0)+\int_0^tw_{k+1}(x_{k+1},\sigma)\mathrm{d}\sigma\\ &\qquad+\int_0^t\tilde I_nL[\dot e_k(\sigma+\tau_1)+\dot e_k(\sigma+\tau_2)]\mathrm{d}\sigma.\end{aligned} \tag{20}$$

Thus, from (20), it is easy to obtain

$$\begin{aligned}&\|x_{k+1}(t)-x_k(t)\|\\ &=\|\int_0^t(f(x_{k+1},\sigma)-f(x_k,\sigma))\mathrm{d}\sigma+\tilde I_nLe_k(0)\\ &\quad+\int_0^t\tilde I_nL(\dot e_k(\sigma+\tau_1)+\dot e_k(\sigma+\tau_2))\mathrm{d}\sigma\\ &\quad+\int_0^tw_{k+1}(x_{k+1},\sigma)\mathrm{d}\sigma\|\\ &\leqslant\int_0^t\|f(x_{k+1},\sigma)-f(x_k,\sigma)\|\mathrm{d}\sigma+\|L\|\|e_k(0)\|\\ &\quad+\|L\|\int_0^t(\|\dot e_k(\sigma+\tau_1)\|+\|\dot e_k(\sigma+\tau_2)\|)\mathrm{d}\sigma\\ &\quad+\int_0^t\|w_{k+1}(x_{k+1},\sigma)\|\mathrm{d}\sigma.\end{aligned} \tag{21}$$

Substituting (3), (12), and (14) into (21), there is

$$\begin{aligned}&\|x_{k+1}(t)-x_k(t)\|\\ &\leqslant\int_0^t[k_{f1}\|x_{k+1}(\sigma)-x_k(\sigma)\|\\ &\quad+k_{f2}\|x_{k+1}(\sigma-\tau)-x_k(\sigma-\tau)\|]\mathrm{d}\sigma\\ &\quad+\|L\|\|e_k(0)\|+\|L\|\int_0^t[m_1\|\Delta x_k(\sigma+\tau_1)\|\\ &\quad+m_2\|\Delta x_k(\sigma+\tau_1-\tau)\|+m_1\|\Delta x_k(\sigma+\tau_2)\|\\ &\quad+m_2\|\Delta x_k(\sigma+\tau_2-\tau)\|+m_0]\mathrm{d}\sigma+n_2.\end{aligned} \tag{22}$$

where $n_2=2T\|L\|b_{\dot v}+Tb_w$.

Taking λ-norms on both sides of (22) and using the properties of λ-norms, there is

$$\begin{aligned}&\|x_{k+1}(t)-x_k(t)\|_\lambda\\ &\leqslant k_{f1}\frac{1-\mathrm{e}^{-\lambda t}}{\lambda}\|x_{k+1}(t)-x_k(t)\|_\lambda\\ &\quad+k_{f2}\frac{\mathrm{e}^{\lambda\tau}[\mathrm{e}^{-\lambda\tau}-\mathrm{e}^{-\lambda(t+\tau)}]}{\lambda}\|x_{k+1}(t)-x_k(t)\|_\lambda\\ &\quad+\|L\|\{m_1\frac{\mathrm{e}^{-\lambda\tau_1}[\mathrm{e}^{\lambda\tau_1}-\mathrm{e}^{-\lambda(t-\tau_1)}]}{\lambda}\|\Delta x_k(t)\|_\lambda\\ &\quad+m_2\frac{[\mathrm{e}^{-\lambda(\tau-\tau_1)}+\mathrm{e}^{-\lambda(t+\tau-\tau_1)}]}{\lambda}\mathrm{e}^{\lambda(\tau-\tau_1)}\|\Delta x_k(t)\|_\lambda\\ &\quad+m_1\frac{\mathrm{e}^{-\lambda\tau_2}[\mathrm{e}^{\lambda\tau_2}-\mathrm{e}^{-\lambda(t-\tau_2)}]}{\lambda}\|\Delta x_k(t)\|_\lambda\\ &\quad+m_2\mathrm{e}^{\lambda(\tau-\tau_2)}\frac{[\mathrm{e}^{-\lambda(\tau-\tau_2)}+\mathrm{e}^{-\lambda(t+\tau-\tau_2)}]}{\lambda}\|\Delta x_k(t)\|_\lambda\\ &\quad+\|e_k(0)\|+\frac{(1-\mathrm{e}^{-\lambda T})m_0}{\lambda}\}+n_2\\ &=\|L\|(2m_1a_2\|\Delta x_k(t)\|_\lambda+2m_2a_2\|\Delta x_k(t)\|_\lambda\\ &\quad+\|e_k(0)\|+a_3m_0)+k_{f1}a_2\|x_{k+1}(t)-x_k(t)\|_\lambda\\ &\quad+k_{f2}a_2\|x_{k+1}(t)-x_k(t)\|_\lambda+n_2,\end{aligned} \tag{23}$$

where $a_2=\dfrac{1-\mathrm{e}^{-\lambda t}}{\lambda}$, $a_3=\dfrac{1-\mathrm{e}^{-\lambda T}}{\lambda}$.

Since

$$\begin{aligned}&\|x_{k+1}(t)-x_k(t)\|_\lambda\\ &=\|x_{k+1}(t)-x_d(t)+x_d(t)-x_k(t)\|_\lambda\\ &=\|\Delta x_k(t)-\Delta x_{k+1}(t)\|_\lambda\\ &\geqslant\|\Delta x_k(t)\|_\lambda-\|\Delta x_{k+1}(t)\|_\lambda,\end{aligned} \tag{24}$$

inequality (23) can be transformed into

$$\begin{aligned}&[1-k_{f1}a_2-k_{f2}a_2-2(m_1+m_2)\|L\|a_2]\|\Delta x_k(t)\|_\lambda\\ &-\|L\|(\|e_k(0)\|+a_3m_0)\\ &\leqslant(1-k_{f1}a_2-k_{f2}a_2)\|\Delta x_{k+1}(t)\|_\lambda+n_2.\end{aligned} \tag{25}$$

Thus, inequality (25) can be rewritten as

$$\begin{aligned}&[1-k_{f1}a_1-k_{f2}a_1-2(m_1+m_2)\|L\|a_1]\|\Delta x_k(t+\tau_*)\|_\lambda\\ &\leqslant(1-k_{f1}a_1-k_{f2}a_1)\|\Delta x_{k+1}(t+\tau_*)\|_\lambda\\ &\quad+\|L\|[\|e_k(0)\|+a_0m_0]+n_1.\end{aligned} \tag{26}$$

Substituting (19) into (26), there is

$$\begin{aligned}&[1-k_{f1}a_1-k_{f2}a_1-4(m_1+m_2)\|L\|a_1]\|\Delta x_k(t+\tau_*)\|_\lambda\\ &\leqslant b_Ba_1\|\Delta u_k(t+\tau_*)\|_\lambda+p_0,\end{aligned} \tag{27}$$

where

$$\begin{aligned}p_0=\;&\|\Delta x_k(0)\|+\|L\Delta e_k(0)\|+\|Le_k(0)\|\\ &+2\|L\|m_0a_0+2n_1.\end{aligned}$$

Thus, inequality (27) becomes

$$\|\Delta x_k(t+\tau_*)\|_\lambda\leqslant k_{\tau_*}\|\Delta u_k(t+\tau_*)\|_\lambda+p_{\tau_*}, \tag{28}$$

where

$$k_{\tau_*}=\frac{b_B\left(1-\mathrm{e}^{-\lambda(t+\tau_*)}\right)}{\lambda-[k_{f1}+k_{f2}+4\,(m_1+m_2)\,\|L\|]\left(1-\mathrm{e}^{-\lambda(t+\tau_*)}\right)}, \tag{29}$$

$$p_{\tau_*}=\frac{p_0\lambda}{\lambda-[k_{f1}+k_{f2}+4\,(m_1+m_2)\,\|L\|]\left(1-\mathrm{e}^{-\lambda(t+\tau_*)}\right)}. \tag{30}$$

Using the properties of Taylor series and λ-norms, it is easy to obtain

$$\|\Delta u_k\,(t+\tau_*)\|_\lambda \leqslant \mathrm{e}^{-\lambda\tau_*}\,\|\Delta u_k(t)\|_\lambda+\omega_{\tau_*}, \tag{31}$$

where

$$\omega_{\tau_*}=b_{\Delta\dot u}\tau_*\mathrm{e}^{-\lambda\tau_*}+0\tau_*\mathrm{e}^{-\lambda(t+\tau_*)}. \tag{32}$$

Therefore, inequality (28) can be converted into

$$\begin{aligned}&\|\Delta x_k\,(t+\tau_*)\|_\lambda\\ &\leqslant k_{\tau_*}\mathrm{e}^{-\lambda\tau_*}\,\|\Delta u_k(t)\|_\lambda+k_{\tau_*}\omega_{\tau_*}+p_{\tau_*},\end{aligned} \tag{33}$$

τ_* is replaced by τ_1, τ_2, $\tau_1-\tau$, $\tau_2-\tau$, respectively, and then, there are

$$\begin{aligned}&\|\Delta x_k\,(t+\tau_1)\|_\lambda\\ &\leqslant k_{\tau_1}\mathrm{e}^{-\lambda\tau_1}\,\|\Delta u_k(t)\|_\lambda+k_{\tau_1}\omega_{\tau_1}+p_{\tau_1},\end{aligned} \tag{34}$$

$$\begin{aligned}&\|\Delta x_k\,(t+\tau_2)\|_\lambda\\ &\leqslant k_{\tau_2}\mathrm{e}^{-\lambda\tau_2}\,\|\Delta u_k(t)\|_\lambda+k_{\tau_2}\omega_{\tau_2}+p_{\tau_2},\end{aligned} \tag{35}$$

$$\begin{aligned}&\|\Delta x_k\,(t+\tau_1-\tau)\|_\lambda\\ &\leqslant k_{\tau_1-\tau}\mathrm{e}^{-\lambda(\tau_1-\tau)}\,\|\Delta u_k(t)\|_\lambda+k_{\tau_1-\tau}\omega_{\tau_1-\tau}+p_{\tau_1-\tau},\end{aligned} \tag{36}$$

$$\begin{aligned}&\|\Delta x_k\,(t+\tau_2-\tau)\|_\lambda\\ &\leqslant k_{\tau_2-\tau}\mathrm{e}^{-\lambda(\tau_2-\tau)}\,\|\Delta u_k(t)\|_\lambda+k_{\tau_2-\tau}\omega_{\tau_2-\tau}+p_{\tau_2-\tau}.\end{aligned} \tag{37}$$

Multiplying both sides of (16) by $\mathrm{e}^{-\lambda t}$, and considering (34)–(37), we have

$$\begin{aligned}&\|\Delta u_{k+1}(t)\|_\lambda\\ &\leqslant \rho\,\|\Delta u_k(t)\|_\lambda+k_1k_{\tau_1}\,\|\Delta u_k(t)\|_\lambda\\ &\quad+k_2k_{\tau_1-\tau}\,\|\Delta u_k(t)\|_\lambda+k_1k_{\tau_2}\,\|\Delta u_k(t)\|_\lambda\\ &\quad+k_2k_{\tau_2-\tau}\,\|\Delta u_k(t)\|_\lambda+k_1k_{\tau_1}\omega_{\tau_1}+k_2k_{\tau_1-\tau}\omega_{\tau_1-\tau}\\ &\quad+k_1k_{\tau_2}\omega_{\tau_2}+k_2k_{\tau_2-\tau}\omega_{\tau_2-\tau}+k_1p_{\tau_1}+k_2p_{\tau_1-\tau}\\ &\quad+k_1p_{\tau_2}+k_2p_{\tau_2-\tau}+k_0+k_3\\ &=(\rho+\rho_1)\,\|\Delta u_k(t)\|_\lambda+k_0',\end{aligned} \tag{38}$$

where

$$\rho_1=k_1k_{\tau_1}+k_2k_{\tau_1-\tau}+k_1k_{\tau_2}+k_2k_{\tau_2-\tau}, \tag{39}$$

$$\begin{aligned}k_0'=&\,k_1k_{\tau_1}\omega_{\tau_1}+k_2k_{\tau_1-\tau}\omega_{\tau_1-\tau}+k_1k_{\tau_2}\omega_{\tau_2}\\ &+k_2k_{\tau_2-\tau}\omega_{\tau_2-\tau}+k_1p_{\tau_1}+k_2p_{\tau_1-\tau}+k_1p_{\tau_2}\\ &+k_2p_{\tau_2-\tau}+k_0+k_3.\end{aligned} \tag{40}$$

According to Assumption 2, we have

$$\lim_{k\to\infty}\sup\|\Delta u_k(t)\|_\lambda \leqslant \frac{k_0'}{1-\rho-\rho_1}. \tag{41}$$

When the uncertain state disturbance and the derivative of the output disturbance tend to zero, it is easy to draw $n_1=0$ (because $n_1=2(T+\tau_*)\|L\|b_{\dot v}+(T+\tau_*)b_w$). Thus, it can be learned from (29) and (32) that there always exists one λ, which is large enough to make $k_{\tau_*}\to 0$ and $\omega_{\tau_*}\to 0$; at the same time, when $k\to\infty$, from (30), we can obtain $p_{\tau_*}\to 0$ ($p_0\to 0$ when $a_0\to 0$ and $n_1=0$). Therefore, it can be seen from (39) and (40) that $\rho_1\to 0$ and $k_0'\to 0$ ($k_0+k_3=0$ when $b_{\dot v}=0$, $b_w=0$, and $k\to\infty$) . According to $\rho<1$ and the above results, a conclusion can be reached as follows:

$$\lim_{k\to\infty}\|\Delta u_k(t)\|=0. \tag{42}$$

From (33) and (4), we can also get

$$\lim_{k\to\infty}\|\Delta x_k(t)\|=0,\ \lim_{k\to\infty}\|\Delta y_k(t)\|=0. \tag{43}$$

When the disturbances of the system are bounded, n_1, p_0, p_{τ_*}, and k_0' are bounded. Thus, the tracking error is uniformly bounded, and the bound depends on $b_{\dot v}$ and b_w. The proof is completed.

4 Simulation results

In order to test the effectiveness of the proposed learning control algorithm, the following nonlinear state-delay system with external uncertain disturbances and output measurement noises can be considered:

$$\begin{aligned}\begin{bmatrix}\dot x_1(t)\\ \dot x_2(t)\end{bmatrix}&=\begin{bmatrix}2x_1(t)+3\sin(x_2\,(t-\tau))\\ 5x_2(t)+2\cos(x_1\,(t-\tau))\end{bmatrix}\\ &\quad+\begin{bmatrix}2t+1 & t^2\\ 1-t & 1\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}+\begin{bmatrix}w_1(t)\\ w_2(t)\end{bmatrix},\\ y(t)&=\begin{bmatrix}2x_1(t)\sin x_2(t)\\ x_2(t)\cos x_1(t)\end{bmatrix}+\begin{bmatrix}v_1(t)\\ v_2(t)\end{bmatrix},\end{aligned}$$

where

$$\begin{bmatrix}w_1(t)\\ w_2(t)\end{bmatrix}=\begin{bmatrix}0.4\cos(0.1\pi t)\\ 0.6\cos(0.1\pi t)\end{bmatrix},\quad \begin{bmatrix}v_1(t)\\ v_2(t)\end{bmatrix}=\begin{bmatrix}0.1\sin(0.1\pi t)\\ 0.2\sin(0.1\pi t)\end{bmatrix}.$$

The following function is chosen as target trajectories:

$$y_{\mathrm d}(t)=\begin{bmatrix}2t^2-3\\ 1+t^3\end{bmatrix},$$

where $t\in[0,10]$ s, and the sampling period is $\Delta T=0.05$ s.

L can be chosen by trial and error based on MATLAB technique, and the chosen L suffices not only for the convergence condition of the theorem but also for the faster convergence speed of the algorithm. For simulation, we chose

$$L=\begin{bmatrix}0.01\\ 0.02\end{bmatrix}B(t),\quad \begin{bmatrix}u_{1,0}(t)\\ u_{2,0}(t)\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix},\quad \begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}.$$

The obtained simulation results are shown in Figs. 1–4. Figs. 1 and 2 are the iterative tracking error curves when $\tau\in[0.02,\ 0.05]$ (the estimated lower and upper bounds of τ are 0.02 and 0.05, respectively). Figs. 3 and 4 are the iterative tracking error curves when $\tau\in[0.05,\ 0.08]$ (the estimated lower and upper bounds of τ are 0.05 and 0.08, respectively). The results concerning the uncertain state disturbances and output measure noises are shown in Figs. 1 and 3, while the results without concerning them are illustrated in Figs. 2 and 4.

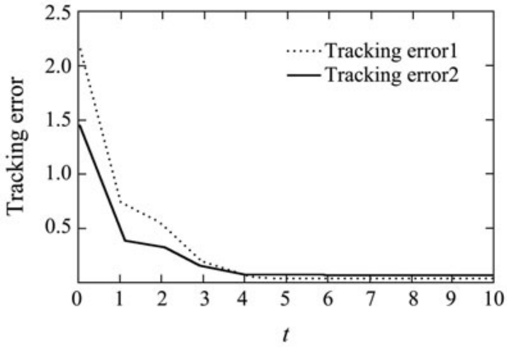

Fig. 1 Curve of iterative tracking error (with initial error and disturbances when $\tau \in [0.02,\ 0.05]$).

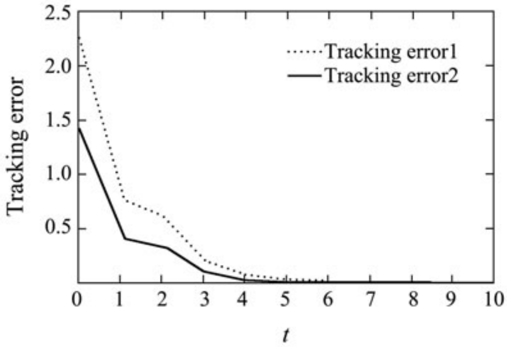

Fig. 2 Curve of iterative tracking error (with initial error and without disturbances when $\tau \in [0.02,\ 0.05]$).

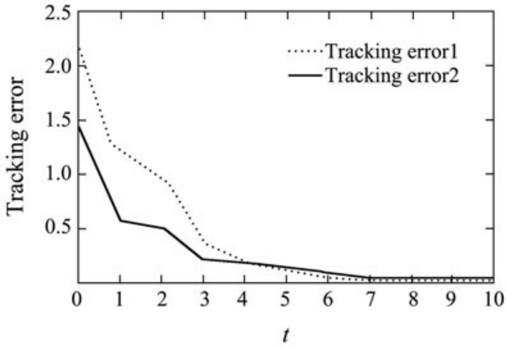

Fig. 3 Curve of iterative tracking error (with initial error and disturbances when $\tau \in [0.05,\ 0.08]$).

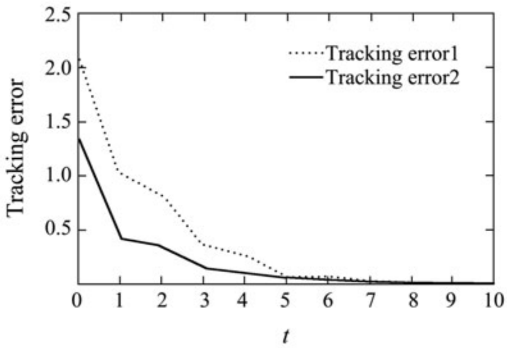

Fig. 4 Curve of iterative tracking error (with initial error and without disturbances when $\tau \in [0.05,\ 0.08]$).

It can be illustrated from the figures that the algorithm proposed has perfect control effects. In Figs.1 and 3, Figs. 2 and 4, respectively, we can see the curves in Fig. 2 have faster convergent speed than that in Fig. 4, and the curves in Fig. 1 have faster convergent speed than Fig. 3. We can come to the conclusion that the influences of the state delay can be almost entirely neglected when its bounds are estimated within one sampling period. Thus, the anticipant tracking performance can be realized by highly estimating the delay or lengthening the sampling period properly.

5 Conclusions

The present paper focuses on the problem of iterative control for nonlinear uncertain state-delay system with arbitrary initial error. A rigorous iterative control algorithm is proposed and, which states that the tracking error under the algorithm can converge to zero if there is no state disturbance or output measurement noise, while the tracking error is uniformly bounded in the presence of unavoidable uncertain state disturbance and output measurement noise. Meanwhile, this algorithm overcomes the defect of former methods which cannot be applied to the system with initial deviation in learning. The effects of the sampling frequency and state-delay on the tracking performance are analyzed. The simulations confirm the effectiveness of the presented method.

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