

An adaptive fuzzy design for fault-tolerant control of MIMO nonlinear uncertain systems

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Abstract: This paper presents a novel control method for accommodating actuator faults in a class of multiple-input multiple-output (MIMO) nonlinear uncertain systems. The designed control scheme can tolerate both the time-varying lock-in-place and loss of effectiveness actuator faults. In each subsystem of the considered MIMO system, the controller is obtained from a backstepping procedure; an adaptive fuzzy approximator with minimal learning parameterization is employed to approximate the package of unknown nonlinear functions in each design step. Additional control effort is taken to deal with the approximation error and external disturbance together. It is proven that the closed-loop stability and desired tracking performance can be guaranteed by the proposed control scheme. An example is used to show the effectiveness of the designed controller.

Keywords: Fault-tolerant control; MIMO nonlinear system; Adaptive fuzzy control; Backstepping

1 Introduction

Fault-tolerant control (FTC) has attracted more and more attention in recent years since it is important for safety and reliability of modern engineering systems. The faults that engineering systems are subject to usually occur in actuators, sensors, and some parts of the plant. As actuator faults may cause undesired system behavior and sometimes lead to instability or even catastrophic accidents, it is necessary to develop FTC methods against actuator faults in system operation.

Generally speaking, FTC can be achieved either passively or actively. In passive FTC, the considered faults are regarded as external disturbances, and robust control can be employed to synthesize the controller. While in active FTC, the controller is designed to be reconfigurable for occurring faults. Since active FTC can usually get better control performances, it has attracted more attentions in recent years and several design methods have been developed, such as multiple-model-based design [1], fault detection and diagnosis (FDD)-based design [2], and neural network-based designs [3, 4]. The common property of these methods is that a FDD mechanism is needed to determine the fault information for the controller reconfiguration; thus, when the information obtained from the FDD mechanism is incorrect, the FTC design will fail. With the learning capability of adaptive approach, FTC can be designed without resorting to FDD. Adaptive FTC has been developed to deal with actuator faults in literature. Ye and Yang [5] considered loss of effectiveness fault of actuator for FTC of linear systems. Tao et al. [6] and Tang et al. [7, 8] studied lock-in-place (stuck at some place) fault of actuator for FTC of nonlinear systems. However, these results require the nonlinearities of the controlled systems to be known. Li and Yang [9, 10] presented adaptive fuzzy approaches for FTC of unknown nonlinear single-input single-output (SISO) systems against both loss

of effectiveness and lock-in-place actuator faults.

Adaptive fuzzy control, as an effective control method for unknown nonlinear systems, has been widely used since Wang proposed stable adaptive fuzzy control for a class of unknown nonlinear systems in [11]. Several control schemes were developed in [12~19] for some unknown nonlinear SISO systems based on adaptive fuzzy approximation theory. Adaptive fuzzy control for unknown nonlinear multiple-input multiple-output (MIMO) systems was studied in [20~23]. For feedback linearizable systems, the control design is easier because the nonlinearities only exist in the last state equation of the system dynamics, see references [11~15, 20]. For strict-feedback nonlinear systems, backstepping technique is employed to obtain a control law, see references [16~19, 21~23]. In [22, 23], adaptive fuzzy control of nonlinear MIMO systems was achieved with less learning parameters. But so far, there have been few results on FTC of unknown nonlinear systems with multiinputs multioutputs and strict-feedback structure.

In this paper, an FTC scheme is proposed for unknown nonlinear MIMO systems with strict-feedback structure. The proposed controller taking the redundant actuation structure in [8] can accommodate both lock-in-place and loss of effectiveness faults of the actuators, although the faults are time-varying. The main contributions of our work are that 1) it develops an FTC approach for MIMO nonlinear systems with unknown nonlinearities and external disturbances, and 2) it enlarges the tolerable fault set to one, which contains both time-varying lock-in-place and loss of effectiveness faults. Therefore, the proposed control scheme can deal with more types of actuator faults in more general nonlinear systems than the existing results. Besides, the adaptive learning for each fuzzy approximator is within minimal learning parameterization as in [22]. However, by a novel design with the help of hyperbolic tangent functions,

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the proposed control law, compared with the one in [22], can obtain the desired tracking performance with smaller range of control amplitude despite actuator faults and external disturbances.

This paper is organized as follows. The problem formulation and preliminaries are presented in Section 2. Section 3 gives the design and analysis of the proposed adaptive fuzzy FTC scheme. In Section 4, a simulation example is employed to illustrate the effectiveness of the control scheme. Finally, Section 5 concludes the paper.

2 Problem formulation and preliminaries

Consider the following uncertain MIMO nonlinear system:

$$\begin{cases} \dot{x}_{ij} = f_{ij}(\bar{x}_{1(j-\rho_{i1})}, \bar{x}_{2(j-\rho_{i2})}, \dots, \bar{x}_{m(j-\rho_{im})}) \\ \quad + g_{ij}(\bar{x}_{1(j-\rho_{i1})}, \bar{x}_{2(j-\rho_{i2})}, \dots, \\ \quad \bar{x}_{m(j-\rho_{im})})x_{i(j+1)} + d_{ij}(t), \\ \dot{x}_{i\rho_i} = f_{i\rho_i}(\bar{x}, \bar{u}_1, \dots, \bar{u}_{i-1}) + \bar{g}_{i\rho_i}^T(\bar{x})\bar{u}_i + d_{i\rho_i}(t), \\ y_i = x_{i1}, \quad 1 \leq i \leq m, \quad 1 \leq j \leq \rho_i - 1, \end{cases} \quad (1)$$

where x_{ij} is the j th state variable of the i th subsystem, and ρ_i is the order of the i th subsystem. $\bar{u}_i = [u_{i1} \ u_{i2} \ \dots \ u_{ik_i}] \in \mathbb{R}^{k_i}$ is the control input vector of i th subsystem, where k_i is the number of its elements, and $y_i \in \mathbb{R}$ is the output of the i th subsystem. $\bar{g}_{i\rho_i}(\bar{x}) \in \mathbb{R}^{k_i}$ is a functional vector. $f_{ij}(\cdot)$, $g_{ij}(\cdot)$ for $j = 1, 2, \dots, \rho_i - 1$, $f_{i\rho_i}(\bar{x})$ and the elements of $\bar{g}_{i\rho_i}(\bar{x})$, which are denoted by $g_{i\rho_i1}(\bar{x}), \dots, g_{i\rho_ik_i}(\bar{x})$, are all unknown smooth nonlinear functions. For simplicity, throughout this paper, the following notations are used: $\bar{x}_{ij} = [x_{i1} \ x_{i2} \ \dots \ x_{ij}]^T$, $\bar{x} = [\bar{x}_{1\rho_1}^T \ \bar{x}_{2\rho_2}^T \ \dots \ \bar{x}_{m\rho_m}^T]^T$ and $\rho_{ij} = \rho_i - \rho_j$, with ρ_i being the order of the i th subsystem.

Remark 1 If $j - \rho_{ir} \leq 0$, $r = 1, \dots, m$, then the corresponding $\bar{x}_{r(j-\rho_{ir})}$ does not exist and does not appear in (1). If $j - \rho_{ir} > 0$, $\bar{x}_{r(j-\rho_{ir})}$ stands for the r th subsystem's state vector, which is embedded in the i th subsystem.

The considered faults are modeled as follows:

1) Lock-in-place fault

$$u_{ij}^F(t) = \bar{u}_{ij}(t), \quad t \geq t_{ij}, \quad j \in \{\bar{j}_1, \bar{j}_2, \dots, \bar{j}_p\}, \quad (2)$$

where $\bar{u}_{ij}(t)$ stands for the place, where the j th actuator in the i th subsystem is stuck. t_{ij} is the time instant at which the lock-in-place fault occurs.

2) Loss of effectiveness fault

$$u_{ik}^F(t) = \phi_{ik}(t)v_{ik}(t), \quad t \geq t_{ik}, \quad k \in \{j_1, j_2, \dots, j_p\}, \quad (3)$$

where $v_{ik}(t)$ is the k th applied control input of the i th subsystem, and t_{ik} is the time instant when the loss of effectiveness fault takes place. $\phi_{ik} \in [\underline{\phi}_{ik}, 1]$ is the effectiveness factor of the corresponding actuator, and $0 < \underline{\phi}_{ik} \leq 1$ is the lower bound of ϕ_{ik} . When $\underline{\phi}_{ik} = 1$, the actuator is normal.

Then, taking (2) and (3) into account, the control input $u_{ij}(t)$ can be written as

$$u_{ij} = (1 - \sigma_{ij})\phi_{ij}(t)v_{ij} + \sigma_{ij}\bar{u}_{ij}(t), \quad (4)$$

$$i = 1, 2, \dots, m, \quad j = 1, 2, \dots, \rho_i,$$

where σ_{ij} is the lock factor defined as follows:

$$\sigma_{ij} = \begin{cases} 1, & \text{if the } j\text{th actuator in the } i\text{th subsystem} \\ & \text{is stucked,} \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

Different from the case in [6] where the fault changes suddenly, the fault signals considered in this paper are continuous with respect to t . This implies that the considered faults can enter into the system either slowly or fast. Also, it can be so abrupt that one can regard it as a sudden change. Note that the faults considered in this paper are time-varying, and the information about them is not available. This kind of faults has not been considered in the existing literature; however, it is more general in the real physical systems.

The control objective is to design a state feedback controller for system (1) such that the desired control performance can be guaranteed, that is, all the signals in the closed-loop system are bounded and the outputs track the given reference signals as closely as possible, although the nonlinearities of the system are unknown and some unknown time-varying actuator faults and external disturbances enter into the controlled system. In order to accomplish the control task, the following assumptions are demanded for our design.

Assumption 1 System (1) is so constructed that, for the i th ($1 \leq i \leq m$) subsystem, when any up to $k_i - 1$ actuators stuck at some unknown places, the other(s) may lose effectiveness but $\phi_{ij} \in [\underline{\phi}_{ij}, 1]$, the closed-loop system can still be driven to achieve the desired control objective.

Assumption 2 For the i th subsystem, there exist constants \underline{a}_{ij} and \bar{a}_{ij} such that $0 < \underline{a}_{ij} \leq |g_{ij}(\cdot)| \leq \bar{a}_{ij}$ for $j = 1, 2, \dots, \rho_i - 1$, there also exist $\underline{a}_{i\rho_i r}$ and $\bar{a}_{i\rho_i r}$ such that $0 < \underline{a}_{i\rho_i r} \leq |g_{i\rho_i r}(\cdot)| \leq \bar{a}_{i\rho_i r}$, $1 \leq r \leq k_i$.

From Assumption 2, it can be seen that the continuous functions g_{ij} and $g_{i\rho_i r}$ are strictly either positive or negative. Without loss of generality, we assume that $g_{ij} > 0$ and $g_{i\rho_i r} > 0$.

Assumption 3 The external disturbances are bounded, that is, $|d_{ij}(t)| \leq \bar{d}_{ij}$, where \bar{d}_{ij} are known constants.

Assumption 4 The time-varying signals $\phi_{ij}(t)$ and $\bar{u}_{ij}(t)$ are continuous with respect to t .

These assumptions are reasonable for real controlled systems. In order to clarify the control design, the following lemmas are introduced.

Lemma 1 For any given real continuous function $F(x)$ on a compact set $\Omega \subseteq \mathbb{R}^n$, there exists a fuzzy logic system $Y(x) = \theta^T \xi(x)$ such that $\forall \varepsilon > 0$,

$$|F(x) - \vartheta^T \xi(x)| < \varepsilon, \quad (6)$$

where $\vartheta = (\vartheta_1, \vartheta_2, \dots, \vartheta_M)^T$ is the weight vector, and $\xi(x) = (\xi_1(x), \xi_2(x), \dots, \xi_M(x))^T$ is the vector of fuzzy basis functions with its elements defined as

$$\xi_j(x) = \frac{\prod_{i=1}^n \mu_{A_i^j}(x_i)}{\sum_{j=1}^M \prod_{i=1}^n \mu_{A_i^j}(x_i)},$$

where n is the dimension of x , M is the number of fuzzy rules, and $\mu_{A_i^j}(x_i)$ is the membership function of fuzzy variable A_i^j . One can refer to reference [11] for more details.

Lemma 2 Let $M(x_1, x_2, \dots, x_n)$ be a real-valued continuous function and satisfy

$$0 < a_m \leq M(x_1, x_2, \dots, x_n) \leq a_M$$

with a_m and a_M being two constants. Define functions $V(t)$

as follows:

$$V(t) = \int_0^{z(t)} \chi M(x_1, x_2, \dots, x_{k-1}, \chi + \beta(t), x_{k+1}, \dots, x_n) d\chi,$$

where $z(t)$ and $\beta(t)$ are real-value functions with $t \in [0, \infty)$. Then, the integral function $V(t)$ has the following properties:

$$\begin{aligned} \frac{1}{2} a_m z^2(t) &\leq V(t) \leq \frac{1}{2} a_M z^2(t), \\ \frac{dV(t)}{dt} &= z(t)M(x_1, \dots, x_{k-1}, z(t) + \beta(t), x_{k+1}, \dots, x_n) \dot{z}(t) \\ &\quad + \dot{\beta}(t)z(t)M(x_1, \dots, x_{k-1}, z(t) + \beta(t), x_{k+1}, \dots, x_n) \\ &\quad + z^2(t)\dot{x}_i(t) \int_0^1 \theta \sum_{i=1, i \neq k}^n \frac{\partial}{\partial x_i} M(x_1, \dots, x_{k-1}, z(t) \\ &\quad + \beta(t), x_{k+1}, \dots, x_n) d\theta - z(t)\dot{\beta}(t) \int_0^1 M(x_1, \dots, \\ &\quad x_{k-1}, \theta z(t) + \beta(t), x_{k+1}, \dots, x_n) d\theta. \end{aligned}$$

The proof of Lemma 2 can be found in [22].

3 Controller design and stability analysis

In this section, we present the design of the adaptive fuzzy fault-tolerant controller by backstepping design procedure. The reason we take fuzzy logic systems but not neural networks is that it can incorporate both numerical data and expert knowledge to achieve faster convergence. For the i th subsystem of system (1), the backstepping design requires ρ_i steps.

Step 1 Define $z_{i1} = x_{i1} - y_{id}$, and it can be obtained that

$$\dot{z}_{i1} = f_{i1} + g_{i1}x_{i2} + d_{i1} - \dot{y}_{id}, \tag{7}$$

where f_{i1} and g_{i1} are short for the unknown functions

$$\begin{aligned} f_{i1}(\bar{x}_{1(1-\rho_{i1})}, \dots, \bar{x}_{m(1-\rho_{im})}), \\ g_{i1}(\bar{x}_{1(1-\rho_{i1})}, \dots, \bar{x}_{m(1-\rho_{im})}). \end{aligned}$$

If $1 - \rho_{ik} \leq 0, k = 1, 2, \dots, m$, the corresponding state vector $\bar{x}_{k(1-\rho_{ik})}$ does not exist in (7). Consider a Lyapunov function candidate as

$$V_{i1} = \int_0^{z_{i1}} \chi M_{i1}(\chi + y_{id}) d\chi + \frac{1}{2\gamma_{i1}} \hat{\theta}_{i1}^2, \tag{8}$$

where

$$\begin{aligned} M_{i1}(\chi + y_{id}) &= g_{i1}^{-1}(\bar{x}_{1(1-\rho_{i1})}, \dots, \bar{x}_{(i-1)(1-\rho_{i(i-1)})}, \chi \\ &\quad + y_{id}, \bar{x}_{(i+1)(1-\rho_{i(i+1)})}, \dots, \bar{x}_{m(1-\rho_{im})}), \end{aligned}$$

and $\tilde{\theta}_{i1} = \hat{\theta}_{i1} - \theta_{i1}^*$, with θ_{i1}^* being an unknown parameter defined later and $\hat{\theta}_{i1}$ is the estimate of θ_{i1}^* , γ_{i1} is a positive constant defined by designer. From Lemma 2, the time derivative of V_{i1} can be obtained as

$$\begin{aligned} \dot{V}_{i1} &= z_{i1}g_{i1}^{-1}\dot{z}_{i1} + \dot{y}_{id}z_{i1}g_{i1}^{-1} \\ &\quad - \dot{y}_{id}z_{i1} \int_0^1 M_{i1}(\theta z_{i1} + y_{id}) d\theta \\ &\quad + \sum_{j=1, j \neq i}^m \sum_{s=1}^{1-\rho_{ij}} z_{i1}^2 \dot{x}_{js} \int_0^1 \theta \frac{\partial M_{i1}(\theta z_{i1} + y_{id})}{\partial x_{js}} d\theta \\ &\quad + \frac{1}{\gamma_{i1}} \tilde{\theta}_{i1} \dot{\hat{\theta}}_{i1}. \end{aligned} \tag{9}$$

Take (7) into (9) and pack the continuous unknown nonlin-

ear functions into \hat{f}_{i1} , one can get

$$\dot{V}_{i1} = z_{i1}(x_{i2} + \hat{f}_{i1} + g_{i1}^{-1}d_{i1}) + \frac{1}{\gamma_{i1}} \tilde{\theta}_{i1} \dot{\hat{\theta}}_{i1} \tag{10}$$

with

$$\begin{aligned} \hat{f}_{i1} &= g_{i1}^{-1}f_{i1} - \dot{y}_{id} \int_0^1 M_{i1}(\theta z_{i1} + y_{id}) d\theta \\ &\quad + \sum_{j=1, j \neq i}^m \sum_{s=1}^{1-\rho_{ij}} z_{i1} \dot{x}_{js} \int_0^1 \theta \frac{\partial M_{i1}(\theta z_{i1} + y_{id})}{\partial x_{js}} d\theta, \end{aligned}$$

\hat{f}_{i1} can be approximated by a fuzzy logic system from Lemma 1. That is, for a given $\varepsilon_{i1} > 0$, we have

$$\hat{f}_{i1} = \vartheta_{i1}^T \xi_{i1}(X_{i1}) + \delta_{i1}(X_{i1}), \tag{11}$$

where $\delta_{i1}(X_{i1})$ is the approximation error satisfying

$$|\delta_{i1}(X_{i1})| \leq \varepsilon_{i1},$$

and X_{i1} defined as

$$\begin{aligned} X_{i1} &= [\bar{x}_{1(1-\rho_{i1})} \cdots \bar{x}_{(i-1)(1-\rho_{i(i-1)})} x_{i1} \\ &\quad \bar{x}_{(i+1)(1-\rho_{i(i+1)})} \cdots \bar{x}_{m(1-\rho_{im})}]^T. \end{aligned}$$

The unknown parameter θ_{i1}^* is defined as $\theta_{i1}^* = \|\vartheta_{i1}\|$. From Assumptions 2 and 3, it is obvious that $|g_{i1}^{-1}d_{i1}| \leq \bar{d}_{i1}/\underline{a}_{i1}$. Let $\omega_{i1} = \varepsilon_{i1} + \bar{d}_{i1}/\underline{a}_{i1}$, and take the virtual control α_{i1} as

$$\begin{aligned} \alpha_{i1} &= -(c_{i1} + \frac{\gamma_{i1}}{2})z_{i1} - \hat{\theta}_{i1}\|\xi_{i1}\| \tanh \frac{z_{i1}\hat{\theta}_{i1}\|\xi_{i1}\|}{\tau_{i1}} \\ &\quad - \omega_{i1} \tanh \frac{z_{i1}\omega_{i1}}{\eta_{i1}}, \end{aligned} \tag{12}$$

where \tanh denotes the hyperbolic tangent function, and c_{i1}, τ_{i1} and η_{i1} are positive parameters. Let $z_{i2} = x_{i2} - \alpha_{i1}$ and design the parameter updating law as

$$\dot{\hat{\theta}}_{i1} = -\frac{1}{2}\hat{\theta}_{i1}\xi_{i1}^T \xi_{i1} - r_{i1}\hat{\theta}_{i1} \tag{13}$$

with $r_{i1} > 0$, the derivative of V_{i1} can be rewritten as

$$\begin{aligned} \dot{V}_{i1} &= z_{i1}(z_{i2} + \alpha_{i1} + \hat{f}_{i1} + g_{i1}^{-1}d_{i1}) - \frac{1}{\gamma_{i1}} \tilde{\theta}_{i1} \\ &\quad \cdot (\frac{1}{2}\hat{\theta}_{i1}\xi_{i1}^T \xi_{i1} + r_{i1}\hat{\theta}_{i1}) \\ &\leq -(c_{i1} + \frac{\gamma_{i1}}{2})z_{i1}^2 - z_{i1}\hat{\theta}_{i1}\|\xi_{i1}\| \tanh \frac{z_{i1}\hat{\theta}_{i1}\|\xi_{i1}\|}{\tau_{i1}} \\ &\quad - z_{i1}\omega_{i1} \tanh \frac{z_{i1}\omega_{i1}}{\eta_{i1}} + |z_{i1}\theta_{i1}^*\|\xi_{i1}\| + |z_{i1}\omega_{i1}| \\ &\quad - \frac{1}{2\gamma_{i1}} \tilde{\theta}_{i1} \hat{\theta}_{i1} \xi_{i1}^T \xi_{i1} - \frac{r_{i1}}{\gamma_{i1}} \tilde{\theta}_{i1} \hat{\theta}_{i1} + z_{i1}z_{i2} \\ &\leq -c_{i1}z_{i1}^2 - \frac{2r_{i1} - \xi_{i1}^T \xi_{i1}}{4\gamma_{i1}} \tilde{\theta}_{i1}^2 + z_{i1}z_{i2} + \kappa(\tau_{i1} + \eta_{i1}) \\ &\quad + \frac{\xi_{i1}^T \xi_{i1} + 2r_{i1}}{4\gamma_{i1}} \theta_{i1}^{*2}, \end{aligned} \tag{14}$$

where the fact that $\forall \epsilon > 0$ and $q \in \mathbb{R}$,

$$0 \leq |q| - q \tanh \frac{q}{\epsilon} \leq \kappa \epsilon$$

with $\kappa = e^{-(\kappa+1)}$ (i.e., $\kappa \approx 0.2785$) has been used.

Step k ($2 \leq k \leq \rho_i - 1$) From the k th state equation of the i th subsystem of system (1), we can get

$$\dot{z}_{ik} = f_{ik} + g_{ik}x_{i(k+1)} + d_{ik} - \dot{\alpha}_{i(k-1)}. \tag{15}$$

Choose the Lyapunov function candidate as

$$V_{ik} = V_{i(k-1)} + \int_0^{z_{ik}} \chi M_{ik}(\chi + \alpha_{i(k-1)}) d\chi + \frac{1}{2\gamma_{ik}} \tilde{\theta}_{ik}^2, \tag{16}$$

where

$$\begin{aligned} \hat{\theta}_{ik} &= \hat{\theta}_{ik} - \theta_{ik}^*, \\ M_{ik} &= g_{ik}^{-1}(\bar{x}_{1(k-\rho_{i1})}, \dots, \bar{x}_{(i-1)(k-\rho_{i(i-1)})}, \bar{x}_{i(k-1)}, \\ &\quad \chi + \alpha_{i(k-1)}, \bar{x}_{(i+1)(k-\rho_{i(i+1)})}, \dots, \bar{x}_{m(k-\rho_{im})}). \end{aligned}$$

The derivative of V_{ik} can be expressed as

$$\begin{aligned} \dot{V}_{ik} &= \dot{V}_{i(k-1)} + z_{ik}(x_{i(k+1)} + \hat{f}_{ik} + g_{ik}^{-1}d_{ik}) \\ &\quad + \frac{1}{\gamma_{ik}} \tilde{\theta}_{ik} \dot{\hat{\theta}}_{ik}, \end{aligned} \tag{17}$$

where

$$\begin{aligned} \hat{f}_{ik} &= g_{ik}^{-1}f_{ik} - \dot{\alpha}_{i(k-1)} \int_0^1 M_{ik}(\theta z_{ik} + \alpha_{i(k-1)})d\theta \\ &\quad + \sum_{j=1, j \neq i}^m \sum_{s=1}^{k-\rho_{ij}} z_{ik} \dot{x}_{js} \int_0^1 \theta \frac{\partial M_{ik}(\theta z_{ik} + \alpha_{i(k-1)})}{\partial x_{js}} d\theta \\ &\quad + \sum_{s=1}^{k-1} z_{ik} \dot{x}_{is} \int_0^1 \theta \frac{\partial M_{ik}(\theta z_{ik} + \alpha_{i(k-1)})}{\partial x_{is}} d\theta. \end{aligned}$$

From Lemma 1, $\hat{f}_{ik} = \vartheta_{ik}^T \xi_{ik}(X_{ik}) + \delta_{ik}(X_{ik})$ with $\delta_{ik}(X_{ik})$ satisfying $|\delta_{ik}(X_{ik})| \leq \varepsilon_{i2}$, where

$$X_{ik} = [\bar{x}_{1(k-\rho_{i1})} \ \dots \ \bar{x}_{(i-1)(k-\rho_{i(i-1)})} \ \bar{x}_{ik} \ \bar{x}_{(i+1)(k-\rho_{i(i+1)})} \ \dots \ \bar{x}_{m(k-\rho_{im})}]^T.$$

Define $\theta_{ik} = \|\vartheta_{ik}\|$ and $\omega_{ik} = \varepsilon_{ik} + \bar{d}_{ik}/\underline{a}_{ik}$, the virtual control in this step and the parameter updating law for $\hat{\theta}_{ik}$ are designed as

$$\begin{aligned} \dot{\alpha}_{ik} &= -(c_{ik} + \frac{\gamma_{ik}}{2})z_{ik} - \hat{\theta}_{ik}\|\xi_{ik}\| \tanh \frac{z_{ik}\hat{\theta}_{ik}\|\xi_{ik}\|}{\tau_{ik}} \\ &\quad - \omega_{ik} \tanh \frac{z_{ik}\omega_{ik}}{\eta_{ik}} - z_{i(k-1)}, \end{aligned} \tag{18}$$

$$\dot{\hat{\theta}}_{ik} = -\frac{1}{2}\hat{\theta}_{ik}\xi_{ik}^T \xi_{ik} - r_{ik}\hat{\theta}_{ik}. \tag{19}$$

With (18) and (19), following the manipulation of (14) but substituting the subscript k for 1, the derivative of V_{ik} can be obtained as

$$\begin{aligned} \dot{V}_{ik} &\leq -\sum_{j=1}^k c_{ij}z_{ij}^2 - \sum_{j=1}^k \frac{2r_{ij} - \xi_{ij}^T \xi_{ij}}{4\gamma_{ij}} \tilde{\theta}_{ij}^2 + z_{ik}z_{i(k+1)} \\ &\quad + \kappa \sum_{j=1}^k (\tau_{ij} + \eta_{ij}) + \sum_{j=1}^k \frac{\xi_{ij}^T \xi_{ij} + 2r_{ij}}{4\gamma_{ij}} \theta_{ij}^{*2}. \end{aligned} \tag{20}$$

Step ρ_i In order to tolerate the lock-in-place faults of the actuators, the proportional actuation structure is taken as in [6]

$$v_{ij} = b_{ij}(\bar{x}_i)u_i, \quad 0 < \underline{b}_{ij} \leq b_{ij}(\bar{x}_i) \leq \bar{b}_{ij}, \tag{21}$$

where $\bar{x}_i = [x_{i1} \ x_{i2} \ \dots \ x_{i\rho_i}]$, u_i is the control signal that will be designed later, and $j = 1, 2, \dots, k_i$. Then, according to the ρ_i th state equation of system (1), we can get

$$\dot{z}_{i\rho_i} = f_{i\rho_i} + \bar{g}_{i\rho_i}^T \bar{u}_i + d_{i\rho_i} - \dot{\alpha}_{i(\rho_i-1)}. \tag{22}$$

Taking the control input (4) into account, the above equation can be rewritten as

$$\dot{z}_{i\rho_i} = g_{i\rho_i}u_i + f'_{i\rho_i} + d_{i\rho_i} - \dot{\alpha}_{i(\rho_i-1)}, \tag{23}$$

where

$$f'_{i\rho_i} = \sum_{j \neq j_1, \dots, j_p} g_{i\rho_i j} \bar{u}_{ij} + f_{i\rho_i} \tag{24}$$

with $g_{i\rho_i j}$ being the short form of

$$g_{i\rho_i}(\bar{x}, \phi_{ij_1}, \dots, \phi_{ij_p}) = \sum_{j=j_1, \dots, j_p} b_{ij} \phi_{ij} g_{i\rho_i j}.$$

From Assumption 4, it can be concluded that the unknown function defined in (24) is continuous. Besides, from Assumption 2 and the boundedness of $b_{ij}(\bar{x}_i)$, it can be seen that $\underline{a}_{i\rho_i} \leq g_{i\rho_i} \leq \bar{a}_{i\rho_i}$, with

$$\begin{aligned} \underline{a}_{i\rho_i} &= \sum_{j=j_1, \dots, j_p} b_{ij} \underline{\phi}_{ij} \underline{a}_{i\rho_i j} > 0, \\ \bar{a}_{i\rho_i} &= \sum_{j=j_1, \dots, j_p} \bar{b}_{ij} \bar{a}_{i\rho_i j}. \end{aligned}$$

Define

$$M_{i\rho_i} = g_{i\rho_i}^{-1}(\bar{x}_1, \dots, \bar{x}_{i-1}, \bar{x}_{i(\rho_i-1)}, \chi + \alpha_{i(\rho_i-1)}, \bar{x}_{i+1}, \dots, \bar{x}_m, \phi_{ij_1}, \dots, \phi_{ij_p}),$$

and choose the Lyapunov function candidate in this step as

$$V_{i\rho_i} = V_{i(\rho_i-1)} + \int_0^{z_{i\rho_i}} \chi M_{i\rho_i}(\chi + \alpha_{i(\rho_i-1)})d\chi + \frac{\tilde{\theta}_{i\rho_i}^2}{2\gamma_{i\rho_i}}. \tag{25}$$

We can get the derivative of $V_{i\rho_i}$ as

$$\dot{V}_{i\rho_i} = \dot{V}_{i(\rho_i-1)} + z_{i\rho_i}(u_i + \hat{f}'_{i\rho_i} + g_{i\rho_i}^{-1}d_{i\rho_i}) + \frac{1}{\gamma_{i\rho_i}} \tilde{\theta}_{i\rho_i} \dot{\hat{\theta}}_{i\rho_i}, \tag{26}$$

where

$$\begin{aligned} \hat{f}'_{i\rho_i} &= f'_{i\rho_i} g_{i\rho_i}^{-1} - \dot{\alpha}_{i(\rho_i-1)} \int_0^1 M_{i\rho_i}(\theta z_{i\rho_i} + \alpha_{i(\rho_i-1)})d\theta \\ &\quad + \sum_{j=1, j \neq i}^m \sum_{s=1}^{\rho_i - \rho_{ij}} z_{i\rho_i} \dot{x}_{js} \int_0^1 \theta \frac{\partial M_{i\rho_i}(\theta z_{i\rho_i} + \alpha_{i(\rho_i-1)})}{\partial x_{js}} d\theta \\ &\quad + \sum_{s=1}^{\rho_i - 1} z_{i\rho_i} \dot{x}_{is} \int_0^1 \theta \frac{\partial}{\partial x_{is}} (\theta z_{i\rho_i} + \alpha_{i(\rho_i-1)})d\theta. \end{aligned}$$

A fuzzy logic system $\vartheta_{i\rho_i}^T \xi_{i\rho_i}(X_{i\rho_i})$ can be used to approximate the packaged unknown function $\hat{f}'_{i\rho_i}$ with the estimate error $\delta_{i\rho_i}(X_{i\rho_i})$ satisfying $|\delta_{i\rho_i}(X_{i\rho_i})| \leq \varepsilon_{i\rho_i}$, where

$$X_{i\rho_i} = [\bar{x}_{1(k-\rho_{i1})} \ \dots \ \bar{x}_{(i-1)(k-\rho_{i(i-1)})} \ \bar{x}_{i\rho_i} \ \bar{x}_{(i+1)(k-\rho_{i(i+1)})} \ \dots \ \bar{x}_{m(k-\rho_{im})}]^T.$$

Define $\theta_{i\rho_i}^* = \|\vartheta_{i\rho_i}\|$ and $\omega_{i\rho_i} = \varepsilon_{i\rho_i} + \bar{d}_{i\rho_i}/\underline{a}_{i\rho_i}$, the control signal u_i and the adaptive law for $\hat{\theta}_{i\rho_i}$ are designed as

$$\begin{aligned} u_i &= -(c_{i\rho_i} + \frac{\gamma_{i\rho_i}}{2})z_{i\rho_i} - z_{i(\rho_i-1)} \\ &\quad - \hat{\theta}_{i\rho_i}\|\xi_{i\rho_i}\| \tanh \frac{z_{i\rho_i}\hat{\theta}_{i\rho_i}\|\xi_{i\rho_i}\|}{\tau_{i\rho_i}} \\ &\quad - \omega_{i\rho_i} \tanh \frac{z_{i\rho_i}\omega_{i\rho_i}}{\eta_{i\rho_i}}, \end{aligned} \tag{27}$$

$$\dot{\hat{\theta}}_{i\rho_i} = -\frac{1}{2}\hat{\theta}_{i\rho_i}\xi_{i\rho_i}^T \xi_{i\rho_i} - r_{i\rho_i}\hat{\theta}_{i\rho_i}. \tag{28}$$

Substituting (27) and (28) into (26), the following expression can be obtained as

$$\begin{aligned} \dot{V}_{i\rho_i} &\leq \sum_{j=1}^{\rho_i} (-c_{ij}z_{ij}^2 - \frac{2r_{ij} - \xi_{ij}^T \xi_{ij}}{4\gamma_{ij}} \tilde{\theta}_{ij}^2) + \kappa \sum_{j=1}^{\rho_i} (\tau_{ij} + \eta_{ij}) \\ &\quad + \sum_{j=1}^{\rho_i} \frac{\xi_{ij}^T \xi_{ij} + 2r_{ij}}{4\gamma_{ij}} \theta_{ij}^{*2}. \end{aligned} \tag{29}$$

Then, we are ready to give the following theorem.

Theorem 1 For nonlinear system (1) with actuator faults (2) and (3), if Assumptions 1~4 are satisfied, the adaptive controllers u_i in (27) together with the adaptive laws (13), (19), and (28), $j = 1, 2, \dots, \rho_i$, $i = 1, 2, \dots, m$, can guarantee that all the signals in the closed-loop system remain bounded and the tracking error of i th subsystem satisfies $\lim_{t \rightarrow \infty} z_{i1}^2 \leq \varepsilon$ for any given $\varepsilon > 0$, although there are unknown nonlinearities and external disturbances in the

controlled system.

Proof Consider the Lyapunov function candidate for the whole system as

$$V = \sum_{i=1}^m V_{i\rho_i}. \tag{30}$$

Then, it can be deduced from (29) and (30) that

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^m \left[\sum_{j=1}^{\rho_i} (-c_{ij} z_{ij}^2 - \frac{2r_{ij} - \xi_{ij}^T \xi_{ij}}{4\gamma_{ij}} \hat{\theta}_{ij}^2) \right. \\ & \left. + \kappa \sum_{j=1}^{\rho_i} (\tau_{ij} + \eta_{ij}) + \sum_{j=1}^{\rho_i} \frac{\xi_{ij}^T \xi_{ij} + 2r_{ij}}{4\gamma_{ij}} \theta_{ij}^{*2} \right]. \end{aligned} \tag{31}$$

Define $\delta_0 = \sum_{i=1}^m \sum_{j=1}^{\rho_i} [\kappa \sum_{j=1}^{\rho_i} (\tau_{ij} + \eta_{ij}) + \frac{\xi_{ij}^T \xi_{ij} + 2r_{ij}}{4\gamma_{ij}} \theta_{ij}^{*2}]$,

it is obvious that δ_0 is bounded. Besides, according to Assumption 2 and the fact that $\underline{a}_{i\rho_i} \leq g_{i\rho_i} \leq \bar{a}_{i\rho_i}$, we have $\bar{a}_{ij}^{-1} \leq g_{ij}^{-1} \leq \underline{a}_{ij}^{-1}$. Then, from the first property of Lemma 2, it can be seen that

$$-\frac{1}{2\underline{a}_{ij}} z_{ij}^2 \leq -\int_0^{z_{ij}} \chi M_{ij}(\chi + \alpha_{(j-1)}) d\chi. \tag{32}$$

For a given positive constant μ , one can always choose the design parameters such that

$$c_{ij} \geq \frac{\mu}{2\underline{a}_{ij}} \text{ and } r_{ij} \geq \mu + \frac{\xi_{ij}^T \xi_{ij}}{2}.$$

Therefore, from (30)~(32) and the definition of $V_{i\rho_i}$, we can conclude that

$$\dot{V} \leq -\mu \sum_{i=1}^m \sum_{j=1}^{\rho_i} V_{ij} + \delta_0 = -\mu V + \delta_0, \tag{33}$$

which implies that

$$V \leq (V(0) - \frac{\delta_0}{\mu}) e^{-\mu t} + \frac{\delta_0}{\mu}. \tag{34}$$

From (34), it can be concluded that z_{ij} and $\hat{\theta}_{ij}$ are bounded and belong to the following compact set

$$\Omega = \{(z_{ij}, \hat{\theta}_{ij}) | V \leq \max(V(0), \frac{\delta_0}{\mu})\}.$$

Also, it can be obtained that $\lim_{t \rightarrow \infty} V = \frac{\delta_0}{\mu}$. Besides, according to Assumption 2 and Lemma 2, for each i ,

$$\frac{1}{2\bar{a}_{i1}} z_{i1}^2 \leq \int_0^{z_{i1}} \chi M_{i1}(\chi + y_{id}) d\chi \leq V_{i1} \leq V.$$

Thus, $z_{i1}^2 \leq 2\bar{a}_{i1} V$. Then, for a given small constant $\varepsilon > 0$, if the design parameters are chosen to meet $2\bar{a}_{i1} (\frac{\delta_0}{\mu}) \leq \varepsilon$, it can be obtained that

$$\lim_{t \rightarrow \infty} z_{i1}^2 \leq \lim_{t \rightarrow \infty} 2\bar{a}_{i1} V = \frac{\delta_0}{2\bar{a}_{i1}\mu} \leq \varepsilon. \tag{35}$$

So far, it has been shown that if the initial value $V(0)$ is finite, the desired control objective can be achieved, that is, all the closed-loop signals remain bounded and the output can track the given reference signal with the tracking error satisfying $\lim_{t \rightarrow \infty} z_{i1}^2 \leq \varepsilon$.

Remark 2 The adaptive fuzzy controller of this paper retains the minimal learning parameterization property of [22] by updating the norm of the parameter vectors. However, it has been qualified the ability of fault tolerance and disturbance rejection. Besides, it can be seen from the control scheme in [22], if one wants to make the output tracking

error small, some parameters should be chosen small. However, this will correspondingly lead a considerably large control input. Generally speaking, very large control signal is not desired during the control process due to the actuator saturation phenomenon. With the help of hyperbolic tangent function, the control law in this paper can guarantee the desired tracking performance within a smaller range of control amplitude by appropriately choosing the design parameters. Thus, the problem of actuator saturation can be avoided effectively.

Remark 3 Compared with the existing advanced FTC methods [2, 5, 8], the contributions of this work are that, first, FTC technique is extended to more general systems, a class of nonlinear strict-feedback systems with unknown nonlinearities and external disturbances are successfully controlled to tolerate actuator faults; second, the fault set that can be tolerated is enlarged to one that contain both time-varying loss of effectiveness and lock-in-place actuator faults. This development is obviously important because FTC is necessarily applicable to more and more general systems and meanwhile should deal with as many faults as possible.

4 Simulation

In this section, the proposed adaptive fuzzy FTC approach is utilized to control a system, and the simulation results will show the effectiveness of the approach.

Consider the following MIMO nonlinear system:

$$\begin{cases} \dot{x}_{11} = f_{11}(x_{11}, x_{21}) + g_{11}(x_{11}, x_{21})x_{12} + d_{11}(t), \\ \dot{x}_{12} = f_{12}(\bar{x}) + g_{121}(\bar{x})u_{11} + g_{122}(\bar{x})u_{12} + d_{12}(t), \\ y_1 = x_{11}, \\ \dot{x}_{21} = f_{21}(x_{11}, x_{21}) + g_{21}(x_{11}, x_{21})x_{22} + d_{21}(t), \\ \dot{x}_{22} = f_{22}(\bar{x}, \bar{u}_1) + g_{221}(\bar{x})u_{21} + g_{222}(\bar{x})u_{22} \\ \quad + d_{22}(t), \\ y_2 = x_{21}. \end{cases} \tag{36}$$

For simulation, the nonlinear functions are chosen as

$$\begin{aligned} f_{11} &= 0.5(x_{11} + x_{21}), \quad f_{12} = x_{11}x_{12} + x_{21}x_{22}, \\ f_{21} &= x_{11}x_{21}, \quad f_{22} = x_{11}x_{22} + x_{12}x_{21} + u_{11}, \\ g_{11} &= 1 + 0.1x_{11}^2x_{21}^2, \quad g_{121} = 2 - \cos(x_{11}x_{12}), \\ g_{122} &= 2 - \sin(x_{11}x_{21}), \quad g_{21} = 2 + \cos x_{11} \sin x_{21}, \\ g_{221} &= e^{-x_{12}} + e^{x_{21}}, \quad g_{222} = e^{x_{12}} + \sin x_{22}. \end{aligned}$$

The external disturbances are assumed to be

$$d_{11}(t) = d_{21}(t) = 0.1 \sin t, \quad d_{12}(t) = d_{22}(t) = 0.05d(t),$$

where $d(t)$ is a square wave signal with the amplitude being 1 and the period being 2. The reference signals are generated from the following system:

$$\begin{cases} \dot{x}_{d1} = x_{d2}, \\ \dot{x}_{d2} = -x_{d1} + 0.001(1 - x_{d1}^2)x_{d2}, \\ y_{im} = x_{di}, \quad i = 1, 2 \end{cases} \tag{37}$$

with $x_{d1}(0) = 1.5$, $x_{d2}(0) = 0.8$. The initial conditions are chosen as $x_{11}(0) = 0.5$, $x_{12}(0) = 2$, $x_{21}(0) = 0.7$, $x_{22}(0) = 1$, $\hat{\theta}_{11}(0) = \hat{\theta}_{12}(0) = \hat{\theta}_{21}(0) = \hat{\theta}_{22}(0) = 0$. Choose the design parameters as $c_{ij} = 1.5$, $\gamma_{ij} = 8$, $r_{ij} = 0.5$, $\tau_{ij} = 0.01$, $\eta_{ij} = 0.01$, $\varepsilon_{ij} = 0.5$ for $i = 1, 2$. $b_{11} = 1$, $b_{12} = 0.5$, $b_{21} = 1$, $b_{22} = 1$. The fuzzy membership functions are chosen the same as those in Example 1

of [22]. The actuator faults are described as follows:

$$u_{12}(t) = \begin{cases} 0.5u_1, & t \leq 7.5, \\ 0.5e^{-50(t-7.5)}u_1, & 7.5 < t \leq 7.5 - 0.02 \ln 0.6, \\ 0.3u_1, & t > 7.5 - 0.02 \ln 0.6, \end{cases} \quad (38)$$

$$u_{22}(t) = \begin{cases} u_2, & t \leq 5u_2(t=5) + (t-5)e^{20}(1-u_2(t=5)), \\ 5 < t \leq 5 + e^{-20}, \\ 1, & t > 5 + e^{-20}. \end{cases} \quad (39)$$

The simulation results are shown in Figs. 1~4. It can be seen that good tracking performance is achieved, although there are some severe and time-varying actuator faults. It should be noted that although the FTC law is designed assuming continuous σ_{ij} , if σ_{ij} changes suddenly from 0 to 1, the designed control law can guarantee good tracking performances as well.

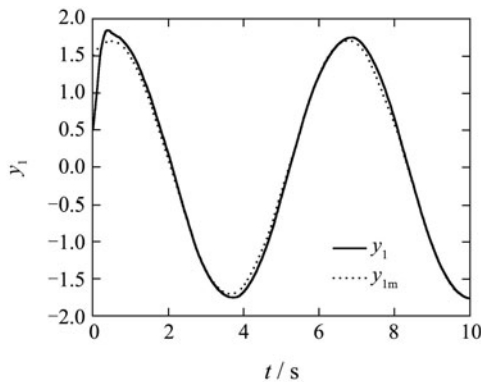


Fig. 1 Output y_1 and the reference y_{1m} .

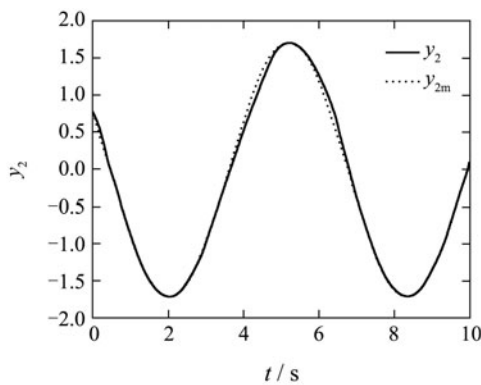


Fig. 2 Output y_2 and the reference y_{2m} .

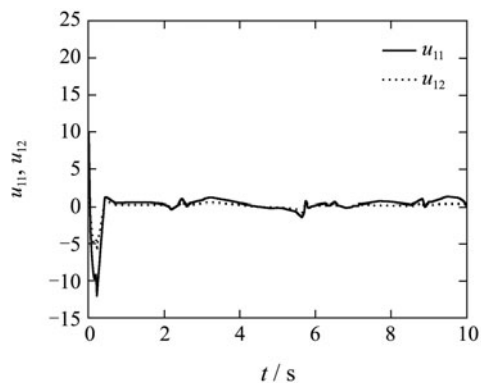


Fig. 3 Control input u_{11} and u_{12} .

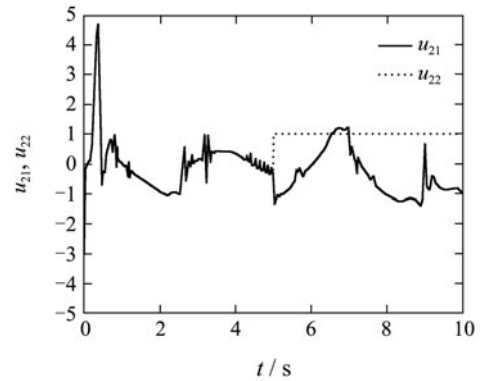


Fig. 4 Control input u_{21} and u_{22} .

We have attempted a few examples; the results obtained are similar to the ones shown in Figs. 1~4. Actually, the lock-in-place fault of $u_{22}(t)$ in (39) is so abrupt that it can be viewed as a sudden jump. Therefore, the fault set that can be tolerated is very large. Fig. 5 displays the output tracking and control input curves with the control method of [22] to system (36) without actuator faults and external disturbances. It can be seen by comparison that our design can get good tracking performance within a smaller control range, although there are severe actuator faults and external disturbances entering into the system.

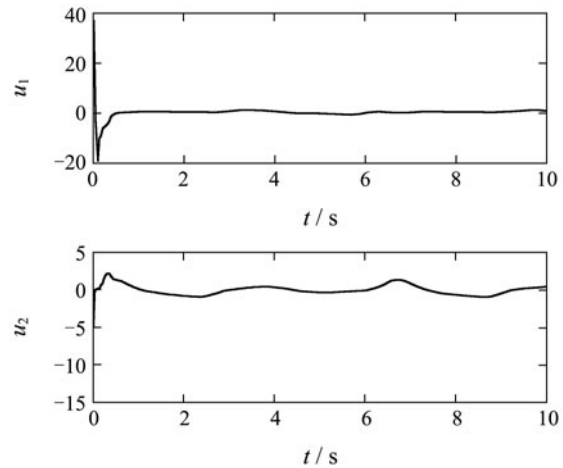


Fig. 5 Comparison of output tracking and control input curves.

5 Conclusions

In this paper, the problem of adaptive fuzzy FTC for output tracking of a class of uncertain nonlinear MIMO systems is considered. By using adaptive fuzzy systems to approximate the packages of unknown nonlinear functions, a novel FTC approach is proposed. Since the considered systems cannot be feedback-linearized, the control law is obtained step by step from a backstepping procedure. It is proven in theory and shown by simulation that the proposed controller guarantees that all the signals of the closed-loop system are bounded and the outputs of the controlled system track the given reference signals closely, although there are actuator faults and external disturbances entering into the controlled systems. The proposed FTC approach can deal with both lock-in-place and loss of effectiveness actuator faults, even though they are time-varying. Meanwhile, the number of adaptive laws are reduced to one just equal to the number of the used fuzzy approximators and the control objective can be achieved within a small range of control

amplitude compared to the existing result. Thus, the computation burden can be alleviated considerably and the potential saturation problem of the actuator can be avoided effectively.

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