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Stabilization for nonlinear systems via a limited capacity communication channel with data packet dropout

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Abstract: This paper addresses the stabilization problem for a class of nonlinear systems. It is assumed that the controller can only receive the transmitted sequence of finite coded signals via a limited digital communication channel. Both state and output feedback coder-decoder-controller procedures are proposed. Stabilization conditions involving the size of coding alphabet, the sampling period, system state growth rate and data packet dropout rate are obtained. Finally, an example is given to illustrate the design procedures and effectiveness of the proposed results.

Keywords: Stabilization; Nonlinear system; Limited information; Data packet dropout; Linear matrix inequality

1 Introduction

In classic control theory, a standard assumption is that the communication channel is perfect, that is, the control signals can be transmitted with infinite precision. However, this is just not the case in some practical situations, such as in networked control systems, where the physical plant and controller are located at different place, and measured control signals are sent via communication networks. Due to the bandwidth constraint, data packet dropout may occur during data communication, which is one of the most important issues in networked control systems and has been extensively studied [1∼8].

For control problem with a limited capacity communication channel, there have been a few important results reported in the literature [9]. One way to reduce the bandwidth is to reduce the rate at which packets are sent [10]. A much more popular method is to reduce the number of bits used to transmit each packet, which is closely related with feedback control under quantization or coded feedback control [11]. Recently, various quantization schemes have been developed, such as logarithmic quantization [12∼14], uniform quantization [15] and so on. In this kind of feedback control, the measured control signal is sampled and quantized into packets with a finite number of bits. So far, many important results have been reported in the literature. For example, based on the idea from the work on quantized feedback stabilization [15], a coding scheme and stabilizing control strategy are developed for linear systems in [16], which is generalized to Lipschitz nonlinear systems in [17]. It is worth noting that no data packet dropout has been considered in these results, which may occur randomly due to bit-rate constraint and transmission error.

Motivated by recent works on limited information control, we, in this paper, consider the stabilization problem for Lipschitz nonlinear systems via a limited communication channel with data packet dropout. It is assumed that control signals are coded and sent via a communication-constrained channel, which may be lost due to the limited bandwidth of the communication channel and transmission error. The controller can only be updated when coded signals are transmitted successfully. Coder-decoder based state feedback and output feedback stabilizing procedures are proposed, and corresponding stabilizing conditions are obtained. In this paper, we obtain a generalization of the main results of [16, 17] to the presence of data packet dropout case. We show that under the similar condition as those of [16, 17], the proposed procedures can stabilize the system at a certain level of data packet loss rate, which make the approach more practical and applicable. Simulation results show the effectiveness of the approaches.

Notation Throughout this paper, $\boldsymbol{R}^n$ denotes the n-dimensional Euclidean space. For

$$x = [x_1\ x_2 \cdots x_n]' \in \boldsymbol{R}^n,$$
$$\|x\| = \sqrt{x'x},$$
$$\|x\|_\infty = \max\{|x_i|, 1 \leqslant i \leqslant n\}.$$

I_m (or I) is the m-dimensional (or appropriately dimensioned) identity matrix, W' denotes the transpose of matrix W, and $W > 0$ means that W is positive definite. Asterisk '$*$' in a symmetric matrix denotes the entry implied by symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

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2 Problem statements and preliminaries

In this paper, we consider a class of nonlinear continuous-time systems in the form:

$$\begin{cases} \dot{x}(t) = Ax(t) + Hf(t, x(t), u(t)) + Bu(t), \\ y(t) = Cx(t), \\ x(0) = x_0, \end{cases} \tag{1}$$

where $x(t) \in \boldsymbol{R}^n$, $u(t) \in \boldsymbol{R}^m$ and $y \in \boldsymbol{R}^q$ are the system state, control input and measured output, respectively. A, B, C and H are constant matrices of appropriate dimensions, and initial condition x_0 is assumed to lie in a bounded set $\mathcal{X}_0$. $f(t, x(t), u(t))$ is a vector-valued nonlinear function with $f(t, 0, 0) = 0, \forall\, t \in \boldsymbol{R}$ and satisfies the following quadratic inequality for all $(t, x, u),\ (t, \bar{x}, \bar{u}) \in \boldsymbol{R} \times \boldsymbol{R}^n \times \boldsymbol{R}^m$,

$$\|f(t, x, u) - f(t, \bar{x}, \bar{u})\| \leqslant \|F(x - \bar{x}) + G(u - \bar{u})\|, \tag{2}$$

where F and G are constant matrices with appropriate dimensions.

Remark 1 The nonlinear function $f(t, x, u)$ with structure of (2) has been extensively discussed in the literature [18∼20].

In our stabilization problem, the controller and the plant are separated by an unreliable communication channel with limited bandwidth. Since only a limited number of bits are available to the controller, a proper information encoding and decoding procedure is necessary. Generally speaking, for a known sampling period $T > 0$ and $j = 1, 2, \cdots$, the encoded signal $h(jT)$, which is generated by the output of the system $y(t)$ and selected from a coding alphabet $\mathcal{H}$ of size l, is transmitted through a digital communication channel at each time jT. At the remote reception, a register and decoder-controller are designed to produce the control input $u(t)$. Due to the limited bandwidth of communication channel, transmitted messages dropout may occur during data communication. The control architecture is shown in Fig.1. When the switch is closed (in mode S_1), codeword $h(jT)$ is transmitted successfully, and then control input $u(jT)$ will be updated accordingly. Whereas when the switch is open (in mode S_2), codeword $h(jT)$ is dropped out; in this case, no update occurs, and $u(jT)$ utilizes the previous value $u(jT - 0)$. This can be fulfilled by the register, which generates a discrete variable $\delta(jT)$ switching between 0 and 1 to denote data dropout and successful transmission, respectively. More specially, we present the following stabilization procedure:

Coder

$$h(jT) = \mathcal{F}_j(y(\cdot)|_0^{jT}); \tag{3}$$

Decoder-controller

$$u(t)|_{jT}^{(j+1)T} = \mathcal{G}_j(\delta(T)h(T), \delta(2T)h(2T), \cdots, \delta(jT)h(jT)). \tag{4}$$

For $j = 1, 2, \cdots$, $\mathcal{F}_j$ and $\mathcal{G}_j$ are coder and decoder functions to be designed, and $\delta(jT) \in \{0, 1\}$ are independent and identically distributed Bernoulli processes with

$$\mathrm{Prob}\{\delta(jT) = 0\} = \sigma. \tag{5}$$

In the proposed procedure, we also assume that the information whether data packet is lost or is transmitted as an acknowledgment message to the decoder-controller is known. In other words, at each time jT, previous switch signal $\delta(jT - T)$ is known to the decoder-controller.

Remark 2 Equation (5) implies that average data packet dropout rate is σ. In addition, the assumption that $\{\delta(jT)\}_{j\geqslant 1}$ are independent means that the previous transmission does not effect the present transmission, which is practically reasonable.

Remark 3 Without consideration of packet dropout, a similar coder-decoder procedure is adopted in [16, 17]. In this paper, we will show that under the similar condition, the proposed procedure can stabilize the system under a certain level of data packet loss rate.

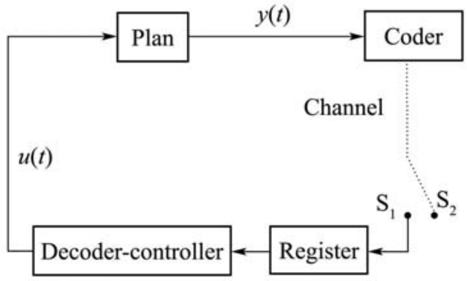

Fig. 1 Stabilization via a communication channel with data packet dropout.

The following definition is a generalization from that in [17].

Definition 1 System (1) is said to be stabilizable via a communication channel of capacity l if there exists a coder-decoder-controller (3), (4) with a coding alphabet of size l such that

$$\lim_{t\to\infty} E\{\|x(t)\|_\infty\} = 0, \quad \lim_{t\to\infty} E\{\|u(t)\|_\infty\} = 0 \tag{6}$$

for any solution to the closed-loop systems (1) and (4).

In the following, we will present two preliminary lemmas which play important roles in the design of our stabilization procedure.

Lemma 1 Given a scalar $\alpha > 0$. Suppose that there exists a positive definite matrix $P > 0$ such that the following LMI is solvable

$$\varOmega := \begin{bmatrix} (A - \alpha I)'P + P(A - \alpha I) + F'F & PH \\ H'P & -I \end{bmatrix} < 0. \tag{7}$$

Then for any solutions $x_1(t)$ and $x_2(t)$ to system (1), the inequality

$$\|x_1(t+T) - x_2(t+T)\|_\infty \leqslant \gamma \mathrm{e}^{\alpha T} \|x_1(t) - x_2(t)\|_\infty \tag{8}$$

holds for all $t > 0$ and $T > 0$, where $\gamma = \sqrt{\dfrac{n\lambda_{\max}(P)}{\lambda_{\min}(P)}}$.

Proof Let

$$z(t) = \mathrm{e}^{-\alpha t}(x_1(t) - x_2(t)),$$
$$\varPhi = \mathrm{e}^{-\alpha t}(f(t, x_1(t), u(t)) - f(t, x_2(t), u(t))),$$

then we have

$$\dot{z}(t) = A_\alpha z(t) + H\varPhi, \tag{9}$$

where $A_\alpha = A - \alpha I$ and $\|\varPhi\| \leqslant \|Fz\|$.

Choosing Lyapunov function $V = z'Pz$, along system (9), we have

$$\begin{aligned} \dot{V} &= (A_\alpha z + H\varPhi)'Pz - z'P(A_\alpha z + H\varPhi) \\ &\leqslant (A_\alpha z + H\varPhi)'Pz - z'P(A_\alpha z + H\varPhi) + z'F'Fz - \varPhi'\varPhi \\ &= [z'\ \varPhi']\ \varOmega\ [z'\ \varPhi']'. \end{aligned} \tag{10}$$

Denoting $\lambda_0 = \lambda_{\min}(-\varOmega) > 0$, $\lambda_1 = \lambda_{\min}(P) > 0$,

$\lambda_2 = \lambda_{\max}(P) > 0$, we can get

$$\dot{V} \leqslant -\lambda_0\|z\|^2 \leqslant -\lambda_0\lambda_2^{-1}V(t).$$

For any $T > 0$, we have

$$V(t+T) \leqslant \mathrm{e}^{-\lambda_0\lambda_2^{-1}T}V(t),$$

which implies

$$\|z(t+T)\| \leqslant \sqrt{\lambda_2\lambda_1^{-1}}\mathrm{e}^{-\frac{1}{2}\lambda_0\lambda_2^{-1}T}\|z(t)\|.$$

Therefore,

$$\|z(t+T)\|_\infty \leqslant \sqrt{n\lambda_2\lambda_1^{-1}}\mathrm{e}^{-\frac{1}{2}\lambda_0\lambda_2^{-1}T}\|z(t)\|_\infty.$$

Then we have $\|z(t+T)\|_\infty \leqslant \gamma\|z(t)\|_\infty$ for any $T > 0$, which completes the proof.

Remark 4 Lemma 1 gives an estimation of growth rate of the system state in sense of $\|x(\cdot)\|_\infty$. In the case of linear system $\dot{x}(t) = Ax(t) + Bu(t)$ discussed in [16], we can choose $\gamma = 1$ and $\alpha > 0$ satisfying $\max\limits_{0\leqslant t\leqslant T} \|\mathrm{e}^{At}\|_\infty \leqslant \mathrm{e}^{\alpha T}$.

The next lemma presents a state feedback controller design in terms of LMI without consideration of the communication channel.

Lemma 2 Suppose that there exist positive definite matrix Q and matrix Y such that

$$\varGamma := \begin{bmatrix} AQ+QA'+BY+Y'B' & H & (FQ+GY)' \\ H' & -I & 0 \\ FQ+GY & 0 & -I \end{bmatrix} < 0. \quad (11)$$

Then the closed-loop system of (1) with state feedback controller $u(t) = YQ^{-1}x(t) =: Kx(t)$ is globally asymptotically stable.

Proof Let

$$u(t) = Kx(t),\ \ g(t, x(t)) = f(t, x(t), Kx(t)),$$

then we have the following closed-loop system:

$$\dot{x}(t) = A_Kx(t) + Hg(t, x(t)), \quad (12)$$

where $A_K = A + BK$ and $\|g(t,x)\| \leqslant \|(F + GK)x\|$.

Choosing Lyapunov function $V = x'Q^{-1}x$, along system (12), we have

$$\begin{aligned}\dot{V} &= (A_Kx + Hg)'Q^{-1}x - x'Q^{-1}(A_Kx + Hg)\\ &\leqslant (A_Kx + Hg)'Q^{-1}x - x'Q^{-1}(A_Kx + Hg)\\ &\quad +g'g - x'(F + GK)'(F + GK)x\\ &= \left[x'\ g'\right] \varTheta \left[x'\ g'\right]', \end{aligned} \quad (13)$$

where

$$\varTheta = \begin{bmatrix} A_K'Q^{-1}+Q^{-1}A_K+(F+GK)'(F+GK) & Q^{-1}H \\ H'Q^{-1} & -I \end{bmatrix}.$$

Since $K = YQ^{-1}$ and $A_K = A + BK$, using Schur Complement Lemma, it is easy to show that LMI (11) is equivalent to $\varTheta < 0$. Thus, the closed-loop system (12) is globally asymptotically stable. This completes the proof.

3 Main results

The objective of this section is to show that if nonlinear system (1) can be stabilized by linear time-invariant feedback, then we can design a coder-decoder-controller procedure (3) and (4) such that the system can also be stabilized via a limited communication channel.

3.1 Uniform state quantization

In the following, we first recall the uniform state quantization method proposed by [17] and then show how to modify their results to design our coder-decoder procedure.

For any given constant $a > 0$ and positive integer q, we can partition the hypercube $\mathcal{B}_a = \{x \in \boldsymbol{R}^n : \|x\|_\infty \leqslant a\}$ into q^n hypercubes $I^1_{i_1}(a) \times I^2_{i_2}(a) \times \cdots \times I^n_{i_n}(a)$, where $i_1, i_2, \cdots, i_n \in \{1, 2, \cdots, q\}$ and

$$\begin{cases} I^i_1(a) := \{x_i : -a \leqslant x_i < -a + \dfrac{2a}{q}\};\\ I^i_2(a) := \{x_i : -a + \dfrac{2a}{q} \leqslant x_i < -a + \dfrac{4a}{q}\};\\ \quad\vdots\\ I^i_q(a) := \{x_i : a - \dfrac{2a}{q} \leqslant x_i \leqslant a\}. \end{cases} \quad (14)$$

Then for any $x \in \mathcal{B}_a$, there exist unique integers $i_1, i_2, \cdots, i_n \in \{1, 2, \cdots, q\}$ such that

$$x \in I^1_{i_1}(a) \times I^2_{i_2}(a) \times \cdots \times I^n_{i_n}(a).$$

The center of the hypercube $I^1_{i_1}(a) \times I^2_{i_2}(a) \times \cdots \times I^n_{i_n}(a)$ is defined as

$$\eta(i_1, i_2, \cdots, i_n) = \begin{bmatrix} -a + \dfrac{(2i_1 - 1)a}{q}\\ -a + \dfrac{(2i_2 - 1)a}{q}\\ \vdots\\ -a + \dfrac{(2i_n - 1)a}{q} \end{bmatrix}. \quad (15)$$

3.2 State feedback stabilization

In this paper, we assume that the first decoder signal $h(T)$ is transmitted successfully. Given constant $\alpha > 0$, suppose that LMI (7) in Lemma 1 is solvable and (8) holds for some $\gamma > 0$ and $T > 0$. In addition, a state feedback controller is designed based on Lemma 2. For convenience, denote

$$m_0 = \sup_{x_0\in\mathcal{X}_0} \|x_0\|_\infty;\ \ a(T) = \gamma\mathrm{e}^{\alpha T}m_0;$$

$$a((j + 1)T) = (1 - (1 - \frac{1}{q})\delta(jT))\gamma\mathrm{e}^{\alpha T}a(jT),\ j \geqslant 1.$$

Now, we present the state feedback coder-decoder-controller pairs as follows:

Coder For $x(jT) - \hat{x}(jT - 0) \in I^1_{i_1} \times I^2_{i_2} \times \cdots \times I^n_{i_n} \subset B_{a(jT)}$,

$$h(jT) = \{i_1, i_2, \cdots, i_n\}. \quad (16)$$

Decoder-controller For $h(jT) = \{i_1, i_2, \cdots, i_n\}$,

$$\begin{cases} \hat{x}(0) = 0;\\ \dot{\hat{x}}(t) = A\hat{x}(t) + Hf(t, \hat{x}(t), u(t)) + Bu(t),\\ t \in [jT, (j + 1)T);\\ \hat{x}(jT) = \hat{x}(jT - 0) + \delta(jT)\eta(i_1, i_2, \cdots, i_n);\\ u(t) = K\hat{x}(t). \end{cases} \quad (17)$$

Remark 5 The proposed procedure relies on the upper bound of the initial state, which can be derived by "zoom-out" method proposed in [16, 17].

Remark 6 From the uniform state quantization (14) and definition of coder $h(jT)$, we can conclude that the size of coding alphabet $\mathcal{H}$ and quantization parameter q satisfy $q^n \leqslant l$, where l is determined by the limited bandwidth of communication channel. In this sense, we say that the proposed problem belongs to the framework of control with limited information.

The following lemma shows that our proposed coder-decoder procedure (16) and (17) is well posed; that is, the decoding condition $x(jT)-\hat{x}(jT-0)\in B_{a(jp)}$ holds for all $j\geqslant 1$.

Lemma 3 For all $j\geqslant 1$, the coder-decoder procedure (16) and (17) satisfies $\|x(jT)-\hat{x}(jT-0)\|_\infty\leqslant a(jT)$.

Proof For case $j=1$, since $h(T)$ is transmitted successfully, which implies that $\delta(T)=1$, then it follows from inequality (8) and $\hat{x}(0)=0$ that

$$\|x(T)-\hat{x}(T-0)\|_\infty\leqslant\gamma e^{\alpha T}m_0=a(T).$$

Suppose that $\|x(jT)-\hat{x}(jT-0)\|_\infty\leqslant a(jT)$ holds. Then for $j+1$, we have

$$\begin{aligned}&\|x((j+1)T)-\hat{x}((j+1)T-0)\|_\infty\\ &\leqslant\gamma e^{\alpha T}\|x(jT)-\hat{x}(jT)\|_\infty\\ &\leqslant\gamma e^{\alpha T}\delta(jT)\|x(jT)-\hat{x}(jT-0)-\eta(i_1,i_2,\cdots,i_n)\|_\infty\\ &\quad+\gamma e^{\alpha T}(1-\delta(jT))\|x(jT)-\hat{x}(jT-0)\|_\infty\\ &\leqslant\gamma e^{\alpha T}\delta(jT)\frac{a(jT)}{q}+\gamma e^{\alpha T}(1-\delta(jT))a(jT)\\ &=a((j+1)T).\end{aligned}$$

Applying the method of mathematical induction, we get $\|x(jT)-\hat{x}(jT-0)\|_\infty\leqslant a(jT)$ holds for all $j\geqslant 1$, which completes the proof.

Theorem 1 Suppose that for some $\alpha>0$, LMI (7) is solvable and

$$\Big(\sigma+\frac{1-\sigma}{q}\Big)\gamma e^{\alpha T}<1 \tag{18}$$

holds. Then the coder-decoder-controller procedure (16) and (17) asymptotically stabilizes system (1).

Proof From the definition of $a(kT)$, we have

$$\begin{aligned}&E\{a((k+1)T)\}\\ &=\gamma e^{\alpha T}E\Big\{\Big(1-\Big(1-\frac{1}{q}\Big)\delta(jT)\Big)\Big\}E\{a(kT)\}\\ &=\Big(\sigma+\frac{1-\sigma}{q}\Big)\gamma e^{\alpha T}E\{a(kT)\}.\end{aligned} \tag{19}$$

Then it follows from (18) that $\lim\limits_{k\to\infty}E\{a(kT)\}=0$, which implies that

$$\lim_{j\to\infty}E\{\|x(jT)-\hat{x}(jT)\|_\infty\}=0. \tag{20}$$

In addition, inequality (8) implies that for any solutions $x_1(t)$ and $x_2(t)$ to systems (1),

$$\|x_1(t+v)-x_2(t+v)\|_\infty\leqslant\gamma e^{\alpha T}\|x_1(t)-x_2(t)\|_\infty \tag{21}$$

holds for $t>0$ and $0\leqslant v\leqslant T$.

Considering the same structure of (1) and (17), we can obtain from (20) and (21)

$$\lim_{t\to\infty}E\{\|x(t)-\hat{x}(t)\|_\infty\}=0.$$

This together with Lemma 2 implies asymptotic stability of the closed-loop system, which completes the proof.

Remark 7 Theorem 1 shows that asymptotic stabilization can be guaranteed if a proper relationship holds between the size of coding alphabet, the sampling period, growth rate of the system and data packet dropout rate.

Remark 8 For the given α, it follows from (18) that $\frac{1}{q}<\frac{1-\sigma\gamma e^{\alpha T}}{(1-\sigma)\gamma e^{\alpha T}}$, which implies that $\sigma\gamma e^{\alpha T}<1$ and $q>\gamma e^{\alpha T}$, which means that the lower bound of coding alphabet size should satisfy $l_{\min}>\gamma^n e^{n\alpha T}$. In addition, inequality (18) also implies that for any $q>\gamma e^{\alpha T}$, the upper bound of admissible data packet dropout rate is $\sigma_{\max}<\frac{q-\gamma e^{\alpha T}}{(q-1)\gamma e^{\alpha T}}$.

Remark 9 The larger the α, the lower the data packet dropout rate and the more code needed to guarantee the asymptotic stabilization. It is practically reasonable that asymptotically stabilizing a system with bad property (corresponding to large α) requires a good communication channel (corresponding to small σ) and a large number of codes.

3.3 Output feedback stabilization

Consider the following Luenberger-like state observer that will be a part of our proposed coder procedure:

$$\begin{cases}\dot{\tilde{x}}(t)=A\tilde{x}+Hf(t,\tilde{x}(t),u(t))-L(y(t)-C\tilde{x}(t)),\\ \tilde{x}(0)=0.\end{cases} \tag{22}$$

As a special case of Theorem 2.1 in [18], we obtain the following observer design approach in terms of LMI.

Lemma 4 Suppose that there exist positive definite matrix $R>0$ and matrix Z such that the following LMI is solvable:

$$\Sigma:=\begin{bmatrix}A'R+RA+C'Z'+ZC+F'F & RH\\ H'R & -I\end{bmatrix}<0. \tag{23}$$

Then there exists observer gain $L=R^{-1}Z$ such that for any solutions $x(t)$ and $\tilde{x}(t)$ to systems (1) and (22), there exist constant $\beta>0$ and time T_0 such that the inequality

$$\|x(t+T)-\tilde{x}(t+T)\|_\infty\leqslant e^{-\beta T}\|x(t)-\tilde{x}(t)\|_\infty \tag{24}$$

holds for all $t>0$ and $T\geqslant T_0$.

Suppose that γ and α satisfy (8), β and T_0 satisfy (24) and $T>T_0$. Then we present the output feedback coder-decoder-controller pairs as follows:

Coder For $\tilde{x}(jT)-\hat{x}(jT-0)\in I^1_{i_1}\times I^2_{i_2}\times\cdots\times I^n_{i_n}\subset B_{a(jT)}$,

$$h(jT)=\{i_1,i_2,\cdots,i_n\}; \tag{25}$$

Decoder-controller For $h(jT)=\{i_1,i_2,\cdots,i_n\}$,

$$\begin{cases}\hat{x}(0)=0;\\ \dot{\hat{x}}(t)=A\hat{x}(t)+Hf(t,\hat{x}(t),u(t))+Bu(t),\\ t\in[jT,(j+1)T);\\ \hat{x}(jT)=\hat{x}(jT-0)+\delta(jT)\eta(i_1,i_2,\cdots,i_n);\\ u(t)=K\hat{x}(t),\end{cases} \tag{26}$$

where $a(jT)$ is defined as

$$\begin{aligned}&a(T)=(e^{-\beta T}+\gamma e^{\alpha T})m_0,\\ &a(jT+T)=\big(e^{-(j+1)\beta T}+\gamma e^{\alpha T-j\beta T}\big)m_0\\ &\qquad+\big(1-\big(1-\frac{1}{q}\big)\delta(jT)\big)\gamma e^{\alpha T}a(jT),\\ &j\geqslant 1.\end{aligned}$$

Remark 10 With the above definition of $a(jT)$ and by similar reasoning to the proof of Lemma 3, we conclude that $\tilde{x}(jT)-\hat{x}(jT-0)\in B_{a(jp)}$ holds for all $j\geqslant 1$.

Following the same line as in the proof of Theorem 1, we present the following theorem without proof.

Theorem 2 Under the same condition as that in Theorem 1, the output feedback coder-decoder-controller procedure (25) and (26) asymptotically stabilizes system (1).

Remark 11 Theorem 2 implies that for $q > \gamma \mathrm{e}^{\alpha T}$, the proposed procedure can stabilize the system under data packet loss rate less than $\sigma_{\max} < \dfrac{q-\gamma \mathrm{e}^{\alpha T}}{(q-1)\gamma \mathrm{e}^{\alpha T}}$. Thus, the result extends Theorem 3.1 of [17] to data packet dropout case.

4 A numerical example

To illustrate the effectiveness of the design procedure, we give a numerical example. Consider the following nonlinear system with

$$A=\begin{bmatrix}-3 & 2 & 1\\ 0 & -2 & 4\\ 1 & 2 & 4\end{bmatrix},\ B=H=\begin{bmatrix}0\\0\\1\end{bmatrix},\ C=\begin{bmatrix}0 & 0 & 1\end{bmatrix},$$

$$f(t,x(t),u(t))=\sin(x_3(t)),$$

$$x(t)=\begin{bmatrix}x_1(t) & x_2(t) & x_3(t)\end{bmatrix}'\in \boldsymbol{R}^3.$$

For $\alpha = 0.7$, applying Lemma 1, we can get that (8) holds for $\gamma = 1.7362$. In addition, the state feedback gain $K = (-0.8438\ -3.3129\ 3.2557)$ can be obtained by Lemma 2. Therefore, choosing sampling period $T = 1$, the stabilization condition by Theorem 1 is $(\sigma + \dfrac{1-\sigma}{q}) < \gamma^{-1}\mathrm{e}^{-\alpha T} = 0.2860$. Let $q = 10$. When no packet loss occurs, the simulation of corresponding coder-decoder-controller procedure (17) and (16) is shown in Fig.2. When $q = 10$, we can get the upper bound of packet loss rate $\sigma_{\max} < 0.2067$. Fig.3 is a simulation of the closed-loop system with $\sigma = 0.2$, which means that averagely 20% of the packets are lost.

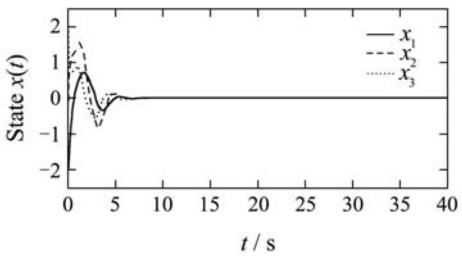

Fig. 2 The state response of the closed-loop system for $q = 10$ without packet loss.

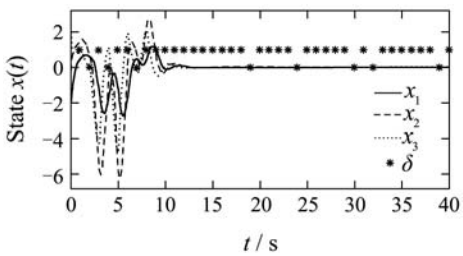

Fig. 3 The state response of the closed-loop system for $q = 10$ with $\sigma = 0.2$.

By using Lemma 3, we can obtain the observer gain $L = (-0.0147\ -6.0804\ -994.3243)'$, and inequality (24) holds for $\beta = 0.1$ and $T_0 = 1.3739$. Choosing $T = 1.5 > T_0$, we get the stabilization condition by Theorem 2 as

$$(\sigma + \frac{1-\sigma}{q}) < \gamma^{-1}\mathrm{e}^{-\alpha T} = 0.2016.$$

When $q = 20$, we can get the upper bound of packet loss rate $\sigma_{\max} < 0.1595$. The simulations of coder-decoder-controller procedure (25) and (26) with no packet loss and $\sigma = 0.15$ are shown in Fig.4 and Fig.5, respectively.

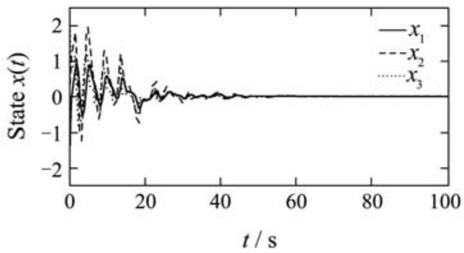

Fig. 4 The state response of the closed-loop system for $q = 20$ without packet loss.

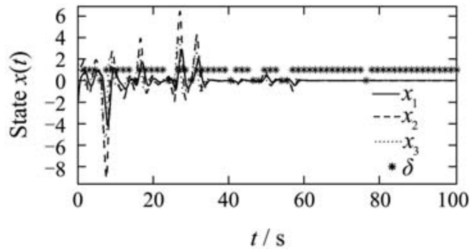

Fig. 5 The state response of the closed-loop system for $q = 20$ with $\sigma = 0.15$.

As shown in the simulations, compared with the case of no data packet dropout, lower convergence rate of the system state is obtained when a certain rate of data packet dropout occurs. Fig.6 is a simulation of the output feedback closed-loop system with $\sigma = 0.25$, which indicates that if data packet dropout rate $\sigma > \sigma_{\max} = 0.1595$, the convergence of state response may not be guaranteed.

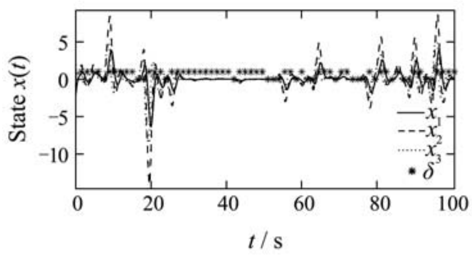

Fig. 6 The state response of the closed-loop system for $q = 20$ with $\sigma = 0.25$.

5 Conclusions

In this paper, taking the data packet dropout of the limited communication channel into consideration, we have proposed the state and output coder-decoder-controller procedures for Lipschitz nonlinear system. Asymptotic stabilization can be guaranteed provided a proper relationship holds between the size of coding alphabet, the sampling period, instability parameter of the system and data packet dropout rate. The approach can also be adopted to consider the stabilization for singular systems and chaos synchronization problems via a limited communication channel with data packet dropout.

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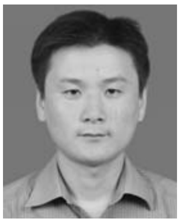

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