

Robust fault-tolerant controller design for linear time-invariant systems with actuator failures: an indirect adaptive method

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Abstract: In this paper, indirect adaptive state feedback control schemes are developed to solve the robust fault-tolerant control (FTC) design problem of actuator fault and perturbation compensations for linear time-invariant systems. A more general and practical model of actuator faults is presented. While both eventual faults on actuators and perturbations are unknown, the adaptive schemes are addressed to estimate the lower and upper bounds of actuator-stuck faults and perturbations online, as well as to estimate control effectiveness on actuators. Thus, on the basis of the information from adaptive schemes, an adaptive robust state feed-back controller is designed to compensate the effects of faults and perturbations automatically. According to Lyapunov stability theory, it is shown that the robust adaptive closed-loop systems can be ensured to be asymptotically stable under the influence of actuator faults and bounded perturbations. An example is provided to further illustrate the fault compensation effectiveness.

Keywords: Fault-tolerant control; Actuator failures; Adaptive robust control; Asymptotic stability

1 Introduction

As we know, the failure of system components (including actuators, sensors, and even the plant itself) may occur at uncertain time with unknown size of faults in practical systems. Some serious faults may result in performance degradation or even instability of the systems. Therefore, the fault-tolerant control (FTC) systems design, which ensures safe operation of the systems and proper performances whenever component is faulted or healthy, has received significant attention over the past two decades (see, e.g., references [1~22]).

According to the existing researches for FTC systems, the FTC design methods can be classified into two types, that is, the passive method [1~7] and the active method [8~22]. Generally, the passive method uses a robust control strategy to construct a fixed control gain to maintain the stability and performance of systems. Many related methods have been introduced in literature, such as the algebraic Riccati equation (ARE)-based approach [1~3], the linear matrix inequality (LMI)-based approach [5, 6], the pole region assignment approach [7], etc. The passive method is easy to obtain a controller for the presumed faults relative to the active method, since it is not relying on online adjustment for control gains. However, it is well known that along with the increase of possible faults and redundancy degree of system, the FTC design becomes conservative. Thus, the fault tolerant capability of the passive method seems limited. Correspondingly, the active method provides more power-

ful fault tolerant capability for compensating for faults of the systems in terms of reconfiguring control strategies online or switching to a more suitable precomputed control law based on the fault information. Nowadays, the adaptive method [8~17, 22] and the fault detection and isolation (FDI)-based method [18~21] can be considered as two typical and effective active methods for fault compensations.

For the FTC system based on FDI method, a more suitable controller can be chosen for the fault case and the better performance can be obtained than the passive FTC system. But it should be pointed out that the FDI mechanism might give incorrect fault information to reconfigure or restructure the controller. Therefore, the adaptive method which needs no mechanism to estimate the exact faults on actuators is worthy to be studied. In references [13~16], the model reference adaptive fault-tolerant control designs were introduced to track the given reference signals. Under considering the fault of loss actuator effectiveness, a perfect performance tracking result was obtained in reference [9]. However, it is worth pointing out that the above FTC systems [9~14] have no perturbation rejection capability, even under some special conditions such as

$$\lim_{t \rightarrow \infty} z(t) = 0$$

($z(t)$ is perturbation) [15] and constant perturbation [16]. On the other hand, direct adaptive methods were developed in references [11, 12] to ensure stability of the FTC systems with constant and time-varying parameterizable stuck-actuator faults. The unparameterizable stuck faults were

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considered in reference [13], but the upper bounds of faults were assumed to be known and asymptotic tracking could not be achieved. The asymptotic stability result was obtained in recent paper [8] via designing an adaptive gain function with direct adaptive method when considering the unparameterizable stuck faults and perturbations without the knowledge of upper bounds of them. In this paper, differing from the direct adaptive method, a robust indirect adaptive scheme is proposed to make sure asymptotic stability of the FTC system with time-varying unparameterizable faults and perturbations.

This paper is concerned with the fault-tolerant compensation control design problem with a general actuator fault model, including normal operation, loss of effectiveness, outage, and unparameterizable stuck. The requirement of knowledge of each control effectiveness is not needed, and the bounds of time-varying stuck faults and perturbations are also assumed to be unknown. For the sake of compensating the faults and the perturbations completely, an indirect adaptive method is developed to construct a class of state feedback controllers. Some adaptive schemes are proposed to estimate the unknown bounds of stuck faults and perturbations and control effectiveness online firstly. Then, based on the updated information of these estimated values, the control gains are adjusted online to compensate for faults and perturbations. In terms of Lyapunov stability theory, the resulting robust adaptive closed-loop FTC system can be ensured to be asymptotically stable even in the case of faults on actuators and perturbations. It should be emphasized that different from the FDI method with the need for a mechanism to provide the exact fault information, the new proposed indirect adaptive design method is not necessary for the estimations to give the exact fault information.

The rest of the paper is organized as follows. Section 2 describes the FTC problem formulation. In Section 3, the indirect adaptive robust state feedback controller is developed. Section 4 gives an example and simulation. Finally, conclusions are included in Section 5.

2 Preliminaries and problem statement

First, notations are introduced as follows. \mathbb{R} denotes the set of real numbers, and for a real matrix D , D^T and D^{-1} denote its transpose, and inverse, respectively. Given matrices $E_k, k = 1, 2, \dots, n$, the notation $\text{diag}_{k=1}^n \{E_k\}$ represents the block diagonal matrix with the diagonal element E_k . For brevity, it is also described by $\text{diag}_k \{E_k\}$. For easing the notation of partitioned symmetric matrices, the symbol $(*)$ stands for each of its symmetric blocks generically.

In this paper, we consider a linear time-invariant continuous-time model captured by the state space equation as

$$\dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t), \quad (1)$$

where $x(t) \in \mathbb{R}^n$ denotes the state, $u(t) \in \mathbb{R}^m$ represents the control input, and $w(t) \in \mathbb{R}^q$ is a continuous vector function which stands for the perturbations for the system. A, B_1, B_2 are known real constant matrices with appropriate dimensions.

Here, we formulate that the faults cover actuator outage, loss of effectiveness and stuck. Let $u_{ij}^F(t)$ stand for the signal from the i th faulted actuator in the j th faulty mode.

Then, we describe the following fault model [3, 13]:

$$u_{ij}^F(t) = \rho_i^j u_i(t) + \sigma_i^j u_{si}(t), \quad (2)$$

where $i = 1, 2, \dots, m, j = 1, 2, \dots, L, \rho_i^j$ and σ_i^j are unknown constants, the index j represents the j th faulty mode, and the total number of faulty modes is L , and $\underline{\rho}_i^j$ and $\bar{\rho}_i^j$ denote respectively the known upper and lower bounds of ρ_i^j ; and $u_{si}(t)$ is the unknown time-varying bounded fault-stuck in the i th actuator. Following the practical case, we have $0 \leq \underline{\rho}_i^j \leq \rho_i^j \leq \bar{\rho}_i^j$, and σ_i^j is defined as

$$\sigma_i^j = \begin{cases} 0, & \rho_i^j > 0, \\ 0 \text{ or } 1, & \rho_i^j = 0. \end{cases} \quad (3)$$

Note that, when $\underline{\rho}_i^j = \bar{\rho}_i^j = 1$, there is no fault on the i th actuator u_i in the j th fault mode. The case of $\underline{\rho}_i^j = \bar{\rho}_i^j = 0$ and $\sigma_i^j = 0$ means that the i th actuator is outage in the j th fault mode. When $\underline{\rho}_i^j = \bar{\rho}_i^j = 0$ and $\sigma_i^j = 1$, the fault-stuck is occurred on the i th actuator in the j th fault mode. The case of $0 < \underline{\rho}_i^j \leq \bar{\rho}_i^j < 1$ denotes that actuator u_i loses its effectiveness in the j th fault mode. Then, Table 1 can be given to illustrate the fault model generally.

Table 1 Fault model.

Fault model	$\underline{\rho}_i^j$	$\bar{\rho}_i^j$	σ_i^j
Normal	1	1	0
Outage	0	0	0
Loss of effectiveness	>0	<1	0
Stuck	0	0	1

Denote

$$u_j^F(t) = [u_{1j}^F(t) \ \dots \ u_{mj}^F(t)]^T = \rho^j u(t) + \sigma^j u_s(t), \quad (4)$$

where

$$\rho^j = \text{diag}\{\rho_1^j, \rho_2^j, \dots, \rho_m^j\}, \quad \rho_i^j \in [\underline{\rho}_i^j, \bar{\rho}_i^j],$$

$$\sigma^j = \text{diag}\{\sigma_1^j, \sigma_2^j, \dots, \sigma_m^j\}, \quad j = 1, 2, \dots, L.$$

Then, the set with above structure is defined by

$$\Delta_{\rho^j} = \{\rho^j : \rho^j = \text{diag}\{\rho_1^j, \dots, \rho_m^j\}, \rho_i^j \in [\underline{\rho}_i^j, \bar{\rho}_i^j]\}, \quad (5)$$

and we can also define a set as follows

$$N_{\rho^j} = \{\rho^j : \rho^j = \text{diag}\{\rho_1^j, \dots, \rho_m^j\},$$

$$\rho_i^j = \underline{\rho}_i^j \text{ or } \rho_i^j = \bar{\rho}_i^j\}, \quad (6)$$

where $i = 1, 2, \dots, m, j = 1, 2, \dots, L$. Thus, the set N_{ρ^j} includes a maximum of 2^m elements.

For convenience, in the following, actuator fault model is uniformly exploited by:

$$u^F(t) = \rho u(t) + \sigma u_s(t), \quad (7)$$

where $\rho = \text{diag}\{\rho_1, \dots, \rho_m\} \in \{\rho^1, \dots, \rho^L\}$.

Therefore, consider actuator fault (7), the dynamics of (1) is written by

$$\dot{x}(t) = Ax(t) + B_2 \rho u(t) + B_2 \sigma u_s(t) + B_1 w(t). \quad (8)$$

To ensure the achievement of the fault-tolerant objective, two standard requirements are that all the states of system are available at every instant for state feedback case, and the system $\{A, B_2 \rho\}$ is uniformly completely controllable for any fault mode $\rho \in \{\rho^1 \dots \rho^L\}$. Besides, the following assumptions in the FTC design are also assumed to be valid.

Assumption 1 For the FTC system (8), there exists a matrix function F of appropriate dimensions such that

$$B_1 = B_2F. \tag{9}$$

Assumption 2 $\text{rank}[B_2\rho] = \text{rank}[B_2]$ for any actuator failure mode $\rho \in \{\rho^1 \cdots \rho^L\}$.

Remark 1 Assumption 1 defines a matching condition about the perturbations. It is necessary to compensate for perturbation completely. Based on the knowledge of linear algebra theory, we should design a control law $K(t)$ to make $B_2\rho(t)K(t)x(t) = B_1w(t)$ for compensating the perturbation according to (8). Thus, there must exist an appropriate dimensions matrix F such that $B_3 = B_2F$ to ensure that the equation holds true. It is a standard assumption for the robust control problem [23]. Assumption 2 introduces a condition of actuator redundancy in the system, and is also necessary for completely compensating the stuck-actuator faults. The reason can be explained from the controllability of system or linear algebra theory. Many practical systems belongs to this class of systems and some studies [8, 10~13] had also been proposed based on the redundant condition. Although it is still under the condition, a novel FTC will be proposed.

Thus, by using adaptive mechanism, the purpose of this paper is to design an adaptive robust state feed-back controller such that the state of closed-loop system (8) converges to zero asymptotically under normal or faulted actuators and perturbations.

3 Indirect adaptive FTC system design

In Section 3, some adaptive schemes are designed to estimate the unknown fault effect factors ρ and σ of actuators, the unknown upper and lower bounds of stuck faults, \bar{u}_s and \underline{u}_s , and also the unknown upper and lower bounds of perturbations, \bar{w} and \underline{w} , respectively. Then, a result of asymptotic stability of the system is introduced in Theorem 1 via constructing an indirect adaptive robust fault-tolerant controller.

Now, the indirect adaptive robust fault-tolerant controller model is described by

$$u(t) = K_1x(t) + \hat{\rho}(t)K_2x(t) + K_3\hat{\sigma}(t)[(I - \tau_u)\hat{u}_s(t) + \tau_u\underline{u}_s(t)] + K_4[(I - \tau_w)\hat{w}(t) + \tau_w\underline{w}(t)], \tag{10}$$

where $\hat{\rho}$, $\hat{\sigma}$ are the estimates of ρ , σ respectively; $\hat{u}_s(t)$ and $\underline{u}_s(t)$ denote respectively the estimates of unknown upper and lower bounds of $u_s(t)$; \bar{w} and \underline{w} represent the unknown upper and lower bounds of $w(t)$, respectively, and $\hat{w}(t)$ and $\underline{w}(t)$ are their estimations; τ_u is defined as

$$\tau_u = \text{diag}\{\tau_{u1}, \dots, \tau_{um}\}, \quad \tau_{ui} = \begin{cases} 0, & x^T P b_{2i} \geq 0, \\ 1, & x^T P b_{2i} < 0, \end{cases} \tag{11}$$

where b_{2i} , $i = 1, 2, \dots, m$ is the i th column of B_2 ; and τ_w is defined as

$$\tau_w = \text{diag}\{\tau_{w1}, \dots, \tau_{wq}\}, \quad \tau_{wk} = \begin{cases} 0, & x^T P B_2 f_k \geq 0, \\ 1, & x^T P B_2 f_k < 0, \end{cases} \tag{12}$$

where f_k , $k = 1, 2, \dots, q$ is the k th column of F which satisfied (9).

Then, substituting (10) into (8), the closed-loop FTC sys-

tem model is captured by

$$\dot{x}(t) = (A + B_2\rho K_1 + B_2\rho\hat{\rho}(t)K_2)x(t) + B_2\rho K_3[(I - \tau_u)\hat{u}_s(t) + \tau_u\underline{u}_s(t)] + B_2\rho K_4[(I - \tau_w)\hat{w}(t) + \tau_w\underline{w}(t)] + B_2\sigma u_s(t) + B_2Fw(t). \tag{13}$$

Before giving our main result, we first introduce the system matrices' decomposed form as

$$F = [f_1 \ f_2 \ \cdots \ f_q] \in \mathbb{R}^{m \times q}, \\ B_2 = [b_{21} \ b_{22} \ \cdots \ b_{2m}] \in \mathbb{R}^{m \times m}, \\ K_3 = [k_{31} \ k_{32} \ \cdots \ k_{3m}] \in \mathbb{R}^{m \times m}, \\ K_4 = [k_{41} \ k_{42} \ \cdots \ k_{4q}] \in \mathbb{R}^{m \times q}.$$

Denote

$$M := x^T P B_2, \\ R := \hat{\sigma}[(I - \tau_u)\hat{u}_s + \tau_u\underline{u}_s], \\ S := F[(I - \tau_w)\hat{w} + \tau_w\underline{w}], \tag{14}$$

where M , R and S are bound vector functions described by

$$M = [M_1 \ M_2 \ \cdots \ M_m] \in \mathbb{R}^{1 \times m}, \\ R = [R_1 \ R_2 \ \cdots \ R_m]^T \in \mathbb{R}^m, \\ S = [S_1 \ S_2 \ \cdots \ S_m]^T \in \mathbb{R}^m,$$

which will be used later; P is a positive symmetric matrix.

In particular, the unknown ρ_i is estimated by $\hat{\rho}_i(t)$ which is adjusted by the adaptive laws such as

$$\frac{d\hat{\rho}_i(t)}{dt} = \begin{cases} 0, & \text{if } \hat{\rho}_i = 0.1 \text{ and } L_{\rho_i} \leq 0, \\ L_{\rho_i}, & \text{otherwise,} \end{cases} \tag{15}$$

where $i = 1, 2, \dots, m$, and

$$L_{\rho_i} = -l_i[\frac{1}{2}\hat{\rho}_i(t)M_i^2 + \hat{\rho}_i^{-1}(t)M_iR_i + \hat{\rho}_i^{-1}(t)M_iS_i]. \tag{16}$$

The unknown σ_i is estimated by $\hat{\sigma}_i(t)$, which is adjusted according to the adaptive laws: $i = 1, 2, \dots, m$,

$$\frac{d\hat{\sigma}_i(t)}{dt} = s_{1i}x^T(t)Pb_{2i}[(I - \tau_u)\hat{u}_s + \tau_u\underline{u}_s]. \tag{17}$$

Besides, \hat{u}_{si} and \underline{u}_{si} are the estimates of upper and lower bounds of $u_{si}(t)$, respectively, which are updated by the adaptive laws:

$$\frac{d\hat{u}_{si}(t)}{dt} = s_{2i}x^T(t)Pb_{2i}, \tag{18}$$

$$\frac{d\underline{u}_{si}(t)}{dt} = s_{2i}x^T(t)Pb_{2i}. \tag{19}$$

Similarly, $\hat{w}_k(t)$ and $\underline{w}_k(t)$ are the estimates of upper and lower bounds of unknown perturbations $w_k(t)$, respectively, updated by the following adaptive laws:

$$\frac{d\hat{w}_k(t)}{dt} = r_kx^T(t)PB_2f_k, \tag{20}$$

$$\frac{d\underline{w}_k(t)}{dt} = r_kx^T(t)PB_2f_k, \tag{21}$$

where $k = 1, 2, \dots, q$. The constants $l_i > 0$, $s_{1i} > 0$ and $s_{2i} > 0$, $r_k > 0$ are the adaptive law gains to be designed based on practical application [24]. Then, the FTC closed-loop system can be drawn as in Fig.1.

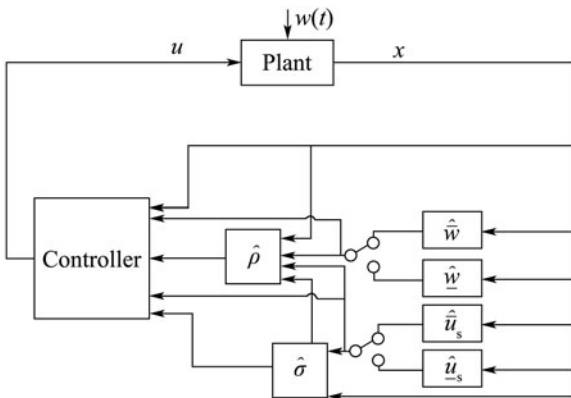


Fig. 1 Active compensation fault-tolerant control schemes.

Let

$$\begin{cases} \tilde{\rho}(t) = \hat{\rho}(t) - \rho, & \tilde{\sigma}(t) = \hat{\sigma}(t) - \sigma, \\ \tilde{u}_{s_i}(t) = \hat{u}_{s_i}(t) - u_{s_i}, & \tilde{u}_s(t) = \hat{u}_s(t) - u_s, \\ \tilde{w}(t) = \hat{w}(t) - w, & \tilde{w}(t) = \hat{w}(t) - w, \end{cases} \quad (22)$$

where

$$\begin{aligned} \rho &= \text{diag}_i\{\rho_i\}, \quad \sigma = \text{diag}_i\{\sigma_i\}, \quad i = 1, 2, \dots, m, \\ \bar{u}_s &= [\bar{u}_{s1} \ \bar{u}_{s2} \ \dots \ \bar{u}_{sm}]^T, \quad \underline{u}_s = [u_{s1} \ u_{s2} \ \dots \ u_{sm}]^T, \\ \bar{w} &= [\bar{w}_1 \ \bar{w}_2 \ \dots \ \bar{w}_q]^T, \quad \underline{w} = [w_1 \ w_2 \ \dots \ w_q]^T. \end{aligned}$$

Because $\rho_i, \sigma_i, \bar{u}_{s_i}, \underline{u}_{s_i}, \bar{w}_k$ and \underline{w}_k are unknown constants, the error system is written by the following equations:

$$\begin{cases} \dot{\tilde{\rho}}_i(t) = \dot{\hat{\rho}}_i(t), & \dot{\tilde{\sigma}}_i(t) = \dot{\hat{\sigma}}_i(t), \\ \dot{\tilde{u}}_{s_i}(t) = \dot{\hat{u}}_{s_i}(t), & \dot{\tilde{u}}_s(t) = \dot{\hat{u}}_s(t), \\ \dot{\tilde{w}}_k(t) = \dot{\hat{w}}_k(t), & \dot{\tilde{w}}_k(t) = \dot{\hat{w}}_k(t). \end{cases} \quad (23)$$

Then, for the FTC system described in (13), the control gain functions K_2, K_3 and K_4 in the controllers (10) are designed by

$$\begin{cases} K_2 = -\frac{1}{2}B_2^T P, \\ K_3 = -\hat{\rho}^{-1}(t), \\ K_4 = -\hat{\rho}^{-1}(t)F, \end{cases} \quad (24)$$

and K_1, P will be given according to the following lemma which comes from reference [25].

Lemma 1 If there exist matrices $Y > 0, H > 0$ and appropriate dimensions matrices W and V such that LMI

$$\begin{bmatrix} \psi_i^j & * & * \\ (B_2 \rho_i^j W)^T & -V - V^T & * \\ 0 & (Y - V)^T & -H \end{bmatrix} < 0 \quad (25)$$

holds true, where

$$\begin{aligned} \psi_i^j &= AY + YA^T + B_2 \rho_i^j W + (B_2 \rho_i^j W)^T + H, \\ i &= 1, 2, \dots, 2^m, \quad j = 1, 2, \dots, L. \end{aligned}$$

Then for $K_1 = WV^{-1}$ and $P = Y^{-1}$, the following matrix inequality

$$(A + B_2 \rho_i^j K_1)^T P + P(A + B_2 \rho_i^j K_1) < 0 \quad (26)$$

is feasible.

Proof Firstly, we give a fact of a matrix inequality addressed in reference [26], namely,

$$Z^T U^{-1} Z \geq Z + Z^T - U, \quad (27)$$

which holds for all $U > 0$ and square matrix Z with appro-

priate dimensions.

Then as (25) means that $V + V^T > 0$, matrix V is a non-singular. By using Schur complement to the third row and the third column of LMI (25) and considering the inequality (27), we yield

$$\begin{bmatrix} \psi_i^j & * \\ (B_2 \rho_i^j W)^T & -V^T [(Y - V)H^{-1}(Y - V^T)]^{-1} V \end{bmatrix} < 0. \quad (28)$$

Then applying again Schur complement to the second row and the second column of that LMI, we have

$$\psi_i^j + B_2 \rho_i^j W V^{-1} (Y - V) H^{-1} (Y - V^T) V^{-T} (B_2 \rho_i^j W)^T < 0. \quad (29)$$

We also follow inequality (27) and let $K_1 = WV^{-1}$. From (29), we have

$$\begin{aligned} 0 &> \psi_i^j + B_2 \rho_i^j W V^{-1} (Y - V) H^{-1} (Y - V^T) V^{-T} (B_2 \rho_i^j W)^T \\ &\geq \psi_i^j - H + B_2 \rho_i^j W V^{-1} (Y - V) + (Y - V^T) V^{-T} (B_2 \rho_i^j W)^T \\ &= \psi_i^j - H + B_2 \rho_i^j K_1 Y - B_2 \rho_i^j W + Y K_1^T (B_2 \rho_i^j)^T \\ &\quad - (B_2 \rho_i^j W)^T \\ &= AY + YA^T + B_2 \rho_i^j K_1 Y + Y K_1^T (B_2 \rho_i^j)^T. \end{aligned} \quad (30)$$

Let $P = Y^{-1}$, and pre- and post-multiplying both sides of (30) by P , we obtain (26).

Hence, the following theorem, which shows the uniform ultimate boundedness of error system (23) and closed-loop system (13), can be stated as follows.

Theorem 1 Consider the closed-loop FTC system described by (13) satisfying Assumptions 1~2. If there exists a symmetric matrix $P > 0$, and by using the state feedback controller $u(t)$ described in (10) with adaptive laws (16)~(21), and control gain functions (24), then all closed-loop system signals are bounded and $\lim_{t \rightarrow \infty} x(t) = 0$ for any $\rho \in \Delta_{\rho^j}, j = 1, 2, \dots, L$.

Proof For the closed-loop system (13), we define the following Lyapunov function first:

$$\begin{aligned} V(t) &= x^T P x + \sum_{i=1}^m \frac{\tilde{\rho}_i^2}{l_i} + \sum_{i=1}^m \frac{\tilde{\sigma}_i^2}{s_{1i}} \\ &\quad + \sum_{i=1}^m \frac{\sigma_i (1 - \tau_{ui}) \tilde{u}_{s_i}^2}{s_{2i}} + \sum_{i=1}^m \frac{\sigma_i \tau_{ui} \tilde{u}_{s_i}^2}{s_{2i}} \\ &\quad + \sum_{k=1}^q \frac{(1 - \tau_{wk}) \tilde{w}_k^2}{r_k} + \sum_{k=1}^q \frac{\tau_{wk} \tilde{w}_k^2}{r_k}. \end{aligned} \quad (31)$$

The time derivative of $V(t)$ with a certain fault mode $\rho \in \Delta_{\rho^j}$ is

$$\begin{aligned} \dot{V}(t) &= x^T [(A + B_2 \rho K_1)^T P + P(A + B_2 \rho K_1)] x \\ &\quad + x^T [(B_2 \rho \hat{\rho} K_2)^T P + P(B_2 \rho \hat{\rho} K_2)] x \\ &\quad + 2x^T P B_2 \sigma u_s(t) + 2x^T P B_2 \rho K_3 \hat{\sigma} (\hat{u}_s + \tau_u \underline{u}_s - \tau_u \hat{u}_s) \\ &\quad + 2x^T P B_1 w + 2x^T P B_2 \rho K_4 (\hat{w} + \tau_w \underline{w} - \tau_w \hat{w}) \\ &\quad + \sum_{i=1}^m \frac{2\tilde{\rho}_i \dot{\tilde{\rho}}_i}{l_i} + \sum_{i=1}^m \frac{2\tilde{\sigma}_i \dot{\tilde{\sigma}}_i}{s_{1i}} \end{aligned}$$

$$\begin{aligned}
 & + \sum_{i=1}^m \frac{2\sigma_i(1-\tau_{ui})\tilde{u}_{si}\dot{\tilde{u}}_{si}}{s_{2i}} + \sum_{i=1}^m \frac{2\sigma_i\tilde{u}_{si}\dot{\tilde{u}}_{si}}{s_{2i}} \\
 & + \sum_{k=1}^q \frac{2(1-\tau_{wk})\tilde{w}_k\dot{\tilde{w}}_k}{r_k} + \sum_{k=1}^q \frac{2\tau_{wk}\tilde{w}_k\dot{\tilde{w}}_k}{r_k} \\
 = & x^T[(A+B_2\rho K_1)^T P + P(A+B_2\rho K_1)]x \\
 & - x^T P B_2(\hat{\rho} - \tilde{\rho})\hat{\rho} B_2^T P x + \sum_{i=1}^m \frac{2\tilde{\rho}_i\dot{\tilde{\rho}}_i}{l_i} + \sum_{i=1}^m \frac{2\tilde{\sigma}_i\dot{\tilde{\sigma}}_i}{s_{1i}} \\
 & + 2x^T P B_2 \sum_{i=1}^m \sigma_i u_{si} + 2x^T P B_2 \sum_{k=1}^q f_k w_k \\
 & + 2x^T P B_2 \rho (\sum_{i=1}^m k_{3i} \hat{\sigma}_i (1-\tau_{ui}) \hat{u}_{si} + \sum_{i=1}^m k_{3i} \hat{\sigma}_i \tau_{ui} \hat{u}_{si}) \\
 & + 2x^T P B_2 \rho (\sum_{k=1}^q k_{4k} (1-\tau_{wk}) \hat{w}_k + \sum_{k=1}^q k_{4k} \tau_{wk} \hat{w}_k) \\
 & + \sum_{i=1}^m \frac{2\sigma_i(1-\tau_{ui})\tilde{u}_{si}\dot{\tilde{u}}_{si}}{s_{2i}} + \sum_{i=1}^m \frac{2\sigma_i\tilde{u}_{si}\dot{\tilde{u}}_{si}}{s_{2i}} \\
 & + \sum_{k=1}^q \frac{2(1-\tau_{wk})\tilde{w}_k\dot{\tilde{w}}_k}{r_k} + \sum_{k=1}^q \frac{2\tau_{wk}\tilde{w}_k\dot{\tilde{w}}_k}{r_k}. \tag{32}
 \end{aligned}$$

Note that

$$\begin{aligned}
 x^T P B_2 \sum_{i=1}^m \sigma_i u_{si} \leq & x^T P B_2 \sum_{i=1}^m \sigma_i ((1-\tau_{ui})\tilde{u}_{si} \\
 & + \tau_{ui} \underline{u}_{si}), \tag{33}
 \end{aligned}$$

$$\begin{aligned}
 x^T P B_2 \sum_{k=1}^q f_k w_k \leq & x^T P B_2 \sum_{k=1}^q f_k ((1-\tau_{wk})\tilde{w}_k \\
 & + \tau_{wk} \underline{w}_k), \tag{34}
 \end{aligned}$$

where τ_{ui} and τ_{wk} are denoted in (11) and (12), respectively. With the adaptive laws chosen in (17)~(21) and controller gain function chosen in (24), (32) can be rewritten by

$$\begin{aligned}
 \dot{V}(t) \leq & x^T[(A+B_2\rho K_1)^T P + P(A+B_2\rho K_1)]x \\
 & + x^T P B_2 \tilde{\rho} \hat{\rho} B_2^T P x + \sum_{i=1}^m \frac{2\tilde{\rho}_i\dot{\tilde{\rho}}_i}{l_i} + \sum_{i=1}^m \frac{2\tilde{\sigma}_i\dot{\tilde{\sigma}}_i}{s_{1i}} \\
 & + 2x^T P B_2 \sum_{i=1}^m \sigma_i (\tilde{u}_{si} + \tau_{ui} \underline{u}_{si} - \tau_{ui} \tilde{u}_{si}) \\
 & + 2x^T P B_2 (\sum_{k=1}^q f_k \tilde{w}_k + \sum_{k=1}^q f_k \underline{w}_k - \sum_{k=1}^q \tau_{wk} f_k \tilde{w}_k) \\
 & + 2x^T P B_2 \rho (\sum_{i=1}^m k_{3i} \hat{\sigma}_i (1-\tau_{ui}) \hat{u}_{si} + \sum_{i=1}^m k_{3i} \hat{\sigma}_i \tau_{ui} \hat{u}_{si}) \\
 & + 2x^T P B_2 \rho (\sum_{k=1}^q k_{4k} (1-\tau_{wk}) \hat{w}_k + \sum_{k=1}^q k_{4k} \tau_{wk} \hat{w}_k) \\
 & + \sum_{i=1}^m \frac{2\sigma_i(1-\tau_{ui})\tilde{u}_{si}\dot{\tilde{u}}_{si}}{s_{2i}} + \sum_{i=1}^m \frac{2\sigma_i\tilde{u}_{si}\dot{\tilde{u}}_{si}}{s_{2i}} \\
 & + \sum_{k=1}^q \frac{2(1-\tau_{wk})\tilde{w}_k\dot{\tilde{w}}_k}{r_k} + \sum_{k=1}^q \frac{2\tau_{wk}\tilde{w}_k\dot{\tilde{w}}_k}{r_k} \\
 = & x^T[(A+B_2\rho K_1)^T P + P(A+B_2\rho K_1)]x \\
 & + x^T P B_2 \tilde{\rho} \hat{\rho} B_2^T P x + \sum_{i=1}^m \frac{2\tilde{\rho}_i\dot{\tilde{\rho}}_i}{l_i} \\
 & + 2x^T P B_2 \sum_{i=1}^m \hat{\sigma}_i (\hat{u}_{si} + \tau_{ui} \underline{u}_{si} - \tau_{ui} \hat{u}_{si})
 \end{aligned}$$

$$\begin{aligned}
 & + 2x^T P B_2 (\sum_{k=1}^q f_k \hat{w}_k + \sum_{k=1}^q f_k \underline{w}_k - \sum_{k=1}^q \tau_{wk} f_k \hat{w}_k) \\
 & + 2x^T P B_2 \hat{\rho} (\sum_{i=1}^m k_{3i} (1-\tau_{ui}) \hat{\sigma}_i \hat{u}_{si} + \sum_{i=1}^m k_{3i} \hat{\sigma}_i \tau_{ui} \hat{u}_{si}) \\
 & - 2x^T P B_2 \tilde{\rho} (\sum_{i=1}^m k_{3i} (1-\tau_{ui}) \hat{\sigma}_i \hat{u}_{si} + \sum_{i=1}^m k_{3i} \hat{\sigma}_i \tau_{ui} \hat{u}_{si}) \\
 & + 2x^T P B_2 \hat{\rho} (\sum_{k=1}^q k_{4k} (1-\tau_{wk}) \hat{w}_k + \sum_{k=1}^q k_{4k} \tau_{wk} \hat{w}_k) \\
 & - 2x^T P B_2 \tilde{\rho} (\sum_{k=1}^q k_{4k} (1-\tau_{wk}) \hat{w}_k + \sum_{k=1}^q k_{4k} \tau_{wk} \hat{w}_k) \\
 = & x^T[(A+B_2\rho K_1)^T P + P(A+B_2\rho K_1)]x \\
 & + x^T P B_2 \tilde{\rho} \hat{\rho} B_2^T P x + \sum_{i=1}^m \frac{2\tilde{\rho}_i\dot{\tilde{\rho}}_i}{l_i} \\
 & + 2x^T P B_2 \tilde{\rho} \hat{\rho}^{-1} \hat{\sigma} [(I-\tau_u)\hat{u}_s + \tau_u \underline{u}_s] \\
 & + 2x^T P B_2 \tilde{\rho} \hat{\rho}^{-1} F [(I-\tau_w)\hat{w} + \tau_w \underline{w}] \\
 = & x^T[(A+B_2\rho K_1)^T P + P(A+B_2\rho K_1)]x \\
 & + \sum_{i=1}^m \tilde{\rho}_i \hat{\rho}_i M_i^2 + 2 \sum_{i=1}^m \tilde{\rho}_i \hat{\rho}_i^{-1} M_i R_i \\
 & + 2 \sum_{i=1}^m \tilde{\rho}_i \hat{\rho}_i^{-1} M_i S_i + \sum_{i=1}^m \frac{2\tilde{\rho}_i\dot{\tilde{\rho}}_i}{l_i}, \tag{35}
 \end{aligned}$$

where M_i , R_i and S_i are denoted in (14).

In terms of the adaptive law chosen in (16), (35) can be rewritten by

$$\dot{V}(t) \leq x^T[(A+B_2\rho K_1)^T P + P(A+B_2\rho K_1)]x. \tag{36}$$

According to Lemma 1, we yield P and K_1 for any $\rho \in \Delta_{\rho_j}$ such that

$$(A+B_2\rho K_1)^T P + P(A+B_2\rho K_1) < 0. \tag{37}$$

Hence, it is easy to see that $\dot{V}(t) < 0$ for any initial value $x(0)$. The solutions to the closed-loop fault tolerant control system (13) are uniformly ultimate bounded, and the system state $x(t)$ converges to zero asymptotically.

Thus, in terms of the above description, an algorithm is introduced to demonstrate the controller design procedure:

Algorithm 1

Step 1 Solving the LMIs (25), we obtain controller parameter K_1 and Lyapunov matrix P .

Step 2 Following (18)~(21), we obtain the estimates of upper and lower bounds of fault-stuck $u_{si}(t)$, i.e., $\hat{u}_{si}(t)$ and $\underline{u}_{si}(t)$, $i = 1, 2, \dots, m$, respectively, and the estimates of upper and lower bounds of perturbations $w_k(t)$, i.e., $\hat{w}_k(t)$ and $\underline{w}_k(t)$, $k = 1, 2, \dots, q$, respectively. On the other hand, in terms of (24), we get K_2 .

Step 3 According to the adaptive laws (17), we obtain the estimate of σ_i , $i = 1, 2, \dots, m$.

Step 4 The estimations of fault effect factors $\hat{\rho}_i(t)$, $i = 1, 2, \dots, m$ are achieved by the adaptive laws (15) with the information from Steps 1~3.

Step 5 After Step 4, the control gain functions K_3 and K_4 are obtained by equation (24).

Thus, all controller parameters have been obtained by Algorithm 1.

Remark 2 Since using $\hat{\rho}_i^{-1}$, $i = 1, 2, \dots, m$ in the adaptive laws (16) and control gain functions (24), it seems that it cannot solve the model of actuator outage or stuck

under the proposed adaptive method if $\hat{\rho}_i(t)$ reaches zero. However, it should be pointed out that it is not necessary to make the estimation of ρ_i convergent to its true value in our adaptive robust FTC design, that is, $\hat{\rho}_i$ is not necessary to be converge to zero in the case of $\rho_i = 0$. Following (15), we can choose l_i sufficiently small to avoid the estimation to reach the lower bound with the initial condition $\hat{\rho}_i(0) = 1$. Therefore, under the proposed control schemes, we may still solve the problem of actuator outage and stuck.

Remark 3 Through estimating the bound of actuator-stuck faults $u_s(t)$, the proposed method can also solve the actuator fault such as unparametrizable stuck faults without the knowledge of bound of faults. Although the similar problem can be solved by the direct adaptive method proposed in reference [8], the novel indirect adaptive method provides another effective technique to deal with it. Furthermore, by comparing to other existing direct adaptive methods introduced in references [10, 11], the proposed method provides more powerful fault tolerant capability for the time-varying unparametrizable fault compensations.

4 An example and simulation results

Here, we consider the linear time-invariant continuous-time system (8) with the following system matrices:

$$A = \begin{bmatrix} -1 & 0.2 \\ 1 & 0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2 & -1.5 & 1 \\ 1 & -1 & -0.5 \end{bmatrix}, \\ B_1 = \begin{bmatrix} 1.5 & 1 \\ -2 & -1 \end{bmatrix}.$$

Consider the following four possible faulty modes.

Normal mode 1 All of the actuators are normal, that is, $\rho_1^1 = \rho_2^1 = \rho_3^1 = 1$.

Fault mode 2 The first actuator is stuck or outage, the second and third actuators can be normal or have lost effectiveness, described by $\rho_1^2 = 0, a_2 \leq \rho_2^2 \leq 1, a_3 \leq \rho_3^2 \leq 1, a_2 = 0.5, a_3 = 0.3$, which denote the maximum loss of effectiveness for the second and third actuators.

Fault mode 3 The second actuator is stuck or outage, the first and the third actuators may lose effectiveness or normal, that is, $\rho_2^3 = 0, b_1 \leq \rho_1^3 \leq 1, b_3 \leq \rho_3^3 \leq 1, b_1 = 0.5, b_3 = 0.3$, which denote the maximum loss of effectiveness for the first and the third actuators.

Fault mode 4 The third actuator is stuck or outage, the first and the second actuators can be normal or have lost effectiveness, namely, $\rho_3^4 = 0, c_1 \leq \rho_1^4 \leq 1, c_2 \leq \rho_2^4 \leq 1, c_1 = 0.5, c_2 = 0.2$, which denote the maximum loss of effectiveness for the first and second actuators.

Following Lemma 1, we obtain

$$P = \begin{bmatrix} 13.6443 & 3.7195 \\ 3.7195 & 10.3080 \end{bmatrix}, \\ K_1 = \begin{bmatrix} -0.1703 & -0.2824 \\ 0.1309 & 0.2553 \\ -0.1761 & -0.2874 \end{bmatrix}.$$

For the sake of verifying the effectiveness of proposed indirect adaptive approach, the following parameters and initial conditions are provided in the simulation:

$$l_i = 0.1, \quad s_{1i} = 10, \quad s_{2i} = 5, \quad r_k = 1, \quad \hat{\rho}_i(0) = 1, \\ \hat{u}_s(0) = [1.2, 0.6, 0.6]^T, \quad \hat{u}_s(0) = [-1.2, -0.6, -0.6]^T,$$

$$\hat{\sigma}_i(0) = 0, \quad \hat{w}_k(0) = 0.5, \quad \hat{u}_k(0) = -0.5, \\ x_1(0) = 0.5, \quad x_2(0) = -0.5, \quad i = 1, 2, 3, \quad k = 1, 2.$$

Now, in the simulation, we consider the following faulty cases, i.e., before the second second, the system operates in the normal case, and the perturbations

$$w(t) = [-0.5, 0.5 \sin(2t)]^T$$

enter into the systems at the beginning ($t \geq 0$). At the fifth second, some faults occurred on actuators, which can be described as $\rho = \text{diag}\{0, 1, 1\}$, and at the fifth second, the first actuator has stuck at

$$u_{s1}(t) = 1 + 0.1 \sin(2t)$$

and the second actuator has lost effectiveness of 50%.

Fig.2 shows the response curves of the states of system with adaptive robust state feed-back controller in the above-mentioned faulty cases. Fig.3 shows the response curves of the estimate of control effectiveness ρ and σ , respectively. Fig.4 denotes the estimate of the upper and lower bounds of stuck fault $u_s(t)$, respectively. Fig.5 shows the estimate of the upper and lower bounds of perturbations \hat{w} and \hat{u} , respectively. It is easy to see that the estimates is convergence. Following Fig.3, $\hat{\rho}$ will never reach the lower bound $\underline{\rho} = 0$; then there is no problem to solve the case of actuator outage using the proposed adaptive method. As the illustration of Figs.2~5, it is obvious that the estimates of $\rho, \sigma, \hat{u}_s, \hat{u}_s$ and \hat{w}, \hat{u} are not necessarily to converge to their exact values in our design.

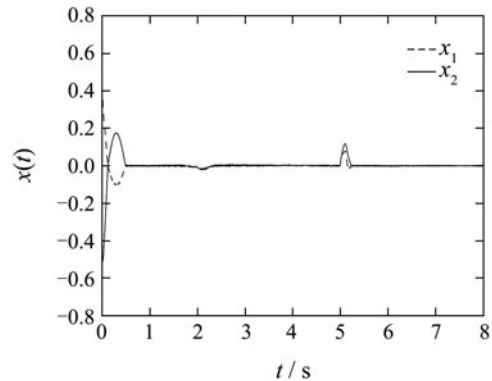


Fig. 2 Response curves of the system states $x(t)$.

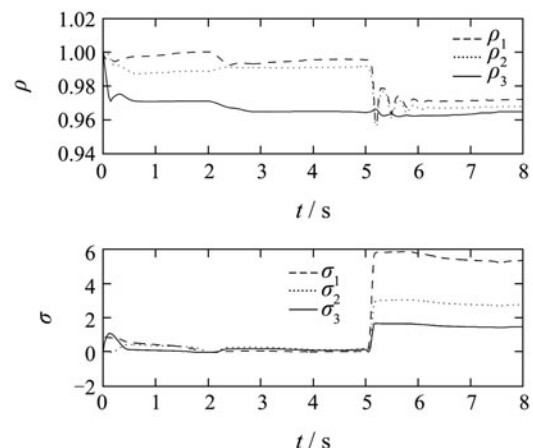


Fig. 3 Response curves of the estimates of actuator fault effect factors ρ and σ .

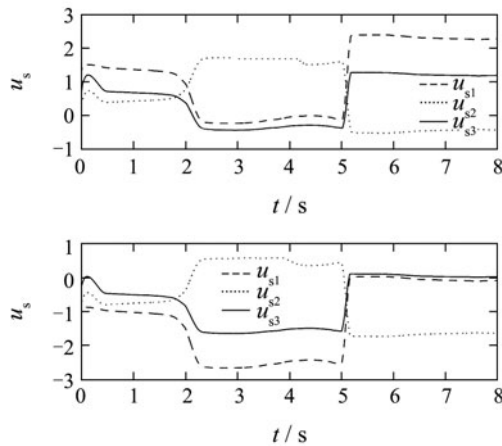


Fig. 4 Response curves of the estimates of stuck faults $u_s(t)$.

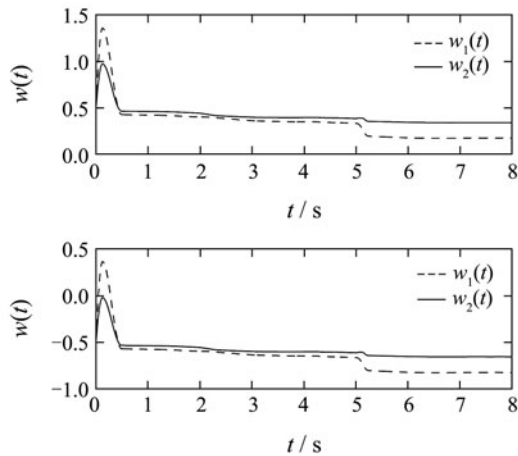


Fig. 5 Response curves of the estimates of upper bound and lower bound of $w(t)$.

5 Conclusions

This paper presented an indirect adaptive method to deal with the robust fault-tolerant compensation control problem with actuator faults and perturbations. A general actuator fault model was adopted, which covers the cases of normal operation, loss of effectiveness, outage and stuck. Based on estimating the unknown bounds of stuck faults and perturbations, and the unknown control effectiveness on actuators on-line, an indirect adaptive state feedback controller was constructed for automatically compensating the fault on actuators and perturbation effects. And the asymptotic stability result of the system was obtained according to Lyapunov stability theory. The simulation results confirm the properties of the proposed method.

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