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Delay-dependent stabilization of singular Markovian jump systems with state delay

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Abstract: This paper deals with the delay-dependent stabilization problem for singular systems with Markovian jump parameters and time delays. A delay-dependent condition is established for the considered system to be regular, impulse free and stochastically stable. Based on the condition, a design algorithm of the desired state feedback controller which guarantees the resultant closed-loop system to be regular, impulse free and stochastically stable is proposed in terms of a set of strict linear matrix inequalities (LMIs). Numerical examples show the effectiveness of the proposed methods.

Keywords: Singular time-delay systems; Markovian jumping parameters; Delay-dependent; Stochastic stability; Linear matrix inequality (LMI)

1 Introduction

Singular systems are an important kind of systems from both the theoretical and practical points of view and have been extensively studied in the past years. Many important and interesting results on the control problems related to singular systems have been proposed by all kinds of methods and a great number of fundamental notions and results in control, and systems theory based on state-space systems have been successfully extended to singular systems. For more details on these results we refer the readers to [1, 2] and the references therein. It is worth noting that when dealing with the related problems of singular systems, not only stability, but also regularity and absence of impulses (for continuous singular systems) and causality (for discrete singular systems) must be considered simultaneously, whereas for state-space systems the latter two issues do not arise. Hence, the study of singular systems is much more difficult and complex than that of state-space systems. Recently, the problems of stability analysis and stabilization for singular systems with time-delay have attracted much attention from many researchers since time delays are often the cause of instability and poor performance of control systems and both delay-independent and delay-dependent conditions have been derived for continuous cases (see [1, 3, 4] and references therein). The corresponding results for discrete cases can be found in [1, 5, 6].

In parallel, there have been also considerable research efforts on the study of Markovian jump systems, which are a special kind of hybrid systems, due to the fact that they can better represent physical systems with abrupt variations. Many important results on such systems have been reported in the literature (see, e.g. [7∼9] and the references therein). When Markovian jump parameters arise in singular systems, the state feedback stabilization and its robustness problems for this kind of systems with norm-bounded uncertainties were tackled in [10] in terms of the linear matrix inequality (LMI) approach. The problems of guaranteed cost control and robust H_∞ control for continuous Markovian jump singular systems were solved in [1] and the desired state feedback controllers can be constructed by solving a set of LMIs. Also, the robust H_∞ control problem for discrete Markovian jump singular systems was discussed in [11]. However, to date only a few results have been reported on singular Markovian jump systems with time delay. In terms of the LMI approach, [12] established sufficient conditions on the stochastic stability and stochastic stabilizability for singular Markovian jump systems with time delays and the stabilization problem was solved via state feedback controller. However, the results of [12] are delay-independent, so they are conservative, especially when time delay is small. The delay-dependent stochastic stabilization problem of singular Markovian jump systems with state delay was discussed in [13] and the design methods for the desired state feedback controllers were given. However, it should be pointed out that in [13] the considered system was assumed to be necessarily regular and impulse free; moreover, a matrix describing the relationship between fast and slow subsystems is needed and an improper choice of the matrix would make the results unreliable. Hence, the delay-dependent stabilization problem for singular Markovian jump time-delay systems has not been well solved in [13]. To the best of our knowledge, very little attention has been paid to the problem of delay-dependent stabilization for singular Markovian jump time-delay systems, which has not been fully discussed and is still open. This motivates the present study.

This paper is concerned with the delay-dependent stabilization problem for singular Markovian jump systems with time delays. Different with the results of [13], the considered system here is not assumed to be necessarily regular and impulse free. In terms of a set of LMIs, we present a delay-dependent sufficient condition which guarantees the

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regularity, absence of impulses, and stochastic stability of such systems. Based on this, a strict LMI-based approach is proposed to solve the delay-dependent stabilization problem and the desired state feedback controllers can be constructed by solving a set of strict LMIs, which can be easily solved using the LMI control toolbox. Several numerical examples are provided to demonstrate the effectiveness and applicability of the established results.

Notations $C_{n,d} = C([-d,0],\mathbb{R}^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-d,0]$ into $\mathbb{R}^n$. $\|\phi(s)\|_d = \sup\limits_{-d\leqslant s\leqslant 0}\|\phi(s)\|$ stands for the norm of a function $\phi(t)\in C_{n,d}$. $(\Omega,\mathcal{F},\mathcal{P})$ is a probability space, Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\mathcal{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$.

2 Problem formulation and preliminaries

Fix a probability space $(\Omega,\mathcal{F},\mathcal{P})$ and consider the following singular time-delay systems with Markovian jump parameters:

$$\begin{cases} E\dot{x}(t) = A(r_t)x(t) + A_d(r_t)x(t-d) + B(r_t)u(t), \\ x(t) = \phi(t),\ \ t\in[-\bar{d},0], \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state, $u(t)\in\mathbb{R}^m$ is the control input. d is an unknown but constant delay satisfying $0\leqslant d\leqslant\bar{d}$, $\phi(t)\in C_{n,\bar{d}}$ is a compatible vector valued initial function. The matrix $E\in\mathbb{R}^{n\times n}$ may be singular and it is assumed that $\operatorname{rank} E = r\leqslant n$, $A(r_t)$, $A_d(r_t)$ and $B(r_t)$ are known real constant matrices with appropriate dimensions. $\{r_t\}$ is a continuous-time Markovian process with right continuous trajectories and taking values in a finite set $\mathcal{S}=\{1,2,\cdots,s\}$ with transition probability matrix $\Pi \triangleq \{\pi_{ij}\}$ given by

$$\Pr\{r_{t+h}=j\,|\,r_t=i\} = \begin{cases} \pi_{ij}h + o(h), & j\neq i, \\ 1+\pi_{ii}h + o(h), & j=i, \end{cases}$$

where $h>0$, $\lim\limits_{h\to 0}\dfrac{o(h)}{h}=0$, and $\pi_{ij}\geqslant 0$, for $j\neq i$, is the transition rate from mode i at time t to mode j at time $t+h$ and $\pi_{ii} = -\sum\limits_{j=1,j\neq i}^{s}\pi_{ij}$.

For notational simplicity, in the sequel, for each possible $r_t = i$, $i\in\mathcal{S}$, a matrix $M(r_t)$ will be denoted by M_i; for example, $A(r_t)$ is denoted by A_i, $A_d(r_t)$ by A_{di} and so on.

Definition 1 1) For a given scalar $\bar{d}>0$, the nominal singular Markovian jump time-delay system

$$\begin{cases} E\dot{x}(t) = A_i x(t) + A_{di}x(t-d), \\ x(t)=\phi(t),\ t\in[-\bar{d},0] \end{cases} \tag{2}$$

is said to be regular and impulse free for any constant time delay d satisfying $0\leqslant d\leqslant\bar{d}$, if the pairs (E,A_i) and (E,A_i+A_{di}) are regular and impulse free for every $i\in\mathcal{S}$.

2) The singular Markovian jump time-delay system (2) is said to be stochastically stable, if there exists a scalar $M(r_0,\phi(\cdot))$ such that

$$\lim_{T\to\infty}\mathcal{E}\{\int_0^T\|x(t)\|^2\mathrm{d}t|r_0,x(s)=\phi(s)\}\leqslant M(r_0,\phi(\cdot)). \tag{3}$$

Our goal in this paper is, for a given scalar $\bar{d}>0$, to design a state feedback controller $u(t)=K_ix(t)$, where K_i is a design parameter that has to be determined for every $i\in\mathcal{S}$, guaranteeing that the resultant closed-loop system is regular, impulse free and stochastically stable for any constant time delay d satisfying $0\leqslant d\leqslant\bar{d}$.

3 Main results

In this section, the LMI method is used to solve the delay-dependent stabilization problem for the singular Markovian jump time-delay system (1). Initially, a delay-dependent condition is proposed for the singular Markovian jump time-delay system (2) to be regular, impulse free, and delay-dependent stochastically stable.

Theorem 1 For a prescribed scalar $\bar{d}>0$, the singular Markovian jump time-delay system (2) is regular, impulse free, and stochastically stable for any constant time delay d satisfying $0\leqslant d\leqslant\bar{d}$, if there exist symmetric positive-definite matrices Q_i, Q, Z and matrices P_i, M_i, N_i, H_i, S_i, R_i and T_i such that for every $i\in\mathcal{S}$,

$$E^{\mathrm{T}}P_i = P_i^{\mathrm{T}}E\geqslant 0, \tag{4a}$$

$$\Xi_i = \begin{bmatrix} \Xi_{i11} & \Xi_{i12} & \Xi_{i13} & \bar{d}S_i \\ * & \Xi_{i22} & \Xi_{i23} & \bar{d}R_i \\ * & * & \Xi_{i33} & \bar{d}T_i \\ * & * & * & -\bar{d}Z \end{bmatrix} < 0, \tag{4b}$$

$$Q_i < Q, \tag{4c}$$

where $\mu = \max\{|\pi_{ii}|, i\in\mathcal{S}\}$ and

$$\begin{aligned}
\Xi_{i11} &= \sum_{j=1}^{s}\pi_{ij}E^{\mathrm{T}}P_j + M_i^{\mathrm{T}}A_i + A_i^{\mathrm{T}}M_i + S_iE + E^{\mathrm{T}}S_i^{\mathrm{T}} \\
&\quad +Q_i + \mu\bar{d}Q, \\
\Xi_{i12} &= P_i^{\mathrm{T}} - M_i^{\mathrm{T}} + A_i^{\mathrm{T}}N_i + E^{\mathrm{T}}R_i^{\mathrm{T}}, \\
\Xi_{i13} &= -S_iE + M_i^{\mathrm{T}}A_{di} + E^{\mathrm{T}}T_i^{\mathrm{T}} + A_i^{\mathrm{T}}H_i, \\
\Xi_{i22} &= -N_i - N_i^{\mathrm{T}} + \bar{d}Z, \\
\Xi_{i23} &= -R_iE + N_i^{\mathrm{T}}A_{di} - H_i, \\
\Xi_{i33} &= -Q_i - T_iE - E^{\mathrm{T}}T_i^{\mathrm{T}} + H_i^{\mathrm{T}}A_{di} + A_{di}^{\mathrm{T}}H_i.
\end{aligned}$$

Proof From (4), it is easy to show that for every $i\in\mathcal{S}$,

$$\tilde{E}^{\mathrm{T}}\tilde{P}_i = \tilde{P}_i^{\mathrm{T}}\tilde{E}\geqslant 0, \tag{5a}$$

$$\begin{bmatrix} \Delta_{i1} & \Delta_{i2} \\ * & \Delta_{i3} \end{bmatrix} < 0, \tag{5b}$$

where

$$\begin{aligned}
\Delta_{i1} &= \pi_{ii}\tilde{E}^{\mathrm{T}}\tilde{P}_i + \tilde{A}_i^{\mathrm{T}}\tilde{P}_i + \tilde{P}_i^{\mathrm{T}}\tilde{A}_i + \tilde{Q}_i + \tilde{S}_i\tilde{E} + \tilde{E}^{\mathrm{T}}\tilde{S}_i^{\mathrm{T}}, \\
\Delta_{i2} &= \tilde{P}_i^{\mathrm{T}}\tilde{A}_{di} - \tilde{S}_i\tilde{E} + \tilde{E}^{\mathrm{T}}\tilde{T}_i^{\mathrm{T}} + \tilde{A}_i^{\mathrm{T}}\tilde{H}_i, \\
\Delta_{i3} &= -\tilde{Q}_i - \tilde{E}^{\mathrm{T}}\tilde{T}_i^{\mathrm{T}} - \tilde{T}_i\tilde{E} + \tilde{A}_{di}^{\mathrm{T}}\tilde{H}_i + \tilde{H}_i^{\mathrm{T}}\tilde{A}_{di},
\end{aligned}$$

$$\tilde{E} = \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix},\ \tilde{A}_i = \begin{bmatrix} 0 & I \\ A_i & -I \end{bmatrix},\ \tilde{A}_{di} = \begin{bmatrix} 0 & 0 \\ A_{di} & 0 \end{bmatrix},$$

$$\tilde{P}_i = \begin{bmatrix} P_i & 0 \\ M_i & N_i \end{bmatrix},\ \tilde{S}_i = \begin{bmatrix} S_i & 0 \\ R_i & 0 \end{bmatrix},\ \tilde{T}_i = \begin{bmatrix} T_i & 0 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{H}_i = \begin{bmatrix} 0 & 0 \\ H_i & 0 \end{bmatrix},\ \tilde{Q}_i = \begin{bmatrix} Q_i & 0 \\ 0 & \bar{d}Z \end{bmatrix}.$$

Since $\operatorname{rank}\tilde{E} = \operatorname{rank}E = r\leqslant n$, there exist nonsingular matrices G and H such that

$$\hat{E} = G\tilde{E}H = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \tag{6}$$

Denote

$$\begin{cases} G\tilde{A}_i H = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}, \ G^{-\mathrm{T}}\tilde{P}_i H = \begin{bmatrix} P_{i11} & P_{i12} \\ P_{i21} & P_{i22} \end{bmatrix}, \\ H^{\mathrm{T}}\tilde{S}_i G^{-1} = \begin{bmatrix} S_{i11} & S_{i12} \\ S_{i21} & S_{i22} \end{bmatrix} \end{cases} \quad (7)$$

for every $i \in \mathcal{S}$. By (5a), it can be shown that $P_{i12} = 0$ for every $i \in \mathcal{S}$. Pre-multiplying and post-multiplying $\Delta_{i1} < 0$ by H^{T} and H, respectively, we have $A_{i22}^{\mathrm{T}} P_{i22} + P_{i22}^{\mathrm{T}} A_{i22} < 0$, which implies A_{i22} is nonsingular for every $i \in \mathcal{S}$ and thus the pair $(\tilde{E}, \tilde{A}_i)$ is regular and impulse free for every $i \in \mathcal{S}$. Since $\det(sE - A_i) = \det(s\tilde{E} - \tilde{A}_i)$, we can easily see that the pair (E, A_i) is regular and impulse free for every $i \in \mathcal{S}$. Now, pre-multiplying and post-multiplying (5b) by $\begin{bmatrix} I & I \end{bmatrix}$ and $\begin{bmatrix} I & I \end{bmatrix}^{\mathrm{T}}$, respectively, we can obtain

$$\pi_{ii}\tilde{E}^{\mathrm{T}}(\tilde{P}_i + \tilde{H}_i) + (\tilde{A}_i + \tilde{A}_{di})^{\mathrm{T}}(\tilde{P}_i + \tilde{H}_i) + (\tilde{P}_i + \tilde{H}_i)^{\mathrm{T}}(\tilde{A}_i + \tilde{A}_{di}) < 0.$$

Using the above approach, we can get the above LMI and

$$\tilde{E}^{\mathrm{T}}(\tilde{P}_i + \tilde{H}_i) = (\tilde{P}_i + \tilde{H}_i)^{\mathrm{T}}\tilde{E} \geqslant 0 \quad (8)$$

implies that, for every $i \in \mathcal{S}$, the pair $(E, A_i + A_{di})$ is regular and impulse free, and matrices P_i is nonsingular for every $i \in \mathcal{S}$. According to Definition 1, the singular Markovian jump time-delay system (2) is regular and impulse free for any constant time delay d satisfying $0 \leqslant d \leqslant \bar{d}$.

Next, we will show the stochastic stability of system (2). Define a new process $\{(x_t, r_t), t \geqslant 0\}$ by $\{x_t = x(t+\theta), -2d \leqslant \theta \leqslant 0\}$, then $\{(x_t, r_t), t \geqslant d\}$ is a Markov process with initial state $(\phi(\cdot), r_0)$. Now, for $t \geqslant d$, define the following stochastic Lyapunov candidate for system (2):

$$V(x_t, r_t, t) = \sum_{k=1}^{4} V_k(x_t, r_t, t), \quad (9)$$

where

$$\begin{aligned} V_1(x_t, r_t, t) &= x(t)^{\mathrm{T}} E^{\mathrm{T}} P(r_t) x(t), \\ V_2(x_t, r_t, t) &= \int_{t-d}^{t} x(\alpha)^{\mathrm{T}} Q(r_t) x(\alpha) \mathrm{d}\alpha, \\ V_3(x_t, r_t, t) &= \int_{-d}^{0}\int_{t+\beta}^{t} \dot{x}(\alpha)^{\mathrm{T}} E^{\mathrm{T}} Z E \dot{x}(\alpha) \mathrm{d}\alpha \mathrm{d}\beta, \\ V_4(x_t, r_t, t) &= \mu \int_{-d}^{0}\int_{t+\beta}^{t} x(\alpha)^{\mathrm{T}} Q x(\alpha) \mathrm{d}\alpha \mathrm{d}\beta. \end{aligned}$$

Let $\mathcal{A}$ be the weak infinitesimal generator of the random process $\{x_t, r_t\}$. Then, for each $i \in \mathcal{S}$, we have

$$\begin{aligned} &\mathcal{A}V(x_t, i, t) \\ &\leqslant x(t)^{\mathrm{T}} E^{\mathrm{T}} P_i \dot{x}(t) - x(t-d)^{\mathrm{T}} Q_i x(t-d) \\ &\quad + x(t)^{\mathrm{T}} \{\sum_{j=1}^{s} \pi_{ij} E^{\mathrm{T}} P_j\} x(t) + x(t)^{\mathrm{T}} Q_i x(t) \\ &\quad + \int_{t-d}^{t} x(\alpha)^{\mathrm{T}} \{\sum_{j=1}^{s} \pi_{ij} Q_j\} x(\alpha) \mathrm{d}\alpha \\ &\quad + \bar{d}\dot{x}(t)^{\mathrm{T}} E^{\mathrm{T}} Z E \dot{x}(t) - \int_{t-d}^{t} \dot{x}(\alpha)^{\mathrm{T}} E^{\mathrm{T}} Z E \dot{x}(\alpha) \mathrm{d}\alpha \\ &\quad + \mu\bar{d} x(t)^{\mathrm{T}} Q x(t) - \mu \int_{t-d}^{t} x(\alpha)^{\mathrm{T}} Q x(\alpha) \mathrm{d}\alpha \\ &\quad \times [x(t)^{\mathrm{T}} S_i + (E\dot{x}(t))^{\mathrm{T}} R_i + x(t-d)^{\mathrm{T}} T_i] \\ &\quad \times [Ex(t) - Ex(t-d) - \int_{t-d}^{t} E\dot{x}(\alpha) \mathrm{d}\alpha] \\ &\quad \times [x(t)^{\mathrm{T}} M_i^{\mathrm{T}} + (E\dot{x}(t))^{\mathrm{T}} N_i^{\mathrm{T}} + x(t-d)^{\mathrm{T}} H_i^{\mathrm{T}}] \\ &\quad \times [-E\dot{x}(t) + A_i x(t) + A_{di} x(t-d)]. \end{aligned}$$

According to Jensen integral inequality [14], one can obtain

$$-\int_{t-d}^{t} \dot{x}(\alpha)^{\mathrm{T}} E^{\mathrm{T}} Z E \dot{x}(\alpha) \mathrm{d}\alpha \leqslant \zeta(t)^{\mathrm{T}} (-\bar{d}Z) \zeta(t), \quad (10)$$

where $\zeta(t) = -\int_{t-d}^{t} \frac{1}{\bar{d}} E\dot{x}(\alpha) \mathrm{d}\alpha$. Noting $Q_i < Q$, $\pi_{ij} > 0$ for $i \neq j$ and $-\mu \leqslant \pi_{ii} < 0$, we have

$$\begin{aligned} &\int_{t-d}^{t} x(\alpha)^{\mathrm{T}} \{\sum_{j=1}^{s} \pi_{ij} Q_j\} x(\alpha) \mathrm{d}\alpha \\ &\leqslant \int_{t-d}^{t} x(\alpha)^{\mathrm{T}} \{\sum_{j=1, j\neq i}^{s} \pi_{ij} Q\} x(\alpha) \mathrm{d}\alpha \\ &= -\pi_{ii} \int_{t-d}^{t} x(\alpha)^{\mathrm{T}} Q x(\alpha) \mathrm{d}\alpha \leqslant \mu \int_{t-d}^{t} x(\alpha)^{\mathrm{T}} Q x(\alpha) \mathrm{d}\alpha. \end{aligned}$$

Hence, we have that, for every $i \in \mathcal{S}$,

$$\mathcal{A}V(x_t, i, t) \leqslant \eta(t)^{\mathrm{T}} \Xi_i \eta(t), \quad (11)$$

where

$$\eta(t) = \begin{bmatrix} x(t) \\ E\dot{x}(t) \\ x(t-d) \\ \zeta(t) \end{bmatrix}.$$

From (4b), it is easy to see that there exits a scalar $\lambda > 0$ such that for every $i \in \mathcal{S}$, $\mathcal{A}V(x_t, i, t) \leqslant -\lambda\|x(t)\|^2$. Therefore, for any $t \geqslant d$, by Dynkin's formula, we get $\mathcal{E}V(x_t, i, t) - \mathcal{E}V(x_d, r_d, d) \leqslant -\lambda\mathcal{E}\int_{d}^{t} \|x(s)\|^2 \mathrm{d}s$, which yields

$$\mathcal{E}\int_{d}^{t} \|x(s)\|^2 \mathrm{d}s \leqslant \lambda^{-1} \mathcal{E}V(x_d, r_d, d). \quad (12)$$

Because of the regularity and the non-impulsiveness of the pair (E, A_i) for every $i \in \mathcal{S}$, we can choose two nonsingular matrices M and N such that

$$MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \ MA_iN = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix},$$

where A_{i4} is nonsingular for every $i \in \mathcal{S}$. Set

$$\hat{M} = \begin{bmatrix} I_r & -A_{i2}A_{i4}^{-1} \\ 0 & A_{i4}^{-1} \end{bmatrix} M.$$

It is easy to get

$$\hat{M}EN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \ \hat{M}A_iN = \begin{bmatrix} \hat{A}_{i1} & 0 \\ \hat{A}_{i3} & I \end{bmatrix},$$

where $\hat{A}_{i1} = A_{i1} - A_{i2}A_{i4}^{-1}A_{i3}$ and $\hat{A}_{i3} = A_{i4}^{-1}A_{i3}$. Denote

$$\hat{M}A_{di}N = \begin{bmatrix} A_{id1} & A_{id2} \\ A_{id3} & A_{id4} \end{bmatrix}.$$

Then, for every $i \in \mathcal{S}$, system (2) is restricted system equivalent to

$$\begin{cases} \dot{\zeta}_1(t) = \hat{A}_{i1}\zeta_1(t) + A_{id1}\zeta_1(t-d) + A_{id2}\zeta_2(t-d), \\ -\zeta_2(t) = \hat{A}_{i13}\zeta_1(t) + A_{id3}\zeta_1(t-d) + A_{id4}\zeta_2(t-d), \\ \psi(t) = N^{-1}\phi(t), \ t \in [-\bar{d}, 0], \end{cases} \quad (13)$$

where

$$\zeta(t) = \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} = N^{-1}x(t).$$

For any $t \geqslant 0$, it follows from (13) that

$$\begin{aligned}\|\zeta_1(t)\| &= \|\zeta_1(0) + \int_0^t [\hat{A}_{i1}\zeta_1(\alpha) + A_{id1}\zeta_1(\alpha-d) \\ &\quad + A_{id2}\zeta_2(\alpha-d)]\mathrm{d}\alpha\| \\ &\leqslant \|\zeta_1(0)\| + k_1\int_0^t [\|\zeta_1(\alpha)\| + \|\zeta_1(\alpha-d)\| \\ &\quad + \|\zeta_2(\alpha-d)\|]\mathrm{d}\alpha, \end{aligned} \tag{14}$$

where $k_1 = \max_{i\in\mathcal{S}}\{\|\hat{A}_{i1}\|, \|A_{id1}\|, \|A_{id2}\|\} \geqslant 0$. Then, for any $0 \leqslant t \leqslant d$, we have

$$\|\zeta_1(t)\| \leqslant (2k_1\bar{d}+1)\|\psi\|_{\bar{d}} + k_1\int_0^t \|\zeta_1(\alpha)\|\mathrm{d}\alpha. \tag{15}$$

Applying the Gronwall-Bellman lemma, we obtain from (15) that for any $0 \leqslant t \leqslant d$, $\|\zeta_1(t)\| \leqslant (2k_1\bar{d}+1)\|\psi\|_{\bar{d}}\,\mathrm{e}^{k_1\bar{d}}$. Thus

$$\sup_{0\leqslant\alpha\leqslant d} \|\zeta_1(\alpha)\|^2 \leqslant (2k_1\bar{d}+1)^2\|\psi\|_{\bar{d}}^2\,\mathrm{e}^{2k_1\bar{d}}. \tag{16}$$

We have from (13) that

$$\sup_{0\leqslant\alpha\leqslant d} \|\zeta_2(\alpha)\|^2 \leqslant k_2^2[(2k_1\bar{d}+1)\,\mathrm{e}^{k_1\bar{d}} + 2]^2\|\psi\|_{\bar{d}}^2, \tag{17}$$

where $k_2 = \max_{i\in\mathcal{S}}\{\|\hat{A}_{i3}\|, \|A_{id3}\|, \|A_{id4}\|\}$. Hence, there exists a scalar $k_3 > 0$ such that $\sup_{0\leqslant\alpha\leqslant d}\|\zeta(\alpha)\|^2 \leqslant k_3\|\psi\|_{\bar{d}}^2$. Therefore,

$$\sup_{0\leqslant\alpha\leqslant d} \|x(\alpha)\|^2 \leqslant k_3\|N\|^2\|\|N^{-1}\|^2\|\phi\|_{\bar{d}}^2. \tag{18}$$

Then we have from (9), (13) and (18) that there exits a scalar ϱ such that $V(x_d, r_d, d) \leqslant \varrho\|\phi\|_{\bar{d}}^2$. This together with (12) and (18) implies there exits a scalar ρ such that

$$\begin{aligned}&\mathcal{E}\int_0^t \|x(s)\|^2\mathrm{d}s \\ &= \mathcal{E}\{\int_0^d \|x(s)\|^2\mathrm{d}s\} + \mathcal{E}\{\int_d^t \|x(s)\|^2\mathrm{d}s\} \leqslant \rho\mathcal{E}\|\phi\|_{\bar{d}}^2.\end{aligned}$$

Considering this and Definition 1, system (2) is stochastically stable for any constant time delay d satisfying $0 \leqslant d \leqslant \bar{d}$. This completes the proof.

Remark 1 A delay-dependent condition is given in Theorem 1 for the singular Markovian jump time-delay system to be regular, impulse free and stochastically stable. The condition here is different from the result of [13], where the considered system was assumed to be regular and impulse free. Hence, Theorem 1 is much more desirable and elegant than the result of [13].

Remark 2 In Theorem 1, the free weighting matrix approach is extended to singular Markovian jump time-delay systems. Such method has been widely applied to deal with the delay-dependent related problem to state-space time-delay systems; see [15∼17] for the continuous case and [18, 19] for the discrete case.

Theorem 2 For a prescribed scalar $\bar{d} > 0$, the singular Markovian jump time-delay system (1) controlled by $u(t) = V_iL_i^{-1}x(t)$ is regular, impulse free and stochastically stable for any constant time delay d satisfying $0 \leqslant d \leqslant \bar{d}$, if there exist symmetric positive-definite matrices $\bar{Q}_i$, $\bar{Q}$, $\bar{Z}$, and matrices L_i, V_i, U_i, G_i, F_i, $\mathcal{S}_i$, $\mathcal{T}_i$, $\mathcal{R}_i$ such that for every $i \in \mathcal{S}$,

$$EL_i = L_i^{\mathrm{T}}E^{\mathrm{T}} \geqslant 0, \tag{19a}$$

$$\begin{bmatrix} \mathcal{V}_{i11} & \mathcal{V}_{i12} & \mathcal{V}_{i13} & \bar{d}\mathcal{S}_i & \mu\bar{d}L_i^{\mathrm{T}} & \bar{d}U_i^{\mathrm{T}} \\ * & \mathcal{V}_{i22} & \mathcal{V}_{i23} & \bar{d}\mathcal{T}_i & 0 & \bar{d}G_i^{\mathrm{T}} \\ * & * & -F_i-F_i^{\mathrm{T}} & \bar{d}\mathcal{R}_i & 0 & \bar{d}F_i^{\mathrm{T}} \\ * & * & * & \mathcal{V}_{i44} & 0 & 0 \\ * & * & * & * & -\mu\bar{d}\bar{Q} & 0 \\ * & * & * & * & 0 & -\bar{d}\bar{Z} \end{bmatrix} < 0, \tag{19b}$$

$$\bar{Q}_i + \bar{Q} - L_i - L_i^{\mathrm{T}} < 0, \tag{19c}$$

where

$$\begin{aligned}\mathcal{V}_{i11} &= U_i + U_i^{\mathrm{T}} + \bar{Q}_i + \mathcal{S}_iE^{\mathrm{T}} + E\mathcal{S}_i^{\mathrm{T}} + \pi_{ii}L_i^{\mathrm{T}}E^{\mathrm{T}} \\ &\quad + \sum_{j=1,j\neq i}^{s}\pi_{ij}L_i^{\mathrm{T}}E^{\mathrm{T}}L_j^{-1}L_i, \\ \mathcal{V}_{i12} &= G_i + E\mathcal{T}_i^{\mathrm{T}} - \mathcal{S}_iE^{\mathrm{T}}, \\ \mathcal{V}_{i13} &= F_i + L_i^{\mathrm{T}}A_i^{\mathrm{T}} + V_i^{\mathrm{T}}B_i^{\mathrm{T}} - U_i^{\mathrm{T}} + E\mathcal{R}_i^{\mathrm{T}}, \\ \mathcal{V}_{i22} &= -\bar{Q}_i - \mathcal{T}_iE^{\mathrm{T}} - \mathcal{T}_iE^{\mathrm{T}}, \\ \mathcal{V}_{i23} &= L_i^{\mathrm{T}}A_{di}^{\mathrm{T}} - G_i^{\mathrm{T}} - E\mathcal{R}_i^{\mathrm{T}}, \\ \mathcal{V}_{i44} &= \bar{d}\bar{Z} - \bar{d}L_i - \bar{d}L_i^{\mathrm{T}}.\end{aligned}$$

Proof Plugging the controller $u(t) = K_ix(t)$ in the system (1) gives

$$E\dot{x}(t) = A_{ki}x(t) + A_{di}x(t-d), \tag{20}$$

where $A_{ki} = A_i + B_iK_i$. Using Theorem 1 to this system and replacing A_i in (4b) by A_{ki}. Taking this into account, the condition $\varXi_i < 0$ is equivalent to

$$\mathcal{I}\varXi_i\mathcal{I} = \varTheta_i^{\mathrm{T}}\varOmega_i + \varOmega_i^{\mathrm{T}}\varTheta_i + \mathcal{G}_i\mathcal{X}^{\mathrm{T}} + \mathcal{X}\mathcal{G}_i^{\mathrm{T}} + \mathcal{Q}_i < 0, \tag{21}$$

where

$$\mathcal{I} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix},\ \varTheta_i = \begin{bmatrix} P_i & 0 & 0 & 0 \\ 0 & P_i & 0 & 0 \\ M_i & H_i & N_i & 0 \\ 0 & 0 & 0 & P_i^{\mathrm{T}} \end{bmatrix},$$

$$\varOmega_i = \begin{bmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \\ A_{ki} & A_{di} & -I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ \mathcal{G}_i = \begin{bmatrix} S_i \\ T_i \\ R_i \\ 0 \end{bmatrix},\ \mathcal{X} = \begin{bmatrix} E^{\mathrm{T}} \\ -E^{\mathrm{T}} \\ 0 \\ \bar{d}I \end{bmatrix},$$

$$\mathcal{Q}_i = \begin{bmatrix} \sum_{j=1}^{s}\pi_{ij}E^{\mathrm{T}}P_j + Q_i + \mu\bar{d}Q & 0 & 0 & 0 \\ 0 & -Q_i & 0 & 0 \\ 0 & 0 & \bar{d}Z & 0 \\ 0 & 0 & 0 & -\bar{d}Z \end{bmatrix}.$$

From Theorem 1, P_i and N_i are nonsingular for every $i \in \mathcal{S}$. Define

$$\varTheta_i^{-1} = \begin{bmatrix} P_i & 0 & 0 & 0 \\ 0 & P_i & 0 & 0 \\ M_i & H_i & N_i & 0 \\ 0 & 0 & 0 & P_i^{\mathrm{T}} \end{bmatrix}^{-1} = \begin{bmatrix} L_i & 0 & 0 & 0 \\ 0 & L_i & 0 & 0 \\ U_i & G_i & F_i & 0 \\ 0 & 0 & 0 & L_i^{\mathrm{T}} \end{bmatrix}.$$

Pre- and post-multiplying (4a) by L_i^{T} and L_i, respectively, we get (19a). Similarly, pre- and post-multiply (21) by $\varTheta_i^{-\mathrm{T}}$ and $\varTheta_i^{-1}$, respectively, and pre- and post-multiply (4c) by L_i^{T} and L_i, respectively, and introduce change of variables such that $\bar{Q}_i = L_i^{\mathrm{T}}Q_i^{-1}L_i$, $\bar{Z} = Z^{-1}$, $\bar{Q} = Q^{-1}$, $V_i = K_iL_i$, and $\left[\mathcal{S}_i^{\mathrm{T}}\ \mathcal{T}_i^{\mathrm{T}}\ \mathcal{R}_i^{\mathrm{T}}\ 0\right]^{\mathrm{T}} = \varTheta_i^{-\mathrm{T}}\mathcal{G}_iL_i^{\mathrm{T}}$. After some manipulation including Schur complement and Lemma 2 of [13], and then we can obtain (19b) and (19c). This completes the proof.

It is noted that the conditions in Theorem 2 are no longer LMI conditions because of the term $\sum_{j=1,j\neq i}^{s} \pi_{ij} L_i^{\mathrm{T}} E^{\mathrm{T}} L_j^{-1} L_i$. In order to obtain an LMI-based design method of the desired state feedback controller, without loss of generality, in the following discussion we assume that $E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$. Then (19a) implies $L_i = \begin{bmatrix} L_{i1} & 0 \\ L_{i2} & L_{i3} \end{bmatrix}$, where $L_{i1} > 0$. After some manipulation, we find

$$\sum_{j=1,j\neq i}^{s} \pi_{ij} L_i^{\mathrm{T}} E^{\mathrm{T}} L_j^{-1} L_i = \sum_{j=1,j\neq i}^{s} \pi_{ij} \begin{bmatrix} L_{i1} \\ 0 \end{bmatrix} L_{j1}^{-1} \begin{bmatrix} L_{i1} & 0 \end{bmatrix}.$$

Now, applying Schur complement to (19b), we get the following result.

Theorem 3 For a prescribed scalar $\bar{d} > 0$, the singular Markovian jump time-delay system (1) controlled by $u(t) = V_i L_i^{-1} x(t)$ is regular, impulse free and stochastically stable for any constant time delay d satisfying $0 \leqslant d \leqslant \bar{d}$, if there exist symmetric positive-definite matrices $\bar{Q}_i$, $\bar{Q}$, $\bar{Z}$, L_{i1} and matrices $L_i = \begin{bmatrix} L_{i1} & 0 \\ L_{i2} & L_{i3} \end{bmatrix}$, V_i, U_i, G_i, F_i, $\mathcal{S}_i$, $\mathcal{T}_i$, $\mathcal{R}_i$ such that for every $i \in \mathcal{S}$, (19c) and (22) hold,

$$\begin{bmatrix} \mathcal{V}_{i11} & \mathcal{V}_{i12} & \mathcal{V}_{i13} & \bar{d}\mathcal{S}_i & \mu\bar{d}L_i^{\mathrm{T}} & \bar{d}U_i^{\mathrm{T}} & \mathcal{Z}_i \\ * & \mathcal{V}_{i22} & \mathcal{V}_{i23} & \bar{d}\mathcal{T}_i & 0 & \bar{d}G_i^{\mathrm{T}} & 0 \\ * & * & -F_i - F_i^{\mathrm{T}} & \bar{d}\mathcal{R}_i & 0 & \bar{d}F_i^{\mathrm{T}} & 0 \\ * & * & * & \mathcal{V}_{i44} & 0 & 0 & 0 \\ * & * & * & * & -\mu\bar{d}\bar{Q} & 0 & 0 \\ * & * & * & * & 0 & -\bar{d}\bar{Z} & 0 \\ * & * & * & * & 0 & 0 & -\mathcal{F}_i \end{bmatrix} < 0, \quad (22)$$

where $\mathcal{V}_{i12}$, $\mathcal{V}_{i13}$, $\mathcal{V}_{i22}$, $\mathcal{V}_{i23}$ and $\mathcal{V}_{i44}$ follow the same definitions as those in Theorem 2 and

$$\mathcal{V}_{i11} = U_i + U_i^{\mathrm{T}} + \bar{Q}_i + \mathcal{S}_i E^{\mathrm{T}} + E\mathcal{S}_i^{\mathrm{T}} + \pi_{ii} L_i^{\mathrm{T}} E^{\mathrm{T}},$$

$$\mathcal{Z}_i = \Bigg[\sqrt{\pi_{i1}} \begin{bmatrix} L_{i1} \\ 0 \end{bmatrix} \cdots \sqrt{\pi_{i(i-1)}} \begin{bmatrix} L_{i1} \\ 0 \end{bmatrix} \sqrt{\pi_{i(i+1)}} \begin{bmatrix} L_{i1} \\ 0 \end{bmatrix} \cdots \sqrt{\pi_{is}} \begin{bmatrix} L_{i1} \\ 0 \end{bmatrix} \Bigg],$$

$$\mathcal{F}_i = \mathrm{diag}\left\{ L_{11} \cdots L_{(i-1)1}\ L_{(i+1)1} \cdots L_{s1} \right\}.$$

Remark 3 Without any additional assumption on the regularity, absence of impulse of the considered system, Theorem 3 provides a sufficient condition for the solvability of the delay-dependent stabilization problem for singular Markovian jump time-delay system in terms of a set of strict LMIs. Moreover, if (19c) and (22) are feasible, it follows from $\bar{d}\bar{Z} - \bar{d}L_i - \bar{d}L_i^{\mathrm{T}} < 0$ that L_i is nonsingular and thus the desired state feedback gain K can be readily obtained.

From the above theorem, the following result on stochastic stabilization of singular Markovian jump systems without time delay can be easily obtained.

Corollary 1 The singular Markovian jump system

$$E\dot{x}(t) = A(r_t)x(t) + B(r_t)u(t) \quad (23)$$

controlled by $u(t) = V_i L_i^{-1} x(t)$ is regular, impulse free and stochastically stable, if there exist symmetric positive-definite matrix L_{i1} and matrices $L_i = \begin{bmatrix} L_{i1} & 0 \\ L_{i2} & L_{i3} \end{bmatrix}$, V_i, U_i, F_i such that for every $i \in \mathcal{S}$,

$$\begin{bmatrix} U_i + U_i^{\mathrm{T}} + \pi_{ii} L_i^{\mathrm{T}} E^{\mathrm{T}} & \Delta_i & \mathcal{Z}_i \\ * & -F_i - F_i^{\mathrm{T}} & 0 \\ * & * & -\mathcal{F}_i \end{bmatrix} < 0, \quad (24)$$

where $\mathcal{Z}_i$ and $\mathcal{F}_i$ follow the same definitions as those in Theorem 3, and $\Delta_i = F_i + L_i^{\mathrm{T}} A_i^{\mathrm{T}} + V_i^{\mathrm{T}} B_i^{\mathrm{T}} - U_i^{\mathrm{T}}$.

4 Numerical examples

Example 1 Consider Markovian jump time-delay system (2) with $E = I$, two modes and the following parameters

$$A_1 = \begin{bmatrix} -3.5 & 0.8 \\ -0.6 & -3.3 \end{bmatrix},\ A_2 = \begin{bmatrix} -2.5 & 0.3 \\ 1.4 & -0.1 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -0.9 & -1.3 \\ -0.7 & -2.1 \end{bmatrix},\ A_{d2} = \begin{bmatrix} -2.8 & 0.5 \\ -0.8 & -1.0 \end{bmatrix},$$

that is, the singular Markovian jump time-delay system (2) reduces to a regular Markovian jump time-delay system. We suppose $\pi_{22} = -0.8$. For given π_{11}, the maximum $\bar{d}$, which satisfies the LMIs (4), can be calculated by using the LMI toolbox of MATLAB. Table 1 presents the comparison results, which show that the stochastic stability result in Theorem 1 is less conservative than those in [7∼9].

Table 1 Comparison of stability conditions.

π_{11}	−0.40	−0.55	−0.70	−0.85	−1.00
[7∼9]	0.5044	0.5025	0.5010	0.4998	0.4987
Theorem 1	0.5754	0.5718	0.5689	0.5598	0.5405

Example 2 Consider singular Markovian jumping time-delay system (2) with two modes, that is, $\mathcal{S} = \{1, 2\}$. The mode switching is governed by the rate matrix $\begin{bmatrix} -\pi_{11} & \pi_{11} \\ 0.3 & -0.3 \end{bmatrix}$. The system parameters are described as follows:

$$A_1 = \begin{bmatrix} 0.4972 & 0 \\ 0 & -0.9541 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.5121 & 0 \\ 0 & -0.7215 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -1.010 & 1.5415 \\ 0 & 0.5449 \end{bmatrix},\ A_{d2} = \begin{bmatrix} -0.8521 & 1.9721 \\ 0 & 0.4321 \end{bmatrix}.$$

The singular matrix $E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Using Theorem 1, when $\pi_{11} = 0.45$, by using the MATLAB LMI Control Toolbox to solve the LMIs (4), it can be shown that the system is regular, impulse free and stochastically stable for any constant time delay d satisfying $0 \leqslant d \leqslant 1.0918$. Table 2 provides the maximum allowed time-delay $\bar{d}$ for different $\pi_{11} > 0$. However, the result of [13] fails to determine the stochastic stability of the above system.

Table 2 Allowed $\bar{d}$ with different π_{11}.

π_{11}	0.60	0.55	0.50	0.45	0.40	0.35
$\bar{d}$	1.0808	1.0842	1.0878	1.0918	1.0960	1.1006

Example 3 Consider singular Markovian jumping system in (1) with two modes. The system parameters are de-

scribed as follows:

$$A_1=\begin{bmatrix}1.0 & 1.5 & 1\\ -1.2 & 1.0 & 2.0\\ 1.0 & 2.0 & -1.5\end{bmatrix},\ A_2=\begin{bmatrix}0.5 & 1.5 & 0\\ -1.0 & -2.0 & 0.5\\ 1.0 & 1.2 & -1.0\end{bmatrix},$$

$$A_{d1}=\begin{bmatrix}0.5 & 1.5 & 0\\ -1.0 & -2.0 & 0.5\\ 1.0 & 1.2 & -1.0\end{bmatrix},\ A_{d2}=\begin{bmatrix}1.0 & 1.2 & 0.5\\ 1.5 & -0.2 & 1.0\\ -1.0 & 2.0 & 1.0\end{bmatrix},$$

$$B_1=\begin{bmatrix}1.5 & -2.0\\ 1.0 & -1.0\\ 1.0 & 2.0\end{bmatrix},\ B_2=\begin{bmatrix}0.5 & 1.0\\ 1.0 & 0.5\\ 1.0 & 1.5\end{bmatrix},\ E=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix},$$

and $\Pi=\begin{bmatrix}-1 & 1\\ 2 & -2\end{bmatrix}$. Using Theorem 3 to the above system, we are able to find a feasible solution for the set of LMIs (19c) and (22) for any constant time delay d satisfying $0 \leqslant d \leqslant 0.36$. Especially when $d = 0.2$, using MATLAB LMI Control Toolbox to solve the LMIs (19c) and (22), we obtain the delay-dependent state feedback controller, which has the following gains:

$$K_1=\begin{bmatrix}-1.9014 & -2.0720 & -0.7458\\ 0.5671 & -0.5851 & 0.1480\end{bmatrix},$$

$$K_2=\begin{bmatrix}1.0789 & 2.3365 & -1.5397\\ -3.8889 & -4.2162 & 0.3111\end{bmatrix}.$$

Although the results of [13] fail to give the desired state feedback controller gains, the result in this paper improves the existing ones.

5 Conclusions

The problem of delay-dependent stabilization for singular Markovian jump time-delay systems is discussed in terms of LMI approach. A sufficient condition is established such that the considered system is regular, impulse free, and stochastically stable. The desired state feedback controller is designed to assure that the resultant closed-loop system is regular, impulse free and stochastically stable. All the given results are formulated in terms of LMIs, which makes the analysis and design procedure relatively simple and reliable. Numerical examples are given to show the effectiveness and reduced conservatism of the proposed conditions.

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