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A stabilizing model predictive control for networked control system with data packet dropout

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Abstract: A constrained model predictive control (MPC) algorithm for networked control system with data packet dropout is proposed in this paper. A buffer is designed to store the predicted control sequence between controller and actuator. It is shown that if the control horizon of MPC is not less than the number of data packets lost continuously, feasibility of MPC at initial time implies asymptotical stability of the closed-loop system. A simulation example illustrates the effectiveness of the proposed approach.

Keywords: Networked control system; Model predictive control; Stability

1 Introduction

A networked control system (NCS) [1∼2] is a feedback control system with network channels used for the communications between spatially distributed system components like sensors, actuators, and controllers, which have brought forth increasing interest of many researchers in the recent decade. Flexibility, reconfigurability, low installation, and ease of maintenance are some of the advantages of NCSs. However, deterministic or non-deterministic delay and/or data packet dropout often occur in the process of data exchange between NCS components, which degrades the performance and stability of the closed-loop control system, if the controller is designed by traditional approaches.

Model predictive control (MPC) [3] is a control strategy applied to process industries broadly. Its control input is obtained by solving an optimization problem that minimizes a predetermined objective function in each sampling period. Typically, a sequence of predicted control actions will be calculated, but only the first one is implemented. At the next sampling time, the optimization problem is solved again with new measurements, and control input is updated. Because a sequence of predicted control actions can be obtained by MPC at each sample time, these future control actions can also be reserved and applied to the plant to compensate for network-induced time delay or data packet dropout in NCS.

There are some research results on MPC for NCS [4∼9]. In [4], a time-stamped model predictive control algorithm is presented for NCS with random delay less than one sample time. Zhang [5] proposed a segmented time-stamped dynamic matrix control (DMC) algorithm to reduce the online calculation burden. Liu [6] proposed using a networked control predictor to take the latest control value from the predictive control sequence available to deal with random communication time delay problem, and Mu [7] presented an improved version to enhance system robustness by using a low-pass filter to filter the error produced between the delayed plant output measurement and its delayed open-loop model output. Tang [8] proposed an MPC control strategy with the buffering of future control sequence to overcome the transmission delay problems between the controller and the actuator, and Tang [9] provided the conditions under which the stability of the constrained model predictive control for NCS with random delay can be guaranteed.

In this paper, we deal with the design of MPC for NCS with data packet dropout. We firstly present a stabilizing MPC algorithm in which a saturated linear feedback controller is used as local stabilizing controller. In contrast with MPC using linear feedback controller as local stabilizing controller, the proposed MPC has a larger terminal constraint set and region of attraction. Then, an NCS control structure based on the presented MPC algorithm will be given, in which a buffer is designed to reserve predicted control sequence computed by the MPC algorithm between controller and actuator. It will be shown that if the control horizon of MPC is not less than the number of data packets lost continuously, feasibility of the optimization problem of MPC at initial time implies asymptotical stability of the closed-loop system. A simulation example is presented to illustrate the effectiveness of proposed control algorithm.

2 A stabilizing MPC algorithm

Consider the following discrete-time linear system:

$$x(k+1) = Ax(k) + Bu(k),\ \ x \in \mathbb{R}^n,\ \ u \in \mathbb{R}^m, \quad (1)$$

where (A, B) is assumed to be stabilizable. Without loss of generality, input constraint is assumed to be

$$\|u\|_\infty \leqslant 1. \quad (2)$$

The initial state $x(0) = x_0$, and the control objective is to regulate the initial state to the origin. At sample time k, MPC aims at minimizing the following cost function with state and input weighting matrix $Q > 0, R > 0$ of adequate dimensions:

$$J(x(k), U(k))$$

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$$= \sum_{i=0}^{N-1}[x^{\mathrm{T}}(k+i|k)Qx(k+i|k)+u^{\mathrm{T}}(k+i|k)Ru(k+i|k)]+x^{\mathrm{T}}(k+N|k)\Psi x(k+N|k), \quad (3)$$

where $U(k)=[u(k|k),u(k+1|k),\cdots,(k+N-1|k)]^{\mathrm{T}}$, $N>0$ is control horizon and $\Psi>0$ is terminal weighting matrix. Let

$$A_1=\begin{bmatrix}A\\A^2\\\vdots\\A^{N-1}\end{bmatrix} B_1=\begin{bmatrix}B & 0 & \cdots 0\\ AB & B & \cdots 0\\ \vdots & \vdots & \vdots\\ A^{N-1}B & A^{N-2}B & \cdots B\end{bmatrix},$$

$$Q_1=\mathrm{diag}\{Q,\cdots,Q,\Psi\},\quad R_1=\mathrm{diag}\{R,R,\cdots,R\},$$

the cost function (3) can be rewritten as

$$J(x(k),U(k)) = x^{\mathrm{T}}(k)Qx(k)+U^{\mathrm{T}}(k)RU(k)+[A_1x(k)+B_1u(k)]^{\mathrm{T}}Q_1[A_1x(k)+B_1u(k)]. \quad (4)$$

The following three ingredients are necessary to guarantee stability of an MPC algorithm: a local stabilizing controller $K(x)$, a terminal constraint set X_{T}, and a terminal cost function Ψ (see [3] and references therein). Here, we consider a saturated linear feedback controller as the local stabilizing controller:

$$K(x)=\mathrm{sat}(Fx(k)), \quad (5)$$
$$\mathrm{sat}(U)=[\mathrm{sat}(u_1),\mathrm{sat}(u_2),\cdots,\mathrm{sat}(u_m)]^{\mathrm{T}},$$
$$\mathrm{sat}(u_j)=\begin{cases}-1, & u_j<-1,\\ 1, & u_j>1,\\ u_j, & -1\leqslant u_j\leqslant 1.\end{cases}$$

In the following, we consider how to construct a terminal constraint set for saturated feedback controller (5). Let D be the set composed of $m\times m$ diagonal matrices whose diagonal elements are either 1 or 0. Obviously, there are 2^m elements in D. Suppose that each element of D is labeled as $D_i, i=1,2,\cdots,2^m$. Define $D_i^-=I-D_i$. Clearly, D_i^- is also an element of D. Let Co denote the convex hull. Given a matrix H, let h_i denote the ith row of H, and we have the following two lemmas.

Lemma 1 [10] Given $F,H\in\mathbb{R}^{m\times n}$. For $x\in\mathbb{R}^n$, if $|h_ix|\leqslant 1$ for any $i\in[1,m]$, then

$$\mathrm{sat}(Fx)=\mathrm{Co}\{(D_iF+D_i^-H)x: i\in[1,2^m]\}. \quad (6)$$

Lemma 2 [10] Given an ellipsoid $E_P=\{x: x^{\mathrm{T}}Px\leqslant 1, P=P^{\mathrm{T}}>0\}$, consider saturated feedback controller $u=\mathrm{sat}(Fx)$ for the system (1), if there exists an $H\in\mathbb{R}^{m\times n}$ such that $|h_ix|\leqslant 1$ for any $x\in E_P, i\in[1,m]$, and

$$(A+B(D_iF+D_i^-H))^{\mathrm{T}}P(A+B(D_iF+D_i^-H))-P\leqslant 0,\ \forall i\in[1,2^m], \quad (7)$$

then E_P is an invariant set for closed-loop system.

From Lemma 2, the invariant set E_P is an appropriate terminal constraint set of MPC. Let $P=W^{-1}$ and $Z=HW$, the optimization problem of maximizing the area of the ellipsoid E_P can be formulated as

$$\min_{W,Z} -\mathrm{logdet(W)}$$
$$\begin{bmatrix}W & *\\ A+BD_iFW+BD_i^-Z & W\end{bmatrix}\geqslant 0,\quad \begin{bmatrix}1 & *\\ z_j^{\mathrm{T}} & W\end{bmatrix}\geqslant 0, \quad (8)$$

where $i\in[1,2^m], j\in[1,m]$, and '$*$' in the matrix is used to induce the symmetric structure. Thus, a terminal constraint set can be given as

$$E_P=\{x: x^{\mathrm{T}}Px\leqslant 1, P=W^{-1}\}. \quad (9)$$

Furthermore, the terminal weighting matrix Ψ needs to satisfy the following condition [3]:

$$(Ax+B\mathrm{sat}(Fx))^{\mathrm{T}}\Psi(Ax+B\mathrm{sat}(Fx))-x^{\mathrm{T}}\Psi x+x^{\mathrm{T}}Qx+\mathrm{sat}(Fx)^{\mathrm{T}}R\mathrm{sat}(Fx)\leqslant 0,\ \forall x\in E_P. \quad (10)$$

Using (6) and let $Y=\Psi^{-1}$ and $Z=HY$, an appropriate terminal weight matrix $\Psi=Y^{-1}$ can be computed by solving the following SDP problem:

$$\min_{Y,Z} -\mathrm{logdet}(Y)$$
$$\begin{bmatrix}Y & * & * & *\\ A+BD_iFY+BD_i^-Z & Y & * & *\\ Q^{\frac12}Y & 0 & I & *\\ R^{\frac12}(D_iFY+D_i^-Z) & 0 & 0 & I\end{bmatrix}\geqslant 0,\quad \begin{bmatrix}1 & *\\ z_j^{\mathrm{T}} & Y\end{bmatrix}\geqslant 0. \quad (11)$$

After obtaining Ψ and E_P, we can transform the optimization problem of constrained MPC into a second-order cone programming (SOCP) problem:

$$\min_{U(k),\gamma} \gamma \quad (12)$$
$$|u_j(k+i-1|k)|\leqslant 1,\ i\in[1,N],\ j\in[1,m], \quad (13)$$
$$\left\|P^{\frac12}(A^Nx(k)+B_2U(k))\right\|_2\leqslant 1, \quad (14)$$
$$\left\|\begin{bmatrix}2Q^{\frac12}x(k)\\ 2R_1^{\frac12}U(k)\\ 2Q_1^{\frac12}(A_1x(k)+B_1U(k))\\ 1-\gamma\end{bmatrix}\right\|_2\leqslant 1+\gamma, \quad (15)$$

where $B_2=[A^{N-1}B,A^{N-2}B,\cdots,B]$. The inequality (14) denotes $x(k+N|k)\in E_P$, and (15) denotes that γ is upper bound of $J(x(k),U(k))$. Note that when a linear local stabilizing controller is used, the maximal terminal constraint set is the maximal output admissive set O_∞ [11]. The proposed algorithm has a larger terminal constraint set, which can enlarge the region of attraction.

3 NCS control structure and stability

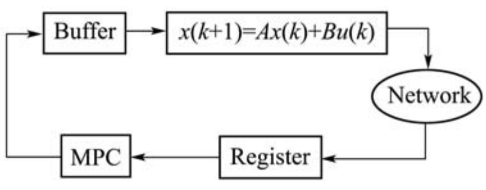

Fig. 1 NCS Control Structure.

Using the above MPC algorithm, the proposed NCS structure is described in Fig.1. We assume that network communication only occurs between the sensor and controller through a communication channel with finite bandwidth. When sensor data are successfully sent to the controller through the communication link, they will be put into a register and replace old data. Then, MPC reads out the content of the register and utilizes the data to compute the new control input. If the register does not receive new data packet, MPC will not work. Between the controller and actuator, a buffer is designed to store the control input $U(k)$ computed by MPC. If data packet dropout does not happen, new control sequence will be obtained to substitute old control sequence in the buffer, and the first control action will be enforced to the actuator, or the control action in the old control sequence corresponding to current sample time will be applied to the plant. For example, assume that the data packet is successfully transmitted to the controller at the sample time $k1$ and $k2$ ($k1 < k2$), and all data packet between $k1$ and $k2$ are lost, the control action applied to the plant from $k1$ to $k2$ is

$$\begin{array}{ccccc} k1 & k1+1 & \cdots & k2-1 & k2 \\ u(k1|k1) & u(k1+1|k1) & \cdots & u(k2-1|k1) & u(k2|k2). \end{array} \tag{16}$$

Without loss of generality, we assume that the data packet at initial time is transmitted to the controller successfully. We also assume that the control horizon N is not less than the number d of data packet lost continuously, which guarantees that there always exists a control action corresponding to current time in buffer when data packet loss occurs continuously. Thus, the proposed MPC algorithm for NCS can be implemented in the following steps:

Off-line: design controller parameter Q, R; design local stabilizing controller $K(x)$ using (5); compute P and Ψ by solving (8) and (11).

Online: when data packet is transmitted successfully, solve the optimization problem (12), and send $U(k)$ to the buffer. Apply control input to the plant in accordance with (16).

We have the following stability theorem for NCS in Fig.1.

Theorem 1 Consider the proposed MPC algorithm for NCS, suppose the control horizon N is not less than the number d of data packet lost continuously and data packet is transmitted successfully at initial time. Feasibility of the optimization problem (12) at initial time $k = 0$ guarantees asymptotic stability of the closed-loop system.

Proof Without loss of generality, assume that the data packet is successfully transmitted to the controller at the sample time $k1$ and $k2$ ($k1 < k2, k2-k1 < N$) and all data packet between $k1$ and $k2$ are lost. Assume the optimization problem is feasible at $k1$ and optimal cost function is

$$\begin{aligned} J^*(k1) = & \sum_{i=0}^{N-1}[x^{*\mathrm{T}}(k1+i|k1)Qx^*(k1+i|k1) \\ & +u^{*\mathrm{T}}(k1+i|k1)Ru^*(k1+i|k1)] \\ & +x^{*\mathrm{T}}(k1+N|k1)\Psi x^*(k1+N|k1). \end{aligned} \tag{17}$$

The related control and state sequences are respectively presented as

$$\begin{aligned} U^*(k1) = & [u^*(k1|k1), u^*(k1+1|k1), \cdots, \\ & u^*(k1+N-1|k1)]^{\mathrm{T}} \end{aligned}$$

and

$$\begin{aligned} X^*(k) = & [x^*(k1|k1), x^*(k1+1|k1), \cdots, \\ & x^*(k1+N|k1)]^{\mathrm{T}}. \end{aligned}$$

In terms of proposed algorithm, the control action $u^*(k1+1|k1), u^*(k1+2|k1), \cdots, u^*(k2-1|k1)$ will be applied to the plant in turn between $k1$ and $k2$. The plant state is $x^*(k2|k1)$ at the sample time $k2$. Because $x^*(k1+N|k1) \in E_P$ and E_P is an invariant set for $u(k) = \mathrm{sat}(Fx(k))$, then a feasible solution to the optimization problem (12) at the sample time $k2$ is

$$\begin{aligned} U(k2) = & [u^*(k2|k1), \cdots, u^*(k1+N-1|k1), \\ & \mathrm{sat}(Fx^*(k1+N|k1)), \mathrm{sat}(Fx^*(k1+N+1|k1)), \\ & \cdots, \mathrm{sat}(Fx^*(k2+N|k1))]^{\mathrm{T}}, \end{aligned}$$

where

$$\begin{aligned} & x^*(k1+N+i|k1) \\ = & Ax^*(k1+N+i-1|k1) + B\mathrm{sat}(Fx^*(k1+N \\ & +i-1|k1)), \quad 1 \leqslant i \leqslant k2-k1. \end{aligned} \tag{18}$$

This shows that if the optimization problem (12) is feasible at $k1$, it is also feasible at $k2$. The cost function corresponds to the feasible solution:

$$\begin{aligned} J(k2) = & \sum_{i=k2-k1}^{N-1}[x^{*\mathrm{T}}(k1+i|k1)Qx^*(k1+i|k1) \\ & +u^{*\mathrm{T}}(k1+i|k1)Ru^*(k1+i|k1)] \\ & + \sum_{i=0}^{k2-k1-1}[x^{*\mathrm{T}}(k1+N+i|k1)Qx^*(k1+N \\ & +i|k1) + u^{*\mathrm{T}}(k1+N+i|k1)Ru^*(k1+N \\ & +i|k1)] + x^{*\mathrm{T}}(k2+N|k1)\Psi x^*(k2+N|k1). \end{aligned} \tag{19}$$

In terms of (10) and (18), for any $0 \leqslant i \leqslant k2-k1-1$,

$$\begin{aligned} & x^{*\mathrm{T}}(k1+N+i|k1)Qx^*(k1+N+i|k1) \\ & +\mathrm{sat}(Fx^{*\mathrm{T}}(k1+N+i|k1))R\mathrm{sat}(Fx^*(k1+N+i|k1)) \\ & \leqslant x^{*\mathrm{T}}(k1+N+i|k1)\Psi x^*(k1+N+i|k1) \\ & \quad -x^{*\mathrm{T}}(k1+N+i-1|k1)\Psi x^*(k1+N+i-1|k1). \end{aligned} \tag{20}$$

Substituting (20) into (19), we have

$$\begin{aligned} J(k2) \leqslant & \sum_{i=k2-k1}^{N-1}[x^{*\mathrm{T}}(k1+i|k1)Qx^*(k1+i|k1) \\ & +u^{*\mathrm{T}}(k1+i|k1)Ru^*(k1+i|k1)] \\ & +x^{*\mathrm{T}}(k1+N|k1)\Psi x^*(k1+N|k1). \end{aligned} \tag{21}$$

Comparing with (17) and (21), we can obtain $J(k2) < J^*(k1)$. Assume the optimal cost function is $J^*(k2)$, we have $J^*(k2) < J(k2)$. Furthermore, $J^*(k2) < J^*(k1)$. This shows convergence of the closed loop system, that is, $x \to 0$ and $u \to 0$ as $k \to \infty$. Therefore, when the optimization problem (12) is feasible at initial time, the cost function $J^*(k)$ can be served as a Lyapunov function for asymptotical stability.

4 Simulation example

Consider the following double-integrator system sampled at a frequency of 2 Hz:

$$x(k+1) = \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 0.125 \end{bmatrix} u(k).$$

Let $Q = I, R = I$, we can obtain an LQR controller $F = [-1.3142, -0.6514]$. Terminal constraint set and terminal weighting matrix are respectively obtained by solving (8) and (11):

$$E_P = \left\{ x : x^{\mathrm{T}}Px \leqslant 1, P = \begin{bmatrix} 0.0807 & 0.0217 \\ 0.0217 & 0.0164 \end{bmatrix} \right\},$$

$$\Psi = \begin{bmatrix} 445.6 & 107.7 \\ 107.7 & 96.7 \end{bmatrix}.$$

The terminal constraint ellipsoid E_P and the maximal output admissible set O_∞ are shown in Fig.2. Obviously, E_P is larger than O_∞, and the proposed MPC has a larger region of attraction. Define p to be the probability of data packet dropout at each sample time. Assume control horizon $N = d$ and initial state $x_0 = [0, -10]$. Fig.3 shows the time responses of the states and the input with $N = d = 5$. Simulation results show that the proposed algorithm guarantees the stability of the closed-loop system and has good control performance. It can also be seen that control input is subject to input constraint (2) and the increase of the p does almost not affect control performance of the proposed algorithm.

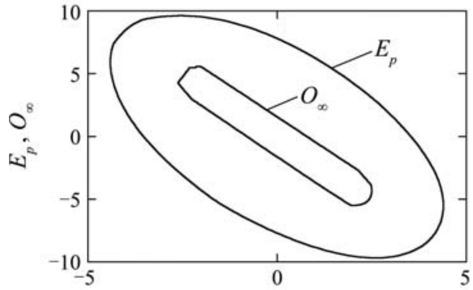

Fig. 2 E_P and O_∞.

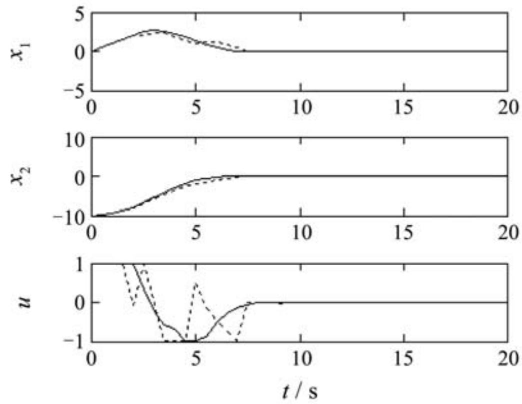

Fig. 3 Time responses of states and input. solid line: $p = 0.1$, dotted line: $p = 0.9$.

5 Conclusions

In this paper, a stabilizing constrained MPC algorithm is proposed for NCS with data packet dropout. The saturated linear feedback controller is used as the local stabilizing controller to enlarge terminal constraint set and region of attraction. A buffer is introduced to reserve the future control action of MPC, which is applied to the plant when packet loss happens. We also presented a sufficient condition for stability of the closed-loop system. Numerical simulation results show the effectiveness of the algorithm.

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