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Robust stabilization for a class of nonlinear networked control systems

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Abstract: The problem of robust stabilization for a class of uncertain networked control systems (NCSs) with nonlinearities satisfying a given sector condition is investigated in this paper. By introducing a new model of NCSs with parameter uncertainty, network-induced delay, nonlinearity and data packet dropout in the transmission, a strict linear matrix inequality (LMI) criterion is proposed for robust stabilization of the uncertain nonlinear NCSs based on the Lyapunov stability theory. The maximum allowable transfer interval (MATI) can be derived by solving the feasibility problem of the corresponding LMI. Some numerical examples are provided to demonstrate the applicability of the proposed algorithm.

Keywords: Networked control systems (NCSs); Robust control; Nonlinearity; Linear matrix inequality (LMI)

1 Introduction

The modern control systems in which sensors, controllers, and plants are often connected over a realtime network medium are called networked control systems (NCSs) [1]. The insertion of the communication network in the feedback control loop of an NCS makes the stability analysis and controller design complex. One of the fundamental issues in an NCS is the effect of network-induced delay that occurs when sensors, actuators and controllers exchange data across the shared network. This delay can degrade the performance of control systems designed without considering it and can even destabilize the system. Many nonlinear physical systems can be represented as a feedback connection of a linear dynamical system and a nonlinear element that satisfies a sector condition [2]. In this paper, the modeling and robust stabilization problems of NCS with inevitable network-induced delay, data packet dropout and nonlinearity are investigated.

The problems of stability analysis and controller design for networked control systems (NCSs) have received much attention in the past decades [3∼5] due to NCSs' low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability [1]. A great number of results on this topic have been reported in [1, 6, 7]. Only the nominal linear NCS with network-induced delay is considered, and a state feedback controller is proposed in [1]. Both state feedback and output feedback controller on model-based control of NCS are presented in [6]. A global exponential stability result for multiple-input-multiple-output (MIMO) NCSs is studied for the first time based on a novel control network protocol, try-once-discard (TOD), in [7]. This approach is also shown to produce stabilizing controllers for linear NCSs in [8]. New results in L_p stability for a large class of protocols in a unified manner are presented based on the classical small gain theorem [5], where a better result of maximum allowable transfer interval (MATI) for the NCSs is provided in [8].

However, only those cases where there are no parameter uncertainties and nonlinearity are considered in the above works. Few results in robust stabilization problems for uncertain or nonlinear NCSs have been presented in the past years [9∼11]. In the discrete context, the state feedback robust stabilization problem for uncertain discrete-time systems via a limited capacity channel is discussed in [10], while in the continuous context, a robust H_∞ controller was designed and the MATI was derived in [11]. For a nonlinear nominal NCS, a two-step design approach is proposed, using standard control methodologies and choosing the network protocol in order to ensure that important closed-loop properties are preserved when a computer network is inserted into the feedback loop [9]. In this paper, we are concerned with the robust stabilization problem for uncertain and nonlinear NCSs, and both network-induced delay and data dropout are considered. We follow the method proposed in [1, 3], where the controller which does not take the network into account is firstly designed, and then in the second step a design parameter MATI is determined such that the closed-loop system remains stable when some control and sensor signals are transmitted via the network. By introducing some slack matrix parameters, an explicit expression of the desired state feedback control law is given, which can be obtained by solving the feasibility problem of a strict linear matrix inequality (LMI). Some numerical examples are given to illustrate the effectiveness of the proposed method.

2 Problem formulation

In an NCS, the controlled plant with parameter uncertainty is a continuous system that can be described as follows:

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$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) + B_w w(t), \\ z(t) = Cx(t), \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $w(t) \in \mathbb{R}^p$, $z(t) \in \mathbb{R}^q$ are the state vector, control input vector, nonlinear input vector and output vector, respectively. A, B, B_w and C are constant matrices with appropriate dimensions. ΔA and ΔB denote real-valued matrix functions representing parameter uncertainties, which are assumed to be $\begin{bmatrix}\Delta A & \Delta B\end{bmatrix} = DF(t)\begin{bmatrix}E_a & E_b\end{bmatrix}$, where D, E_a, E_b are known constant matrices with appropriate dimensions, and $F(t)$ is an unknown matrix with Lebesgue-measurable elements and satisfies

$$F^{\mathrm{T}}(t)F(t) \leqslant I. \quad (2)$$

And the nonlinear feedback conjunction is described as follows

$$w(t) = -\varphi(t, z(t)), \quad (3)$$

where $\varphi(t, z(t)) : [0, \infty) \times \mathbb{R}^q \mapsto \mathbb{R}^p$ is a class of nonlinear functions located in a generic sector $[N_1, N_2]$, that is to say,

$$\left[\varphi(t, z) - N_1 z(t)\right]^{\mathrm{T}} \left[\varphi(t, z) - N_2 z(t)\right] \leqslant 0, \quad \forall t \geqslant 0. \quad (4)$$

The robust stabilization problem to be addressed is to construct a state feedback control law $u(t) = Kx(t)$, $K \in \mathbb{R}^{m\times n}$ that robustly stabilizes system (1) for all admissible parameter uncertainties. Actually, the real input $u(t)$ is a piecewise continuous function. Furthermore, if we consider the effect of the network-induced delay and network packet dropout on the NCSs, the real control system can be modeled as

$$\begin{cases} \dot{x}(t) = \bar{A}x(t) + \bar{B}u(t) + B_w w(t) \\ \qquad = (A + \Delta A)x(t) + (B + \Delta B)u(t) + B_w w(t), \\ t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}); \\ z(t) = Cx(t), \\ u(t^+) = Kx(t - \tau_k), \;\; t \in \{i_k h + \tau_k, k = 1, 2, \cdots\}; \\ \phi(t) = x(t_0 - \theta)\mathrm{e}^{A(t-t_0+\theta)}, \;\; t \in [t_0 - \theta, t_0]; \end{cases} \quad (5)$$

where h is the sampling period, $i_k (k = 1, 2, 3, \cdots)$ are some integers and $\{i_1, i_2, i_3, \cdots\} \subset \{0, 1, 2, 3, \cdots\}$, τ_k is the time delay which denotes the time from the instant $i_k h$ when sensor nodes sample sensor data from a plant to the instant when actuators transfer data to the plant. $\phi(t)$ is the compatible initial condition of the system, θ is a constant satisfying $(i_{k+1} - i_k)h + \tau_{k+1} \leqslant \theta (k = 1, 2, \cdots)$. The initial time $t_0 \geqslant 0$ is supposed. Therefore,

$$\bigcup_{k=1}^{\infty} [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}) = [t_0, \infty).$$

It is not required that $i_{k+1} > i_k$, which implies that some packet dropout is occurring. Especially, when $\{i_1, i_2, i_3, \cdots\} = \{0, 1, 2, \cdots\}$, it means that no packet dropout occurs in the transmission. So (5) can be viewed as a general form of the NCS model.

For system (5), we give the following assumption, lemma and definition, which will be used in the proof of our main results.

Assumption 1

1) We assume the system with clock-driven sensors samples the plant outputs periodically at sampling instants.

2) An event-driven controller exists which can be implemented by an external event interrupt mechanism and which calculates the control signal as soon as the sensor data arrives.

3) Event-driven actuators are presented which means the plant inputs are changed as soon as the data become available.

4) Before the first control signal reaches the controlled plant, we have $u(t) = 0$.

Lemma 1 [12] Given matrices $\varGamma$, $\varXi$ and symmetric matrix $\varOmega$, then $\varOmega + \varGamma F \varXi + \varXi^{\mathrm{T}} F^{\mathrm{T}} \varGamma^{\mathrm{T}} < 0$ holds for any F satisfying $F^{\mathrm{T}}F \leqslant I$, if and only if there exists a scalar $\varepsilon > 0$ such that $\varOmega + \varepsilon^{-1}\varGamma\varGamma^{\mathrm{T}} + \varepsilon\varXi^{\mathrm{T}}\varXi < 0$.

Definition 1 The nonlinear NCS described by (5) is said to be robustly stabilizable in the sector $[N_1, N_2]$ if the system is asymptotically stabilizable for any nonlinear function $\varphi(t, z)$ satisfying (4) and all admissible uncertainties.

3 Main results

In this section, we give a solution to the robust stabilization problem formulated previously. We assume that the full state variables are available for measurements. Firstly, We consider a special class of nonlinear functions $\varphi(t, z)$ which are located in a sector $[0, N]$, that is

$$\varphi^{\mathrm{T}}(t, z)[\varphi(t, z) - Nz] \leqslant 0. \quad (6)$$

For this class of nonlinear connections, we have the following result.

Based on the Lyapunov functional method, we give a controller design method for nominal system (5), that is, $\Delta A = 0$, $\Delta B = 0$.

Theorem 1 The control law $u(t) = X\hat{P}^{-1}x(t)$ asymptotically stabilizes the nominal NCS (5) with nonlinear function satisfying (6) for any τ_k satisfying $0 < \tau_k \leqslant \bar{\tau}_k$, if there exist matrices $\hat{P} > 0$, $\hat{Q} > 0$, and matrices X, $\hat{Y}$, $\hat{W}$ such that the following LMI (7) holds,

$$\begin{bmatrix} \varXi_{11} & \varXi_{12} & \varXi_{13} & -\bar{\tau}_k\hat{Y} & \bar{\tau}_k\hat{P}A^{\mathrm{T}} \\ * & -\hat{Q} - \hat{W} - \hat{W}^{\mathrm{T}} & 0 & -\bar{\tau}_k\hat{W} & \bar{\tau}_k X^{\mathrm{T}}B^{\mathrm{T}} \\ * & * & -2I & 0 & \bar{\tau}_k B_w^{\mathrm{T}} \\ * & * & * & -\bar{\tau}_k\hat{P} & 0 \\ * & * & * & * & -\bar{\tau}_k\hat{P} \end{bmatrix} < 0, \quad (7)$$

where

$$\varXi_{11} = A^{\mathrm{T}}\hat{P} + \hat{P}A + \hat{Y} + \hat{Y}^{\mathrm{T}} + \hat{Q},$$

$$\varXi_{12} = BX - \hat{Y} + \hat{W}^{\mathrm{T}}, \;\; \varXi_{13} = \bar{\tau}_k B_w - \hat{P}C^{\mathrm{T}}N^{\mathrm{T}}.$$

Proof Construct a Lyapunov functional candidate as

$$\begin{aligned} V(x_t) &= V_1(x_t) + V_2(x_t) + V_3(x_t) \\ &= x^{\mathrm{T}}(t)Px(t) + \int_{t-\tau_k}^{t} x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s \\ &\quad + \int_{-\tau_k}^{0}\int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(\alpha)Z\dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta, \end{aligned} \quad (8)$$

where $P > 0$, $Z > 0$, $Q > 0$, and

$$V_1(x_t) = x^{\mathrm{T}}(t)Px(t),$$

$$V_2(x_t) = \int_{-\tau_k}^{0}\int_{t+\beta}^{t} \dot{x}(\alpha)^{\mathrm{T}}Z\dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta,$$

$$V_3(x_t) = \int_{t-\tau_k}^{t} x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s.$$

After taking the time derivative of $V(x_t)$ for $t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1})$ along the solution to the nominal system (5) and using the Newton-Leibniz formula $x(t - \tau_k) =$

$x(t)-\int_{t-\tau_k}^{t}\dot{x}(s)\mathrm{d}s$, we introduce the slack matrix variables W and Y to yield,

$$\begin{aligned}
&\dot{V}_1(x_t)\\
&=2x^{\mathrm{T}}(t)P\big[Ax(t)+BKx(t-\tau_k)+B_w w(t)\big]\\
&=2x^{\mathrm{T}}(t)P(A+BK)x(t)+2x^{\mathrm{T}}(t)PB_w w(t)\\
&\quad -2x^{\mathrm{T}}(t)PBK\int_{t-\tau_k}^{t}\dot{x}(\alpha)\mathrm{d}\alpha\\
&=2x^{\mathrm{T}}(t)P(A+BK)x(t)+2x^{\mathrm{T}}(t)PB_w w(t)\\
&\quad +2x^{\mathrm{T}}(t)(Y-PBK)\int_{t-\tau_k}^{t}\dot{x}(\alpha)\mathrm{d}\alpha\\
&\quad +2x^{\mathrm{T}}(t-\tau_k)W\int_{t-\tau_k}^{t}\dot{x}(\alpha)\mathrm{d}\alpha-\Big[2x^{\mathrm{T}}(t)Y\\
&\quad \times\int_{t-\tau_k}^{t}\dot{x}(\alpha)\mathrm{d}\alpha+2x^{\mathrm{T}}(t-\tau_k)W\int_{t-\tau_k}^{t}\dot{x}(\alpha)\mathrm{d}\alpha\Big]\\
&=\frac{1}{\tau_k}\int_{t-\tau_k}^{t}\Big[2x^{\mathrm{T}}(t)(PA+Y)x(t)\\
&\quad +2x^{\mathrm{T}}(t)(PBK-Y+W^{\mathrm{T}})x(t-\tau_k)\\
&\quad -2x^{\mathrm{T}}(t-\tau_k)Wx(t-\tau_k)-2\tau_k x^{\mathrm{T}}(t)Y\dot{x}(\alpha)\\
&\quad -2\tau_k x^{\mathrm{T}}(t-\tau_k)W\dot{x}(\alpha)+2\tau_k x^{\mathrm{T}}(t)PB_w w(t)\Big]\mathrm{d}\alpha,
\end{aligned}$$

$$\begin{aligned}
&\dot{V}_2(x_t)\\
&=\int_{-\tau_k}^{0}\big[\dot{x}^{\mathrm{T}}(t)Z\dot{x}(t)-\dot{x}^{\mathrm{T}}(t+\beta)Z\dot{x}(t+\beta)\big]\mathrm{d}\beta\\
&=\int_{t-\tau_k}^{t}[\dot{x}^{\mathrm{T}}(t)Z\dot{x}(t)-\dot{x}^{\mathrm{T}}(\alpha)Z\dot{x}(\alpha)]\mathrm{d}\alpha\\
&=\frac{1}{\tau_k}\int_{t-\tau_k}^{t}\Big[\tau_k x^{\mathrm{T}}(t)A^{\mathrm{T}}ZAx(t)+2\tau_k x^{\mathrm{T}}(t)A^{\mathrm{T}}ZBK\\
&\quad \times x(t-\tau_k)+\tau_k x^{\mathrm{T}}(t-\tau_k)(BK)^{\mathrm{T}}ZBKx(t-\tau_k)\\
&\quad +2\tau_k w^{\mathrm{T}}(t)B_w^{\mathrm{T}}ZAx(t)+2\tau_k w^{\mathrm{T}}(t)B_w^{\mathrm{T}}ZBKx(t-\tau_k)\\
&\quad +\tau_k w^{\mathrm{T}}(t)B_w^{\mathrm{T}}ZB_w w(t)-\tau_k\dot{x}^{\mathrm{T}}(\alpha)Z\dot{x}(\alpha)\Big]\mathrm{d}\alpha,
\end{aligned}$$

$$\begin{aligned}
&\dot{V}_3(x_t)\\
&=\frac{1}{\tau_k}\int_{t-\tau_k}^{t}\big[x^{\mathrm{T}}(t)Qx(t)-x^{\mathrm{T}}(t-\tau_k)Qx(t-\tau_k)\big]\mathrm{d}\alpha\\
&=x^{\mathrm{T}}(t)Qx(t)-x^{\mathrm{T}}(t-\tau_k)Qx(t-\tau_k).
\end{aligned}$$

Since (6) holds, then

$$\varphi^{\mathrm{T}}(t,z)[Nz-\varphi(t,z)]\geqslant 0.$$

Therefore, we have $\dot{V}(x_t)<0$, if

$$\dot{V}(x_t)+2\varphi^{\mathrm{T}}(t,z)[Nz-\varphi(t,z)]<0.$$

That is to say,

$$\begin{aligned}\dot{V}(x_t)\leqslant{}&\dot{V}_1(x_t)+\dot{V}_2(x_t)+\dot{V}_3(x_t)\\&+2\varphi^{\mathrm{T}}(t,z)[-\varphi(t,z)+Nz].\end{aligned}$$

It then follows that

$$\dot{V}(x_t)\leqslant\frac{1}{\tau_k}\int_{t-\tau_k}^{t}\xi^{\mathrm{T}}(t,\alpha)\varPhi(\tau_k)\xi(t,\alpha)\mathrm{d}\alpha, \tag{9}$$

where $\xi(t,\alpha)$ and $\varPhi(\tau_k)$ are defined as follows,

$$\xi(t,\alpha)=\begin{bmatrix}x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-\tau_k) & w^{\mathrm{T}}(t) & \dot{x}^{\mathrm{T}}(\alpha)\end{bmatrix}^{\mathrm{T}}$$

$$\varPhi(\tau_k)=\begin{bmatrix}\varPhi_{11} & \varPhi_{12} & \varPhi_{13} & -\tau_k Y\\ * & \varPhi_{22} & \tau_k K^{\mathrm{T}}B^{\mathrm{T}}ZB_w & -\tau_k W\\ * & * & -2I+\tau_k B_w^{\mathrm{T}}ZB_w & 0\\ * & * & * & -\tau_k Z\end{bmatrix}$$

$$\begin{aligned}
&\varPhi_{11}=PA+A^{\mathrm{T}}P+Y+Y^{\mathrm{T}}+Q+\tau_k A^{\mathrm{T}}ZA,\\
&\varPhi_{12}=PBK-Y+W^{\mathrm{T}}+\tau_k A^{\mathrm{T}}ZBK,\\
&\varPhi_{13}=\tau_k PB_w-C^{\mathrm{T}}N^{\mathrm{T}}+\tau_k A^{\mathrm{T}}ZB_w,\\
&\varPhi_{22}=\tau_k(BK)^{\mathrm{T}}ZBK-Q-W-W^{\mathrm{T}}.
\end{aligned}$$

Since $Z>0$, it follows that for all τ_k satisfying $0<\tau_k\leqslant\bar{\tau}_k$, $\varPhi(\tau_k)<0$ is equivalent to (10) in sense of the Schur complement.

$$\varPsi=\begin{bmatrix}\varPsi_{11} & \varPsi_{12} & \varPsi_{13} & -\bar{\tau}_k Y & \bar{\tau}_k A^{\mathrm{T}}Z\\ * & -Q-W-W^{\mathrm{T}} & 0 & -\bar{\tau}_k W & \bar{\tau}_k K^{\mathrm{T}}B^{\mathrm{T}}Z\\ * & * & -2I & 0 & \bar{\tau}_k B_w^{\mathrm{T}}Z\\ * & * & * & -\bar{\tau}_k Z & 0\\ * & * & * & * & -\bar{\tau}_k Z\end{bmatrix}<0, \tag{10}$$

where

$$\begin{aligned}
&\varPsi_{11}=PA+A^{\mathrm{T}}P+Y+Y^{\mathrm{T}}+Q,\\
&\varPsi_{12}=PBK-Y+W^{\mathrm{T}},\\
&\varPsi_{13}=\bar{\tau}_k PB_w-C^{\mathrm{T}}N^{\mathrm{T}}.
\end{aligned}$$

Pre- and post-multiplying both sides of (10) by $\mathrm{diag}\{P^{-1},P^{-1},I,P^{-1},Z^{-1}\}$, and defining $\hat{P}=P^{-1}$, $\hat{Y}=P^{-1}YP^{-1}$, $\hat{Q}=P^{-1}QP^{-1}$, $\hat{W}=P^{-1}WP^{-1}$, $X=KP^{-1}, Z=P$, we can show that the feasibility of (7) implies that of (10). By solving the corresponding LMI (7), the solutions of $\hat{P}$, $\hat{Y}$, $\hat{Q}$, $\hat{W}$ and X can be obtained. So, we have $P=\hat{P}^{-1}$, $Y=\hat{P}^{-1}\hat{Y}\hat{P}^{-1}$, $Q=\hat{P}^{-1}\hat{Q}\hat{P}^{-1}$, $W=\hat{P}^{-1}\hat{W}\hat{P}^{-1}$, $K=X\hat{P}^{-1}, Z=\hat{P}^{-1}$. Then the Lyapunov functional (8) is determined.

Combining (9), it is easy to see that $\dot{V}(x_t)\leqslant-\alpha\|x(t)\|^2$, where $\alpha=\lambda_{\min}(-\varPhi(\tau_k))>0$. Finally, along a similar line as in the proof of Theorem 1 in [8], it follows from the Lyapunov–Krasovskii stability theory that this nonlinear NCS is asymptotically stabilizable for any network-induced delay τ_k satisfying $0<\tau_k\leqslant\bar{\tau}_k$. This completes the proof.

For the nonlinearity $\varphi(t,z)$ satisfying the more general sector condition (4), by applying an idea known as loop transformation [13], we can conclude that the stability of the nominal system of (5) in the sector $[N_1,N_2]$ is equivalent to the stability of the following system

$$\begin{cases}\dot{x}(t)=(A-B_wN_1C)x(t)+Bu(t)+B_w w(t),\\ t\in[i_kh+\tau_k,i_{k+1}h+\tau_{k+1});\\ z(t)=Cx(t),\ \ u(t^+)=Kx(t-\tau_k),\\ t\in\{i_kh+\tau_k,k=1,2,\cdots\};\\ \phi(t)=x(t_0-\theta)\mathrm{e}^{A(t-t_0+\theta)},\\ t\in[t_0-\theta,t_0]\end{cases} \tag{11}$$

in the sector $[0,N_2-N_1]$. Similar to the proof of Theorem 1, we have:

Theorem 2 The control law $u(t)=X\hat{P}^{-1}x(t)$ asymptotically stabilizes the nominal NCS of (5) with a nonlinear function satisfying (4) for any τ_k satisfying $0<\tau_k\leqslant\bar{\tau}_k$, if there exist matrices $\hat{P}>0$, $\hat{Q}>0$, and matrices X, $\hat{Y}$, $\hat{W}$ such that the following LMI (12) holds,

$$\begin{bmatrix}\hat{\varXi}_{11} & \varXi_{12} & \hat{\varXi}_{13} & -\bar{\tau}_k\hat{Y} & \hat{\varXi}_{15}\\ * & -\hat{Q}-\hat{W}-\hat{W}^{\mathrm{T}} & 0 & -\bar{\tau}_k\hat{W} & \bar{\tau}_k X^{\mathrm{T}}B^{\mathrm{T}}\\ * & * & -2I & 0 & \bar{\tau}_k B_w^{\mathrm{T}}\\ * & * & * & -\bar{\tau}_k\hat{P} & 0\\ * & * & * & * & -\bar{\tau}_k\hat{P}\end{bmatrix}<0, \tag{12}$$

where

$$\hat{\Xi}_{11} = (A - B_w N_1 C)^{\mathrm{T}} \hat{P} + \hat{P}(A - B_w N_1 C) + \hat{Y} + \hat{Y}^{\mathrm{T}} + \hat{Q},$$
$$\hat{\Xi}_{13} = -\bar{\tau}_k B_w - \hat{P} C^{\mathrm{T}} (N_2 - N_1)^{\mathrm{T}},$$
$$\hat{\Xi}_{15} = \bar{\tau}_k \hat{P} (A - B_w N_1 C)^{\mathrm{T}}.$$

Theorem 2 presents a sufficient condition for stabilization of nominal nonlinear NCS (5) in a general sector $[N_1, N_2]$. When time-varying norm-bounded parameter uncertainties appear in system (5), we provide a verifiable characterization on the feasibility of inequality (13).

Theorem 3 The uncertain nonlinear NCS (5) is robustly asymptotically stabilizable with control law $u(t) = X\hat{P}^{-1}x(t)$ for any τ_k satisfying $0 < \tau_k \leqslant \bar{\tau}_k$, if there exist a scalar $\epsilon > 0$, matrices $\hat{P} > 0$, $\hat{Q} > 0$, and matrices X, $\hat{Y}$, $\hat{W}$ such that LMI (13) is true,

$$\begin{bmatrix} \bar{\Xi}_{11} & \Xi_{12} & \hat{\Xi}_{13} & -\bar{\tau}_k \hat{Y} & \bar{\Xi}_{15} & \bar{\tau}_k \hat{P} E_a^{\mathrm{T}} \\ * & \bar{\Xi}_{22} & 0 & -\bar{\tau}_k \hat{W} & \bar{\tau}_k X^{\mathrm{T}} B^{\mathrm{T}} & \bar{\tau}_k X^{\mathrm{T}} E_b^{\mathrm{T}} \\ * & * & -2I & 0 & \bar{\tau}_k B_w^{\mathrm{T}} & 0 \\ * & * & * & -\bar{\tau}_k \hat{P} & 0 & 0 \\ * & * & * & * & \bar{\Xi}_{55} & 0 \\ * & * & * & * & * & -\epsilon I \end{bmatrix} < 0, \quad (13)$$

where

$$\bar{\Xi}_{11} = \hat{\Xi}_{11} + \epsilon DD^{\mathrm{T}},$$
$$\bar{\Xi}_{15} = \bar{\tau}_k \hat{P} (A - B_w N_1 C)^{\mathrm{T}} + \epsilon \bar{\tau}_k DD^{\mathrm{T}},$$
$$\bar{\Xi}_{22} = -\hat{Q} - \hat{W} - \hat{W}^{\mathrm{T}},$$
$$\bar{\Xi}_{55} = -\bar{\tau}_k \hat{P} + \epsilon \bar{\tau}_k^2 DD^{\mathrm{T}}.$$

Proof Replacing A and B in (10) with $A + \Delta A$ and $B + \Delta B$, respectively, then the following inequality is obtained,

$$\Psi + \Pi_m^{\mathrm{T}} F(t) \Pi_n + \Pi_n^{\mathrm{T}} F^{\mathrm{T}}(t) \Pi_m < 0 \quad (14)$$

where

$$\Pi_m = \begin{bmatrix} D^{\mathrm{T}} P & 0 & 0 & 0 & \tau D^{\mathrm{T}} Z \end{bmatrix}, \ \Pi_n = \begin{bmatrix} E_a & E_b K & 0 & 0 & 0 \end{bmatrix}.$$

Using Lemma 1, a sufficient and necessary condition guaranteeing (14) is that there exists a scalar $\epsilon > 0$ such that

$$\Psi + \epsilon \Pi_m^{\mathrm{T}} \Pi_m + \epsilon^{-1} \Pi_n^{\mathrm{T}} \Pi_n < 0. \quad (15)$$

Applying the Schur complement shows that (15) is equivalent to the following condition,

$$\begin{bmatrix} \bar{\Psi}_{11} & \bar{\Psi}_{12} & \bar{\Psi}_{13} & -\bar{\tau}_k Y & \bar{\Psi}_{15} & \bar{\tau}_k E_a^{\mathrm{T}} \\ * & \bar{\Psi}_{22} & 0 & -\bar{\tau}_k W & \bar{\Psi}_{25} & \bar{\tau}_k K^{\mathrm{T}} E_b^{\mathrm{T}} \\ * & * & -2I & 0 & \bar{\Psi}_{35} & 0 \\ * & * & * & -\bar{\tau}_k Z & 0 & 0 \\ * & * & * & * & \bar{\Psi}_{55} & 0 \\ * & * & * & * & * & -\epsilon I \end{bmatrix} < 0, \quad (16)$$

where

$$\bar{\Psi}_{11} = \Psi_{11} + \epsilon PDD^{\mathrm{T}} P,$$
$$\bar{\Psi}_{12} = PBK - Y + W^{\mathrm{T}},$$
$$\bar{\Psi}_{13} = \bar{\tau}_k PB_w - C^{\mathrm{T}} (N_2 - N_1)^{\mathrm{T}},$$
$$\bar{\Psi}_{15} = \bar{\tau}_k (A - B_w N_1 C)^{\mathrm{T}} Z + \epsilon \bar{\tau}_k PDD^{\mathrm{T}} Z,$$
$$\bar{\Psi}_{22} = -Q - W - W^{\mathrm{T}},$$
$$\bar{\Psi}_{25} = \bar{\tau}_k K^{\mathrm{T}} B^{\mathrm{T}} Z,$$
$$\bar{\Psi}_{35} = -\bar{\tau}_k B_w^{\mathrm{T}} Z,$$
$$\bar{\Psi}_{55} = -\bar{\tau}_k Z + \epsilon \bar{\tau}_k^2 ZDD^{\mathrm{T}} Z.$$

Pre- and post-multiplying both sides of (16) by $\mathrm{diag}\{P^{-1}, P^{-1}, I, P^{-1}, Z^{-1}, I\}$, respectively, and defining $\hat{P} = P^{-1}$, $\hat{Y} = P^{-1} Y P^{-1}$, $\hat{Q} = P^{-1} Q P^{-1}$, $\hat{W} = P^{-1} W P^{-1}$, $X = KP^{-1}$, $Z = P$, we can show that (13) implies (16). The desired result follows immediately. This completes the proof.

When the nonlinearity and uncertainty are not considered, system (5) becomes a nominal NCS (17), being the same as in [8]. We employ the same method as in Theorem 1, and then the following result is derived.

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \ t \in [i_k h + \tau_k, i_{k+1} h + \tau_{k+1}); \\ u(t^+) = Kx(t - \tau_k), \ t \in \{i_k h + \tau_k, k = 1, 2, \cdots\}. \end{cases} \quad (17)$$

Theorem 4 The control law $u(t) = X\hat{P}^{-1}x(t)$ asymptotically stabilizes the nominal NCS (17) for any τ_k satisfying $0 < \tau_k \leqslant \bar{\tau}_k$, if there exist matrices $\hat{P} > 0$, $\hat{Q} > 0$, and matrices X, $\hat{Y}$, $\hat{W}$ such that LMI (18) is true,

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & -\bar{\tau}_k \hat{Y} & \bar{\tau}_k \hat{P} A^{\mathrm{T}} \\ * & -\hat{Q} - \hat{W} - \hat{W}^{\mathrm{T}} & -\bar{\tau}_k \hat{W} & \bar{\tau}_k X^{\mathrm{T}} B^{\mathrm{T}} \\ * & * & -\bar{\tau}_k \hat{P} & 0 \\ * & * & * & -\bar{\tau}_k \hat{P} \end{bmatrix} < 0. \quad (18)$$

where

$$\Xi_{11} = A^{\mathrm{T}} \hat{P} + \hat{P} A + \hat{Y} + \hat{Y}^{\mathrm{T}} + \hat{Q},$$
$$\Xi_{12} = BX - \hat{Y} + \hat{W}^{\mathrm{T}}.$$

4 Numerical examples

In this section, we present some numerical examples to illustrate the proposed results of Section 3.

Example 1 Consider system (5) with parameters uncertainties (2) and nonlinearities satisfying (4) described as follows,

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \ B_w = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix},$$
$$C = [0.4 \ \ 0.6], \ D = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix}, \ \alpha \geqslant 0,$$
$$E_a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \ E_b = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ N_1 = 0.$$

The objective is to find an upper bound $\bar{\tau}_k$ such that the considered uncertain nonlinear NCS (5) is robustly stabilizable by $u(t) = X\hat{P}^{-1}x(t)$ for all allowable uncertainties and nonlinearities satisfying (4). Table 1 lists the MATIs for stabilizing system (5) by Theorem 3. At worst, when $\alpha = 0.60$ and $N_2 = 100$, by solving the corresponding LMI (13), the solutions $\hat{P}$, $\hat{Q}$, $\hat{Y}$, $\hat{W}$ are given as follows.

$$\hat{P} = \begin{bmatrix} 0.3507 & -0.2104 \\ -0.2104 & 0.1329 \end{bmatrix}, \ \hat{Q} = \begin{bmatrix} 1.1134 & -0.6806 \\ -0.6806 & 0.4161 \end{bmatrix},$$
$$\hat{Y} = \begin{bmatrix} -8.5532 & 5.1317 \\ 5.1316 & -3.2404 \end{bmatrix}, \ \hat{W} = \begin{bmatrix} 8.5534 & -5.1319 \\ -5.1317 & 3.2405 \end{bmatrix}.$$

And

$$X = \begin{bmatrix} -0.4680 & -0.4546 \end{bmatrix},$$

then the control law

$$u(t) = X\hat{P}^{-1}x(t) = \begin{bmatrix} -67.9127 & -110.9692 \end{bmatrix} x(t)$$

is obtained to stabilize the uncertain nonlinear NCS (5).

Table 1 The obtained MATIs for Example 1.

α	0.00	0.15	0.30	0.45	0.60
$N_2 = 0.5$	0.9999	0.9925	0.9772	0.9545	0.9271
$N_2 = 50$	0.1052	0.1023	0.0956	0.0878	0.0802
$N_2 = 100$	0.0444	0.0441	0.0434	0.0423	0.0410

Example 2 The nominal system (17) with the same parameters as the example in [1] is considered, where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},$$

The nonnetworked controller is designed as $u(t) = Kx(t)$, where the feedback gain $K = [-3.75 \ \ -11.5]$. From [1] and [14], the MATI that guarantees the stability of system (17) controlled over a network is 4.5×10^{-4} and 0.0538, respectively. By using Theorem 1 of [15] and Corollary 1 of [8], it can be computed that the MATI is 0.7805 and 0.8695, respectively. On this note, by applying Theorem 4, it shows that the MATI is 1.0758. The results are summarized in Table 2. Obviously, our result is less conservative than the ones based on the reported methods.

Table 2 The obtained MATIs using different methods.

Methods	MATI
Zhang et al. (2001)	4.5×10^{-4}
Park et al. (2002)	0.0538
Kim et al. (2003)	0.7805
Yue et al. (2004)	0.8695
Theorem 4	1.0758

5 Conclusions

The problem of robust stabilization for a class of uncertain nonlinear NCSs with network-induced delay and data packet dropout is investigated based on the Lyapunov-Krasovskii functional method. A strict LMI condition of the state feedback controller design is proposed for the considered NCS. By introducing some slack matrix parameters, a less conservative MATI can be derived by solving the corresponding LMI. Some numerical examples are provided to demonstrate the feasibility and superiority of the proposed approach.

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