Journal of Control Theory and Applications 2007 5 (1) 28–36 DOI 10.1007/s11768-005-5062-3

Design of a bilinear fault detection observer for singular bilinear systems

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Abstract: A bilinear fault detection observer is proposed for a class of continuous time singular bilinear systems subject to unknown input disturbance and fault. By singular value decomposition on the original system, a bilinear fault detection observer is proposed for the decomposed system via an algebraic Riccati equation, and the domain of attraction of the state estimation error is estimated. A design procedure is presented to determine the fault detection threshold. A model of flexible joint robot is used to demonstrate the effectiveness of the proposed method.

Keywords: Singular bilinear systems (SBS); Bilinear observer; Fault detection; State estimation; Domain of attraction

1 Introduction

The process of fault detection and isolation of system has been of considerable interest during the last two decades [1∼3]. Research is still under way into the development of more efficient methods for fault detection and isolation (FDI) in automatic control system. Although some researches on FDI for nonlinear system are available in recent years [4∼7], many researches on FDI still concentrate on linear system [3]. Bilinear system is a class of nonlinear system in which the control quantity appears in both additive and multiplicative terms. Bilinear systems arise in a variety of physical situations, for example, heat exchangers and many biomedical processes, which are known to be described by bilinear systems [8, 9]. The problems of fault detection for bilinear systems were extensively discussed in [10∼12]. In contrast, for the case of singular bilinear systems (SBS), which is a more general nonlinear system than the normal bilinear systems and linear singular systems [13∼15], few results were available. In [16], an unknown input residual generator (UIRG) was designed to fulfill the fault detection for SBS, but the UIRG cannot provide the state estimation information and decouple the fault from the external disturbance. In this paper, we will design a bilinear fault detection observer (BFDO) for SBS subject to unknown input disturbances and faults. Firstly, by singular value decomposition technique (SVD), the original SBS is decomposed into two parts: dynamical system and static system, and the existence of the solution to the decomposed system is presented. Secondly, a BFDO is proposed for the decomposed system based on an algebraic Riccati equation (ARE), and the domain of attraction of the state estimation error is also estimated. Thirdly, the effect of the threshold on the fault detection performance is discussed and a design procedure is given to determine the threshold. Finally, a model of single-link flexible joint robot is used to demonstrate the effectiveness of the proposed method.

2 System formulation

Consider the following continuous time SBS

$$E_a\dot{x}(t) = A_a x(t) + G_a g(x(t), u(t)) + B_a u(t) + D_a d(t) + F_a f(t), \tag{1}$$

$$y(t) = C_s x(t) + F f(t) + D d(t), \tag{2}$$

where

$$g(x(t), u(t)) = \sum_{i=1}^{h} u^i(t) A_{si} x(t), \tag{3}$$

$x(t) \in \mathbb{R}^n$ is the singular state vector, $u(t) \in \mathbb{R}^m (m \leqslant n)$ is the input vector, $d(t) \in \mathbb{R}^{n_1}$ is the unknown input disturbance vector, $f(t) \in \mathbb{R}^{n_2}$ is the unknown fault vector, and $y(t) \in \mathbb{R}^l$ is the output vector. The matrices A_a, G_a, A_{si}, B_a, D_a, F_a, C_s, F and D are all compatibly dimensioned. E_a is square and possibly singular, i.e. rank(E_a) = p ⩽ n.

For convenience, we will drop out the time t for variables in system (1) and (2) in the sequel.

The existence of the observer design depends partly on whether or not some conditions are met on the system (1) and (2). Hence, we need the following assumption.

Assumption 1 [15] a) The triple (E_a, A_a, B_a) is R-

Received 30 March 2005; revised 18 July 2006.

This work was supported in part by National Nature Science Foundation of China (No. 60325311, 60534010, 60572070), the Funds for Creative Research Groups of China (No. 60521003) and the Program for Changjiang Scholars and Innovative Research Team in University (No. IRT0421).

controllable and Y-controllable; b) The triple (E_a, A_a, C_s) is R-observable and Y-observable.

2.1 Transformation of the SBS system

There exist two orthogonal matrices U and V such that

$$U^{\mathrm{T}} E_a V = \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix}, \tag{4}$$

where $\Sigma = \mathrm{diag}(\mu_1, \cdots, \mu_p)$ is a diagonal matrix, $\mu_i (i = 1, 2, \cdots, p)$ are the singular values of matrix E_a. Now define new variables as follows,

$$z = V^{\mathrm{T}} x = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix},\ U^{\mathrm{T}} B_a = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$

$$U^{\mathrm{T}} A_a V = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},\ C_s V = \begin{bmatrix} C_1 & C_2 \end{bmatrix},$$

$$U^{\mathrm{T}} G_a = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix},\ U^{\mathrm{T}} D_a = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix},\ U^{\mathrm{T}} F_a = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}.$$

Equations (1) (2) can now be expressed in SVD form as a p-th order dynamic system and a q-th ($q = n - p$) order static system,

$$\Sigma \dot{z}_1 = A_{11} z_1 + A_{12} z_2 + B_1 u + D_1 d + F_1 f + G_1 g(Vz, u), \tag{5}$$

$$0 = A_{21} z_1 + A_{22} z_2 + B_2 u + D_2 d + F_2 f + G_2 g(Vz, u), \tag{6}$$

$$y = C_1 z_1 + C_2 z_2 + Ff + Dd, \tag{7}$$

where $z_1 \in \mathbb{R}^p$, $z_2 \in \mathbb{R}^q$. If z_2 can be expressed by z_1, matrix A_{22} must be nonsingular, which is called causal in [13,17]. This condition can be guaranteed by the following lemma.

Lemma 1 [17] The triple (E_a, A_a, B_a) is Y-controllable if $\mathrm{rank}[A_{22}\ \ B_2] = q$. $\mathrm{rank}[A_{22}\ \ B_2] = q$ if and only if there exists a compatible matrix K_2 such that $A_{22} - B_2 K_2$ is nonsingular.

But for the general case where A_{22} is singular, a control action is necessary to make the system causal. Without loss of generality, we take the controller with the form

$$u = -K_3 y + v. \tag{8}$$

Then the system (5)∼(7) can be written as follows,

$$\Sigma \dot{z}_1 = (A_{11} - B_1 K_3 C_1) z_1 + (A_{12} - B_1 K_3 C_2) z_2 + B_1 v + (D_1 - B_1 K_3 D) d + (F_1 - B_1 K_3 F) f + G_1 g(z_1, z_2, v), \tag{9}$$

$$0 = (A_{21} - B_2 K_3 C_1) z_1 + (A_{22} - B_2 K_3 C_2) z_2 + B_2 v + (D_2 - B_2 K_3 D) d + (F_2 - B_2 K_3 F) f + G_2 g(z_1, z_2, v), \tag{10}$$

$$y = C_1 z_1 + C_2 z_2 + Ff + Dd. \tag{11}$$

According to Lemma 1, there exists a K_3 to make the matrix $A_{22} - B_2 K_3 C_2$ nonsingular.

Let $L = A_{22} - B_2 K_3 C_2$ be a known matrix, then equations (9)(10) can be rewritten as

$$\dot{z}_1 = a_{11} z_1 + a_{12} z_2 + b_1 v + d_1 d + f_1 f + g_1 g(z_1, z_2, v), \tag{12}$$

$$0 = L(a_{21} z_1 + z_2 + b_2 v + d_2 d + f_2 f + g_2 g(z_1, z_2, v)), \tag{13}$$

$$y = C_1 z_1 + C_2 z_2 + Ff + Dd. \tag{14}$$

where

$$a_{11} = \Sigma^{-1}(A_{11} - B_1 K_3 C_1),\ a_{12} = \Sigma^{-1}(A_{12} - B_1 K_3 C_2),$$
$$d_1 = \Sigma^{-1}(D_1 - B_1 K_3 D),\ f_1 = \Sigma^{-1}(F_1 - B_1 K_3 F),$$
$$g_1 = \Sigma^{-1} G_1,\ b_1 = \Sigma^{-1} B_1,\ a_{21} = L^{-1}(A_{21} - B_2 K_3 C_1),$$
$$b_2 = L^{-1} B_2,\ g_2 = L^{-1} G_2,\ d_2 = L^{-1}(D_2 - B_2 K_3 D),$$
$$f_2 = L^{-1}(F_2 - B_2 K_3 F).$$

2.2 System solutions

Given that the system (12)∼(14) is well posed. To ensure that the total system has the same properties, the following assumption is required.

Assumption 2 a) The unknown functions d and f are continuously bounded, i.e. $d : \mathbb{R}^l \to \mathbb{R}^{l_1}$, $f : \mathbb{R}^{n_1} \to \mathbb{R}^{n_2}$, $\|d\| \leqslant d_b$, $\|f\| \leqslant f_b$, and the set $(d_b\ \ f_b)$ is known. b) The control v is bounded with $\|v\| \leqslant v_b$. In the presence of disturbance and fault, the output is also bounded, i.e. $\|y\| \leqslant y_b$, and the set $(v_b\ \ y_b)$ is known. c) There exist known non-negative real constants α_1 and α_2 such that the function $g(t, z_1, z_2, v)$, $g : \mathbb{R}^1 \times \mathbb{R}^p \times \mathbb{R}^q \times \mathbb{R}^m \to \mathbb{R}^{n_3}$ satisfies the following Lipschitz condition,

$$\|g(t, z_1, z_2, v) - g(t, z_3, z_4, v)\| \leqslant \alpha_1 \|z_1 - z_3\| + \alpha_2 \|z_2 - z_4\|, \tag{15}$$

for all $z_1, z_3 \in \mathbb{R}^p$ and $z_2, z_4 \in \mathbb{R}^q$, where

$$\alpha_1 = \alpha_2 = (\|K_3\| y_b + b_v) \sum_{i=1}^{h} \|A_{si}\|. \tag{16}$$

Multiplying the static equation (13) by $a_{12} L^{-1}$ and subtracting the result from the dynamic equation (12), the following system is derived

$$\dot{z}_1 = A_c z_1 + B_c v + D_c d + F_c f + G_c g(z_1, z_2, v), \tag{17}$$

$$0 = a_{21} z_1 + z_2 + b_2 v + d_2 d + f_2 f + g_2 g(z_1, z_2, v), \tag{18}$$

$$y = C_{2c} z_1 + F_{2c} f + D_{2c} d - G_{2c} g(z_1, z_2, v) - B_{2c} v, \tag{19}$$

where

$$A_c = (a_{11} - a_{12} a_{21}),\ B_c = (b_1 - a_{12} b_2),$$
$$D_c = (d_1 - a_{12} d_2),\ F_c = (f_1 - a_{12} f_2),$$
$$G_c = (g_1 - a_{12} g_2),\ C_{2c} = (C_1 - C_2 a_{21}),$$
$$F_{2c} = (F - C_2 f_2),\ D_{2c} = (D - C_2 d_2) G_{2c} = C_2 g_2,$$
$$B_{2c} = C_2 b_2.$$

3 Design of a BFDO

Consider the following BFDO to estimate the state z_1 and z_2 in system (17)∼(19),

$$\dot{\hat{z}}_1 = A_c\hat{z}_1 + B_c v + G_c g(\hat{z}_1, \hat{z}_2, v) + L_1(y - \hat{y}), \quad (20)$$
$$0 = a_{21}\hat{z}_1 + \hat{z}_2 + b_2 v + g_2 g(\hat{z}_1, \hat{z}_2, v) + L_2(y - \hat{y}), \quad (21)$$
$$\hat{y} = C_1\hat{z}_1 + C_2\hat{z}_2, \quad (22)$$

where L_1 and L_2 are the gain matrices to be designed. Define the state error $e_1 = z_1 - \hat{z}_1$, $e_2 = z_2 - \hat{z}_2$, then from (14), (17), (18) and (20)∼(22) it yields,

$$e_2 = -S[(a_{21} - L_2C_1)e_1 + g_2\tilde{g} + (d_2 - L_2D)d + (f_2 - L_2F)f], \quad (23)$$
$$\dot{e}_1 = (A_c - L_1C_1)e_1 + G_c\tilde{g} + (D_c - L_1D)d + (F_c - L_1F)f - L_1C_2e_2 = (A_c - L_1C_c)e_1 + G_{cg}\tilde{g} + D_{cd}d + F_{cf}f, \quad (24)$$

where

$$S = (I - L_2C_2)^{-1},$$
$$\tilde{g} = g(z_1, z_2, \hat{z}_1, \hat{z}_2, v) = g(z_1, z_2, v) - g(\hat{z}_1, \hat{z}_2, v),$$
$$C_c = C_1 - C_2S(a_{21} - L_2C_1),$$
$$D_{cd} = D_c - L_1D + L_1C_2S(d_2 - L_2D),$$
$$G_{cg} = G_c + L_1C_2Sg_2,$$
$$F_{cf} = F_c - L_1F + L_1C_2S(f_2 - L_2F).$$

The design of a BFDO is to choose L_1 and L_2 (in which L_2 also guarantees the matrix S nonsingular) such that: a) The BFDO exists; b) The error e_1 converges to zero when $f = 0$ and $d = 0$, and has a degree of robust when $d \neq 0$; c) For any bounded disturbance, if $e_1(t_0) \leqslant \varepsilon_0$ for $t_0 > 0$, then $e_1(t) > \varepsilon_0$ for $f \neq 0$ and $t > t_0$, ε_0 is the given threshold to judge whether the fault occurs or not.

From (23) and (24) we may get the following result, which will be used in the sequel.

Lemma 2 Given Assumption 2 holds. Then 1) the observer system (20)∼(22) is causal and solutions $\hat{z}_1$ and $\hat{z}_2$ exist if $\|g_2\|\alpha_2 < 1$; 2) the error system (23) (24) is also causal and the solutions e_1 and e_2 exist if $\|Sg_2\|\alpha_2 < 1$, and the bounds of e_2 and $\tilde{g}$ satisfy

$$\|e_2\| \leqslant \beta_1\|e_1\| + \beta_2\|d\| + \beta_3\|f\|, \quad (25)$$
$$\|\tilde{g}\| \leqslant \gamma_1\|e_1\| + \gamma_2\|d\| + \gamma_3\|f\|, \quad (26)$$

where

$$\beta_1 = \beta_4(\|S(a_{21} - L_2C_1)\| + \alpha_1\|Sg_2\|),$$
$$\beta_2 = \beta_4\|S(d_2 - L_2D)\|, \ \gamma_3 = \alpha_2\beta_3,$$
$$\beta_3 = \beta_4\|S(f_2 - L_2F)\|, \ \beta_4 = (1 - \|Sg_2\|\alpha_2)^{-1},$$
$$\gamma_1 = (\alpha_1 + \alpha_2\beta_1), \ \gamma_2 = \alpha_2\beta_2.$$

Proof Part 1) For fixed $t, \hat{z}_1, v, d$ and f, the static equation (21) can be expressed in the form $-\hat{z}_2 = \bar{g}(\hat{z}_2) = a_{21}\hat{z}_1 + b_2v + g_2g(\hat{z}_1, \hat{z}_2, v) + L_2(y - \hat{y})$. By Assumption 2, $\bar{g}(\hat{z}_2)$ is a global contraction mapping with constant $\|g_2\|\alpha_2 < 1$, so there exists a unique solution $\hat{z}_2$ in equation (21). This also implies that equation (20) is equivalent to a nonsingular causal, and differential system. By Assumption 2, there exists a well defined solution to equation (20) by the existence theorem of differential equation.

Part 2) For the existence of e_1 and e_2, it can be proved in the similar way as part 1). Therefore the error e_1 and e_2 exist and the system (23) (24) is causal. Taking the norm on both sides of (23), it gives

$$\|e_2\| \leqslant \|S(a_{21} - L_2C_1)\|\,\|e_1\| + \|Sg_2\|\,\|\tilde{g}\| + \|S(d_2 - L_2D)\|\,\|d\| + \|S(f_2 - L_2F)\|\,\|f\|. \quad (27)$$

By Assumption 2, $\|\tilde{g}\| \leqslant \alpha_1\|e_1\| + \alpha_2\|e_2\|$, we have

$$\|e_2\| \leqslant (1 - \|Sg_2\|\alpha_2)^{-1}[(\|S(a_{21} - L_2C_1)\| + \alpha_1\|Sg_2\|)\|e_1\| + \|S(d_2 - L_2D)\|\,\|d\| + \|S(f_2 - L_2F)\|\,\|f\|].$$

Then the equation (25) holds. Consider equation (25) and $\|\tilde{g}\| \leqslant \alpha_1\|e_1\| + \alpha_2\|e_2\|$ again, equation (26) is obviously true. This completes the proof of Lemma 2.

When disturbances and faults are not present, it is clear from (23) and (24) that e_2 converge to zero if the dynamic error e_1 converges to zero. Given L and L_2 (so does S), then the remained problem is to determine matrix L_1. Now we consider a special form of an ARE, which is used in the stability analysis of the BFDO design,

$$A_cP + PA_c^{\mathrm{T}} + \varepsilon Q + (G_c + \frac{1}{\varepsilon}PC_c^{\mathrm{T}}C_2Sg_2)(G_c + \frac{1}{\varepsilon}PC_c^{\mathrm{T}}C_2Sg_2)^{\mathrm{T}} + P(\gamma^2N^{\mathrm{T}}N - \frac{1}{\varepsilon}C_c^{\mathrm{T}}C_c)P = 0, \quad (28)$$

where $\varepsilon > 0$, $\gamma > 0$, $Q = Q^{\mathrm{T}} > 0$, N is an adjustable matrix with compatible dimension to make the ARE (28) hold. Therefore, for a given parameter set $(\varepsilon, \gamma, Q, N)$, if ARE (28) has a positive definite symmetric matrix P, then the matrix L_1 can be specified as

$$L_1 = \frac{1}{\varepsilon}PC_c^{\mathrm{T}}. \quad (29)$$

Substituting (29) into (28), we obtain

$$(A_c - L_1C_c)P + P(A_c - L_1C_c)^{\mathrm{T}} + \varepsilon Q + G_{cg}G_{cg}^{\mathrm{T}} + P(\gamma^2N^{\mathrm{T}}N + \frac{1}{\varepsilon}C_c^{\mathrm{T}}C_c)P = 0, \quad (30)$$

where $G_{cg} = G_c + L_1C_2Sg_2$ is the same as that in (24).

Equation (30) cannot be directly used in the stability analysis of BFDO, so multiplying both sides of (30) by P^{-1}/ρ, and define $P_0 = P^{-1}/\rho^2$, we get the following form,

$$P_0(A_c - L_1C_c) + (A_c - L_1C_c)^{\mathrm{T}}P_0 + \rho^2P_0(\varepsilon Q + G_{cg}G_{cg}^{\mathrm{T}})P_0 + (\frac{\gamma^2}{\rho^2}N^{\mathrm{T}}N + \frac{1}{\varepsilon\rho^2}C_c^{\mathrm{T}}C_c) = 0. \quad (31)$$

For the case of $f \neq 0$, we have the following main result provided that the system is still stable after fault occurs.

Theorem 1 Assume that there exist L and L_2 such that Lemma 2 holds, and that $\delta_1 > 0$, $\delta_2 > 0$, $\delta_3 > 0$, $\delta_4 > 0$ and $\delta_5 > 0$ are specified in advance. If for some specified parameters Q,$\gamma > 0$, $\varepsilon > 0$ and N, ARE (31) exists a positive definite symmetric matrix P_0 such that the following

M_1 is positive definite,

$$M_1 = \frac{\gamma^2}{\rho^2}N^{\mathrm{T}}N + \frac{1}{\varepsilon\rho^2}C_c^{\mathrm{T}}C_c + \rho^2\varepsilon P_0QP_0 -\delta_1 P_0D_{cd}D_{cd}^{\mathrm{T}}P_0 - \delta_4 P_0F_{cf}F_{cf}^{\mathrm{T}}P_0 -\frac{1}{\delta_2}I, \tag{32}$$

where

$$\rho^2 = \delta_2\gamma_1^2 + \delta_3\gamma_2^2 + \delta_5\gamma_3^2,$$

γ_1, γ_2 and γ_3 are given in Lemma 2. Then 1) $\dot{W}(t)$ satisfies the following inequality

$$\dot{W}(t) \leqslant -e_1^{\mathrm{T}}M_1e_1 + (\frac{1}{\delta_1} + \frac{1}{\delta_3})\|d\|^2 +(\frac{1}{\delta_4} + \frac{1}{\delta_5})\|f\|^2, \tag{33}$$

2) the observer error equation (24), i.e., e_1 is asymptotically stable when $d = 0$ and $f = 0$; 3) the error trajectories e_1 are ultimately bounded and eventually enter the ellipsoid defined by

$$e_1^{\mathrm{T}}P_0e_1 = (\frac{1}{\delta_1} + \frac{1}{\delta_3})\frac{\|d\|^2}{0.5\lambda_{\min}(P_0^{-1}M_1 + M_1P_0^{-1})} + (\frac{1}{\delta_4} + \frac{1}{\delta_5})\frac{\|f\|^2}{0.5\lambda_{\min}(P_0^{-1}M_1 + M_1P_0^{-1})}, \tag{34}$$

where $\lambda_{\min}(P_0)$ is the smallest eigenvalue of P_0, $W(t)$ is a Lyapunov function defined in the later proof, $\dot{W}(t)$ is the time derivative of $W(t)$.

Proof Consider Lyapunov function $W(t) = e_1^{\mathrm{T}}P_0e_1$, where P_0 is defined in (31). Along the trajectories of system (24), the derivative of Lyapunov function $W(t)$ is

$$\begin{aligned}\dot{W}(t) &= \dot{e}_1^{\mathrm{T}}P_0e_1 + e_1^{\mathrm{T}}P_0\dot{e}_1 \\ &= e_1^{\mathrm{T}}[(A_c - L_1C_c)^{\mathrm{T}}P_0 + P_0(A_c - L_1C_c)]e_1 \\ &\quad +2e_1^{\mathrm{T}}P_0(D_{cd}d + G_{cg}\tilde{g} + F_{cf}f).\end{aligned} \tag{35}$$

Using Lemma 2 and the following inequality

$$\pm 2X^{\mathrm{T}}Y \leqslant \sigma X^{\mathrm{T}}X + \frac{1}{\sigma}Y^{\mathrm{T}}Y, \tag{36}$$

where X and Y are column vectors with appropriate dimensions, $\sigma > 0$ is a scalar, it follows

$$2e_1^{\mathrm{T}}P_0D_{cd}d \leqslant \delta_1 e_1^{\mathrm{T}}P_0D_{cd}D_{cd}^{\mathrm{T}}P_0e_1 + \frac{1}{\delta_1}d^{\mathrm{T}}d, \tag{37}$$

$$2e_1^{\mathrm{T}}P_0F_{cf}f \leqslant \delta_4 e_1^{\mathrm{T}}P_0F_{cf}F_{cf}^{\mathrm{T}}P_0e_1 + \frac{1}{\delta_4}f^{\mathrm{T}}f, \tag{38}$$

$$\begin{aligned}2e_1^{\mathrm{T}}P_0G_{cg}\tilde{g} &\leqslant \left\|2e_1^{\mathrm{T}}P_0G_{cg}\tilde{g}\right\| \leqslant 2\left\|e_1^{\mathrm{T}}P_0G_{cg}\right\|\|\tilde{g}\| \\ &\leqslant 2\left\|e_1^{\mathrm{T}}P_0G_{cg}\right\|(\gamma_1\|e_1\| + \gamma_2\|d\| + \gamma_3\|f\|) \\ &\leqslant (\delta_2\gamma_1^2 + \delta_3\gamma_2^2 + \delta_5\gamma_3^2)\left\|e_1^{\mathrm{T}}P_0G_{cg}\right\|^2 \\ &\quad + \frac{1}{\delta_2}\|e_1\|^2 + \frac{1}{\delta_3}\|d\|^2 + \frac{1}{\delta_5}\|f\|^2,\end{aligned} \tag{39}$$

where $\delta_i > 0, i = 1, \cdots, 5$, are positive constants.

Substituting (37)~(39) into (35), it yields,

$$\begin{aligned}\dot{W}(t) &\leqslant e_1^{\mathrm{T}}[(A_c - L_1C_c)^{\mathrm{T}}P_0 + P_0(A_c - L_1C_c)]e_1 \\ &\quad +\delta_1 e_1^{\mathrm{T}}P_0D_{cd}D_{cd}^{\mathrm{T}}P_0e_1 + (\frac{1}{\delta_1} + \frac{1}{\delta_3})\|d\|^2 \\ &\quad +(\delta_2\gamma_1^2 + \delta_3\gamma_2^2 + \delta_5\gamma_3^2)\left\|e_1^{\mathrm{T}}P_0G_{cg}\right\|^2 + \frac{1}{\delta_2}\|e_1\|^2 \\ &\quad +\delta_4 e_1^{\mathrm{T}}P_0F_{cf}F_{cf}^{\mathrm{T}}P_0e_1 + (\frac{1}{\delta_4} + \frac{1}{\delta_5})\|f\|^2 \\ &= -e_1^{\mathrm{T}}[\frac{\gamma^2}{\rho^2}N^{\mathrm{T}}N - \frac{3}{\varepsilon\rho^2}C_c^{\mathrm{T}}C_c - \frac{1}{\delta_2}I]e_1 \\ &\quad +(\frac{1}{\delta_1} + \frac{1}{\delta_3})\|d\|^2 + (\frac{1}{\delta_4} + \frac{1}{\delta_5})\|f\|^2 \\ &\quad +(P_0e_1)^{\mathrm{T}}[-\rho^2\varepsilon Q + \delta_1 D_{cd}D_{cd}^{\mathrm{T}} + (\delta_2\gamma_1^2 + \delta_3\gamma_2^2 \\ &\quad +\delta_5\gamma_3^2 - \rho^2)G_{cg}G_{cg}^{\mathrm{T}} + \delta_4 F_{cf}F_{cf}^{\mathrm{T}}](P_0e_1) \\ &= -e_1^{\mathrm{T}}[\frac{\gamma^2}{\rho^2}N^{\mathrm{T}}N - \frac{3}{\varepsilon\rho^2}C_c^{\mathrm{T}}C_c + \rho^2\varepsilon P_0QP_0 \\ &\quad -\delta_1 P_0D_{cd}D_{cd}^{\mathrm{T}}P_0 - \delta_4 P_0F_{cf}F_{cf}^{\mathrm{T}}P_0 - \frac{1}{\delta_2}I]e_1 \\ &\quad +(\frac{1}{\delta_1} + \frac{1}{\delta_3})\|d\|^2 + (\frac{1}{\delta_4} + \frac{1}{\delta_5})\|f\|^2 \\ &= -e_1^{\mathrm{T}}M_1e_1 + (\frac{1}{\delta_1} + \frac{1}{\delta_3})\|d\|^2 + (\frac{1}{\delta_4} + \frac{1}{\delta_5})\|f\|^2 \\ &\leqslant -\lambda_{\min}(M_1)\|e_1\|^2 + (\frac{1}{\delta_1} + \frac{1}{\delta_3})\|d\|^2 \\ &\quad +(\frac{1}{\delta_4} + \frac{1}{\delta_5})\|f\|^2.\end{aligned} \tag{40}$$

Therefore, part 1) is proved. The part 2) is obvious from the proof of part 1) if $f = 0$ and $d = 0$. For part 3), by dividing above inequality (33) by $W(t) = e_1^{\mathrm{T}}P_0e_1$, it follows

$$\frac{\dot{W}(t)}{W(t)} \leqslant -0.5\lambda_{\min}(P_0^{-1}M_1 + M_1P_0^{-1}) +(\frac{1}{\delta_1} + \frac{1}{\delta_3})\frac{\|d\|^2}{e_1^{\mathrm{T}}P_0e_1} + (\frac{1}{\delta_4} + \frac{1}{\delta_5})\frac{\|f\|^2}{e_1^{\mathrm{T}}P_0e_1}, \tag{41}$$

which implies that the trajectories of error equation (24) starting outside the ellipsoid (34) will eventually enter this ellipsoid and remain there. So the trajectories of error equation (24) are ultimately bounded. It is end for part 3). This completes the proof of Theorem 1.

4 Fault detection

Define $\varepsilon = y - \hat{y}$, then

$$\begin{aligned}r &= H(y - \hat{y}) = H(C_1e_1 + C_2e_2 + Ff + Dd) \\ &= H(C_1e_1 - C_2S[(a_{21} - L_2C_1)e_1 + g_2\tilde{g} \\ &\quad +(d_2 - L_2D)d + (f_2 - L_2F)f] + Ff + Dd) \\ &= H[(C_1 - C_2S(a_{21} - L_2C_1))e_1 - C_2Sg_2\tilde{g} + (D \\ &\quad -C_2S(d_2 - L_2D))d + (F - C_2S(f_2 - L_2F)f)] \\ &= H[(C_1 - C_2S(a_{21} - L_2C_1))e_1 \\ &\quad -[C_2Sg_2 \quad -(D - C_2S(d_2 - L_2D))]\begin{bmatrix}\tilde{g} \\ d\end{bmatrix} \\ &\quad +(F - C_2S(f_2 - L_2F)f)],\end{aligned} \tag{42}$$

where H is designed to make the residual r insensitive to disturbance d but sensitive to fault f. If the complete decoupling between d and f is realized, then residual r directly reflects the action of fault f, and the residual threshold $r_{\rm th}$ can be simply set to zero. In practice it is difficult to decouple d and f completely, then the threshold $r_{\rm th}$ is a key parameter to be designed.

Consider the error equation (24), it can be rewritten as

$$\dot{e}_1 = (A_c - L_1C_c)e_1 + [G_{cg} \;\; D_{cd}]\begin{bmatrix}\tilde{g}\\ d\end{bmatrix} + F_{cf}f. \quad (43)$$

Then the transfer function from generalized disturbance $d_c = \begin{bmatrix}\tilde{g}\\ d\end{bmatrix}$ to residual r is

$$\begin{aligned}G_{rd_c} = {} & H(C_1 - C_2S(a_{21} - L_2C_1))(sI - A_c \\ & +L_1C_1)^{-1}\left[G_{cg}\; D_{cd}\right] \\ & -H[C_2Sg_2 - (D - C_2S(d_2 - L_2D))]. \quad (44)\end{aligned}$$

This is the standard form of linear transfer function, so the residual threshold can be formulated explicitly by the generalized disturbance magnitude

$$\begin{aligned}r_{\rm th} &= \max\|G_{rd_c}(j\omega)\|\,\|d_c\| \\ &= \max\|G_{rd_c}(j\omega)\|\max(d_b, \|\tilde{g}\|). \quad (45)\end{aligned}$$

If the norm bound of the nonlinear term $\tilde{g}$ is less than the bound of disturbance d, then the threshold can correctly detect the fault. Otherwise, it may miss alarm and decrease the sensitivity of residual to fault.

Remark 1 From (15) and (45) we can further see that large Lipschitz constants in SBS system can generally reduce the set of detectable faults as defined in [2]. On the contrary, if the Lipschitz constants in SBS system are small, it can usually enlarge the set of detectable faults. This argument about the effects of Lipschitz constant on the sensitivity to the fault detection is a similar problem as stated in [18].

As usual, the following simple decision logic can be employed,

$$\|r\| \geqslant r_{\rm th} \Rightarrow \text{a fault has occurred}, \quad (46)$$
$$\|r\| \leqslant r_{\rm th} \Rightarrow \text{no fault has occurred}, \quad (47)$$

where $r_{\rm th}$ is a threshold to be designed.

In order to overcome the effects of $\tilde{g}$ on $r_{\rm th}$, one can proceed as follows. From (42) and (26) it yields,

$$\begin{aligned}\|r\| \leqslant {} & \|H(C_1 - C_2S(a_{21} - L_2C_1))\|\,\|e_1\| \\ & + \|HC_2Sg_2\|\,\|\tilde{g}\| \\ & + \|H(D - C_2S(d_2 - L_2D))\|\,\|d\| \\ & + \|H(F - C_2S(f_2 - L_2F))\|\,\|f\| \\ = {} & \|H(C_1 - C_2S(a_{21} - L_2C_1))\|\,\|e_1\| \\ & + \|HC_2Sg_2\|\,(\gamma_1\|e_1\| + \gamma_2\|d\| + \gamma_3\|f\|) \\ & + \|H(D - C_2S(d_2 - L_2D))\|\,\|d\| + \\ & \|H(F - C_2S(f_2 - L_2F))\|\,\|f\| \\ = {} & [\|H(C_1 - C_2S(a_{21} - L_2C_1))\| \\ & + \|HC_2Sg_2\|\,\gamma_1]\,\|e_1\| + [\gamma_3\|HC_2Sg_2\| \\ & + \|H(F - C_2S(f_2 - L_2F))\|)\,\|f\|] \\ & +[\|HC_2Sg_2\|\,\gamma_2 \\ & + \|H(D - C_2S(d_2 - L_2D))\|]\,\|d\|. \quad (48)\end{aligned}$$

From (24) one can have

$$\begin{aligned}e_1(t) = {} & {\rm e}^{(A_c - L_1C_c)t}e_1(0) + \int_0^t {\rm e}^{(A_c - L_1C_c)(t-\tau)} \\ & \cdot (G_{cg}\tilde{g}(\tau,\cdot) + D_{cd}d(\tau) + F_{cf}f(\tau)){\rm d}\tau. \quad (49)\end{aligned}$$

Taking norms on both sides of (49) yields

$$\begin{aligned}\|e_1({\rm t})\| \leqslant {} & \left\|{\rm e}^{(A_c - L_1C_c)t}e_1(0)\right\| \\ & + \left\|\int_0^t {\rm e}^{(A_c - L_1C_c)(t-\tau)}G_{cg}\tilde{g}(\tau,\cdot){\rm d}\tau\right\| \\ & + \left\|\int_0^t {\rm e}^{(A_c - L_1C_c)(t-\tau)}D_{cd}d(\tau){\rm d}\tau\right\| \\ & + \left\|\int_0^t {\rm e}^{(A_c - L_1C_c)(t-\tau)}F_{cf}f(\tau){\rm d}\tau\right\|, \quad (50)\end{aligned}$$

or

$$\begin{aligned}\|e_1({\rm t})\| \leqslant {} & \left\|{\rm e}^{(A_c - L_1C_c)t}e_1(0)\right\| \\ & + \int_0^t \left\|{\rm e}^{(A_c - L_1C_c)(t-\tau)}\right\|{\rm d}\tau * \|G_{cg}\|\,\|\tilde{g}\| \\ & + \int_0^t \left\|{\rm e}^{(A_c - L_1C_c)(t-\tau)}\right\|{\rm d}\tau * \|D_{cd}\|\,\|d\| \\ & + \int_0^t \left\|{\rm e}^{(A_c - L_1C_c)(t-\tau)}\right\|{\rm d}\tau * \|F_{cf}\|\,\|f\|. \quad (51)\end{aligned}$$

Because $A_c - L_1C_c$ is stable, there exist two positive real constants β_4 and β_5 such that

$$\left\|{\rm e}^{(A_c - L_1C_c)t}\right\| \leqslant \beta_4{\rm e}^{-\beta_5 t}. \quad (52)$$

β_5 can be chosen as $\beta_5 = -\max({\rm Re}(\lambda_S)) > 0$, where λ_S denotes the eigenvalue set of $A_c - L_1C_c$, ${\rm Re}(\lambda)$ denotes the real part of eigenvalue λ. Therefore , (51) can be rewritten as

$$\begin{aligned}\|e_1({\rm t})\| \leqslant {} & \beta_4{\rm e}^{-\beta_5 t}\,\|e_1(0)\| \\ & +\beta_4\int_0^t \left\|{\rm e}^{-\beta_5(t-\tau)}\right\|{\rm d}\tau * \|G_{cg}\|\,\|\tilde{g}\| \\ & +\beta_4\int_0^t \left\|{\rm e}^{-\beta_5(t-\tau)}\right\|{\rm d}\tau * \|D_{cd}\|\,\|d\| \\ & +\beta_4\int_0^t \left\|{\rm e}^{-\beta_5(t-\tau)}\right\|{\rm d}\tau * \|F_{cf}\|\,\|f\| \\ = {} & \beta_4{\rm e}^{-\beta_5 t}\,\|e_1(0)\| + \frac{\beta_4\|G_{cg}\|\,\|\tilde{g}\|}{\beta_5}(1 - {\rm e}^{-\beta_5 t}) \\ & +\frac{\beta_4\|D_{cd}\|\,\|d\|}{\beta_5}(1 - {\rm e}^{-\beta_5 t}) \\ & +\frac{\beta_4\|F_{cf}\|\,\|f\|}{\beta_5}(1 - {\rm e}^{-\beta_5 t}). \quad (53)\end{aligned}$$

Now we assume that there is no fault after a period time of operation $0 < \mathrm{t} \leqslant \mathrm{t}_1$. It is obvious that $0 \leqslant \beta_6 < \mathrm{e}^{-\beta_5 t} < 1$ holds. Then,

$$\|e_1\| \leqslant (1 - \frac{\gamma_1\beta_4(1-\beta_6)}{\beta_5}\|G_{cg}\|)^{-1}[\beta_4\beta_6\|e_1(0)\| + \frac{\beta_4(1-\beta_6)}{\beta_5}(\gamma_2\|G_{cg}\| + \|D_{cd}\|)\|d\| + \frac{\beta_4(1-\beta_6)}{\beta_5}(\gamma_3\|G_{cg}\| + \|F_{cf}\|)\|f\|]. \quad (54)$$

In accordance with the small gain theorem, the following inequality holds,

$$0 < 1 - \frac{\gamma_1\beta_4(1-\beta_6)}{\beta_5}\|G_{cg}\| < 1. \quad (55)$$

Inserting (54) into (48), we can obtain

$$\|r\| \leqslant \sigma_1\|e_1(0)\| + \sigma_2\|d\| + \sigma_3\|f\|, \quad (56)$$

where

$$\sigma_1 = (\|H(C_1 - C_2S(a_{21} - L_2C_1))\| + \|HC_2Sg_2\|\gamma_1)(1 - \frac{\gamma_1\beta_4(1-\beta_6)}{\beta_5}\|G_{cg}\|)^{-1}\beta_4\beta_6,$$

$$\sigma_2 = (\|H(C_1 - C_2S(a_{21} - L_2C_1))\| + \|HC_2Sg_2\|\gamma_1)(1 - \frac{\gamma_1\beta_4(1-\beta_6)}{\beta_5}\|G_{cg}\|)^{-1} \cdot\frac{\beta_4(1-\beta_6)}{\beta_5}(\gamma_2\|G_{cg}\| + \|D_{cd}\|) + (\|HC_2Sg_2\|\gamma_2 + \|H(D - C_2S(d_2 - L_2D))\|),$$

$$\sigma_3 = (\|H(C_1 - C_2S(a_{21} - L_2C_1))\| + \|HC_2Sg_2\|\gamma_1) \cdot(1 - \frac{\gamma_1\beta_4(1-\beta_6)}{\beta_5}\|G_{cg}\|)^{-1}\frac{\beta_4(1-\beta_6)}{\beta_5} \cdot(\gamma_3\|G_{cg}\| + \|F_{cf}\|) + (\gamma_3\|HC_2Sg_2\| + \|H(F - C_2S(f_2 - L_2F))\|).$$

The threshold can then be defined by the upper bound of $\|r\|$, i.e.

$$r_{\mathrm{th}} = \sup_{d,f=0}\|r\| = \sigma_1\|e_1(0)\| + \sigma_2 d_{\mathrm{b}} \quad (57)$$

This threshold guarantees that no false alarm will occur.

The design procedure of a BFDO is as follows.

1) By singular value decomposition technique, system (1)$\sim$(3) is converted into the form (5)$\sim$(7);

2) Check whether A_{22} is singular or not. If A_{22} is singular, take control action (8), otherwise, take $u = -K_3z$. Then system (5)$\sim$(7) is converted into system (17)$\sim$(19).

3) Construct an observer in the form of (20)$\sim$(22) for system (17)$\sim$(19).

4) Construct an ARE (28).

5) Choose L and L_2 such that Lemma 2 holds. Assume that $\delta_i > 0$ $(i = 1, \cdots, 5)$ are known constants, check whether ARE (28) exists a positive definite symmetric matrix P or P_0 such that (32) holds for a set of parameters $(\varepsilon, \gamma, Q, N)$. If there doesn't exist a P, then adjust matrix N or scalars γ and ε. Otherwise $L_1 = \frac{1}{\varepsilon}PC_c^{\mathrm{T}}$.

6) Compute the detection threshold (57).

5 Illustrative example

A single-link flexible joint robot powered by a DC motor is considered, and it is assumed that the fast dynamics can be modeled by a nonlinear static equation. For physical reasons, while motor position and motor velocity can be easily measured, the measurement of other states is not easy and an observer is needed to estimate these states for fault detection. A state space description of the system is given as follows, which is a slightly modified version of that in [17],

$$\begin{aligned}
&\dot{x}_1(t) = x_2(t) + \beta(t - T_0)f(t),\\
&\dot{x}_2(t) = -\frac{k}{J}x_1(t) - \frac{B_k}{J}x_2(t) + \frac{k}{J}x_3(t) - \frac{K_k}{J}u(t),\\
&\dot{x}_3(t) = x_4(t),\\
&\dot{x}_4(t) = \frac{k}{J_0}x_1(t) - \frac{k}{J_0}x_3(t) + \gamma x_5(t) + \frac{mgh_k}{J_0}x_3(t)u(t) + \frac{mgh_k}{J_0}d(t),\\
&0 = x_1(t) + x_3(t) + x_5(t) + d_0x_3(t)u(t) + d_1d(t),\\
&y_1(t) = x_1(t),\ y_2(t) = x_2(t),\\
&\beta(t - T_0) = \begin{cases} 0, & t < T_0\\ 1, & t \geqslant T_0\end{cases}.
\end{aligned}$$

where x_1, x_2, x_3, x_4, x_5, $d(t)$ and $f(t)$ are the angular rotation of the motor, the angular velocity of the motor, the angular position of the link, the angular velocity of the link, the fast subsystem perturbation, the scalar unknown disturbance and the unknown fault. Without loss of generality, we assume no fault at present. The form is already in the transformed form (5)$\sim$(7), and for illustration purpose, a set of model parameters $\{k, J, B_k, K_k, J_0, m, g, h_k, \gamma, d_1, d_2, T_0\}$ is chosen to give a model of the form (5)$\sim$(7) with the partition state $z_1 = [x_1\ \ x_2\ \ x_3\ \ x_4]^{\mathrm{T}}$ and $z_2 = x_5$, and

$$A_{11} = \begin{bmatrix} 0 & 1 & 0 & 0\\ -48.5 & -1.25 & 48.5 & 0\\ 0 & 0 & 0 & 1\\ 19.5 & 0 & -19.5 & 0\end{bmatrix},\ A_{12} = \begin{bmatrix}0\\0\\0\\-0.1\end{bmatrix},$$

$$B_1 = \begin{bmatrix}0\\21.5\\0\\0\end{bmatrix},\ G_1 = D_1 = \begin{bmatrix}0\\0\\0\\-0.353\end{bmatrix},$$

$$A_{22} = 1,\ B_2 = 0,\ A_{21} = [\,1\ \ 0\ \ 1\ \ 0\,],\ F_2 = 0,$$

$$F = 0,\ D = 0,\ D_2 = 0.1,\ G_2 = 0.2,$$

$$g(Vz, u) = G(t, z_1, u) = [\,0\ \ 0\ \ 1\ \ 0\,]z_1u = V_1z_1u,$$

$$y = C_1z_1 = \begin{bmatrix}1\,0\,0\,0\\0\,1\,0\,0\end{bmatrix}z_1,\ C_2 = 0.$$

Because $\mathrm{rank}[A_{22}\ B_2] = q = 1$, so the uncontrolled system is causal and the equations (9)$\sim$(11) with $K_3 = 0$ have the same form as (5)∼(7). The observer is given by (20)∼(22) where

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.5 & -1.25 & 48.5 & 0 \\ 0 & 0 & 0 & 1 \\ 19.4 & 0 & -19.6 & 0 \end{bmatrix}, D_c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.343 \end{bmatrix},$$

$$G_c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.333 \end{bmatrix}, \ B_c = B_1, \ C_{2c} = C_1,$$

L_1 and L_2 are to be designed. Since G is only a function of z_1, an explicit observer exists and we can simply choose $L_2 = 0$ in (21) with $S = I$. A bounded control $v(t)$ and disturbance $d(t)$ with $d_{\rm b} = 0.1$ is implemented to produce the bounded output. By Assumption 2 we have $\alpha_1 = 1$ and $\alpha_2 = 0$. From (25) we have $\beta_4 = 1, \beta_3 = 0, \beta_2 = 0.1$ and $\beta_1 = 1.6142$. From (26) we have $\gamma_1 = 1$, $\gamma_2 = \gamma_3 = 0$. Taking $\varepsilon = 0.1$, $\gamma = 1, Q = I$, $N = 2I$, the solution to equation (28) is

$$P = \begin{bmatrix} 0.6706 & 0.3241 & 0.7044 & 0.3857 \\ 0.3241 & 1.1841 & 0.4389 & -0.0332 \\ 0.7044 & 0.4389 & 0.7818 & 0.4278 \\ 0.3857 & -0.0332 & 0.4278 & 0.6322 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 6.7056 & 3.2414 \\ 3.2414 & 11.8409 \\ 7.0443 & 4.3886 \\ 3.8572 & -0.3321 \end{bmatrix}.$$

Taking $\delta_1 = 0.01$, $\delta_2 = 0.5$, $\delta_3 = 100$, $\delta_4 = 0.05$, $\delta_5 = 1$, then $\rho = 1/\sqrt{2}$,

$$P_0 = \frac{P^{-1}}{\rho^2} = \begin{bmatrix} 68.6637 & 6.5101 & -67.9439 & 4.4292 \\ 6.5101 & 3.2838 & -8.9407 & 2.2510 \\ -67.9439 & -8.9407 & 73.6359 & -8.8479 \\ 4.4292 & 2.2510 & -8.8479 & 6.5669 \end{bmatrix}$$

and M_1 in equation (32) is

$$M_1 = \begin{bmatrix} 259.8957 & 31.9290 & -254.9790 & 32.2107 \\ 31.9290 & 30.7834 & -35.3582 & 5.0466 \\ -254.9790 & -35.3582 & 284.9309 & -36.4191 \\ 32.2107 & 5.0466 & -36.4191 & 12.2731 \end{bmatrix}.$$

The eigenvalues of M_1 are 536.7717, 26.4523, 17.2578 and 7.4012, respectively. Hence, matrix M_1 is positive definite.

Now consider the fault with form of $f(t) = 0.1x_1(t)$, and the disturbance with form of $d(t) = 0.01(\sin x_1)^2 + 0.09\cos x_2$. The external input $u(t) = 0.5\sin t$, and the initial states are $z_1 = [x_1 \ \ x_2 \ \ x_3 \ \ x_4]^{\rm T} = [0 \ \ 8 \ \ 6 \ \ 0.6]^{\rm T}$ and $\hat{z}_1 = [\hat{x}_1 \ \ \hat{x}_2 \ \ \hat{x}_3 \ \ \hat{x}_4]^{\rm T} = [10 \ \ -5 \ \ -0.34 \ \ 1]^{\rm T}$.

By the method proposed above, without loss of generality, taking $H = 2 \times 2$ unit matrix, and we can easily obtain following parameters:

$$\beta_5 = 8.3487, \ \beta_4 = 13, \ \sigma_1 = 0.0162,$$
$$\sigma_2 = 1.1079, \ r_{\rm th} = 0.1109, \ d_{\rm b} = 0.1,$$
$$1 - \frac{\gamma_1\beta_4(1-\beta_6)}{\beta_5}\|G_{cg}\| = 0.5182 > 0,$$
$$v_b = 1,$$

and equation (59) holds. When entering the stable stage, β_6 may become very small. Here we take $\beta_6 = 0$. In the interval of 0∼10 seconds, no fault occurs. Just from the 10th second on, fault occurs.

Fig. 1 and Fig. 2 show the state curve and residual curve of x_2 without fault, respectively. It can be seen that the estimated state $\hat{x}_2$ can correctly reflect the real state x_2, and the residual curve of x_2 is in the range of the given threshold $r_{\rm th}$=0.1109. Fig.3∼Fig.6 depict the residual curve of x_i with fault, $i = 1, \cdots, 4$. After 10th second, the fault occurs and the residual is out of the range of the given threshold, which indicates that the fault occurs.

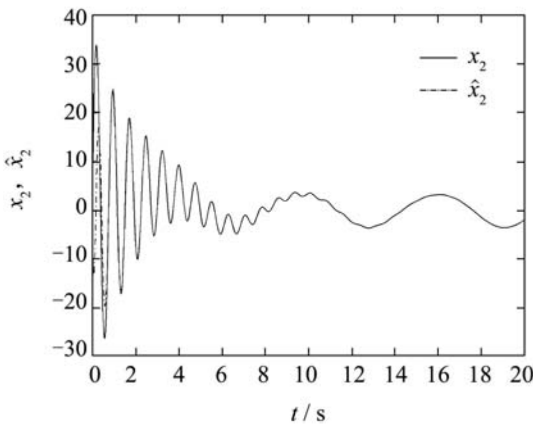

Fig. 1 The state trajectory of x_2 without fault.

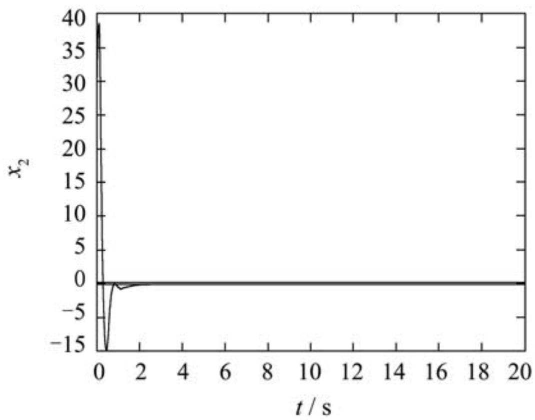

Fig. 2 Residual curve of x_2 without fault and threshold $r_{\rm th}$=0.1109.

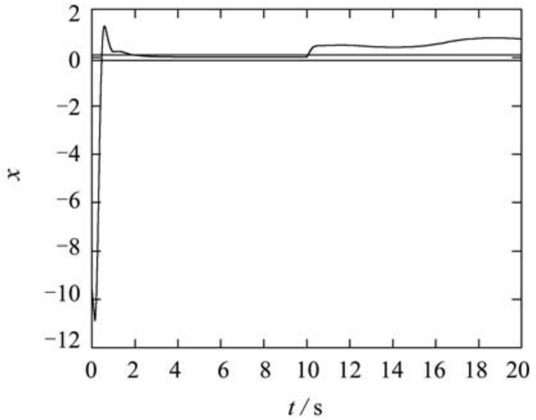

Fig. 3 Residual curve of x_1 with fault and threshold r_{th}=0.1109.

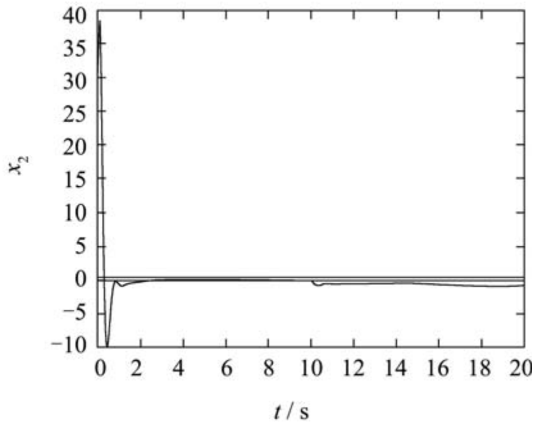

Fig. 4 Residual curve of x_2 with fault and threshold r_{th}=0.1109.

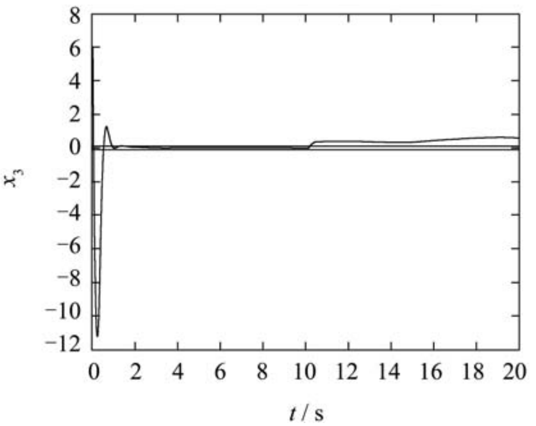

Fig. 5 Residual curve of x_3 with fault and threshold r_{th}=0.1109.

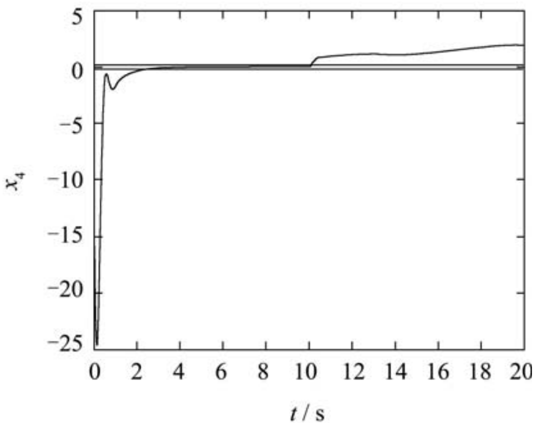

Fig. 6 Residual curve of x_4 with fault and threshold r_{th}=0.1109.

6 Conclusions

In the paper we provide both theoretical basis and design procedure for the design of BFDO for a class of nonlinear system, which is singular bilinear system subject to unknown input disturbance and fault. Conditions for the observer design of a well-posed system are given and the robustness of the detection system to unknown disturbance is discussed. The design scheme is used to the fault detection for a flexible joint robot model where the fast dynamics are modeled as a static system, and the simulation results demonstrate the effectiveness of the proposed method.

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