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Time variant multi-objective linear fractional interval-valued transportation problem

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Abstract. This paper studies a time-variant multi-objective linear fractional transportation problem. In reality, transported goods should reach in destinations within a specific time. Considering the importance of time, a time-variant multi-objective linear fractional transportation problem is formulated here. We take into account the parameters as cost, supply and demand are interval valued that involved in the proposed model, so we treat the model as a multi-objective linear fractional interval transportation problem. To solve the formulated model, we first convert it into a deterministic form using a new transformation technique and then apply fuzzy programming to solve it. The applicability of our proposed method is shown by considering two numerical examples. At last, conclusions and future research directions regarding our study is included.

§1 Introduction

Transportation experts encountered challenges at the beginning of the 21st century to meet the growing difficulty of shipping in a competitive market scenario. Transportation specialists then wanted to use the capacity restriction during transportation. They wanted to reduce environmental pollution, to recover poor safety record, unreliability, and to utilize the wasted energy. Transportation systems are generally complex systems involving a large number of components and different objectives, each having different and often conflicting objectives.
Transportation Problem (TP) can be explained as a specific case of linear programming problem and its model is used to decide how many units of items to be transferred from each origin to various destinations, satisfying source availabilities, and destination demands, while minimizing the total cost of transportation along with striking down the costs per unit item for the purchasers. The transportation model was first investigated by Kantorovich [18], who has defined an incomplete algorithm for achieving the solution of the TP. Hitchcock [15] considered the

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problem of minimizing the cost of distribution of products from different factories to a number of customers. Liu [23] presented a method for solving the cost minimization TP with varying demand and supply. A good number of researches for solving TP in different directions have been developed by several researchers such as (cf., [1], [27], [30], [35]).

The fractional TP is another type of TP with the ratio of objective functions. It is concerned with transporting goods from various sources to destinations along with keeping good relationships among some parameters. These parameters of TPs may occur in the form such as; actual transportation cost/total standard transportation cost, shipping cost/preferred route, total return/total investment, etc. The fractional programming problem, which is a problem of optimizing one or several ratios of functions, has been widely discussed from both methodological and practical perspectives (see, for example, the review of [42]). Swarup [44] introduced linear fractional functional programming. Few studies had been discussed on fractional network flows in the literature. Gupta et al. [10] investigated a paradox in linear fractional TPs with mixed constraints. A sufficient condition for the existence of a paradox is established. Lin [22] proposed iterative labelling algorithms to determine the type-II sensitivity ranges of the fractional assignment problem. Tirkolaee et al. [49] studied multi-objective optimization for the reliable pollution-routing problem with cross-dock selection using Pareto-based algorithms. Xu et al. [48] developed a new algorithm to deal with the linear fractional minimal cost flow problem on the network. Bharati [2] solved the trapezoidal intuitionistic fuzzy fractional TP. Mahmoodirad et al. [25] solved fuzzy linear fractional set covering problem by a goal programming based solution approach. A solution proposal to the interval fractional TP was proposed by Guzel et al. [14]. Schell [40] discussed the distribution of s products with several properties, in: Proceedings of 2nd Symposium in Linear Programming. Garg et al. [8] solved fractional two-stage transshipment problem under uncertainty and provided the application based on the extension principle approach. Schaible [41] described whole bibliography in fractional programming. Zimmermann [52] applied fuzzy programming and linear programming to solve several objective functions. Ravi and Reddy [34] applied fuzzy linear fractional goal programming to solve refinery operations planning. Bitran and Novaes [3] solved linear programming with a fractional objective function. Chadha [4] introduced fractional programming with absolute-value functions. Gupta and Arora [11] solved linear plus linear fractional capacitated transportation problems with restricted flow. Jain and Saksena [16] solved time minimizing TP with fractional bottleneck objective function. Chang [5] introduced polynomial mixed 0-1 fractional programming problems. Wolf [46] depicted an approach for determining the optimal solution of the linear fractional programming problem using parametric method. Radhakrishnan and Anukokila [33] established the fuzzy fractional transportation problem with compensatory approach. Wu [47] wrote a note on a global approach for general 0-1 fractional programming. A comparison with different fractional TPs is given in Table 1. A good number of researchers have studied linear programming problem in different directions such as (cf. [13], [17], [21], [51]).

The modern aggressive market scenario indicates that the single objective TP is not enough to

Table 1. Comparison among different fractional TPs.

Article	Type	Parameters	Approach	Time environment
Porchelvi and Sheela [32]	Single objective	Interval	With and without considering budgetary constraints	
Liu [24]	Single objective	Fuzzy	Two-level mathematical programming	
Kocken et al. [19]	Single objective	Interval	Taylor series approach	
Bharati [2]	Single objective	Fuzzy	Charnes and Cooper transformation	
Radhakrishnan and Anukokila [33]	Single objective	Fuzzy	Compensatory approach	
Guzel et al. [14]	Single objective	Interval	Taylor series approach	
Proposed method	Multi-objective	Interval	Taylor series approach	✓

handle the real-life decision-making problem. To deal with practical situations on TP, we introduce here the multi-objective concept on TP in which the objective functions are conflicting to each other. Maity and Roy [27] solved multi-objective TP using utility function approach. Vincent et al. [50] studied on an interactive approach for multi-objective TP with interval parameters. Maity and Roy [28] solved a multi-objective TP with non-linear cost and multi-choice demand. Niroomand et al. [31] solved an intuitionistic fuzzy two stage supply chain network design problem with multi-mode demand and multi-mode transportation. Recently, Roy et al. [36] studied on multi-choice multi-objective TP using conic scalarization approach; and Roy et al. [39] introduced a new approach for solving the intuitionistic fuzzy multi-objective transportation problem. Gupta et al. [12] solved parameter estimation and optimization of multi-objective capacitated stochastic transportation problem for Gamma distribution. Maity et al. [29] analyzed multi-modal transportation problem and discussed its application to artificial intelligence. Goli et al. [9] developed accelerated cuckoo optimization algorithm for capacitated vehicle routing problem in competitive conditions. The optimal solution of each objective function is not always derivable in a solution of Multi-Objective Transportation Problem (MOTP) due to the conflicting nature of the objective functions and hence we find Pareto optimal solution. Pareto optimality is one approach to obtain a better solution for MOTP, and it is named after economist Vilfredo Pareto (1848-1923). Pareto marked that many economic solutions helped some people while hurting others. "Pareto optimality" is a formally defined concept applied to determine when an allocation is optimal. If an allocation is not a Pareto optimal that means there is an alternative allocation and we can improve at least one participant's well-being without reducing any other participant's well-being. If there exists an alternative allocation that satisfies this condition, then the reallocation is called "Pareto improvement". When no further Pareto improvements are possible, then the allocation is "Pareto optimum". In real-life, various distributions are applied to the minimization of the ratio of the total cost to profit. The problem derived by such type of two linear functions gets its name as a linear fractional TP. The linear fractional TP is applied in different areas such as the financial sector, inventory management, production planning, banking sector, and others. It is used for modelling real-life problems with one or more objective functions such as debt/equity, profit/cost, inventory/sales, actual cost/standard cost, output/employees, nurses/patient ratios, etc. concerning some constraints. In many real-world situations, decisions are often made in the presence of multiple, conflicting, non-commensurable objectives. Then, it gets its name as a multi-objective linear fractional transportation problem (MLFTP). It deals with the distribution of goods at a time by considering the ratio of several objective functions. Multi-objective form of linear fractional programming problem has been studied in numerous studies and some of them are [[20], [43]]. Chakraborty and Gupta [7] developed a solution for multi-objective linear fractional programming problems. The parameters associated with the MLFTP are not deterministic or fixed value. Chang [6] developed a goal programming approach for fuzzy multi-objective fractional programming problems. In [45], a fuzzy multi-objective linear fractional programming problem is reduced to a single objective problem using the Taylor series and its approximation solution is obtained. However, several research papers were available in the literature to analyze fractional TP, but to the best of our knowledge, till now no one solved fractional TP in time environment. In this paper, a new mathematical model is proposed for solving fractional TP by incorporating time in the transporting system. The main contributions of our proposed study are summarized as follows:

- Design MLFTP model when the parameters in the objective functions, and supply and demand are interval numbers.
- We convert interval parameters like cost, supply and demand into crisp form using a new transformation technique.
- We linearize the fractional TP using Taylor series expansion.
- Develop a solution procedure to solve the proposed model (i.e., Model 3).
- Two real-life examples are incorporated to test the applicability of the proposed model.

The residue of this paper can be depicted as follows: the mathematical model of the study is presented in Section 2. Section 3 contains the solution procedure along with three subsections. A reduction procedure from interval to real number is presented in Section 3.1. A linearization technique for multi-objective fractional functions is presented in Section 3.2. An algorithm for solving the proposed MLFITP is offered in Section 3.2. Two numerical examples are included to justify our present study in Section 4, and sensitivity analysis is carried out in Section 4.1. Finally, concluding remarks and outlook of the study are described in Section 5.

§2 Formulation of the problem

We employ the following notations and list of abbreviations throughout the paper to formulate the mathematical formulation of MLFITP.

Notations:

c^1_{ij}: unit transporting cost due to the travelled route from i^{th} origin to j^{th} destination,

l^1_{ij}: unit transporting cost due to preferring route from i^{th} origin to j^{th} destination,

c^2_{ij}: unit transporting damage cost due to the travelled route from i^{th} origin to j^{th} destination,

l^2_{ij}: unit transporting damage cost due to preferring route from i^{th} origin to j^{th} destination.

Abbreviations:

TP: Transportation Problem,

MOTP: Multi-Objective Transportation Problem,

MLFTP: Multi-objective Linear Fractional Transportation Problem,

MLFITP: Multi-objective Linear Fractional Interval-valued Transportation Problem.

Considering a fractional TP with m origins having s_i $(i = 1, 2, \ldots, m)$ units of supply to be transported among n destinations with d_j $(j = 1, 2, \ldots, n)$ units of demand. Here we choose two fractional objective functions, which are

• Units transporting cost c^1_{ij} due to the travelled route and unit transporting cost due to preferring route l^1_{ij}, for transporting the product from i^{th} origin to j^{th} destination.

• Unit transporting damage cost c^2_{ij} (lost of quality and quantity of transportation) due to the travelled route and unit transporting damage cost due to preferring route l^2_{ij}, for transporting the product from i^{th} origin to j^{th} destination.

The problem is to determine the transportation schedule of transporting the available quantity of products, to satisfy the demand that minimizes the total transportation cost and damage charges. Let x_{ij} be the number of units transported from i^{th} origin to j^{th} destination. Then, the mathematical model for the MLFTP [6] which can be expressed as follows:

Model 1

$$\begin{aligned}
\text{minimize} \quad & Z_1 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c^1_{ij}x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} l^1_{ij}x_{ij}} \\
\text{minimize} \quad & Z_2 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c^2_{ij}x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} l^2_{ij}x_{ij}} \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \le s_i,\ (i = 1, 2, \ldots, m), \\
& \sum_{i=1}^{m} x_{ij} \ge d_j,\ (j = 1, 2, \ldots, n), \\
& x_{ij} \ge 0\ \forall\ i, j.
\end{aligned}$$

The first set of constraints stipulates that the sum of the shipments from a source does not exceed its supply; the second set requires that the sum of the shipments to a destination must satisfy its demand. The above problem implies that the total supply $\sum_{i=1}^{m} s_i$ must be greater than or equal to total demand $\sum_{j=1}^{n} d_j$.

Due to real-life situations in the market c^k_{ij} and l^k_{ij} $(k = 1, 2)$ are not always crisp numbers so here we consider these as interval numbers, then the mathematical model reduces to the following form (Model 2) as:

Model 2

$$\text{minimize} \quad Z_k = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}[c^{kl}_{ij}, c^{ku}_{ij}]x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n}[l^{kl}_{ij}, l^{ku}_{ij}]x_{ij}}\ (k = 1, 2)$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq [s_i^l, s_i^u], \; (i = 1, 2, \ldots, m), \tag{1}$$

$$\sum_{i=1}^{m} x_{ij} \geq [d_j^l, d_j^u], \; (j = 1, 2, \ldots, n), \tag{2}$$

$$x_{ij} \geq 0 \; \forall \; i, j. \tag{3}$$

In above model cost, supply and demand parameters all are interval numbers. The objective function in Model 2 is of non-linear type, and non-linearity occurs due to the effect of extra cost in the cost parameter in the TP. In this situation, the TP (see Model 2) is treated as TP with non-linear cost. The single objective TP is not adequate to formulate all real-life TPs. TPs with multiple objective functions are considered as MOTP. However, we deal with those kinds of objective functions that are conflicting and non-commensurable with each other.
In TP, time of transportation, especially for transporting the goods considering the sustainable development of nature, it is an important factor. In most of the governments and private industrial systems, the main aim for transporting goods is to reduce transportation cost and to optimize benefits within a quicker time. However, in many situations, no one would like to consider the optimal situation of controlling pollution during manufacturing as well as transportation. Therefore, in a faster world, to keep our nature in the best condition, it is the time to think not only of the benefit but also of the sustainability of nature during the system. As time is an important factor, we construct another objective function to minimize the transportation time as follows:

$$\text{minimize} \quad T = \sum_{i=1}^{m} \sum_{j=1}^{n} T_{ij} \chi_{ij}$$

$$\text{where} \quad \chi_{ij} = \begin{cases} 0, & \text{if } x_{ij} = 0 \text{ in } X \\ 1, & \text{if } x_{ij} \neq 0 \text{ in } X. \end{cases} \tag{4}$$

Here, T_{ij} is the time of transporting the goods from i^{th} node to j^{th} destination and $X \in F'$; where F', the set of all points satisfying constraints (1)-(4), is the feasible region of Model 2. Hence, in our proposed model, we include multiple objective functions along with objective function of time to be minimized and is defined as follows (see Model 3):

Model 3

$$\text{minimize} \quad Z_k = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}^{kl}, c_{ij}^{ku}] x_{ij}}{\sum_{i=1}^{m} \sum_{j=1}^{n} [l_{ij}^{kl}, l_{ij}^{ku}] x_{ij}}$$

$$\text{minimize} \quad T = \sum_{i=1}^{m} \sum_{j=1}^{n} T_{ij} \chi_{ij}$$

$$\text{subject to} \quad \text{the constraints } (1) - (4).$$

Fractional transportation problem has been studied in different environments by several researchers. But there is a gap of study of fractional transportation problem in time window. In this paper we design Model 3 which includes multi-objective interval valued fractional transportation problem in the time environment. Then we optimize the objective functions of fractional transportation problem, and we are not directly minimized the transportation time from the objective function of time. To minimize the transportation time in Model 3, here we apply new solution methodology which mentions in the next section, and then Model 4 is obtained. In Model 4 there is no time function whereas in Model 3 there is a time function.

§3 Solution methodology

The mathematical model of MLFITP described in this paper (see Model 3) cannot be solved directly due to presence of interval-valued parameters. Therefore, at first, we transfer the problem into MLFITP with crisp penalties and thereafter we linearize the fractional objective function by using first order Taylor series expansion. After that, fuzzy programming is imposed to determine the optimal solution of the reduced MLFITP. Now, we define a function which depends on time of which the value lies in the interval [0,1], and this issued is to reduce the interval valued cost parameter to the real valued parameter. The interval valued supply and demand parameters are also converted to real numbers by using parameters that do not necessarily depend on time. This procedure is depicted in the first subsection of this section. After that, an algorithm is presented to solve the proposed MLFITP in later subsection.

3.1 General transformation technique used to convert interval number into real number

The cost parameters involved in Model 3 are of the interval type, i.e., $[c_{ij}^{kl}, c_{ij}^{ku}]$ and $[l_{ij}^{kl}, l_{ij}^{ku}]$ $(k = 1, 2)$; this indicates that within the interval the cost parameters may take any value. Now we construct two new parameters $\lambda^{c_{ij}^k}$, $\lambda^{l_{ij}^k}$ which depend on time. Let the minimum time range period that assigns by DM is T_0. If the delivery occurred within the minimum range period, then the minimum costs c_{ij}^{kl} and l_{ij}^{kl} have to be paid. Due to delay of delivery the product, the cost becomes $\tilde{c}_{ij}^k = c_{ij}^{kl} + \lambda^{c_{ij}^k}(c_{ij}^{ku} - c_{ij}^{kl})$, $\tilde{l}_{ij}^k = l_{ij}^{kl} + \lambda^{l_{ij}^k}(l_{ij}^{ku} - l_{ij}^{kl})$ $(k = 1, 2)$. Here, $\lambda^{c_{ij}^k}$ and $\lambda^{l_{ij}^k}$ are parameters for each k such that

$$\lambda^{c_{ij}^k} = \begin{cases} 0, \text{ if } T_{ij} < T_0 \\ \frac{T_{ij} - T_0}{T_{ij}}, \text{ if } T_{ij} \geq T_0. \end{cases} \qquad \lambda^{l_{ij}^k} = \begin{cases} 0, \text{ if } T_{ij} < T_0 \\ \frac{T_{ij} - T_0}{T_{ij}}, \text{ if } T_{ij} \geq T_0. \end{cases} \tag{5}$$

where T_{ij} is the time of transportation from i^{th} origin to j^{th} destination, and then $\lambda^{c_{ij}^k}$ and $\lambda^{l_{ij}^k}$ are the increasing functions of time. Furthermore, all the objective functions in the considered MLFITP are not the type to deliver goods at a scheduled time. It may also reduce the damaging cost of delivering the goods. When goods are required on an urgent basis there is a lot of chance to increase the rate of the damaging cost of the goods; consequently, DM expects a less damaging cost as the requirement fulfils in hurry.

DM can also choose the function according to this choice, but it should be time-dependent as per our consideration in this paper. Again, the supply $\bar{s}_i(= [s_i^l, s_i^u])$ and demand $\bar{d}_j(= [d_j^l, d_j^u])$ parameters are also considered as interval numbers. The interval numbers reduce into real numbers by the following way: $\bar{s}_i = s_i^l + \lambda^{s_i}(s_i^u - s_i^l)$ and $\bar{d}^j = d_j^l + \lambda^{d_j}(d_j^u - d_j^l)$. Here, λ^{s_i} and λ^{d_j} are the parameters not necessarily related to time but may be linear or stochastic or fuzzy depending upon the choice of DM.

3.2 Linearization technique for multi-objective fractional functions

To apply the Taylor series approach, we need to specify an initial single point from the feasible region of Model 3. At first, interval supply and demand quantities and interval objective function coefficients of the numerator and denominator are converted into deterministic ones using the technique mentioned in Section 3.1. Then the corresponding Model 3 reduces into the form in Model 4 as follows:

Model 4

$$\text{minimize} \quad Z_k = \frac{\sum_{i=1}^m \sum_{j=1}^n \left[c_{ij}^{kl} + \lambda^{c_{ij}^l}(c_{ij}^{ku} - c_{ij}^{kl})\right] x_{ij}}{\sum_{i=1}^m \sum_{j=1}^n \left[l_{ij}^{kl} + \lambda^{l_{ij}^l}(l_{ij}^{ku} - l_{ij}^{kl})\right] x_{ij}} \; (k = 1, 2)$$

$$\text{subject to} \quad \sum_{j=1}^n x_{ij} \leq \left[s_i^l + \lambda^{s_i}(s_i^u - s_i^l)\right], \; (i = 1, 2, \ldots, m), \tag{6}$$

$$\sum_{i=1}^m x_{ij} \geq \left[d_j^l + \lambda^{d_j}(d_j^u - d_j^l)\right], \; (j = 1, 2, \ldots, n), \tag{7}$$

$$x_{ij} \geq 0, \; \lambda^{s_i} \;, \lambda^{d_j} \in [0, 1] \; \forall \; i, j. \tag{8}$$

The main purpose here is to specify an initial feasible point, not an optimal one. An initial basic feasible solution of Model 4 can be determined by Northwest Corner Method which ignores the objective function coefficients and compute a basic feasible solution of a TP, where the basic variables are selected from the North West corner (i.e., top left corner). Let denote the initial feasible solution as $X^{(0)} = (x^{(0)}, \lambda_{(0)}^{s_i}, \lambda_{(0)}^{d_j})$.

Thereafter, using the first order Taylor series about at the feasible point $X^{(0)}$, the objective function of Model 4 can be constructed approximately as follows:

$$Z_k \approx Z_k(X^{(0)}) + \sum_{i=1}^m \sum_{j=1}^n \frac{\partial Z_k}{\partial x_{ij}}\bigg|_{X^{(0)}} (x_{ij} - x_{ij}^{(0)}) \tag{9}$$

Hence the constant terms do not change the direction of minimization, these can be eliminated. The first order partial derivatives with respect to the variables x_{ij} in the Taylor series expansion is:

$$\frac{\partial Z_k}{\partial x_{ij}} = \frac{\frac{\partial Z_{1k}}{\partial x_{ij}} Z_{2k} - \frac{\partial Z_{2k}}{\partial x_{ij}} Z_{1k}}{(Z_{2k})^2} = \frac{\bar{c}_{ij} Z_{2k} - \bar{l}_{ij} Z_{1k}}{(Z_{2k})^2}$$

Now the denominator of the partial derivative $(Z_{2k})^2$ can be eliminated similarly to a constant value. Thus, an equivalent form of Model 4 can be constructed as follows:

Model 5

$$\text{minimize} \quad \overline{Z}_k \approx \sum_{i=1}^m \sum_{j=1}^n (\bar{c}_{ij} Z_{2k} - \bar{l}_{ij} Z_{1k})\big|_{X^{(0)}} x_{ij}$$

$$\text{subject to} \quad \text{the constraints } (6) - (8).$$

3.3 Fuzzy programming for solving time variant MLFITP

The steps for the solution of MLFITP using fuzzy programming are as follows:

Step 1: First, convert the objective functions of Model 4 and the constraints (1)-(4) into crisp numbers using the procedure mentioned in Section 3.1.

Step 2: Find the basic feasible solution of Model 4 using the North West Corner method and then using the linearization technique presented in Section 3.2 for converting Model 4 into Model 5.

Step 3: Solve each of the objective function of the multi-objective TP in Model 5 as single objective TP using each time only one objective function and ignoring other.

Step 4: We find the lower bound L_k and the upper bound U_k for the k^{th} objective function $\overline{Z}_k$ $(k = 1, 2)$, where L_k is the aspired level of achievement for the k^{th} objective function, U_i is the highest acceptable level of achievement for k^{th} objective function and $d_k = [U_k - L_k]$ is the degradation allowance for the k^{th} objective function. When the aspiration level and degradation allowance for each objective function are specified, a fuzzy model is formed and then it is converted into a crisp model.

Step 5: From the results of Step 3, determine the corresponding value for every objective function at each solution derived.

Step 6: From Step 4, we find the best L_k and the worst U_k values for each objective function corresponding to the set of solutions. The initial fuzzy model can then be stated, in terms of the aspiration level of each objective function, as follows:
Find x_{ij}, so as to satisfy $\overline{Z}_k \leq L_k,\ k = 1, 2$ with given constraints (5)-(8). For the multi-objective fractional TP, a membership function $\mu_k(x)$ corresponding to k^{th} objective function is defined as:

$$\mu_k(x) = \begin{cases} 1, & \text{if } \overline{Z}_k \leq L_k, \\ 1 - \left(\frac{\overline{Z}_k - L_k}{U_k - L_k}\right), & \text{if } L_k \leq \overline{Z}_k \leq U_k \ (k = 1, 2), \\ 0, & \text{if } \overline{Z}_k \geq U_k. \end{cases} \tag{10}$$

From the results of Step 5, determine the corresponding value for every objective function at each solution derived.
The equivalent linear programming problem for the minimization problem may then be written as:

$$\begin{aligned} &\text{maximize} && \lambda \\ &\text{subject to} && \lambda \leq \left(\frac{\overline{Z}_k - L_k}{U_k - L_k}\right) \ (k = 1, 2), \\ &&& \text{the constraints } (5) - (8), \\ &&& 0 \leq \lambda \leq 1. \end{aligned}$$

Here $\lambda = \min\{\mu_k(x) : k = 1, 2\}$. This linear programming problem can further be simplified as follows:

$$\begin{aligned} &\text{maximize} && \lambda \\ &\text{subject to} && \overline{Z}_k + \lambda(U_k - L_k) \leq U_k \ (k = 1, 2), \\ &&& \text{the constraints } (5) - (8), \\ &&& 0 \leq \lambda \leq 1. \end{aligned}$$

Step 7: From Step 5, we find the optimal allocation is $X^{(*)} = (x^{(*)}, \lambda_{(*)}^{s_i}, \lambda_{(*)}^{d_j})$, so we calculate the fractional objective values of Model 4 i.e., Z_1 and Z_2 with the minimum transportation time T by the following way:

$$\text{minimize} \ \ T = \sum_{i=1}^{m} \sum_{j=1}^{n} T_{ij} \chi_{ij}, \quad \text{where } \chi_{ij} = \begin{cases} 0, \text{ if } x_{ij} = 0 \text{ in } X^{(*)} \\ 1, \text{ if } x_{ij} \neq 0 \text{ in } X^{(*)}. \end{cases}$$

§4 Numerical example

We present here two examples for proving the effectiveness of the proposed methodologies. The first example is selected based on the real-life problem and the second example is viewed due to our preference to explain the applicability when there are more number of origins and destinations.

4.1 Example 1

A mobile company delivers mobiles from three stores located in S_1, S_2 and S_3, to dealers in three places located in D_1, D_2 and D_3. Here we consider two fractional objective functions, which are

• Unit transporting cost c_{ij}^1 due to the travelled route and unit transporting cost l_{ij}^1 due to preferring route r_{ij}, for transporting the product from i^{th} origin to j^{th} destination.

• Unit transporting damage cost c_{ij}^2 (lost of quality and quantity of transportation) due to the travelled route and unit transporting damage cost l_{ij}^2 due to preferring route r_{ij}, for transporting the product from i^{th} origin to j^{th} destination.

The transportation cost per unit of product from each source to the various destinations, c_{ij}^1 and unit transporting cost l_{ij}^1 due to preferring route r_{ij} listed in cost matrices in equation (11); the unit transportation damage cost of mobiles per unit of product c_{ij}^2 and unit transporting damage cost l_{ij}^2 due to preferring route r_{ij} listed in cost matrices in equation (12). The transportation time from i^{th} origin to j^{th} destination is listed in time matrix in equation (13). There is a basic cost incurred by the supplier for the delivery of a minimum order when the total number of mobiles purchased by the dealers from a source, otherwise extra charges have to be paid according to the desired rule of the company in each case.

The capacities of the stores of origin S_i's are $s_1 = [40, 43]$, $s_2 = [35, 40]$, $s_3 = [52, 56]$, which are maximum amounts, and the demands at destinations (D_j's) are $d_1 = [30, 32]$, $d_2 = [29, 36]$, $d_3 = [33, 37]$. The company wishes to find a compromise solution that minimizes the cost of each objective at a time according to the supplied cost matrices. Since the proposed problem is a minimization problem, the value of the objective functions is always minimized and the minimum demand value will be taken for giving the optimized solution.

$$c_{ij}^1 = \begin{pmatrix} [5,7] & [5,9] & [4,8] \\ [7,10] & [6,9] & [8,10] \\ [6,8] & [6,9] & [5,8] \end{pmatrix},\ l_{ij}^1 = \begin{pmatrix} [2,6] & [3,7] & [1,5] \\ [5,8] & [1,4] & [3,6] \\ [2,5] & [2,6] & [1,3] \end{pmatrix} \tag{11}$$

$$c_{ij}^2 = \begin{pmatrix} [1,4] & [1,5] & [1,3] \\ [2,5] & [3,6] & [4,6] \\ [1,5] & [1,6] & [2,4] \end{pmatrix},\ l_{ij}^2 = \begin{pmatrix} [0,3] & [0,4] & [0,2] \\ [1,3] & [2,5] & [2,5] \\ [1,4] & [0,2] & [0,3] \end{pmatrix} \tag{12}$$

$$T_{ij} = \begin{pmatrix} 8 & 9 & 7 \\ 10 & 12 & 11 \\ 7 & 8 & 6 \end{pmatrix} \tag{13}$$

The construction of MLFITP mathematical model from the numerical example is given below:

$$\text{minimize} \qquad Z_1(x) = \frac{Z_{11}(x)}{Z_{12}(x)} \tag{14}$$

$$\text{minimize} \qquad Z_2(x) = \frac{Z_{21}(x)}{Z_{22}(x)} \tag{15}$$

$$\text{subject to} \quad x_{11} + x_{12} + x_{13} \leq [40, 43], \tag{16}$$
$$x_{21} + x_{22} + x_{23} \leq [35, 40], \tag{17}$$
$$x_{31} + x_{32} + x_{33} \leq [52, 56], \tag{18}$$
$$x_{11} + x_{21} + x_{31} \geq [30, 32], \tag{19}$$
$$x_{12} + x_{22} + x_{32} \geq [29, 36], \tag{20}$$
$$x_{13} + x_{23} + x_{33} \geq [33, 37], \tag{21}$$
$$x_{ij} \geq 0, \text{ all are integers } (i = 1, 2, 3).$$

Where the numerator and denominator of each of the objective function are as follows:

$$\begin{aligned}
Z_{11}(x) &= [5,7]x_{11} + [5,9]x_{12} + [4,8]x_{13} + [7,10]x_{21} + [6,9]x_{22} + \\
&\quad [8,10]x_{23} + [6,8]x_{31} + [6,9]x_{32} + [5,8]x_{33}. \\
Z_{12}(x) &= [2,6]x_{11} + [3,7]x_{12} + [1,5]x_{13} + [5,8]x_{21} + [1,4]x_{22} + [3,6]x_{23} + \\
&\quad [2,5]x_{31} + [2,6]x_{32} + [1,3]x_{33}. \\
Z_{13}(x) &= [1,4]x_{11} + [1,5]x_{12} + [1,3]x_{13} + [2,5]x_{21} + [3,6]x_{22} + [4,6]x_{23} + \\
&\quad [1,5]x_{31} + [1,6]x_{32} + [2,4]x_{33}. \\
Z_{14}(x) &= [0,3]x_{11} + [0,4]x_{12} + [0,2]x_{13} + [1,3]x_{21} + [2,5]x_{22} + [2,5]x_{23} + \\
&\quad [1,4]x_{31} + [0,2]x_{32} + [0,3]x_{33}.
\end{aligned}$$

After converting the interval costs involved in the objective functions (14), (15) and the constraints (16)-(21) into the crisp numbers using the solution procedure mentioned in Section 3.1, we form the following MLFITP.

Model 6

$$\text{minimize} \quad \overline{Z}_1(x) = \frac{\overline{Z}_{11}(x)}{\overline{Z}_{12}(x)} \tag{22}$$
$$\text{minimize} \quad \overline{Z}_2(x) = \frac{\overline{Z}_{21}(x)}{\overline{Z}_{22}(x)} \tag{23}$$
$$\text{subject to} \quad x_{11} + x_{12} + x_{13} \leq 40(1 - \lambda^{a_1}) + 43\lambda^{a_1} \tag{24}$$
$$x_{21} + x_{22} + x_{23} \leq 35(1 - \lambda^{a_2}) + 40\lambda^{a_2} \tag{25}$$
$$x_{31} + x_{32} + x_{33} \leq 52(1 - \lambda^{a_3}) + 56\lambda^{a_3} \tag{26}$$
$$x_{11} + x_{21} + x_{31} \geq 30(1 - \lambda^{b_1}) + 32\lambda^{b_1} \tag{27}$$
$$x_{12} + x_{22} + x_{32} \geq 29(1 - \lambda^{b_2}) + 36\lambda^{b_2} \tag{28}$$
$$x_{13} + x_{23} + x_{33} \geq 33(1 - \lambda^{b_3}) + 37\lambda^{b_3} \tag{29}$$
$$0 \leq \lambda^{a_i} \leq 1, (i = 1, 2, 3), \tag{30}$$
$$0 \leq \lambda^{b_j} \leq 1, (i = 1, 2, 3), \tag{31}$$
$$x_{ij} \geq 0, \text{ all are integers } (i = 1, 2, 3). \tag{32}$$

The costs involved in the objective functions (22) and (23) are crisp numbers and $\overline{Z}_{11}(x)$, $\overline{Z}_{12}(x)$, $\overline{Z}_{21}(x)$ and $\overline{Z}_{22}(x)$ are given by the following expressions.

$$\begin{aligned}
\overline{Z}_{11}(x) &= \left(7 - \frac{T_0}{4}\right)x_{11} + \left(9 - \frac{4T_0}{9}\right)x_{12} + \left(8 - \frac{4T_0}{7}\right)x_{13} + \left(10 - \frac{3T_0}{7}\right)x_{21} + \left(9 - \frac{T_0}{4}\right)x_{22} \\
&\quad + \left(10 - \frac{2T_0}{11}\right)x_{23} + \left(8 - \frac{2T_0}{7}\right)x_{31} + \left(9 - \frac{3T_0}{8}\right)x_{32} + \left(8 - \frac{T_0}{2}\right)x_{33}.
\end{aligned}$$

$$\begin{aligned}
\overline{Z}_{12}(x) &= \left(6-\frac{T_0}{2}\right)x_{11}+\left(7-\frac{4T_0}{9}\right)x_{12}+\left(5-\frac{4T_0}{7}\right)x_{13}+\left(8-\frac{3T_0}{10}\right)x_{21}+\left(4-\frac{T_0}{4}\right)x_{22}\\
&\quad+\left(6-\frac{3T_0}{11}\right)x_{23}+\left(5-\frac{3T_0}{7}\right)x_{31}+\left(6-\frac{T_0}{2}\right)x_{32}+\left(3-\frac{T_0}{3}\right)x_{33}.\\
\overline{Z}_{21}(x) &= \left(4-\frac{3T_0}{8}\right)x_{11}+\left(5-\frac{4T_0}{9}\right)x_{12}+\left(3-\frac{2T_0}{7}\right)x_{13}+\left(5-\frac{3T_0}{10}\right)x_{21}+\left(6-\frac{T_0}{4}\right)x_{22}\\
&\quad+\left(6-\frac{2T_0}{11}\right)x_{23}+\left(5-\frac{4T_0}{7}\right)x_{31}+\left(6-\frac{5T_0}{8}\right)x_{32}+\left(4-\frac{T_0}{3}\right)x_{33}.\\
\overline{Z}_{22}(x) &= \left(3-\frac{3T_0}{8}\right)x_{11}+\left(4-\frac{4T_0}{9}\right)x_{12}+\left(2-\frac{2T_0}{7}\right)x_{13}+\left(3-\frac{T_0}{5}\right)x_{21}+\left(5-\frac{T_0}{4}\right)x_{22}\\
&\quad+\left(5-\frac{3T_0}{11}\right)x_{23}+\left(4-\frac{3T_0}{7}\right)x_{31}+\left(2-\frac{T_0}{4}\right)x_{32}+\left(3-\frac{T_0}{2}\right)x_{33}.
\end{aligned}$$

The basic feasible solution set $X^{(0)}$ of Model 6 using Northwest Corner Method is given as follows:

$$x^{(0)}=\begin{pmatrix}30 & 10 & 0\\ 0 & 19 & 16\\ 0 & 0 & 17\end{pmatrix},\ \lambda_{(0)}^{s_i}=\begin{pmatrix}0 & 0 & 0\end{pmatrix},\ \lambda_{(0)}^{d_j}=\begin{pmatrix}0 & 0 & 0\end{pmatrix}.$$

For the point $X^{(0)}$, the values of numerator and the denominator of the fractional objectives are calculated as $Z_{11}\big|_{X^{(0)}}=598.37,\ Z_{12}\big|_{X^{(0)}}=267.65,\ Z_{21}\big|_{X^{(0)}}=273.87,\ Z_{22}\big|_{X^{(0)}}=156.15$ and so $Z_1\big|_{X^{(0)}}=\frac{Z_{11}\big|_{X^{(0)}}}{Z_{12}\big|_{X^{(0)}}}=\frac{598.37}{267.65}=2.23,\ Z_2\big|_{X^{(0)}}=\frac{Z_{21}\big|_{X^{(0)}}}{Z_{22}\big|_{X^{(0)}}}=\frac{273.87}{156.15}=1.75$

Using the first order Taylor series about at the feasible point $X^{(0)}$, the objective functions of Model 6 can be constructed approximately as follows:

Model 7

$$\begin{aligned}
&\text{minimize} && \overline{Z}_1 \approx \sum_{i=1}^{m}\sum_{j=1}^{n}(\overline{c}_{ij}^1 Z_{12}-\overline{l}_{ij}^1 Z_{11})\big|_{X^{(0)}}x_{ij}\\
&\text{minimize} && \overline{Z}_2 \approx \sum_{i=1}^{m}\sum_{j=1}^{n}(\overline{c}_{ij}^2 Z_{22}-\overline{l}_{ij}^2 Z_{21})\big|_{X^{(0)}}x_{ij}\\
&\text{subject to} && \text{the constraints (24)} - (32).
\end{aligned}$$

Where $\overline{c}_{ij}^1,\ \overline{l}_{ij}^1,\ \overline{c}_{ij}^2$ and $\overline{l}_{ij}^2$ are the partial derivatives of $Z_{12},\ Z_{11},\ Z_{21}$ and Z_{22} respectively with respect to variable x_{ij}, i.e., $\overline{c}_{ij}^1=\frac{\partial Z_{12}}{\partial x_{ij}},\ \overline{l}_{ij}^1=\frac{\partial Z_{11}}{\partial x_{ij}},\ \overline{c}_{ij}^2=\frac{\partial Z_{21}}{\partial x_{ij}}$ and $\overline{l}_{ij}^2=\frac{\partial Z_{22}}{\partial x_{ij}}$.

Now we solve Model 7 using the fuzzy programming procedure presented in subsection 3.3.

To solve the MLFITP (i.e., Model 7) we apply the steps from Step 1 to Step 7 mentioned in subsection 3.3, we get $L_1=0,\ U_1=229.22,\ L_2=0,\ U_2=762.41$ and then construct the membership function. Finally the mathematical model is designed as follows:

$$\begin{aligned} \text{maximize} \quad & \lambda \\ \text{subject to} \quad & \mu_1(x) + \lambda(U_1 - L_1) \leq U_1, \\ & \mu_2(x) + \lambda(U_2 - L_2) \leq U_2, \\ & 0 \leq \lambda \leq 1, \\ & \text{the constraints } (24) - (32). \end{aligned}$$

The Pareto optimal solution $X^{(*)}$ of Model 6 using LINGO 14 iterative scheme is

$$x^{(*)} = \begin{pmatrix} 29 & 2 & 11 \\ 0 & 22 & 0 \\ 3 & 9 & 37 \end{pmatrix}, \ \lambda^{s_i}_{(*)} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}, \ \lambda^{d_j}_{(*)} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}.$$

Now we calculate $Z_{11}\big|_{X^{(*)}} = 737.37, \ Z_{12}\big|_{X^{(*)}} = 329.82, \ Z_{21}\big|_{X^{(*)}} = 347.68, \ Z_{22}\big|_{X^{(*)}} = 198.23$ and so $Z_1\big|_{X^{(*)}} = \frac{Z_{11}|_{X^{(*)}}}{Z_{12}|_{X^{(*)}}} = \frac{737.37}{329.82} = 2.23, \ Z_2\big|_{X^{(*)}} = \frac{Z_{21}|_{X^{(*)}}}{Z_{22}|_{X^{(*)}}} = \frac{347.68}{198.23} = 1.75$ and total time $T = 57$ days.

Taking different schedule time for transportation by using the fuzzy programming, the obtained solutions are presented in Table 2 with objective values and total transportation time.

Table 2. Pareto optimal solutions of the first example of time variant MLFITP.

Schedule time for transportation	**Optimal solution**	**Optimal value**
$T_0 = 8$	$x_{11} = 15, \ x_{12} = 1, \ x_{13} = 4$ $x_{21} = 10, \ x_{22} = 3, \ x_{23} = 12$ $x_{31} = 5, \ x_{32} = 25, \ x_{33} = 17.$	$Z_1 = 1.87, \ Z_2 = 1.91, \ T = 78$
$T_0 = 7$	$x_{11} = 1, \ x_{12} = 4, \ x_{13} = 10$ $x_{21} = 16, \ x_{22} = 4, \ x_{23} = 16$ $x_{31} = 15, \ x_{32} = 23, \ x_{33} = 10.$	$Z_1 = 2.21, \ Z_2 = 2.1, \ T = 78$
$T_0 = 6$	$x_{11} = 17, \ x_{12} = 11, \ x_{13} = 8$ $x_{21} = 1, \ x_{22} = 12, \ x_{23} = 24$ $x_{31} = 18, \ x_{32} = 11, \ x_{33} = 13.$	$Z_1 = 2.23, \ Z_2 = 1.72, \ T = 78$
$T_0 = 5$	$x_{11} = 5, \ x_{12} = 1, \ x_{13} = 24$ $x_{21} = 3, \ x_{22} = 11, \ x_{23} = 8$ $x_{31} = 25, \ x_{32} = 19, \ x_{33} = 10.$	$Z_1 = 2.23, \ Z_2 = 1.75, \ T = 78$
$T_0 = 4$	$x_{11} = 29, \ x_{12} = 2, \ x_{13} = 11$ $x_{21} = 0, \ x_{22} = 22, \ x_{23} = 0$ $x_{31} = 3, \ x_{32} = 9, \ x_{33} = 37.$	$Z_1 = 2.23, \ Z_2 = 1.75, \ T = 57$

4.2 Example 2

Let us consider another example of time variant MLFITP with three supply points and four destinations whose interval costs c^1_{ij}, l^1_{ij}, c^2_{ij} and l^2_{ij} and transportation time from different supply points to different destinations are as follows:

$$c^1_{ij}=\begin{pmatrix}[6,8] & [13,15] & [12,19] & [10,12]\\ [12,16] & [11,13] & [14,17] & [9,12]\\ [15,17] & [7,10] & [6,10] & [15,19]\end{pmatrix},\ l^1_{ij}=\begin{pmatrix}[5,6] & [12,14] & [11,14] & [9,11]\\ [10,15] & [9,12] & [12,16] & [8,14]\\ [13,17] & [6,9] & [5,12] & [14,18]\end{pmatrix},$$

$$c^2_{ij}=\begin{pmatrix}[3,4] & [7,9] & [6,8] & [7,9]\\ [5,10] & [5,11] & [8,12] & [5,8]\\ [4,6] & [6,8] & [7,9] & [12,15]\end{pmatrix},\ l^2_{ij}=\begin{pmatrix}[2,3] & [6,8] & [4,7] & [5,6]\\ [6,9] & [6,8] & [7,9] & [4,8]\\ [3,7] & [5,7] & [6,10] & [11,14]\end{pmatrix},$$

$$s_i=\begin{pmatrix}[50,60] & [70,75] & [80,90]\end{pmatrix},\ d_j=\begin{pmatrix}[30,35] & [50,60] & [42,48] & [35,45]\end{pmatrix}.$$

$$T_{ij}=\begin{pmatrix}6 & 8 & 12 & 7\\ 9 & 11 & 10 & 7\\ 8 & 7 & 9 & 14\end{pmatrix}. \tag{33}$$

The basic feasible solution set $X^{(0)}$ of the given problem using Northwest Corner Method is given as follows:

$$x^{(0)}=\begin{pmatrix}30 & 20 & 0 & 0\\ 0 & 30 & 40 & 0\\ 0 & 0 & 2 & 35\end{pmatrix},\ \lambda^{s_i}_{(0)}=\begin{pmatrix}0 & 0 & 0\end{pmatrix},\ \lambda^{d_j}_{(0)}=\begin{pmatrix}0 & 0 & 0 & 0\end{pmatrix}.$$

The Pareto optimal solution $X^{(*)}$ of the above problem using LINGO 14 iterative scheme with schedule transportation time 5 days is

$$x^{(*)}=\begin{pmatrix}5 & 3 & 0 & 33\\ 22 & 50 & 0 & 1\\ 4 & 21 & 61 & 3\end{pmatrix},\ \lambda^{s_i}_{(*)}=\begin{pmatrix}1 & 1 & 1\end{pmatrix},\ \lambda^{d_j}_{(*)}=\begin{pmatrix}0 & 0 & 0 & 0\end{pmatrix}.$$

Thereafter, we calculate $Z_{11}\big|_{X^{(*)}}=2085.69$, $Z_{12}\big|_{X^{(*)}}=1936.85$, $Z_{21}\big|_{X^{(*)}}=1748.46$, $Z_{22}\big|_{X^{(*)}}=1399.26$ and so $Z_1\big|_{X^{(*)}}=\frac{Z_{11}\big|_{X^{(*)}}}{Z_{12}\big|_{X^{(*)}}}=\frac{2085.69}{1936.85}=1.07$, $Z_2\big|_{X^{(*)}}=\frac{Z_{21}\big|_{X^{(*)}}}{Z_{22}\big|_{X^{(*)}}}=\frac{1748.46}{1399.26}=1.24$ and total time $T=86$ days.

Again choosing the different schedule times for transportation by using the fuzzy programming approach the obtained solutions are presented in Table 3 with objective values and total transportation time.

4.3 Sensitivity Analysis

We solve Examples 1 and 2 with the help of the presented algorithm; the solutions are obtained for different schedule transportation times, and these are listed in Table 2 and Table 3 respectively. From Table 2 and Table 3, we see that the obtained solutions are satisfactory based on the relative importance of the scheduled transportation time. Although we list fewer, one can set schedule transportation time for their preferred transportation time to get better optimal solution. We see that when scheduled transportation times are 4 days and 8 days for the first and second examples respectively then we derive better compromise solutions and the total times of transportation are 57 days and 78 days respectively. The time is calculated through the procedure given in the algorithm, and we do not consider any objective function

Table 3. Pareto optimal solutions of the second example of time variant MLFITP.

Schedule time for transportation	**Optimal solution**	**Optimal value**
$T_0 = 8$	$x_{11} = 0,\ x_{12} = 41,\ x_{13} = 2,\ x_{14} = 1$ $x_{21} = 45,\ x_{22} = 27,\ x_{23} = 0,\ x_{24} = 0$ $x_{31} = 1,\ x_{32} = 0,\ x_{33} = 43,\ x_{34} = 37.$	$Z_1 = 1.07,\ Z_2 = 1.24,\ T = 78$
$T_0 = 7$	$x_{11} = 3,\ x_{12} = 31,\ x_{13} = 0,\ x_{14} = 14$ $x_{21} = 26,\ x_{22} = 42,\ x_{23} = 3,\ x_{34} = 1$ $x_{31} = 2,\ x_{32} = 4,\ x_{33} = 56,\ x_{34} = 28.$	$Z_1 = 1.07,\ Z_2 = 1.24,\ T = 96$
$T_0 = 6$	$x_{11} = 0,\ x_{12} = 29,\ x_{13} = 6,\ x_{14} = 12$ $x_{21} = 40,\ x_{22} = 33,\ x_{23} = 0,\ x_{24} = 0$ $x_{31} = 1,\ x_{32} = 0,\ x_{33} = 39,\ x_{34} = 23.$	$Z_1 = 1.07,\ Z_2 = 1.24,\ T = 78$
$T_0 = 5$	$x_{11} = 5,\ x_{12} = 3,\ x_{13} = 0,\ x_{14} = 33$ $x_{21} = 22,\ x_{22} = 50,\ x_{23} = 0,\ x_{24} = 1$ $x_{31} = 4,\ x_{32} = 21,\ x_{33} = 61, x_{34} = 3.$	$Z_1 = 1.07,\ Z_2 = 1.24,\ T = 86$
$T_0 = 4$	$x_{11} = 7,\ x_{12} = 0,\ x_{13} = 3,\ x_{14} = 18$ $x_{21} = 22,\ x_{22} = 47,\ x_{23} = 3,\ x_{24} = 0$ $x_{31} = 4,\ x_{32} = 15,\ x_{33} = 36,\ x_{34} = 27.$	$Z_1 = 1.08,\ Z_2 = 1.24,\ T = 93$

corresponding to time. In the proposed model, the optimal solution minimizes total time with satisfactory schedule transportation time. From Table 2 and Table 3, it is clear that when the number of allocations is maximum i.e., transportation is done in maximum destination points, then the total transportation time is more as routes are involved in transporting the goods. Since there are many routes (between each origin and destination) and DM would like to prefer the routes so that transportation is made as per their experience. So in the numerical examples, we consider an extra charge in transportation cost and damage cost in our study. If a customer wants to reduce the rate of damage of goods and if he/she wants to reduce transportation cost then DM prefers such routes where the condition of the routes is good so that both the conditions are satisfied. Since in different routes, transportation times are different which are known by DM experience and listed in equation (13) for example 1 and in equation (33) for example 2 so the total transportation time is dependent on the routes which are allocated after solving the problem.

In this study, the most relevant considerations are how time is associated with the other objective functions and how to find the optimal compromise solution. Time in an MLFITP essentially makes a distinction between transportation cost and damage cost for transportation. We include here how time plays a significant role in MLFITP. The time reduces the transportation parameters in such a way that the reduced values serve as neutral on both the seller and buyer sides. The suggested model in this paper removes the complexity of optimizing the objective functions together with reducing the time.

To justify that the obtained solution of our proposed methodology is a better solution, let us introduce a utility function in the following form as:

$$R(\sigma) = \sigma_1 \frac{\overline{Z}_1 - Z_1}{\overline{Z}_1 - \underline{Z}_1} + \sigma_2 \frac{\overline{Z}_2 - Z_2}{\overline{Z}_2 - \underline{Z}_2} + \sigma_3 \frac{\overline{T} - T}{\overline{T} - \underline{T}} \tag{34}$$

where $\overline{Z}_k$= maximum value of Z_k for k^{th} $(k = 1, 2)$ objective function, $\overline{T}$ = maximum possible value of T, $\underline{Z}_k$ = minimum value of Z_k for k^{th} objective function, $\underline{T}$ = minimum transportation time, and σ_k $(k = 1, 2)$ and σ_3 are weights for the objective function Z_k $(k = 1, 2)$ and time T. The value of the utility function R lies between 0 and 1. For Example 1, the different values of R for different schedule transportation times are listed in Table 4. The bigger value of R proposes a better compromise solution of the MLFTP. If we consider the objective values for Example 1 obtained by taking schedule transportation time $T_0 = 4$ days with equal weights in equation (34), then we get $R(\sigma) = 0.63$. This value suggests that the obtained solution is a better compromise solution. Similarly if we consider the objective values for Example 2 obtained by taking schedule transportation time $T_0 = 6$ or 8 days with equal weights in equation (34), then we get $R(\sigma) = 0.66$. This value indicates that the obtained solution is a better compromise solution.

Table 4. Values of $R(\sigma)$ for different schedule transportation time for Example 1.

Schedule transportation time	**Value of $R(\sigma)$**	**Weights**
$T_0 = 8$	0.50	$\sigma_1 = \sigma_2 = \sigma_3 = \frac{1}{3}$
$T_0 = 7$	0.01	$\sigma_1 = \sigma_2 = \sigma_3 = \frac{1}{3}$
$T_0 = 6$	0.33	$\sigma_1 = \sigma_2 = \sigma_3 = \frac{1}{3}$
$T_0 = 5$	0.30	$\sigma_1 = \sigma_2 = \sigma_3 = \frac{1}{3}$
$T_0 = 4$	0.63	$\sigma_1 = \sigma_2 = \sigma_3 = \frac{1}{3}$

4.4 Importance and advantage of the study with the existing studies

We observe from the literature review that many researchers developed several methods for solving fractional TP in different environments. However in literature there is a gap of study in time environment for the fractional TP. In this study we remove this gap by introducing time in the fractional TP. In the transportation system there is minimum transportation time to reach the goods in proper destination to smoothly flow the business. However there is a delay of time for transportation due to several problems. So depending on this fact, here we introduce new parameters. Then we utilize these parameters as transportation costs in such a way that if the delivery occurs within schedule transportation time then the businessman will pay minimum costs which already fixed before delivery otherwise he/she pays less amount that depends on delay of time. The main advantages of our proposed method are given as:

- In MLFITP, all the parameters and variables are interval numbers which accommodate more information from real-life scenario.

- We transform the interval-valued cost parameters into crisp parameters with new transformation technique.

- The Pareto-optimal solution generates that depends on schedule transportation time.

§5 Conclusion and outlook

The time-variant multi-objective fractional transportation problem with the coefficients of the objective functions and the supply and the demand parameters have been chosen as interval type and then it has been solved. Initially, the time variant fractional transportation problem has been linearized using the Taylor series expansion then the solution has been obtained by fuzzy programming. We have considered two numerical examples to test the effectiveness of our proposed methodology. The first and second examples have been solved using different schedules of transportation times then we have compared the obtained solutions. In this work, we have determined a Pareto optimal solution of MLFITP using fuzzy programming that minimizes the total time without considering the objective function of time in the model of time-variant MLFITP. Several researchers have developed several methods for solving fractional TP in different environments. However in literature there is a gap of study in time environment for the fractional TP. This study has been removed this gap by introducing time in the fractional TP. The research contributions of the proposed study are summarized below:

- It applies new transformation technique to convert interval costs into crisp numbers.
- It minimizes total transportation time without considering any objective function.
- Pareto-optimality is guaranteed, which depends on schedule transportation time.

A few research directions can be opened based on our proposed methodology. MLFITP should be integrated in various regions of study, for example, economics and production planning. We must emphasize that in association with this paper, there is a line of future research directions to include the two-phase fractional TP under the time window. Interested readers can formulate a multi-objective two-stage fractional transportation model [37] and multi-objective fixed-charge solid fractional transportation problem with product blending under intuitionistic fuzzy environment [38] under the time window and tackle it by our methodology or some other appropriate methodology. In addition to the above, the proposed study can be implemented in several uncertain ([8], [25]) circumstances to accommodate real-life situations for selecting optimal decisions considering the sustainable development of the atmosphere.

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