Appl. Math. J. Chinese Univ.
2021, 36(4): 554-563

Exact solutions of conformable time fractional Zoomeron equation via IBSEFM

Ulviye Demirbilek[1] Volkan Ala[1] Khanlar R. Mamedov[2]

Abstract. The nonlinear conformable time-fractional Zoomeron equation is an important model to describe the evolution of a single scalar field. In this paper, new exact solutions of conformable time-fractional Zoomeron equation are constructed using the Improved Bernoulli Sub-Equation Function Method (IBSEFM). According to the parameters, 3D and 2D figures of the solutions are plotted by the aid of Mathematics software.The results show that IBSEFM is an efficient mathematical tool to solve nonlinear conformable time-fractional equations arising in mathematical physics and nonlinear optics.

§1 Introduction

Seeking the exact travelling solutions of nonlinear partial differential equations is very important to understand the nonlinear process that appears in many areas of science. To find the exact solutions of these nonlinear equations, many powerful methods have been applied by mathematicians ([5],[15-17],[20]). The investigation of exact solutions for fractional differential equations (FDEs) is nowadays one of the main research topics. This importance is due to the fact that FDEs are widely used to describe several important processes in many fields, such as biology [33], physics [26], biomedicine [13], finance [10]. Several powerful methods have been applied in the literature to obtain the exact solutions of FDEs. Some of these effective methods are; (G'/G) [30], fractional Riccati expansion [12], generalized projective Riccati equation [28], functional variable [23], the exp-function [8], modified Kudryashov [14], extended direct algebraic [32], modified trial equation [25], modified $exp(-\Omega(\xi))$ function [34] and Improved Bernoulli Sub-Equation Function Method (IBSEFM) ([4],[9],[36],[37]).

In [19], a new significant definition of the fractional derivative called conformable fractional derivative is introduced. The conformable fractional derivative is theoretically easier than fractional derivative to handle. In addition, the conformable fractional derivative satisfies many

Received: 2020-05-11. Revised: 2021-03-17.
MR Subject Classification: 35C08, 34K20, 32W50.
Keywords: conformable time-fractional derivative, zoomeron equation, IBSEFM.
Digital Object Identifier(DOI): https://doi.org/10.1007/s11766-021-4145-3.

known features that can't be satisfied by the existing fractional derivatives, for instance; the chain rule [2].

The conformable fractional derivative has the weakness that the fractional derivative of differentiable function at the point zero is equal to zero. So that in [7] a suitable fractional derivative is proposed that allows us to escape the lack of the conformable fractional derivative. During the last years, many of techniques where applied to find exact solutions for conformable fractional nonlinear partial differential equations in literature ([3],[27],[29],[35],[36]).

In this paper, we obtain the exact solutions of (2+1)-dimensional conformable time fractional Zoomeron equation via IBSEFM. We consider

$$\frac{D^{2\alpha}u}{D_t^{2\alpha}u}\left[\frac{u_{xy}}{u}\right]-\frac{D^2u}{Dx^2}\left[\frac{u_{xy}}{u}\right]+2\frac{D^{\alpha}u}{D_t^{\alpha}u}\left[u^2\right]_x=0, \quad 0<\alpha\leq 1, \tag{1}$$

where $u(x,y,t)$ shows amplitude of the relevant wave model. This equation is a convenient model to present the novel phenomena related with boomerons and trappons and it describes the evolution of a single scalar field [21]. Many authors investigated the exact solutions of Zoomeron equation with different methods ([1],[6],[22],[24],[31]). Before beginning to the solution procedure, we should give some important and efficient properties of conformable fractional derivative.

§2 Conformable Fractional Derivative

In this section, we give some basic definition, properties and theorems about the conformable fractional derivative.

The conformable derivative of order α with respect to the independent variable t is defined as in [19]

$$D_t^a(y(t))=\lim_{\tau\to 0}\frac{y(t+\tau t^{1-\alpha})-y(t)}{\tau},\ t>0,\ \alpha\in(0,1],$$

for a function $y=y(t):[0,\infty)\to\mathbb{R}$.

Theorem 1. *Assume that the order of the derivative $\alpha\in(0,1]$ and suppose that $f=f(t)$ and $g=g(t)$ are $\alpha-$differentiable for all positive t. Then,*

1. $D_t^a(c_1f+c_2g)=c_1D_t^a(f)+c_2D_t^a(g),\ \forall\ c_1,c_2\in\mathbb{R}$.

2. $D_t^a(t^k)=kt^{k-\alpha},\ \forall\ k\in\mathbb{R}$.

3. *For all constant function* $f(t)=\lambda,\ D_t^a(\lambda)=0$.

4. $D_t^a(fg)=fD_t^a(g)+gD_t^a(f)$.

5. $D_t^a\left(\frac{f}{g}\right)=\frac{gD_t^a(f)-fD_t^a(g)}{g^2}$.

Conformable fractional differential operator satisfies certain basic features like the chain rule, Taylor series expansion and Laplace transform.

Theorem 2. *Let $f=f(t)$ be an $\alpha-$conformable differentiable function and assume that g is differentiable and defined in the range of f. Then,*

$$D_t^a(f\circ g)(t)=t^{1-\alpha}g'(t)f'(g(t)).$$

The proofs of these theorems are given in [7] and in [2] respectively.

§3 Description of the IBSEFM

In this part, let us give the fundamental properties of the IBSEFM ([4],[9],[11],[18]). We present the six main steps of the IBSEFM below the following:

Step 1: Let us take account of the following conformable time fractional partial differential equation of the style

$$P(v, D_t^{(\mu)} v, D_x^{(\mu)} v, D_{xt}^{(2\mu)} v, D_{xxt}^{(3\mu)} v, ...) = 0, \tag{2}$$

where $D_t^{(\mu)}$ is the conformable derivative operator, $v(x,t)$ is an unknown function, P is a polynomial in v and its partial derivatives contain fractional derivatives. The aim is to convert (2) with a suitable fractional transformation into the nonlinear ordinary differential equation. The wave transformation as

$$v(x,t) = V(\xi), \quad \xi = \left(x - kt^{\alpha}\alpha^{-1}\right), \tag{3}$$

where k is an arbitrary constant and different from zero. Using the properties of conformable derivative, it enables us to convert (3) into a nonlinear ordinary differential equation in the form

$$N(V, V', V'', ...) = 0. \tag{4}$$

Step 2: If we integrate (4) term to term once or more, we acquire integration constant(s) which may be determined then.

Step 3: We hypothesize that the solution of (4) may be presented as below

$$V(\xi) = \frac{\sum_{i=0}^{n} a_i F^i(\xi)}{\sum_{j=0}^{m} b_j F^j(\xi)} = \frac{a_0 + a_1 F(\xi) + a_2 F^2(\xi) + ... a_n F^n(\xi)}{b_0 + b_1 F(\xi) + b_2 F^2(\xi) + ... b_m F^m(\xi)}, \tag{5}$$

where $a_0, a_1, ..., a_n$ and $b_0, b_1, ..., b_m$ are coefficients which will be determined later. $m \neq 0, n \neq 0$ are chosen arbitrary constants to balance principle and considering the form of Bernoulli differential equation below the following;

$$F'(\xi) = \sigma F(\xi) + dF^M(\xi), \; d \neq 0, \sigma \neq 0, \; M \in \mathbb{R}/\{0,1,2\}, \tag{6}$$

where $F(\xi)$ is polynomial.

Step 4: The positive integer m, n, M (are not equal to zero) which is found according to the balance principle that is both nonlinear term and the highest order derivative term of (4).

Substituting (5) and (6) in (3) it yields us an equation of polynomial $\Theta(F)$ of F as following;

$$\Theta(F(\xi)) = \rho_s F(\xi)^s + ... + \rho_1 F(\xi) + \rho_0 = 0,$$

where $\rho_i, i = 0, ..., s$ are coefficients and will be determined later.

Step 5: The coefficients of $\Theta(F(\xi))$ which will give us a system of algebraic equations, whole be zero.

$$\rho_i = 0, i = 0, .., s.$$

Step 6: When we solve (4), we get the following two cases with respect to σ and d,

$$F(\xi)=\left[\frac{-de^{\sigma(\epsilon-1)}+\epsilon\sigma}{\sigma e^{\sigma(\epsilon-1)\xi}}\right]^{\frac{1}{1-\epsilon}},\quad d\neq\sigma, \tag{7}$$

$$F(\xi)=\left[\frac{(\epsilon-1)+(\epsilon+1)\tanh\left(\sigma(1-\epsilon)\frac{\xi}{2}\right)}{1-\tanh\left(\sigma(1-\epsilon)\frac{\xi}{2}\right)}\right], d=\sigma,\ \epsilon\in\mathbb{R}. \tag{8}$$

Using a complete discrimination system for polynomial of $F(\xi)$, we obtain the analytical solutions of (4) via mathematics software and categorize the exact solutions of (4). To achieve better results, we can plot two and three dimensional figures of analytical solutions by considering proper values of parameters.

§4 Application of the Improved Bernoulli Sub-Equation Function Method (IBSEFM)

In this section, the application of the IBSEFM to the conformable time fractional Zoomeron equation is given. Let us consider the following wave transform:

$$u\left(x,y,t\right)=U\left(\xi\right),\ \xi=kx+my-l\left(\frac{t^{\alpha}}{\alpha}\right), \tag{9}$$

where k,m,l are nonzero constants. Substituting (9) into (1), we obtain the following equation:

$$kml^{2}\left[\frac{U''}{U}\right]''-k^{3}m\left[\frac{U''}{U}\right]-2kl\left[U^{2}\right]''=0. \tag{10}$$

If we integrate the equation (10) with respect to ξ twice, we get

$$km(k^{2}-l^{2})U''+2klU^{3}+sU=0, \tag{11}$$

where s is a nonzero constant of integration and the second constant of integration vanishes.

When we reconsider (11) for balance principle, considering between U'' and U^3 we obtain the following relationship for m,n and M :

$$M=n-m+1. \tag{12}$$

(12) gives us different cases of the solution of (11) and we can obtain some analytical solutions as follows:

If we take $M=n=3, m=1$ for (5) and (6), then we can write the following equations;

$$U(\xi)=\frac{a_{0}+a_{1}F(\xi)+a_{2}F^{2}(\xi)+a_{3}F^{3}(\xi)}{b_{0}+b_{1}F(\xi)}=\frac{\Upsilon(\xi)}{\Psi(\xi)}, \tag{13}$$

$$U'(\xi)=\frac{\Upsilon'(\xi)\Psi(\xi)-\Upsilon(\xi)\Psi'(\xi)}{\Psi^{2}(\xi)}, \tag{14}$$

and

$$U''(\xi)=\frac{\Upsilon'(\xi)\Psi(\xi)-\Upsilon(\xi)\Psi'(\xi)}{\Psi^{2}(\xi)}-\frac{\left[\Upsilon(\xi)\Psi'(\xi)\right]'\Psi^{2}(\xi)-2\Upsilon(\xi)[\Psi'(\xi)]^{2}\Psi(\xi)}{\Psi^{4}(\xi)}, \tag{15}$$

where $F'=\sigma F+dF^{3}$, $a_{2}\neq0,\ b_{1}\neq0,\ \sigma\neq0,\ d\neq0$. Using (13)-(15) in (11), we obtain a system of algebraic equations from coefficients of F.

$Constant: -2kla_{0}^{3}-sa_{0}b_{0}^{2}=0,$

$F: -6kla_0^2a_1 - sa_1b_0^2 - k^3m\sigma^2a_1b_0^2 + kl^2m\sigma^2b_0^2 - 2sa_0b_0b_1 + k^3ma_0b_0b_1 - kl^2m\sigma^2a_0b_0b_1 = 0,$
$F^2: -6kla_0^2a_1^2 - 6kla_0^2a_2 - sa_2b_0^2 - 4k^3m\sigma^2a_2b_0^2 + 4kl^2m\sigma^2a_2b_0^2 - 2sa_1b_0b_1 + k^3m\sigma^2a_1b_0b_1$
$-sa_0b_1^2 - k^3m\sigma^2a_0b_1^2 + kl^2m\sigma^2a_0b_1^2 = 0,$
$F^3: -2kla_1^3 - 12kla_0a_1a_2 - 6kla_0^2a_3 - 4dk^3m\sigma a_2b_0^2 + 4dkl^2m\sigma a_1b_0^2 - sa_3b_0^2 - 9k^3m\sigma^2a_3b_0^2$
$+9kl^2m\sigma^2a_3b_0^2 + 4dk^3m\sigma a_0b_0b_1 - 4dkl^2m\sigma a_0b_0b_1 - 2sa_2b_0b_1 - 3k^3m\sigma^2a_2b_0b_1$
$+3kl^2m\sigma^2a_2b_0b_1 - sa_1b_1^2 = 0,$
$F^4: -6kla_1^2a_2 - 6kla_0a_2^2 - 12kla_0a_1a_3 - 12dk^3m\sigma a_2b_0^2 + 12dkl^2m\sigma a_2b_0^2 - 2sa_3b_0b_1$
$-11k^3m\sigma^2a_3b_0b_1 + 11kl^2m\sigma^2a_3b_0b_1 - sa_2b_0b_1^2 - k^3m\sigma^2a_2b_1^2 + kl^2m\sigma^2a_2b_1^2 = 0,$
$F^5: -6kla_1a_2^2 - 6kla_1^2a_3 - 12kla_0a_2a_3 - 3d^2k^3m\sigma a_1b_0^2 + 3dkl^2m\sigma a_1b_0^2 - 24dkl^2m\sigma a_3b_0^2$
$+3d^2k^3ma_0b_0b_1 - 3d^2kl^2ma_0b_0b_1 - 12dk^3ma_2b_0b_1 + 12dkl^2m\sigma a_2b_0b_1 - sa_3b_1^2 - 4k^3m\sigma^2a_3b_1^2$
$+4kl^2m\sigma^2a_3b_1^2 = 0,$
$F^6: -2kla_2^3 - 12kla_1a_2a_3 - 6kla_0a_3^2 - 8d^2k^3ma_2b_0^2 + 8d^2kl^2ma_2b_0^2 - d^2k^3ma_1b_0b_1$
$+d^2kl^2ma_1b_0b_1 - 32dk^3m\sigma a_3b_0b_1 + 32dkl^2m\sigma a_3b_0b_1 + d^2k^3ma_0b_1^2 - d^2kl^2ma_0b_1^2$
$-4dk^3m\sigma a_2b_1^2 + 4dkl^2m\sigma a_2b_1^2 = 0,$
$F^7: -6kla_2^3a_3 - 6kla_1a_3^2 - 15d^2k^3ma_3b_0^2 + 15d^2kl^2ma_3b_0^2 - 9d^2k^3ma_2b_0b_1 + 9d^2kl^2ma_2b_0b_1$
$-12dk^3m\sigma a_3b_1^2 + 12dkl^2m\sigma a_3b_1^2 = 0,$
$F^8: -6kla_2a_3^2 - 21d^2k^3ma_3b_0b_1 + 21d^2kl^2ma_3b_0b_1 - 3d^2k^3ma_2b_1^2 + 3d^2kl^2ma_2b_1^2 = 0,$
$F^9: -2kla_3^3 - 8d^2k^3ma_3b_1^2 + 8d^2kl^2ma_3b_1^2 = 0.$

Solving the above system of equations with the help of mathematics programme, it yields us the following coefficients:

Case 1: For $\sigma \neq d$;

$$a_0 = \frac{i\sqrt{s}b_0}{\sqrt{2}\sqrt{k}\sqrt{l}}; a_1 = \frac{i\sqrt{s}b_1}{\sqrt{2}\sqrt{k}\sqrt{l}}; a_2 = \frac{i\sqrt{2}d\sqrt{s}b_0}{\sqrt{k}\sqrt{l}\sigma}; a_3 = \frac{i\sqrt{2}d\sqrt{s}b_1}{\sqrt{k}\sqrt{l}\sigma}; m = \frac{s}{2k(k^2-l^2)\sigma^2}.$$

Substituting these coefficients along with (7) in (13) we obtain the complex exponential function solution of the conformable time fractional Zoomeron equation as follows:

$$u_1(x,y,t) = \frac{i\sqrt{s}\left(1 + \frac{2d}{-d+\epsilon\sigma exp\left(-2kx\sigma + \frac{2lt^\alpha\sigma}{\alpha} - \frac{sy}{k^3\sigma - kl^2\sigma}\right)}\right)}{\sqrt{2kl}}.$$

Case 2: For $\sigma \neq d$;

$$a_0 = \frac{\sqrt{-k^2+l^2}\sqrt{m}\sqrt{s}b_0}{\sqrt{2l}\sqrt{k^3m-kl^2m}}; a_1 = \frac{\sqrt{-k^2+l^2}\sqrt{m}\sqrt{s}b_1}{\sqrt{2}\sqrt{l}\sqrt{k^3m-kl^2m}}; a_2 = -\frac{2d\sqrt{-k^2+l^2}\sqrt{m}b_0}{\sqrt{l}};$$
$$a_3 = -\frac{2d\sqrt{-k^2+l^2}\sqrt{m}b_1}{\sqrt{l}}; \sigma = -\frac{\sqrt{s}}{\sqrt{2}\sqrt{k^3m-kl^2m}}.$$

Putting these coefficients along with (7) in (13) we obtain the exact solution of (1) as follows:

$$u_2(x,y,t) = \frac{\sqrt{-k^2+l^2}\sqrt{m}\left(\sqrt{2}\sqrt{s} - \frac{4d\sqrt{k(k-1)(k+1)m}}{\frac{\sqrt{2}d\sqrt{k(k-1)(k+1)m}}{\sqrt{s}} + \epsilon exp\left(\frac{\sqrt{2}\sqrt{s}\left(kx+my-\frac{lt^\alpha}{\alpha}\right)}{\sqrt{k(k-1)(k+1)m}}\right)}\right)}{2\sqrt{l}\sqrt{k(k-1)(k+1)m}}.$$

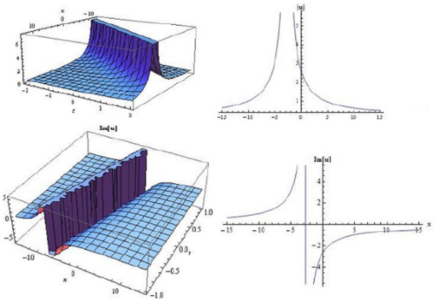

Figure 1. The 3D and 2D graphs of $|u_1(x,y,t)|$ and $u_1(x,y,t)$ considering the values $y = 0.5$; $k = 0.1$; $s = 0.01$; $d = 0.4$; $\sigma = 0.5$; $\alpha = 1$.

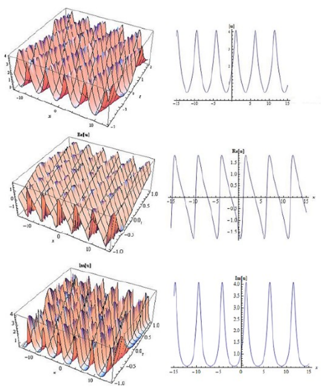

Figure 2. The 3D and 2D graphs of the solution of $|u_2(x,y,t)|$, real and imaginary part of $u_2(x,y,t)$ considering the values $y = 0.2$; $k = 0.2$; $s = 0.3$; $d = 0.6$; $m = 0.7$; $l = 0.4$; $\alpha = 1$; $\epsilon = 0.1$; $-15 < x < 15$, $-1 < t < 1$ for 3D and $t = 0.2$ for 2D.

Case 3: For $\sigma \neq d$;

$$a_0 = \frac{i\sqrt{s}b_0}{\sqrt{2kl}}; a_1 = \frac{i\sqrt{s}b_0}{\sqrt{2kl}}; a_3 = \frac{a_2 b_1}{b_0}; m = \frac{a_2^2}{4d^2(-k^2+l^2)b_0^2}; \sigma = -\frac{i\sqrt{2}d\sqrt{s}b_0}{\sqrt{kl}a_2}.$$

Substituting above the coefficients along with (7) in (13) we obtain the exact solutions of (1) as follows:

$$u_3(x,y,t) = \frac{i\sqrt{s}}{\sqrt{2kl}} + \frac{1}{\frac{i\sqrt{kl}}{\sqrt{2s}} + \frac{e^{-\frac{2id\sqrt{2s}\left(kx - \frac{lt^\alpha}{\alpha} + \frac{lya_s^2}{4d^2(-k^2+l^2)b_0^2}\right)b_0}{\sqrt{kl}a_2}}}{a_2}\varepsilon b_0}.$$

Now, let us show the 3D and 2D figures of the solutions plotted with the help of mathematics software:

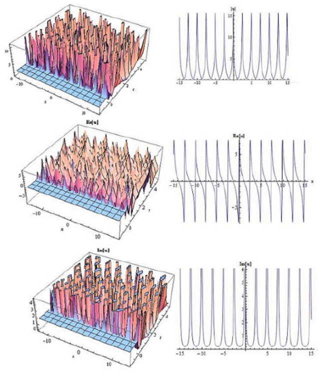

Figure 3. The 3D and 2D graphs of the solution of $|u_3(x, y, t)|$, real and imaginary part of $u_3(x, y, t)$ $y = 0.1$; $k = 0.2$; $b_0 = 1$; $d = 0.6$; $s = 0.7$; $l = 0.5$; $\alpha = 0.5$; $\epsilon = 0.2$; $-15 < x < 15$, $-1 < t < 1$ for 3D and $t = 0.1$ for 2D.

§5 Discussion, comparision and physical explanations

In this article, we have successfully applied the IBSEFM to the nonlinear conformable time fractional Zoomeron equation to investigate some new exact solutions. It has been observed that all analytical solutions examined in this paper verify the nonlinear ordinary differential equation (11) which is obtained from nonlinear conformable time-fractional Zoomeron equation under the terms of wave transformation. All necessary computational calculations and graphs have been acquired by using mathematics software.

Figures 1-3 give a good impression about the change of amplitude and width of the soliton due to the variation of the fractional order. Remarkably, 3D graphs describe the behavior of $u(x, y, t)$ in space x and y at time t corresponding to the value of the fractional order.The behavior represents that an increase of the fractional parameter changes the nature of the solitary wave solution. The nature of the solitary wave solution of the fractional order is confirmed by 2D line plots. Therefore, the fractional order derivative can be used to modulate the shape of the waves.

Furthermore, the nature of the waves are affected from the value of coefficients of the linear and nonlinear term of (11). According to the figures, one can see that the formats of travelling wave solutions in two and three dimensional surfaces are similar to the physical meaning of results. If we take more values of coefficients, we can obtain more travelling wave solutions.

§6 Conclusion

By using the Improved Bernoulli Sub-Equation Function method, we obtain exact traveling wave solutions for the (2+1)-dimensional conformable timefractional Zoomeron equation under the given parameter conditions. For this equation many exact solutions have been obtained which include hyperbolic function solutions, Jacobi elliptic function solutions, trigonometric function solutions and rational function solutions in literature. Compared with the previous works, the solution method obtained in this paper has not been reported. Hence, this method is very reliable, efficient and submits new travelling wave solutions. Therefore, the IBSEFM can be applied to the other nonlinear fractional differential models in mathematical physics and nonlinear optics.

References

[1] R Abazari. *The solitary wave solutions of Zoomeron equation*, Appl Math, 2011, 59(5): 2943-2949.

[2] T Abdeljawad. *On conformable fractional calculus*, J Comput Appl Math, 2015, 279: 57-66.

[3] A Akbulut, M Kaplan. *Auxiliary equation method for time-fractional differential equations with conformable derivative*, Comput Math Appl, 2018, 75: 876-882.

[4] V Ala, U Demirbilek, K Mamedov. *An application of improved Bernoulli sub-equation function method to the nonlinear conformable time-fractional SRLW equation*, AIMS Mathematics, 2020, 5(4): 3751-3761.

[5] A T Ali, E R Hassan. *General Exp_a-function method for nonlinear evolution equations*, Applied Mathematics and Computation, 2010, 217: 451-459.

[6] M Alquran, K Al-Khaled. *Mathematical methods for a reliable treatment of the (2+1)- dimensional Zoomeron equation*, Math Sci, 2012, 6(11), https://doi.org/10.1186/2251-7456-6-11.

[7] A Atangana, D Baleanu, A Alsaedi. *New properties of conformable derivative*, Open Math, 2015, 13: 1-10.

[8] Z Bin. *Exp-function method for solving fractional partial differential equations*, The Sci World J, 2013, 2013: 1-8.

[9] H Bulut, G Yel, H M Baskonuş. *An Application of Improved Bernoulli Sub-Equation Function Method to The Nonlinear Time-Fractional Burgers Equation*, Turk J Math Comput Sci, 2016, 5: 1-7.

[10] W Chen. *Nonlinear dynamics and chaos in a fractional-order financial system*, Chaos, Solitons and Fractals, 2008, 36(5): 14305-1314.

[11] F Duşunceli, E Çelik, M Aşkın, H Bulut. *New exact solutions for the doubly dispersive equation using the improved Bernoulli sub-equation function method*, Indian J Phys, 2021, 95 (2): 309-314.

[12] A Emad, B Abdel-Salam, A Y Eltayeb. *Solution of nonlinear space-time fractional differential equations using the fractional Riccati expansion method*, Math Probl Eng, 2013, 2013: 1-6.

[13] Y Ferdi. *Some applications of fractional order calculus to design digital filters for biomedical signal processing*, Journal of Mechanics in Medicine and Biology, 2012, 12(2), https://doi.org/10.112/S0219519412400088.

[14] K Hosseini, R Ansari. *New exact solutions of nonlinear conformable time fractional Boussinesq equations using the modified Kudryashov method*, Waves Random Complex Media, 2017, 27: 628-636.

[15] K Hosseini, M Mirzazadeh, J F G Aguilar. *Soliton solutions of the Sasa-Satsuma equation in the monomode optical fibers including the beta-derivatives*, Optik, 2020, 224: 165425.

[16] K Hosseini, M Mirzazadeh, J Vahidi, R Asghari. *Optical wave structures to the Fokas-Lenells equation*, Optik, 2020, 207: 164450.

[17] K Hosseini, M S Osman, M Mirzazadeh, F Rabiei. *Investigation of different wave structures to the generalized third order nonlinear Schrodinger equation*, Optik, 2020, 206: 164259.

[18] M E Islam, M A Akbar. *Stable wave solutions to the Landau-Ginzburg-Higgs equation and the modified equal width wave equation using the IBSEF method*, Arab Journal of Basic and Applied Sciences, 2020, 27 (1): 270-278.

[19] R Khalil, M Al Horani, A Yousef, M Sababheh. *A new definition of fractional derivative*, J Comput Appl Math, 2014, 264: 65-70.

[20] N A Kudryashov. *Method for finding highly dispersive optical solitons of nonlinear differential equations*, Optik, 2020, 206: 163550.

[21] D Kumar, M Kaplan. *New analytical solutions of (2+1)- dimensional conformable time fractional Zoomeron equation via two distinct techniques*, Chinese J Phys, 2018, 53: 2173-2185.

[22] Z Li, T Han. *Bifurcation and exact solutions for the (2+1)-dimensional conformable time-fractional Zoomeron equation*, Advances in Difference Equations, 2020, 2020: 656, https://doi.org /10.1186/s13662-020-03119-5.

[23] W Liu, K Chen. *The functional variable method for finding exact solutions of some nonlinear time fractional differential equations*, Pramana, 2013, 81: 377-384.

[24] M Odabaşı. *Traveling wave solutions of conformable time fractional Zakharov-Kuznetsov and Zoomeron equations*, Chinese J Phys, 2020, 64: 194-202.

[25] M Odabaşı, E Mısırlı. *On the solutions of the nonlinear fractional differential equations via the modified trial equation method*, Mathematical Methods in the Applied Sciences, 2018, 41: 904-911.

[26] Z Odibat, S Momani. *The variational iteration method: an efficient scheme for handling fractional partial differential equations in fluid mechanics*, Comput Math Appl, 2009, 58: 2199-2208.

[27] H Rezazadeh, D Kumar, T A Soulaiman, H Bulut. *New complex hyperbolic and trigonometric solutions for the generalized conformable fractional Gardner equation*, Modern Physics Letters, 2019, 33(17), https://doi.org/10.1142/S0217984919501963.

[28] H Rezazadeh, A Korkmaz, M Eslami, J Vahidi, R Ashgari. *Traveling wave solution of conformable fractional generalized reaction Duffing model by generalized projective Riccati equation method*, Opt Quant Electron, 2018, 50(150), https://doi.org/10.1007/s11082-018-1416-1.

[29] M Şenol. *New analytical solutions of fractional symmetric regularized-long-wave equation*, Revista Mexicana de Fisica, 2020, 66(3): 297-307.

[30] N Shang, B Zheng. *Exact solutions for three fractional partial differential equations by the (G'/G) method*, Inter Journal of Appl Math, 2013, 43(3).

[31] M Topsakal, F Taşcan. *Exact Travelling Wave Solutions for Space-Time Fractional Klein-Gordon Equation and (2+1)-Dimensional Time-Fractional Zoomeron Equation via Auxiliary Equation Method*, Applied Mathematics and Nonlinear Sciences, 2020, 5(1):437-446.

[32] A Tozar, A Kurt, O Taşbozan. *New Wave Solutions of Time Fractional Integrable Dispersive Wave Equation Arising in Ocean Engineering Models*, Kuwait J Sci, 2020, 47(2): 22-33.

[33] H Xu. *Analytical approximations for a population growth model with fractional order*, Communications in Nonlinear Science and Numerical Simulation, 2009, 14(5): 1978-1983.

[34] G Yel, H M Başkonuş. *Solitons in conformable time-fractional Wu-Zhang system arising in coastal design*, Pramana, 2019, 93, https://doi.org/10.1007/s12043-019-1818-z.

[35] G Yel. *On the new travelling wave solution of a neural communication model*, Journal of Balikesir University Institute of Science and Technology, 2019, 21(2): 666-678.

[36] B Zheng, C Wen. *Exact solutions for fractional partial differential equations by a new fractional sub-equation method*, Advances in Difference Equations, 2013, 199.

[36] V Ala, U Demirbilek, K Mamedov. *On the Exact Solutions to Conformable Equal width Wave Equation by Improved Bernoulli Sub-Equation Function Method*, Bulletin of South Ural Univ, Series Math, Mech, Phys, 2021, 13(3).

[37] U Demirbilek, V Ala, K Mamedov. *An application of improved Bernoulli sub-equation function method to the nonlinear conformable time-fractional equation*, Tbilisi Math J, 2021, 14(3): 59-70.

[1]Department of Mathematics, Faculty of Science and Letters, Mersin University, Mersin 33343, Turkey.
Email: udemirbilek@mersin.edu.tr, volkanala@mersin.edu.tr

[2]Department of Mathematics, Faculty of Science and Letters, Igdir University, Igdir 76000, Turkey.
Email: hanlar.residoglu@igdir.edu.tr