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When q theory meets large losses risks and agency conflicts

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Abstract. We incorporate large losses risks into the DeMarzo et al.(2012) model of dynamic agency and the q theory of investment. The large losses risks induce losses costs and losses arising from agency conflicts during the large losses prevention process. Both of them reduce firm's value, distort investment policy and generate a deeper wedge between the marginal and average q. In addition, we study the implementation of the contract to enhance the practical utility of our model. The agent optimally manages the firm's cash flow and treats the cash reservation and credit line as the firm's financial slack, and hedges the productivity shocks and large losses shocks via futures and insurance contracts, respectively.

§1 Introduction

As one of the most important sources of financial market frictions, the agency problem affects firm's value and investment decisions by unobservable actions.[1] The investors can provide the agent with compensation according to the cash flow realizations of the firm so as to motivate the agent, or terminate the firm by withdrawing their financial support.

Since all firms inevitably face large losses risks, industrial firms may be exposed to severe accidents, financial firms may suffer sudden sharp drops in the value of financial assets. Lots of facts and papers proved that for preventing these losses it requires managerial efforts.[2] In practice it is often impossible to make the agent bear the costs that the losses generated, since total damages often exceed the extent the agent can bear and also it is protected by the

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[1] Agent's hidden actions involve (i) concealing and diverting cash flows for his own consumption, see Demarzo and Fishman (2007b), and/or (ii) stopping providing costly effort, such as Biais et al. (2010), Demarezo et al. (2012). Some papers consider both cases, for instance, Demarzo and Sannikov (2006), Sannikov (2008). Our paper adopts the second type of action.

[2] During the recent financial crisis, the large losses incurred by financial firms were in part due to insufficient risk control. Systematic analyses of industrial accidents point to the role of human deficiencies and inadequate levels of care, see Gordon, Flin, Mearns, and FLeming (1996), and Hollnagel (2002).

limited liability. This curbs managers' incentives to reduce the losses risks. What's worse, to a large extent, these activities are unobservable by external parties. Those factors also induce an agency conflicts.

Therefore, we incorporate large losses risks into the DeMarzo et al. (2012) model(DFHW hereafter). We study how to design optimal incentive contract to mitigate those conflicts and how the agency problem arising from large losses affects firm's value and investment decision. To enhance practical utility, we also explore how to implement our contract to hedge the large losses risks.

By comparing our conclusions with those of DFHW and Benchmark case, our paper characterizes the effects of the large losses imposed on the firm's value, optimal investment policies, as well as on the marginal and average q. The wedges between the DFHW case and our model reflect large losses effects including the losses costs and the losses arising from agency problem on losses prevention. Both of them reduce firm's value and distort investment decision. To enhance the practical utility of our model, we study the implementation of the optimal contract in practice. The agent optimally manages firm's cash flow and treats the cash reservation and a credit line as the firm's financial slack. The firm is terminated as soon as the cash exhausted. To maximize firm's value, the agent holds futures and insurance contracts to hedge the productivity shocks and large losses shocks, respectively.

Our paper is related to a growing body of literature on optimal contracting theory, such as DeMarzo and Fishman (2003), DeMarzo and Fishman (2006), DeMarzo and Fishman (2007b). He (2008) considered a similar model where the agent's hidden actions have an impact over the scale of the firm rather than its current cash flow. Hoffman and Pfeil (2010) developed a dynamic agency model, in which firm's profitability experiences observable shock.

Lorenzoni and Walentin (2007) and Schmid (2008) used a discrete-time model to analyze the relationship between the agency problems and the q theory of investment. In contrast we use the continuous-time recursive contracting methodology to derive the optimal contract. Other related investment papers include Albuquerque and Hopenhayn (2004), Quadrini (2004), Clementi and Hopenhayn (2006), Demarzo and Fishman (2007a). Our model differs from these models in that we focus on the effects that large losses imposed on the investment. Biais et al. (2010) also consider the losses risks. However, we focus on its interaction with the investment.

This paper is organized as follows: Section 2 presents the model, which includes firm's production technology, the agency problem and the incentive contract. Section 3 formulates the incentive compatibility and derives the model solution. Section 4 analyzes the economic implications based on the contract. Section 5 discusses how to implement the contract. Section 6 concludes.

§2 The model

2.1 Firm's production technology

The firm employs physical capital for production. Let K and I denote the level of capital stock and the gross investment rate, respectively. As is standard in capital accumulation models, the firm's capital stock K evolves according to

$$dK_t = (I_t - \delta K_t)dt, \qquad t \geq 0, \tag{1}$$

where $\delta \geq 0$ is depreciation rate.

Investment entails adjustment costs. Following the neoclassical investment literature (Hayashi (1982)), we assume the adjustment costs function $G(I, K)$ with $G(0, K) = 0$, is convex and s-mooth in I and is homogeneous of degree one in I and K. Given the homogeneity of the adjustment costs, the total investment costs can be written as

$$I + G(I, K) = c(i)K, \tag{2}$$

where $c(i)$ is an increasing convex function and represents the total cost per unit of capital required for the firm to grow at rate $i = \frac{I}{K}$ before depreciation.

While our analysis does not depend on the specific functional form of $c(i)$, we adopt for simplicity the special case of a quadratic form,

$$c(i) = i + \frac{1}{2}\theta i^2, \tag{3}$$

where the parameter θ measures the degree of the adjustment cost.

We suppose that the incremental gross output over time increment dt is proportional to the capital stock, and so can be represented as $K_t dA_t$, where A_t is the cumulative productivity process. We model the instantaneous productivity dA_t in the next subsection, where we introduce the agency problem. After accounting for investment, adjustment cost and the large losses costs, the dynamics of the firm's incremental cash flow dY_t over time increment dt can be written as

$$dY_t = K_t(dA_t - c(i_t)dt - CdN_t). \tag{4}$$

The investors have the option to terminate the contract at any time and enjoy lK_τ, where $l \in [0, 1]$ is a constant. The termination can be interpreted as the firm's liquidation or the replacement of the agent.

2.2 The agency problem

Investors possess the firm and hire the agent to manage it. The agency problem just arises from the separation of the firm's ownership and control. In this paper, we suppose the agent's private efforts lie in two aspects: the productivity process and the large losses prevention process.

Specifically, agent's private efforts lie in production process are denoted by $a_t \in [0, 1]$, which influence the expected rate of output per unit of capital,

$$dA_t = a_t \mu dt + \sigma dB_t, \qquad t \geq 0, \tag{5}$$

where$\mu > 0$ is firm's expected rate of production, $\sigma > 0$ is the constant volatility, $B = \{B_t : 0 \leq t < \infty\}$ is a standard Brownian motion. While the agent chooses proper effort level a_t in the productivity process, the agent enjoys private benefits at the rate $\lambda(1-a_t)\mu dt$ $(0 \leq \lambda \leq 1)$. The action can be interpreted as choice of the effort.

Compared with daily operation, large losses are rare events. It's natural to model the number of large losses occurrence as a Poisson process $N = \{N_t\}_{t\geq 0}$ whose intensity $\mathscr{A}$ depends on the level of risk prevention. For similarity, we suppose $\mathscr{A}_t = \{\alpha, \alpha + \Delta\alpha\}$, with $\Delta\alpha > 0$. Specifically, the agent's efforts c_t influence the large losses prevention process, if the agent shirks, the intensity goes to the higher level $\mathscr{A}_t = \alpha + \Delta\alpha$, and the agent obtains private benefits $\Delta\alpha K_t bdt$ in $[t, t+dt)$. Otherwise, $\mathscr{A}_t \alpha$ and the agent enjoys no private benefits. Once the losses happen, firm's value drops by CK_t. C is the losses rate of firm's capital.

In the absence of fixed investment costs and no financial market frictions, the firm optimally chooses investment to equate the marginal value of capital with the marginal cost of capital (adjustment costs). With the homogeneous production technology, the marginal value of capital, that is, marginal q, equals the average value of capital, that is, average q. This result motivates the widespread use of average q, which is relatively easy to measure, as an empirical proxy for marginal q, which is relatively difficult to measure. Following DeMarzo and Sannikov (2006), the agent (firm management) must be continually provided with the incentive to choose the appropriate action. The optimal contract between investors and the agent minimizes the cost of the agency problem and has implications for the dynamics of investment and firm value.

2.3 Formulating the optimal contracting problem

The optimal contracting problem is to find an incentive-compatible contract to maximize investors' profit subject to delivering the agent an initial required payment W_0. Denote the contract as $\Phi = (I, U, \tau)$, which specifies the firm's investment policy I_t, the agent's cumulative compensation U_t, and termination time τ, all of which depend on the profit history that are affected by the agent's performance.

After the contract is initiated, the agent whose discount rate is $\gamma > r > 0$ chooses proper effort level a_t, c_t, $0 \leq t \leq \tau$, so as to maximize the objective function

$$W(\Phi) = \max_{a_t, c_t} E^a\left[\int_0^\tau e^{-\gamma t}(dU_t + \lambda(1-a_t)\mu K_t dt + I_{\{c_t<1\}}\Delta\alpha K_t bdt)\right]. \tag{6}$$

$E^a(\cdot)$ is the expectation operator under the probability measure P that is induced by the action process.

The investors' purpose is to maximize the value function $F(K, W)$ by choosing an incentive-compatible contract,

$$\begin{aligned} F(K_0, W_0) \quad &= \quad \max_{\Phi} E\left[\textstyle\int_0^\tau e^{-rt} dY_t + e^{-r\tau} lK_\tau - \int_0^\tau e^{-rt} dU_t\right] \\ &\qquad \text{s.t. } \Phi \text{ is incentive compatible and } W(\Phi) = W_0. \end{aligned} \tag{7}$$

The agent's expected payoff W_0 is determined by the relative bargaining power between the agent and the investors, by varying W_0 we can obtain the entire feasible contract curve.

§3 Model solution

3.1 Incentive condition

An incentive-compatible contract Φ is the one which induces the agent to choose full efforts level. Define agent's discounted expected value of future compensation W_t as continuation payoff at time $t > 0$

$$W_t(\Phi) = E_t\left[\int_t^\tau e^{-\gamma(s-t)}dU_s\right].$$

The following result provides a useful representation of W_t.

Lemma 3.1. *At any moment of time $t > 0$, agent's continuation payoff W_t satisfies the following stochastic differential equation*

$$dW_t = \gamma W_t dt - dU_t + \beta_t(dA_t - \mu dt)K_t + H_t(dN_t - \alpha dt). \tag{8}$$

the sensitivity β_t and H_t of the agent's continuation payoff are determined by the agent's past efforts $a_s, c_s, 0 \le s \le t$ imposed on the productivity process and large losses prevention process, respectively.

Proof. To characterize how the agent's continuation payoff evolves over time, it is useful to consider the lifetime expected continuation payoff, evaluated conditionally on the information available at time t. Thus, we construct a stochastic process $\{V_t, t \ge 0\}$ from the agent's continuation payoff W_t as follows:

$$V_t = \int_0^t e^{-\gamma s}dU_s + e^{-\gamma t}W_t = E_t\left[\int_0^\tau e^{-\gamma s}dU_s\right].$$

Since V_t is the expectation of a given random variable conditional on the history up to t, the process V_t is a martingale. Applying Itô formula to the process V_t given in above equation, we obtain

$$dV_t = e^{-\gamma t}dU_t - \gamma e^{-\gamma t}W_t + e^{-\gamma t}dW_t.$$

To maintain agent's compensation is incentive compatible, agent's compensation must be sensitive to the firm's output. Recall that firm's output is driven by two shocks: productivity shock (the Brownian motion B_t) and large losses shock (the Poisson process N_t) in our model. Relying on martingale representation theorem (Stochastic Calculus for Finance II: Continuous-Time Models, Steven E. Shreve (2004)) that there exits adaptive processes β_t and H_t such that

$$dV_t = e^{-\gamma t}\beta_t(dA_t - \mu dt)K_t + e^{-\gamma t}H_t(dN_t - \alpha dt)$$

Thus, we can write the stochastic differential equation for dW_t as the sum of: i) the expected change term $E_{t^-}[dW_t]$; ii) a martingale term driven by the Bownian motion dB_t; and iii) a martingale term driven by the Compound Poisson process $dN_t - \alpha dt$:

$$dW_t = \gamma W_t dt - dU_t + \beta_t(dA_t - \mu dt)K_t + H_t(dN_t - \alpha dt).$$

Thus (8) holds. □

Lemma 3.2. *The necessary and sufficient condition for the contract to be incentive-compatible* *is*

$$a_t = 1, c_t = 1 \quad \Leftrightarrow \quad \beta_t \geq \lambda, \quad h_t \geq b, \quad t \geq 0. \tag{9}$$

where $h_t = H_t/K$.

Proof. One important aspect of large losses lies in their timing. Large losses are relatively rare events that contrast with day-to-day firm productivity process and cash flows. Thus, the firm's outputs from the production are much more directly observed than the cash flow caused by the large losses. The investors are more sensitive to the productivity process than the large losses. Therefore, from investors' perspective, so that the incentive conditions on the two efforts process should be calculated independently.

we then turn to determine the incentive coefficients β_t. According to (8), β_t denotes agent's compensation sensitivity to the efforts in the productivity process, When the agent shirks in the production, the the instantaneous cost to the agent is the expected reduction of the compensation, $\beta_t(1-a_t)\mu K_t dt$, the benefits she get is $\lambda(1-a_t)\mu K_t dt$. Thus, to induce the agent to choose full efforts, $\lambda(1-a_t)\mu K_t dt - \beta_t(1-a_t)\mu K_t dt \leq 0$. So the sensitivity coefficient on the productivity process β_t satisfies the following condition:

$$\beta_t \geq \lambda.$$

h_t stands for the sensitivity to agent's efforts in the large losses prevention process, When the agent deviates and chooses shirking, the the instantaneous cost to the agent is the expected reduction of the compensation, $\Delta\alpha H_t dt$, the benefits getting is $\Delta\alpha K_t b dt$. Thus, to induce the agent to choose full efforts, we should set $\Delta\alpha K_t b dt - \Delta\alpha H_t dt \leq 0$. Thus the sensitivity coefficient on the productivity process h_t should satisfies

$$h_t \geq b. \qquad \square$$

Combining the fact that the agent's continuation payoff must remain nonnegative according to the limited liability constraint and the fact that it must be reduced by H_t if there is a large loss at time $t \in [0, \tau]$, we should set

$$W_t \geq H_t. \tag{10}$$

3.2 The optimal contract

Now we use the dynamic programming approach to determine the most profitable way for the investors to deliver the agent any value W. Denote by $F(K, W)$ the investors' value function. There are two state variables in our model: the capital stock K_t and agent's continuation payoff W_t. By using the scale invariance of the firm's technology $F(K, W) = Kf(\omega)$, the problem reduces to a one dimensional problem with the single state variable $\omega = \frac{W}{K}$.

We begin with a number of key properties of investors' scaled value function $f(\omega)$. First, it cannot exceed the first best, $f(\omega) \leq f^{FB}(\omega)$, where $f^{FB}(\omega)$ is the maximum value that investors can get when there is no agency problem, the agent always takes full efforts and gets a constant payoff. The details about the full-commitment benchmark are presented in Appendix. Second, from (9) and (10), we have $\omega_t \geq b$, for all $t \geq 0$. When $\omega_t \in [0, b]$, the contract must be terminated, the investors get the termination value, so that

$$f(\omega) = l, \quad \omega \in [0, b]. \tag{11}$$

Third, f is concave. This property can be derived from the following economic interpretation. As argued above, termination is inefficient. It is necessary to provide incentives to the agent when the agent's scaled continuation payoff ω is low. Under this circumstance, the investors' value reacts strongly to the bad performance, because the latter significantly raises the risk of costly termination. By contrast, when ω is large, bad performance has a more limited impact on the termination risk. Therefore, the sensitivity to shocks is higher for smaller ω and lower for larger w; this results in the concavity of the investors' scaled value function f.

Because the investors have the option to provide the agent with ω by making a lump-sum transfer of $dU > 0$ to the agent and then moving to the optimal contract with payoff $\omega - dU$,

$$f(\omega) \geq f(\omega - dU) - dU.$$

This equation suggests that $f'(\omega) \geq -1$ for all ω, which means the marginal cost of compensating the agent can never exceed the cost of an immediate transfer. Define ω^1 as the lowest value such that

$$f'(\omega^1) = -1. \tag{12}$$

At ω^1, the firm is indifferent to allocating or keeping one dollar. Since ω^1 is optimally chosen, we also have the following "super contract" condition of ω^1[3]

$$f''(\omega^1) = 0. \tag{13}$$

The point ω^1 serves as cash payout boundary. It is optimal to pay the agent with cash when $\omega_t > \omega^1$ and to defer compensation otherwise. Thus, we set

$$dU = \max(\omega - \omega^1, 0), \tag{14}$$

which means $f(\omega_t) = f(\omega_1) - (\omega_t - \omega_1)$ for $\omega_t > \omega^1$.

When $\omega_t \in [b, \omega^1]$, the agent's compensation is deferred ($dU_t = 0$). According to (8), we get the evolution of $\omega = \frac{W}{K}$ as follows:

$$d\omega_t = (\gamma - i + \delta)\omega_t dt + \beta_t \sigma dB_t - h(dN_t - \alpha dt). \tag{15}$$

According to the dynamic programming principle, the corresponding Hamilton-Jacobi-Bellman equation for $f(\omega)$ is given by

$$\begin{aligned} rf(\omega) \quad = \quad & \max_{i,h,\beta} \Big\{ \mu - \alpha C - c(i) + f(\omega)(i-\delta) - \omega f'(\omega)(i-\delta) + f'(\omega)(\gamma\omega + h\alpha) \\ & + \tfrac{1}{2} f''(\omega)\sigma^2\beta^2 - \alpha(f(\omega) - f(\omega - h)) \Big\}. \end{aligned} \tag{16}$$

Since f is concave, the mapping $\beta \mapsto \frac{1}{2} f''(\omega)\sigma^2\beta^2$ is decreasing for positive β. According to (9) we get

$$\beta = \lambda. \tag{17}$$

In the same way, it is optimal to take h as low as possible,

$$h = b. \tag{18}$$

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[3] Dixit(1993) stated that the super contract condition essentially requires that the second derivatives match at the boundary.

Taking the first order condition of the HJB equation (16) with respect to i, we have the optimal investment-capital ratio $i(\omega)$ which satisfies the following equation:

$$c'(i) = f(\omega) - \omega f'(\omega). \tag{19}$$

From the investor's perspective, this equation shows that the marginal cost of investing equals the marginal value of investing. The marginal value of investing equals the current per unit value of the firms to investors $f(\omega)$ plus the marginal effect of decreasing the agent's per unit of payoff ω as the firm grows.

Our main results on the optimal contract are summarized in the following theorem.

Theorem 3.1. *The investors' value function $F(K,W)$ is proportional to capital stock K, $F(K,W) = K \cdot f(\omega)$, where $f(\omega)$ is the investors' scaled value function. For $\omega_t \in [b, \omega^1]$, $f(\omega)$ is concave and is the unique solution of ODE (16) with boundary conditions (11), (12) and (13). The agent's scaled continuation payoff ω evolves according to (15). For $\omega > \omega^1$, $f(\omega) = f(\omega^1) - (\omega - \omega^1)$. Cash payments $du_t = dU_t/K_t$ reflects ω_t back to ω^1. The contract is terminated at time τ, which is the smallest τ such that $\omega_\tau = b$. Optimal investment-capital ratio is given by (19).*

We present a formal verification theorem for the optimal contract in Appendix B. And prove that $f(\omega)$ represents the investor's optimal profits, which can be achieved by the contract outlined in the proposition.

§4 Model implications and analysis

4.1 Parameter choices and calibration

We take the widely used value for the agent's discount rate $\gamma = 5\%$, the annual risk free interest rate as investors' discount rate $r = 4.6\%$. According to Eberly, Rebelo and Vincent (2009) which provides empirical evidence in support of Hayashi (1982), we set the expected productivity rate $\mu = 20\%$, and the volatility of productivity process $\sigma = 20\%$. To fit the first-best values of q^{FB} and i^{FB} to the sample averages, we install the adjustment cost parameter $\theta = 2$ and the capital depreciation rate $\delta = 12.5\%$. For the parameters depicting the large losses, the intensity of the large losses $\alpha = 0.02$, the cost of losses per capital $C = 0.35$, the incentive parameters $\lambda = 0.2$ and $b = 0.1$. Finally, we choose the firm's termination value per capital $l = 0.9$, in line with some empirical estimates.[4]

4.2 Investor's scaled value function

According to the solution of optimal contract in Section 3, we plot the investors' scaled value function $f(\omega)$ for $\omega \in (b, \omega^1)$ in three cases. The gap between the benchmark solution

[4] See Li, Whited and Wu (2014) for the empirical estimates of l. The averages are 1.2 for Tobin's q and 0.1 for the investment-capital ratio, respectively, for the sample used by Eberly, Rebelo and Vincent (2009). The imputed value for the adjustment cost parameter θ is 2 broadly in the range of estimates used in the literature. See Hall (2004), Riddick and Whited (2009) and Eberly, Rebelo and Vincent (2009).

Table 1: Summary of Parameters

The parameter values used for numerical illustration		
Parameters	Symbol	Value
Investors' discount rate	r	4.6%
Agent's discount rate	γ	5%
Expected productivity rate	μ	20%
Volatility of productivity process	σ	20%
Adjustment costs parameter	θ	2
Depreciation rate	δ	12.5%
Intensity of the large losses	α	2%
The large losses cost per capital	C	35%
Fraction of private benefits derived from the productivity process	λ	20%
Private benefits caused by the large losses per capital	b	10%
Termination value per capital	l	90%

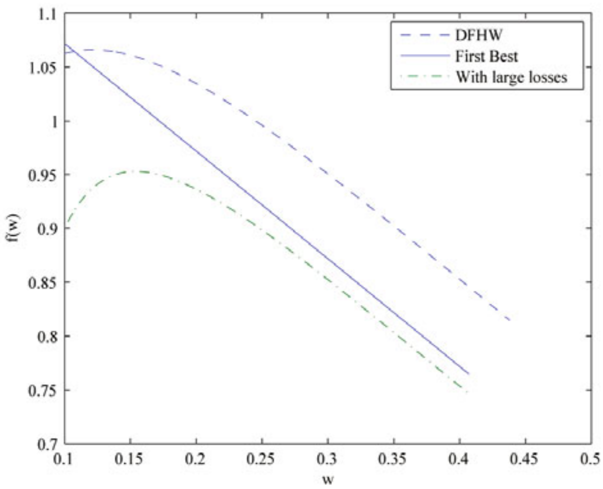

Figure 1: The investors'scaled value function $f(\omega)$ in different model

and our model captures the total losses due to the agency conflicts. The wedge between the DFHW model and the large losses model characterizes the large losses costs and the loss arising from agency problem during the large losses prevention.

Note that, the cash payout boundary ω^1 is the end point of the curves, it's valuable to notice that the cash payout boundary in our model is lower than that in the DFHW. The result is very intuitive since the large losses risks imposed more risks on the agent and also increases the risk of inefficient termination, all of these make the agent more impatient. So it's optimal for the agent to get cash payoff earlier. Obviously, $f(\omega)$ in the DFHW model is much higher than the rest, which implies large losses act as a shock creating great destruction on the firm is an important factor in the investors' value composition, and taking such losses into account has practical significance.

4.3 Average q and marginal q

Based on $f(\omega)$ we now derive the effect of the large losses risks on the marginal q_m and average q_a. The average q is defined as the ratio between the firm's value and capital stock,

$$q_a(\omega) = \frac{F(K,W)+W}{K} = f(\omega) + \omega. \tag{20}$$

This is consistent with the definition of q in the benchmark condition. The marginal q_m captures the marginal impact of an incremental unit of capital on the firm value and can be represented by

$$q_m(\omega) = \frac{\partial(F(K,W)+W)}{\partial K} = f(\omega) - \omega f'(\omega). \tag{21}$$

Recall $f'(\omega) \geq -1$, we have the following relationship:

$$q^{FB} > q_a(\omega) > q_m(\omega). \tag{22}$$

So the average q is always above the marginal q and the difference between them varies over time. Thess results are consistent with the existing research in DFHW. In our model, we focus on the effect of the large losses play on q_a and q_m.

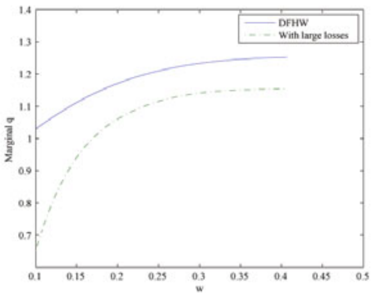

Figure 2: Marginal q_a

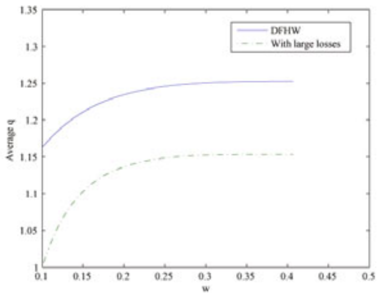

Figure 3: Average q_a

Figure 2 and 3 plot the marginal q and average q for the DFHW and our model. Obviously, both q_a and q_m with large losses are smaller than the values in DFHW model. As in the analysis above, the gap reflects the firm's losses when the losses happened and the costs due to agent's conflicts during the large losses prevention process. Both of them cause a reduction in firm's value. As marginal q is a forward looking measure which captures future investment opportunities. So Figure 2 implies the large losses influence the evaluation of firm's investment opportunities.

4.4 Investment

In this part, we discuss the firm's investment decision. According to the expression of $c(i)$ (3), the first order condition (19) and the definition of marginal q (2), the optimal investment-capital ratio $i(\omega)$ can be written as

$$i = \frac{1}{\theta}\big(f(\omega) - \omega f'(\omega) - 1\big). \tag{23}$$

Taking the first-order derivative with respect to ω, we obtain the following measure of $i(\omega)$ sensitivity, which is $i'(\omega)$. From the convexity of $c(i)$ and the monotonicity of q_m, we have

$$i'(\omega) = -\frac{\omega f''(\omega)}{c''(i(\omega))} \geq 0, \tag{24}$$

the equality holds only at termination and payout boundaries.

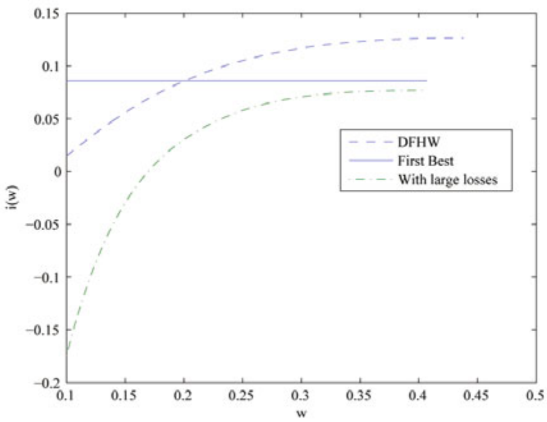

Figure 4: Comparing investment ratio in different model.

The gap between $i(\omega)$ in the DFHW case and our model reflects large loss effects, which contains two factors, the loss cost and the loss arising from agency problem on loss prevention. Figure 4 implies that investment-capital ratio in our model is less than that in DFHW, it is optimal to invest less when large loss risk exists. It is noteworthy that $i(\omega)$ is higher in benchmark case than that in DFHW model when ω is lower, which implies the agency conflict cost is higher than the large loss cost when agency problem occupies dominated position. When ω is higher, which means the agency problem relaxes, large loss occupies the main position, the investment ratio in the DFHW model is higher than that in the benchmark case.

§5 Model implementation

In this section, we examine the liquidity and risk management through firm's financial slack and financial instruments. Specifically, the agent optimally manages the firm's cash flow, and hedges the firm's risk by the futures and insurance contract. It is well known that the implementation is not unique, we adopt this intuitive way in this paper.

5.1 Financial slack

During the liquidity management, credit line can be used as a source to fund the firm. A credit line is an important alternative source of liquidity. We set it as a fraction of the firm's capital stock. The economic interpretation is that the firm must be able to post collateral to secure a credit line and the value of the highest quality collateral does not exceed the fraction m of the firm's capital stock, where $m > 0$ is a constant. mK can be interpreted as the firm's short-term debt capacity. For simplicity, we assume m as exogenously specified in this paper.

Recall that the firm potentially generates operating losses and needs access to cash or credit to operate. Financial slack may correspond to the firm's cash reserves, line of credit, or a combination of these two. Rather than describing all possibilities, we adopt the combination of the firm's cash reserves and the credit line as financial slack, the largest short-run loss the firm can sustain before the contract is terminated. Once its cash reserves is exhausted, the firm cannot continue to operate, the contract is terminated. The agent controls the firm and maximizes the firm's cash reserve after payout. Let S_t denote the level of cash reserves at time t, which satisfies

$$S_t \geq -mK_t. \tag{25}$$

When $S_t = -mK_t$, the firm exhausts all the cash reserves and is terminated.

5.2 Risk hedging

In addition to cash liquidity management, the agent reduces the firm's risks through financial hedging. One financial instrument, which the agent can use to hedge the productivity risk B_t, is a standard futures contract. And insurance is used to hedge the risk with respect to the large losses.

First, we characterize the futures contract. In the standard asset pricing framework, futures have zero initial value and its payoff has zero mean under the risk-neutral measure. The futures price P_t evolves according to

$$dP_t = \sigma_m P_t dZ_t, \tag{26}$$

where σ_m is the volatility of the aggregate market portfolio, and Z_t is a standard Brownian motion that is partially correlated with firm productivity shocks driven by the Brownian motion B_t, with correlation coefficient ρ. Given any admissible futures position ϕ_t that the agent takes to hedge the firm's risk exposure to the productivity risk B_t, the instantaneous payoff is $\phi_t \sigma_m S_t dZ_t$

Then, the agent takes an insurance to hedge the large losses risk. If the agent takes a unit long position in the contingent claim, he pays an insurance premium α per unit of time and receives a unit payment from the insurer when the losses happen. Let πK denote the agent's demand for this insurance contract. When the large losses happen, $dN_t = 1$, the total stochastic exposure of this insurance is $\pi K_t(dN_t - \alpha dt)$. Because investors are risk neutral, there is no risk premium in the insurance contract.

When the agent takes hedging position in futures contract and insurance, firm's cash reservation evolves as follow:

$$dS_t = rS_tdt + dY_t - dU_t + \phi_t\sigma_m S_t dZ_t + \pi K_t(dN_t - \alpha dt). \tag{27}$$

The first term on the right side comes from the interest rate r earned by the cash reserves. The term $dY_t - dU_t$ refers to the net cash flow derived by the productivity process after the payout, the last two terms are benefits which result from hedging the two shocks.

Next, by optimally choosing the investment, payout policy and hedging position, the agent maximizes the firm's value $P(K, S)$. The firm's decisions and value depend on the value of S_t, and the firm suffers termination if $S_t < -mK_t$. Similarly, there also exists a payout threshold $\overline{S}$. As we discussed before, $\overline{S}$ satisfies

$$P'(K, \overline{S}) = -1,\ P''(K, \overline{S}) = 0. \tag{28}$$

Thus, $S_t > \overline{S}$ corresponds to firm's payout region, so that,

$$P(K, S) = P(K, \overline{S}) - (\overline{S} - dU).$$

When $S_t \in [-mK_t, \overline{S}]$, the firm is in the internal financing region. The HJB equation of the firm's value function $P(K, S)$ is as follows:

$$\begin{aligned} rP(K,S) = \max_{I,\pi,\phi}\Big\{ & Y_t - c(i)K - \alpha C + (I - \delta K)P_K + [rS + K(\mu - c(i) - \alpha C - \alpha\pi)]P_S \\ & + \tfrac{1}{2}(\sigma^2K^2 + \sigma_m^2\phi^2S^2 + 2\rho\phi\sigma\sigma_m SK)P_{SS} + \alpha(P(K, S + \pi K) - P(K, S))\Big\}. \end{aligned} \tag{29}$$

Similarly, the firm's value function is homogeneous with respect to K, that is, $P(K, S) = K \cdot p(s)$, where $s = \frac{S}{K}$ is the firm's cash-capital ratio. The dynamics of s and $p(s)$ can be obtained via the same calculation as in Section 3, so we omit it here.

The first-order condition with respect to ϕ is

$$\phi^*(s) = -\frac{\rho\sigma}{\sigma_m s}. \tag{30}$$

Thus, the firm's total hedge position of the futures contract is $|\phi \cdot W| = (\rho\sigma/\sigma_m)K$, which is linearly increasing with the firm capital K. In futures contract, we do not consider the marginal account; our model can be extended so as to incorporate this factor. See Bolton, Chen and Wang (2011).

§6 Conclusion

In this paper, we extend DeMarzo et al.(2012) by incorporating large losses risks, an important factor in practice, to investigate the corresponding effects on firm value, investment strategies. Therefore, our model is able to link large losses to firm value, average q, marginal q and corporate investment decisions. To enhance the practical utility of our model, we study the implementation of the optimal contract. The agent optimally manages the firm's cash flow and treats the cash reservation and credit line as the firm's financial slack, and hedges the productivity shocks and large losses shocks via futures and insurance contracts, respectively. This paper shows that, except the agency problem in productivity process, the large loss costs and the losses arising from the agency conflicts in losses prevention process result in a loss of firm value due to distorted investment and payout decisions.

Appendix A: The full-commitment benchmark

To highlight the dynamic effects of agency problem, we examine the optimal investment under standard neoclassical setting without the agency problem. Under full commitment, the agent's payments and investment decisions are separated. Investors optimally choose investment I to maximize the firm's value

$$Q(K_t) = \max_I E_t\left[\int_t^\infty e^{-r(s-t)} Y_s ds\right]. \tag{31}$$

Using the dynamic programming approach, we obtain

$$rQ(K) = \max_I K(\mu - c(i) - \alpha C) + Q_K(I - \delta K). \tag{32}$$

According to the homogeneity property, the firm's value function $Q^{FB}(K)$ is given by

$$Q^{FB}(K) = q^{FB} \cdot K. \tag{33}$$

By substituting (33) into (32), we obtain the following HJB equation of q^{FB},

$$(r+\delta)q^{FB} = \max_i \left(\mu - \alpha C - c(i) + iq^{FB}\right). \tag{34}$$

There exists an optimal investment-capital ratio i^{FB} that maximizes the present value of the firm's cash flow. Tobin's q is expressed via the first-order condition for investment. By jointly solving (3), (34), we obtain the values of q^{FB} and i^{FB} as follows:

$$q^{FB} = c'(i^{FB}) = 1 + \theta i^{FB}, \tag{35}$$

$$i^{FB} = r + \delta - \sqrt{(r+\delta)^2 + \frac{2(r+\delta+\alpha C-\mu)}{\theta}}. \tag{36}$$

The agent's and investors' payoffs on a per unit of capital as $\omega = \frac{W}{K}$ and

$$f^{FB}(\omega) = F^{FB}(K, W)/K = q^{FB} - \omega. \tag{37}$$

The above results show that, in absence of the agency problem, the first-best investment is constant over time and independent of the firm's history or the volatility of its cash flows. After taking agency problem into account, we will find these conclusions no longer hold.

Appendix B: Proof of Proposition 3.1

We prove that f represents the investors' optimal profit, which is achieved by the contract outlined in the proposition. Define

$$G_t = \int_0^t e^{-rs}(dY_s - dU_s) + e^{-rt}F(K_t, W_t), \tag{38}$$

where W_t evolves according to (8). Under an arbitrary incentive-compatible contract Φ^*, using Itô's formula, we obtain for $t < \tau$

$$\begin{aligned} e^{rt}dG_t = \ & K_t\Big\{\big[-rf(\omega) + \mu - c(i) - \alpha C + (i-\delta)(f(\omega) - \psi f'(\omega)) + (\gamma\omega + \alpha h)f'(\omega) \\ & +\tfrac{1}{2}\sigma^2\beta^2 f''(\omega)\big]dt + (-1 - f'(\omega))dU_t/K_t + \sigma(1+\beta f'(\omega))dB_t\Big\}. \end{aligned} \tag{39}$$

Under the optimal investment policy $i'(\omega)$ and the optimal incentive policy (9), the coefficient of dt always equals zero. The second term captures the optimality of the cash payment policy. It is equals zero under the optimal contract for $\omega \in [b, \omega^1]$. G_t is a super-martingale. Because any other contracts make first term non-positive, the second term is non-positive by $f'(\omega) \geq -1$, and the expectation of the last term equals zero. It is a martingale if and only if under the optimal contract and $\omega \in [b, \omega^1]$. And it is increasing only when $\omega > \omega^1$.

Now we evaluate the investors' value for an arbitrary incentive-compatible contract. For all $t < \infty$, the investors' expected payoff is

$$\begin{aligned} G(\Phi) &= E\left[\int_0^\tau e^{-rs}(dY_s - dU_s) + e^{-r\tau} l K_\tau\right] \\ &= E\left[G_{t\wedge\tau} + 1_{t\leq\tau}\left(\int_t^\tau e^{-rs}(dY_s - dU_s) + e^{-r\tau} l K_\tau - e^{-rt} F(K_t, W_t)\right)\right] \\ &= E[G_{t\wedge\tau}] + e^{-rt} E\left\{1_{t\leq\tau}\left[\int_t^\tau e^{-r(s-t)}(dY_s - dU_s) + e^{-r(\tau-t)} l K_\tau - F(K_t, W_t)\right]\right\} \\ &\leq G_0 + (q^{FB} - l) E[e^{-rt} K_t]. \end{aligned}$$

The super-martingale property of G_t leads to the first term on the right hand side. The second term in the inequality follows from

$$E_t\left[\int_t^\tau e^{-r(s-t)}(dY_s - dU_s) + e^{-r(\tau-t)} l K_\tau\right] + W_t \leq q^{FB} K_t, \tag{40}$$

and

$$q^{FB} K_t - W_t - F(K_t, W_t) < (q^{FB} - l) K_t. \tag{41}$$

Therefore, letting $t \to \infty$,

$$G(\Phi) \leq G_0. \tag{42}$$

For a contract that satisfies the conditions of the proposition, G_t is a martingale until time τ. Therefore, the payoff G_0 is achieved with equality. Q.E.D.

References

[1] R Albuquerque, H Hopenhayn. *Optimal lending contracts and firm dynamics*, Rev Econom Stud, 2004, 71: 85-315.

[2] B Biais, T Mariotti, G Plantin, J C Rochet. *Dynamic security design: Convergence to continuous time and asset pricing implications*, Rev Econom Stud, 2007, 74: 345-390.

[3] B Biais, T Mariotti, J C Rochet, S Villeneuve. *Large risks, limited liability, and dynamic moral hazard*, Econometrica, 2010, 78: 73-118.

[4] P Bolton, H Chen, N Wang. *A unified theory of Tobin's q, coporate investment, financing and risk management*, J Finance, 2011, 66(5): 1545-1578.

[5] P Bolton, H Chen, N Wang. *Market timing, investment and risk management*, J Financ Econ, 2013, 109(1): 40-62.

[6] P Bolton, D Scharfstein. *A theory of predation based on agency problems in financial contracting*, Amer Econ Rev, 1990, 80: 94-106.

[7] G L Clementi, H A Hopenhayn. *A theory of financing constraints and firm dynamics*, Quart J Econ, 2006, 121: 229-265.

[8] P Demarzo, M J Fishman. *Optimal long-term financial contracting with privately observed cash flows*, Working paper, Stanford University.

[9] P Demarzo, M J Fishman. *Agency and optimal investment dynamics*, Rev Financ Stud, 2007, 20(1): 151-188.

[10] P Demarzo, M J Fishman. *Agency and optimal investmnet dynamics*, Rev Financ Stud, 2007a, 20: 151-188.

[11] P Demarzo, M J Fishman. *Optimal long-term financial contracting*, Rev Financ Stud, 2007b, 20(6): 2079-2128.

[12] P Demarzo, M J Fishman, Z G He, N Wang. *Dynamic agency and the q theory of investment*, J Finance, 2012, 67(6): 2295-2340.

[13] R P E Gordon, R H Flin, K Mearns, M T Fleming. *Assessing the human factors causes of accidents in the offshore oil industry*, In: *Proceedings of the 3rd International Conference on Health, Safety and Environment in Oil and Gas Exploration and Production*, 1996.

[14] F Hyashi. *Tobin's marginal q and average q: a neoclassical interpretation*, Econometrica, 1982, 50: 213-224.

[15] Z G He. *Optimal executive compensation when firm size follows geometric Brownian motion*, Rev Financ Stud, 2009, 22(2): 859-892.

[16] F Hoffmann, S Pfeil. *Reward for luck in a dynamic agency model*, Rev Financ Stud, 2010, 23(9): 3329-3345.

[17] E Hollnagel. *Understanding accidents–from root causes to performance variability*, In: *Proceedings of the 7th IEEE Conference on Human Factors and Power Plants*, 2002.

[18] B Holmström, J Tirole. *Financial intermediation, Loanable funds, and the real sector*, Quart J Econ, 1997, 80: 663-691.

[19] V Quadrini. *Ivestment and liquidation in renegotiation-proof contracts with moral hazard*, J Monetary Econ, 2004, 51: 713-751.

[20] Y Sannikov. *A continuous-time version of the principal-agent problem*, Rev Econom Stud, 2008, 79: 957-984.

[21] E S Spear, S Srivastava. *On repeated moral hazard with discounting*, Rev Econom Stud, 1987, 54: 599-617.

[21] S E Shreve. *Stochastic Calculus for Finance II: Continuous-Time Models*, Springer, 2004.

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