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PRICING EUROPEAN OPTION IN A DOUBLE EXPONENTIAL JUMP-DIFFUSION MODEL WITH TWO MARKET STRUCTURE RISKS AND ITS COMPARISONS

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Abstract. Using Fourier inversion transform, P.D.E. and Feynman-Kac formula, the closed-form solution for price on European call option is given in a double exponential jump-diffusion model with two different market structure risks that there exist CIR stochastic volatility of stock return and Vasicek or CIR stochastic interest rate in the market. In the end, the result of the model in the paper is compared with those in other models, including BS model with numerical experiment. These results show that the double exponential jump-diffusion model with CIR-market structure risks is suitable for modelling the real-market changes and very useful.

§1 Introduction

Option pricing has played a central role in the general theory of asset pricing since the celebrated work of Black and Scholes[1](hereafter BS model). However,option pricing formulas in the BS model do not perform well empirically in practice because of the asymmetric leptokurtic features and the volatility smile. Therefore,considerable attention has been focused on extending the BS model to represent reasonably the asset return dynamics recently. These extensions can be grouped as three approaches: (1) to allow for stochastic volatility[2-5]. That is, the volatility of asset return is assumed to be a stochastic process correlated with the asset process itself; (2) to allow for stochastic interest rates[6-10]. Typically, the interest rates are assumed to be a mean-reverting diffusion process(for example, Vasicek or Cox-Ingersoll-Ross model); (3) to capture the jump behavior of asset return[11-13]. It is noted that [11] is also a log-normal distribution,and is in contradiction with the asymmetric leptokurtic features. As discussed in Bakshi, et al.[14], there are many factors influencing financial economy, such as the excess return rates, volatility of risky asset, stochastic interest rates and the inflation uncertainty, etc., which are called the market structure risks(hereafter MSR). Thus, the most reasonable model

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of asset dynamics would likely include both the market structure risks and jump-diffusion as in the model by [15-18], etc. The results in the mentioned literature, however,are much less comprehensive.

In this paper, we present two different MSR in jump-diffusion model, where the MSR are described as a combination of stochastic interest rate and stochastic volatility, and the jump sizes of the asset return are followed by double exponential distribution(i.e., it can explains the asymmetric leptokurtic feature). We first derive the closed-form solutions of European call option in this model by applying Fourier inversion transform, P.D.E., and Feynman-Kac formula. The solutions are very fast for data calculation and are potentially useful for empirical analysis, then we compare the results of our framework with those of another case including BS model by using numerical experiment, and also examine the hedge ratio and implied volatility of option. As a result, this paper proposes the most reasonable model with both CIR-MSR and double exponential jump-diffusion model.

§2 Problem formulation

Let be given on a complete probability space$(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ three Brownian motions $W = \{W_t : t \in [0,T]\}$,$W^v = \{W_t^v : t \in [0,T]\}$ and $W^r = \{W_t^r : t \in [0,T]\}$, a Poisson process $N = \{N_t, t \in [0,T]\}$with constant intensity $\lambda > 0$, and a sequence of independent identically distributed nonnegative random variables $J = (J_i)_{i\geq 1}$. Assume that the processes $W, W^v, W^r$are independent of N and J, the correlation

$$\mathrm{Cov}(\mathrm{d}W^v, \mathrm{d}W) = \rho \mathrm{d}t, \mathrm{Cov}(\mathrm{d}W^r, \mathrm{d}W) = \mathrm{Cov}(\mathrm{d}W^r, \mathrm{d}W^v) = \mathrm{Cov}(\mathrm{d}N, J) = 0.$$

We consider an arbitrage-free, frictionless financial market where two assets (B, S) are traded continuously up to a fixed horizon date T. Throughout this paper, we assume that there exists a martingale probability measure Q being equivalent to P.

The first asset B is a riskless asset, or bond. Its price is given by

$$\mathrm{d}B_t = r_t B_t \mathrm{d}t, \quad B_0 = 1, \tag{2.1}$$

where r_t is the short interest rate followed under Q by

$$\mathrm{d}r_t \quad = \quad (\theta_r - \alpha_r r_t)\mathrm{d}t + \sigma_r^\delta \mathrm{d}W_t^r, \quad r_0 = r > 0, \tag{2.2}$$

where $\theta_r, \alpha_r, \sigma_r$ are nonnegative constants, $0 \leq \delta \leq 1$ is also a constant. Here, we consider two cases: $\delta = 0$(Vasicek model) or $\delta = 0.5$(Cox,Ingersoll and Ross, hereafter CIR model).

Under Q, the risky asset S is called stock, whose dynamics is described by

$$\begin{aligned} \frac{\mathrm{d}S_t}{S_{t-}} &= (r_t - \lambda\kappa)\mathrm{d}t + \sigma\sqrt{V_t}\mathrm{d}W_t + (J-1)\mathrm{d}N_t, \quad S_0 = s_0 > 0; & (2.3) \\ \mathrm{d}V_t &= (\theta_v - \alpha_v V_t)\mathrm{d}t + \sigma_v\sqrt{V_t}\mathrm{d}W_t^v, \quad V_0 = 1, & (2.4) \end{aligned}$$

without dividend yield, where $\alpha_v, \sigma_v, \sigma, \theta_v$ are all nonnegative constants. The jump sizes of stock price are represented by random variables J such that $Y = \ln J$ has an asymmetric double exponential distribution with the density

$$f_Y(y) = p\eta_1 \mathrm{e}^{-\eta_1 y}\mathbf{1}_{(y\geq 0)} + q\eta_2 \mathrm{e}^{\eta_2 y}\mathbf{1}_{(y<0)}, \qquad (2.5)$$
$$\eta_1 > 1, \quad \eta_2 > 0, \quad p,q > 0 \ \text{ and } \ p+q=1,$$

where p, q represent the probability of upward and downward jumps respectively, $\kappa = E[J-1] = \frac{p\eta_1}{\eta_1-1} + \frac{q\eta_2}{\eta_2+1} - 1$. The stochastic economy (2.1-2.5) is called double exponential Jump-diffusion model with MSR(hereafter SVSI_DexpJ model).The model has many other types as special cases. For instance, (i)the model with constant interest rate and constant volatility, namely the BS model(with $\lambda = 0$) and jump-diffusion model(hereafter DexpJ, see [12]);(ii)the model with constant interest rates, namely stochastic volatility model(hereafter SV_DexpJ , see [5]) with $\lambda = 0$);(iii)the model with $\delta = 0$ is very similar to [18];(iv)the model with constant volatility, namely stochastic interest rate model(hereafter SI_DexpJ, see [7]), etc. Therefore, this model is a general setup of usual market.

Lemma 2.1. When the short interest rate r_t evolves according to (2.2), the price of a risk-free zero-couple bond $P(t,T)$ with maturity T at time t is given by

$$P(t,T) = \begin{cases} \exp\Big\{A(T-t) - \frac{1-\mathrm{e}^{-\alpha_r(T-t)}}{\alpha_r} r_t\Big\}, & \delta = 0, \\ \exp\Big\{A(T-t) - \frac{2[1-\mathrm{e}^{-\gamma(T-t)}]}{2\gamma + (\alpha_r-\gamma)[1-\mathrm{e}^{-\gamma(T-t)}]} r_t\Big\}, & \delta = 0.5, \end{cases} \qquad (2.6)$$

where $\gamma = \sqrt{\alpha_r^2 + 2\sigma_r^2}$, $\quad B(\tau) = \frac{1-\mathrm{e}^{-\alpha_r\tau}}{\alpha_r}$and

$$A(\tau) = \begin{cases} \frac{\sigma_r^2 - 2\alpha_r\theta_r}{2\alpha_r^2}\tau + \frac{\alpha_r\theta_r - \sigma_r^2}{\alpha_r^2}B(\tau) + \frac{\sigma_r^2}{4\alpha_r^2}B(2\tau), & \delta = 0, \\ \frac{(\alpha_r-\gamma)\theta_r\tau}{\sigma_r^2} + \frac{2\theta_r}{\sigma_r^2}\ln\frac{2\gamma}{2\gamma+(\alpha_r-\gamma)(1-\mathrm{e}^{-\gamma\tau})}, & \delta = 0.5. \end{cases} \qquad (2.7)$$

Using the martingale pricing ,we can represent option value as the integral of a discounted probability density times the payoff function and employ Feynman-Kac formula to derive the partial integro-differential equation(hereafter P.I.D.E.). Applying Feynman-Kac formula for the prototype price dynamics (2.1-2.5), we obtain that the value of European-style claim $F(t,r,x,V)$ satisfies the following P.I.D.E.,

$$\frac{\partial F}{\partial t} + (r - \lambda\kappa - \frac{1}{2}\sigma^2 V)\frac{\partial F}{\partial x} + \frac{1}{2}\sigma^2 V\frac{\partial^2 F}{\partial x^2} + (\theta_r - \alpha_r r)\frac{\partial F}{\partial r} + \frac{1}{2}\sigma_r^2 r^{2\delta}\frac{\partial^2 F}{\partial r^2} + (\theta_v - \alpha_v V)\frac{\partial F}{\partial V}$$
$$+ \frac{1}{2}\sigma_v^2 V\frac{\partial^2 F}{\partial V^2} + \rho\sigma\sigma_v V\frac{\partial^2 F}{\partial x\partial V} + \lambda\int_{\mathbf{R}}[F(t,r,x+y,V) - F(t,r,x,V)]f_Y(y)\mathrm{d}y = rF, \ (2.8)$$

subject to the boundary condition: $F(T,r,x,V) = (\mathrm{e}^x - K)^+ \text{or} (K - \mathrm{e}^x)^+$.

§3 Solution of the pricing problem

In this section, we apply Fourier inversion transform and Feynman-Kac approach to solve (2.8) and gain the explicit solution of price on the European call option. Denote by $C_E(t,T)$ the price of an European call option at time t with maturity T, stock price S and strike price K.

In a general equilibrium framework, the value of an European option is the expected discounted terminal payoffs under Q, i.e.,

$$\begin{aligned} C_E(t,T) &= E_t^Q\Big[\mathrm{e}^{-\int_t^T r_s \mathrm{d}s}(S_T-K)^+\Big] \\ &= E_t^Q\Big[\mathrm{e}^{-\int_t^T r_s \mathrm{d}s} S_T \mathbf{1}_{(\ln S_T\geq \ln K)}\Big] - K E_t^Q\Big[\mathrm{e}^{-\int_t^T r_s \mathrm{d}s}\mathbf{1}_{(\ln S_T\geq \ln K)}\Big]. \end{aligned} \tag{3.1}$$

According to [5] and others, we apply changing numeraire to simplify calculation. For the first term, we choose the stock price S as numeraire and switch from measure Q to Q_1. For the second term, we use the T-forward measure $P(t,T)$ to switch from Q to Q_2. The Radon-Nikodym derivatives are then given by

$$\frac{\mathrm{d}Q_1}{\mathrm{d}Q}|_{\mathcal{F}_T} = \mathrm{e}^{-\int_t^T r_s \mathrm{d}s}\frac{S_T}{S_t}, \qquad \text{and} \qquad \frac{\mathrm{d}Q_2}{\mathrm{d}Q}|_{\mathcal{F}_T} = \mathrm{e}^{-\int_t^T r_s \mathrm{d}s}\frac{1}{P(t,T)},$$

respectively. It is easy to say that the above two measures Q_1 and Q_2 are well-defined probability measures, because of $E_t^Q[\mathrm{e}^{-\int_t^T r_s \mathrm{d}s}\frac{S_T}{S_t}] = 1$ and $E_t^Q[\mathrm{e}^{-\int_t^T r_s \mathrm{d}s}\frac{1}{P(t,T)}] = 1$. Under the new measures Q_1, Q_2, the pricing option (3.1) can be restated as

$$\begin{aligned} C_E(t,T) &= S_t E_t^{Q_1}\Big[\mathbf{1}_{(\ln S_T\geq \ln K)}\Big] - K P(t,T) E_t^{Q_2}\Big[\mathbf{1}_{(\ln S_T\geq \ln K)}\Big] \\ &= S_t Q_1(\ln S_T \geq \ln K) - K P(t,T) Q_2(\ln S_T\geq \ln K). \end{aligned} \tag{3.2}$$

In order to get the explicit solution, we use their corresponding characteristic functions which are defined by

$$\phi_j(u) = E_t^{Q_j}[\mathrm{e}^{iu\ln S_T}], \qquad j=1,2.$$

Using the above two Radon-Nikodym derivatives, we obtain expression for $\phi_j(u)$ under the original measure Q,

$$\phi_1(u) = E_t^{Q_1}[\mathrm{e}^{iu\ln S_T}] = E_t^Q\Big[\mathrm{e}^{-\int_t^T r_s \mathrm{d}s}\frac{S_T}{S_t}\mathrm{e}^{iu\ln S_T}\Big] = \frac{\psi(1+iu)}{\psi(1)}, \tag{3.3}$$

$$\phi_2(u) = E_t^{Q_2}[\mathrm{e}^{iu\ln S_T}] = E_t^Q\Big[\mathrm{e}^{-\int_t^T r_s \mathrm{d}s}\frac{1}{P(t,T)}\mathrm{e}^{iu\ln S_T}\Big] = \frac{\psi(iu)}{\psi(0)}, \tag{3.4}$$

where $\psi(z) = E_t^Q\Big[\mathrm{e}^{-\int_t^T r_s \mathrm{d}s + z\ln S_T}\Big]$, z is any complex, and $\psi(1) = S_t, \psi(0) = P(t,T)$.

It is well known that the probability distribution functions can be calculated by using Fourier inversion formula, i.e.,

$$P(X\leq x) = \frac{1}{2} + \frac{1}{2\pi}\int_0^{+\infty}\frac{\phi(-u)\mathrm{e}^{iux} - \phi(u)\mathrm{e}^{-iux}}{iu}\mathrm{d}u \tag{3.5}$$

Based on (3.5), the above two probabilities Q_1 and Q_2 in (3.2) are given by

$$Q_1(\ln S_T\geq \ln K) = \frac{1}{2} + \frac{1}{\pi}\int_0^{+\infty}\Re\Big[\frac{\phi_1(u)\mathrm{e}^{-iu\ln K}}{iu}\Big]\mathrm{d}u, \tag{3.6}$$

$$Q_2(\ln S_T\geq \ln K) = \frac{1}{2} + \frac{1}{\pi}\int_0^{+\infty}\Re\Big[\frac{\phi_2(u)\mathrm{e}^{-iu\ln K}}{iu}\Big]\mathrm{d}u, \tag{3.7}$$

where $\Re[.]$ represents real part. We have then our main result.

Theorem 3.1. Given the stochastic processes S_t, V_t, r_t and ε_t defined in SVSI_DexpJ model, then the price of an European call option is written as
$C_E(t,T) = S_t\Big\{\frac{1}{2}+\frac{1}{\pi}\int_0^{+\infty}\Re\Big[\frac{\phi_1(u)\mathrm{e}^{-iu\ln K}}{iu}\Big]\mathrm{d}u\Big\} - KP(t,T)\Big\{\frac{1}{2}+\frac{1}{\pi}\int_0^{+\infty}\Re\Big[\frac{\phi_2(u)\mathrm{e}^{-iu\ln K}}{iu}\Big]\mathrm{d}u\Big\}.$

Following Theorem 3.1, we need to compute $\psi(z)$. We then develop $\ln S_T$ with Brownian motion W_t expressed as $\mathrm{d}W_t = \rho \mathrm{d}W_t^v + \sqrt{1-\rho^2}\mathrm{d}Z_t$ with Z_t also a Brownian motion being independent of $W_t^v, W_t^r, W_t^\varepsilon$, N_t, and random variables J. We now have

$$\begin{aligned}
\ln S_T &= \ln S_t - \lambda\kappa\tau + \int_t^T r_s\mathrm{d}s + \Big\{\sigma\rho\int_t^T\sqrt{V_s}\mathrm{d}W_s^v - \frac{1}{2}\sigma^2\rho^2\int_t^T V_s\mathrm{d}s\Big\} \\
&\quad + \Big\{\sigma\sqrt{1-\rho^2}\int_t^T\sqrt{V_s}dZ_s - \frac{1}{2}\sigma^2(1-\rho^2)\int_t^T V_s\mathrm{d}s\Big\} + \sum_{i=N_t+1}^{N_T} Y_i \\
&= \ln S_t - \lambda\kappa\tau + \int_t^T r_s\mathrm{d}s + \zeta(\tau) + \xi(\tau) + \sum_{i=N_t+1}^{N_T} Y_i,
\end{aligned} \tag{3.8}$$

and

$$\begin{aligned}
\psi(z) &= \mathrm{e}^{z(\ln S_t - \lambda\kappa\tau)} \cdot E_t^Q\Big(\mathrm{e}^{z\sum_{i=N_t+1}^{N_T} Y_i}\Big) \cdot E_t^Q\Big[\mathrm{e}^{(z-1)\int_t^T r_s\mathrm{d}s + z\zeta(\tau) + z\xi(\tau)}\Big] \\
&= \mathrm{e}^{z(\ln S_t - \lambda\kappa\tau) + \lambda\tau\left(\frac{p\eta_1}{\eta_1 - z} + \frac{q\eta_2}{z+\eta_2} - 1\right) - \frac{z\sigma\rho}{\sigma_v}(\theta_v\tau + V_t)} \cdot E_t^Q\Big[\mathrm{e}^{(z-1)\int_t^T r_s\mathrm{d}s}\Big] \\
&\quad \cdot E_t^Q\Big[\mathrm{e}^{k_1\int_t^T V_s\mathrm{d}s + k_2 V_T}\Big],
\end{aligned} \tag{3.9}$$

where $k_1 = \Big\{\frac{z\sigma\rho\alpha_v}{\sigma_v} + \frac{z\sigma^2}{2}[z(1-\rho^2)-1]\Big\}, k_2 = \frac{z\sigma\rho}{\sigma_v}, \tau = T-t.$

In the following Lemmas, we use Feynman-Kac formula to obtain the above two conditional expectations in (3.9).

Lemma 3.2. Assume that the dynamics of spot interest rates r_t is given by(2.2), then

$$E_t^Q\Big[\mathrm{e}^{(z-1)\int_t^T r_s\mathrm{d}s}\Big] = \mathrm{e}^{C(T-t) - \frac{2(1-z)(1-\mathrm{e}^{-\gamma_\delta(T-t)})}{2\gamma_\delta + (\alpha_r - \gamma_\delta)(1-\mathrm{e}^{-\gamma_\delta(T-t)})} r}, \tag{3.10}$$

where $\gamma_\delta = \sqrt{\alpha_r^2 + 4\sigma_r^2\delta(1-z)}$, and

$$C(\tau) = \begin{cases} (1-z)\Big[\frac{\sigma_r^2(1-z) - 2\alpha_r\theta_r}{2\alpha_r^2}\tau + \frac{\alpha_r\theta_r - \sigma_r^2(1-z)}{\alpha_r^2}B(\tau) + \frac{\sigma_r^2(1-z)}{4\alpha_r^2}B(2\tau)\Big], & \delta = 0; \\ \frac{\theta_r}{\sigma_r^2}\Big[(\alpha_r - \gamma_\delta)\tau + 2\ln\frac{2\gamma_\delta}{2\gamma_\delta + (\alpha_r-\gamma_\delta)(1-\mathrm{e}^{-\gamma_\delta\tau})}\Big], & \delta = 0.5. \end{cases}$$

Proof. Because of the affine structure of (2.2) and guided by [16], we guess a solution of the form

$$E_t^Q\Big[\mathrm{e}^{(z-1)\int_t^T r_s\mathrm{d}s}\Big] = \exp\{C(t,T) - D(t,T)r_t\}. \tag{3.11}$$

Using Feynman-Kac formula, denote $E_t^Q\Big[\mathrm{e}^{(z-1)\int_t^T r_s\mathrm{d}s}\Big] := y(r,t,T)$, then $y(r,t,T)$ is a solution of the following backward P.D.E.

$$\begin{cases} \frac{\partial y}{\partial t}+\frac{1}{2}\sigma_r^2\, r^{2\delta}\frac{\partial^2 y}{\partial r^2}+(\theta_r-\alpha_r r)\frac{\partial y}{\partial r}+(z-1)ry=0, \\ y(r,T,T)=1. \end{cases} \tag{3.12}$$

Substituting (3.11) into (3.12), we have

$$C_t(t,T)-D_t(t,T)r+\frac{1}{2}\sigma_r^2\, r^{2\delta}D^2(t,T)-(\theta_r-\alpha_r\, r)D(t,T)+(z-1)r=0. \tag{3.13}$$

Solving (3.13), we get the following solutions

$$D(t,T) = \frac{2(1-z)[1-\mathrm{e}^{-\gamma_\delta(T-t)}]}{2\gamma_\delta+(\alpha_r-\gamma_\delta)[1-\mathrm{e}^{-\gamma_\delta(T-t)}]}, \tag{3.14}$$

$$C(t,T) = \frac{\theta_r}{\sigma_r^2}\Big[(\alpha_r-\gamma_1)(T-t)+2\ln\frac{2\gamma_1}{2\gamma_1+(\alpha_r-\gamma_1)[1-\mathrm{e}^{-\gamma_1(T-t)}]}\Big]. \tag{3.15}$$

Lemma 3.3. Assume that the volatility factor V_t satisfies (2.4), then

$$E_t^Q\Big[\mathrm{e}^{k_1\int_t^T V_s\mathrm{d}s+k_2V_T}\Big]=\Big[\frac{2\gamma_2\mathrm{e}^{(\alpha_v-\gamma_2)(T-t)/2}}{g_2(T-t)}\Big]^{\frac{2\theta_v}{\sigma_v^2}}\mathrm{e}^{\frac{2k_2\gamma_2-k_3[1-\mathrm{e}^{-\gamma_2(T-t)}]}{g_2(T-t)}V_t}, \tag{3.16}$$

where $\gamma_2=\sqrt{\alpha_v^2-2\sigma_v^2k_1}$, $k_3=k_2\alpha_v+k_2\gamma_2-2k_1$, $g_2(\tau)=2\gamma_2+(\alpha_v-\gamma_2-k_2\sigma_v^2)(1-\mathrm{e}^{-\gamma_2\tau})$.

Proof. Similarly to Lemma 3.2, let $G(V,t,T)=E_t^Q\Big[\mathrm{e}^{k_1\int_t^T V_s\mathrm{d}s+k_2V_T}\Big]$, then $G(V,t,T)$ is a solution to the following P.D.E.,

$$\begin{cases} \frac{\partial G}{\partial t}+\frac{1}{2}\sigma_v^2V\frac{\partial^2 G}{\partial V^2}+(\theta_v-\alpha_vV)\frac{\partial G}{\partial V}+k_1VG=0, \\ G(V,T,T)=\mathrm{e}^{k_2V_T}, \end{cases} \tag{3.17}$$

We also conjecture $G(V,t,T)=\mathrm{e}^{H(t,T)-E(t,T)V_t}$.Following (3.12-3.13), we have

$$E(t,T) = \frac{-2k_2\gamma_2+(k_2\alpha_v+k_2\gamma_2-2k_1)(1-\mathrm{e}^{-\gamma_2(T-t)})}{2\gamma_2+(\alpha_v-\gamma_2-k_2\sigma_v^2)(1-\mathrm{e}^{-\gamma_2(T-t)})}, \tag{3.18}$$

$$H(t,T) = \frac{2\theta_v}{\sigma_v^2}\ln\Big[\frac{2\gamma_2\mathrm{e}^{(\alpha_v-\gamma_2)(T-t)/2}}{2\gamma_2+(\alpha_v-\gamma_2-k_2\sigma_v^2)(1-\mathrm{e}^{-\gamma_2(T-t)})}\Big]. \tag{3.19}$$

By recalling (3.9-3.10) and (3.16) the explicit expresses of characteristic functions in Theorem 3.1 are obtained with tedious calculation as follows

Lemma 3.4. The characteristic functions, $\phi_1(u),\phi_2(u)$ associated to probability Q_1,Q_2 respectively, are given by

$$\phi_1(u)=\begin{cases} \mathrm{e}^{iu\ln S_t}\Big[\frac{2\gamma_v}{2\gamma_v+[\alpha_v-\gamma_v-(1+iu)\rho\sigma\sigma_v](1-\mathrm{e}^{-\gamma_v\tau})}\Big]^{\frac{2\theta_v}{\sigma_v^2}}\exp\Big\{\frac{\theta_v(\alpha_v-\gamma_v)\tau}{\sigma_v^2}-\frac{(1+iu)\theta_v\sigma\rho\tau}{\sigma_v} \\ +\lambda\tau\Big[\frac{p\eta_1}{\eta_1-1-iu}+\frac{q\eta_2}{\eta_2+1+iu}-\kappa(1+iu)-1\Big]+iuB(\tau)r \\ +\Big[\frac{-\sigma_r^2u^2+2iu\alpha_r\theta_r}{2\alpha_r^2}\tau+\frac{-iu\alpha_r\theta_r+\sigma_r^2u^2}{\alpha_r^2}B(\tau)-\frac{\sigma_r^2u^2}{4\alpha_r^2}B(2\tau)\Big]+C_3V\Big\}, & \delta=0, \\ \mathrm{e}^{iu\ln S_t}\Big[\frac{2\gamma_v}{2\gamma_v+[\alpha_v-\gamma_v-(1+iu)\rho\sigma\sigma_v](1-\mathrm{e}^{-\gamma_v\tau})}\Big]^{\frac{2\theta_v}{\sigma_v^2}}\exp\Big\{\frac{\theta_v(\alpha_v-\gamma_v)\tau}{\sigma_v^2}-\frac{(1+iu)\theta_v\sigma\rho\tau}{\sigma_v} \\ +\lambda\tau\Big[\frac{p\eta_1}{\eta_1-1-iu}+\frac{q\eta_2}{\eta_2+1+iu}-\kappa(1+iu)-1\Big]+C_1+\frac{2ui(1-\mathrm{e}^{-\gamma_r\tau})}{2\gamma_r+(\alpha_r-\gamma_r)(1-\mathrm{e}^{-\gamma_r\tau})}r+C_3V\Big\}, & \delta=0.5. \end{cases}$$

$$\phi_2(u)=\begin{cases} \mathrm{e}^{iu\ln S_t}\Big[\frac{2\bar{\gamma}_v}{2\bar{\gamma}_v+[\alpha_v-\bar{\gamma}_v-iu\rho\sigma\sigma_v](1-\mathrm{e}^{-\bar{\gamma}_v\tau})}\Big]^{\frac{2\theta_v}{\sigma_v^2}}\exp\Big\{\frac{\theta_v(\alpha_v-\bar{\gamma}_v)\tau}{\sigma_v^2}-\frac{iu\theta_v\sigma\rho\tau}{\sigma_v} \\ +\Big[\frac{-\sigma_r^2u^2+2iu(\alpha_r\theta_r-\sigma_r^2)}{2\alpha_r^2}\tau+\frac{-iu(\alpha_r\theta_r-2\sigma_r^2)+\alpha_r\theta_r-\sigma_r^2+\sigma_r^2u^2}{\alpha_r^2}B(\tau)-\frac{2iu\sigma_r^2+\sigma_r^2u^2}{4\alpha_r^2}B(2\tau)\Big] \\ +\lambda\tau\Big[\frac{p\eta_1}{\eta_1-iu}+\frac{q\eta_2}{\eta_2+iu}-iu\kappa-1\Big]+iuB(\tau)r+C_4V\Big\}, & \delta=0, \\ \mathrm{e}^{iu\ln S_t}\Big[\frac{2\bar{\gamma}_v}{2\bar{\gamma}_v+[\alpha_v-\bar{\gamma}_v-iu\rho\sigma\sigma_v](1-\mathrm{e}^{-\bar{\gamma}_v\tau})}\Big]^{\frac{2\theta_v}{\sigma_v^2}}\exp\Big\{\frac{\theta_v(\alpha_v-\bar{\gamma}_v)\tau}{\sigma_v^2}-\frac{iu\theta_v\sigma\rho\tau}{\sigma_v} \\ +\lambda\tau\Big[\frac{p\eta_1}{\eta_1-iu}+\frac{q\eta_2}{\eta_2+iu}-iu\kappa-1\Big]+\Big[C_2-C_0\Big] \\ +\Big[\frac{2(1-\mathrm{e}^{-\gamma\tau})}{2\gamma+(\alpha_r-\gamma)(1-\mathrm{e}^{-\gamma\tau})}-\frac{2(1-iu)(1-\mathrm{e}^{-\bar{\gamma}_r\tau})}{2\bar{\gamma}_r+(\alpha_r-\bar{\gamma}_r)(1-\mathrm{e}^{-\bar{\gamma}_r\tau})}\Big]r+C_4V\Big\}, & \delta=0.5. \end{cases}$$

where$\gamma_r=\sqrt{\alpha_r^2-2iu\sigma_r^2}$, $\gamma_v=\sqrt{[\alpha_v-(1+iu)\rho\sigma\sigma_v]^2-iu(1+iu)\sigma^2\sigma_v^2}$,

$$\begin{aligned} \bar{\gamma}_r &= \sqrt{\alpha_r^2+2\sigma_r^2(1-iu)},\quad \bar{\gamma}_v=\sqrt{[\alpha_v-iu\rho\sigma\sigma_v]^2+iu(1-iu)\sigma^2\sigma_v^2}, \\ C_0 &= \frac{\theta_r}{\sigma_r^2}\Big[(\alpha_r-\gamma)\tau+2\ln\frac{2\gamma}{2\gamma+(\alpha_r-\gamma)(1-\mathrm{e}^{-\gamma\tau})}\Big], \\ C_1 &= \frac{\theta_r}{\sigma_r^2}\Big[(\alpha_r-\gamma_r)\tau+2\ln\frac{2\gamma_r}{2\gamma_r+(\alpha_r-\gamma_r)(1-\mathrm{e}^{-\gamma_r\tau})}\Big], \\ C_2 &= \frac{\theta_r}{\sigma_r^2}\Big[(\alpha_r-\bar{\gamma}_r)\tau+2\ln\frac{2\bar{\gamma}_r}{2\bar{\gamma}_r+(\alpha_r-\bar{\gamma}_r)(1-\mathrm{e}^{-\bar{\gamma}_r\tau})}\Big], \\ C_3 &= \frac{iu(1+iu)\sigma^2(1-\mathrm{e}^{-\gamma_v\tau})}{2\gamma_v+[\alpha_v-\gamma_v-(1+iu)\rho\sigma\sigma_v](1-\mathrm{e}^{-\gamma_v\tau})}, \\ C_4 &= \frac{iu(iu-1)\sigma^2(1-\mathrm{e}^{-\bar{\gamma}_v\tau})}{2\bar{\gamma}_v+[\alpha_v-\bar{\gamma}_v-iu\rho\sigma\sigma_v](1-\mathrm{e}^{-\bar{\gamma}_v\tau})}. \end{aligned}$$

Remark 1. Lemma 3.4 is similar to that in [14,15] with $\delta=0.5$, and in [18] with $\delta=0$.

Remark 2. The solution to (2.8) is given by Lemma 3.4 and Theorem 3.1.

Remark 3. The results of Theorem 3.1 and Lemma 3.4 are the more general solution. Moreover, the closed-form solution has a clear and tractable expression. Hence, the hedge ratios Δ and other Greek letters can be given analytically by

Proposition 3.5. Under the same assumptions of Theorem 3.1,the hedge ratios Δ and some Greeks letters are as follows:

$$\begin{aligned} \Delta_S(S;t,T,r,V) &= \frac{\partial C_E(S,t;T,r,V)}{\partial S}=F_1(S,t;T,r,V), \\ \Delta_V(S;t,T,r,V) &= \frac{\partial C_E(St,;T,r,V)}{\partial V}=S(t)\frac{\partial F_1(S,t;T,r,V)}{\partial V}-KP(t,T)\frac{\partial F_2(S,t;T,r,V)}{\partial V}, \\ \Delta_r(S;t,T,r,V) &= \frac{\partial C_E(S,t;T,r,V)}{\partial r}=S(t)\frac{\partial F_1(S,t;T,r,V)}{\partial r}-KP(t,T)\Big[\frac{\partial F_2(S,t;T,r,V)}{\partial r} \\ &\quad -B^*(t,T)F_2(S,t;T,r,V)\Big], \end{aligned}$$

$$\begin{aligned} \Gamma_S(S;t,T,r,V) &= \frac{\partial^2 C_E(S,t;T,r,V)}{\partial S^2}=\frac{\partial F_1(S,t;T,r,V)}{\partial S}, \\ \Gamma_V(S;t,T,r,V) &= \frac{\partial^2 C_E(S,t;T,r,V)}{\partial V^2}=S(t)\frac{\partial^2 F_1(S,t;T,r,V)}{\partial V^2}-KP(t,T)\frac{\partial^2 F_2(S,t;T,r,V)}{\partial V^2}, \end{aligned}$$

$$\begin{aligned}\Gamma_r(S;t,T,r,V) &= \frac{\partial^2 C_E(S,t;T,r,V)}{\partial r^2} = S(t)\frac{\partial^2 F_1(S,t;T,r,V)}{\partial r^2} - KP(t,T)\Big[\frac{\partial^2 F_2(S,t;T,r,V)}{\partial r^2}\\ &\quad -2B^*(t,T)\frac{\partial F_2(S,t;T,r,V)}{\partial r} + B^{*2}(t,T)F_2(S,t;T,r,V)\Big],\end{aligned}$$

where for $h=S,V,r$, and $j=1,2$,

$$\frac{\partial F_j(S,t;T,r,V)}{\partial h} = \frac{1}{\pi}\int_0^{+\infty}\Re\Big[\frac{\mathrm{e}^{-iu\ln K}}{iu}\frac{\partial\phi_j(u)}{\partial h}\Big]\mathrm{d}u, \tag{3.20}$$

$$\frac{\partial^2 F_j(S,t;T,r,V)}{\partial h^2} = \frac{1}{\pi}\int_0^{+\infty}\Re\Big[\frac{\mathrm{e}^{-iu\ln K}}{iu}\frac{\partial^2\phi_j(u)}{\partial h^2}\Big]\mathrm{d}u, \tag{3.21}$$

and $B^*(t,T)=\frac{2(1-\mathrm{e}^{-\gamma_0(T-t)})}{2\gamma_0+(\alpha_r-\gamma_0)(1-\mathrm{e}^{-\gamma_0(T-t)})}$, $\gamma_0=\sqrt{\alpha_r^2+4\sigma_r^2\delta}$.

Next, we give two corollaries from Lemma 3.4.

Corollary 3.6. If the volatility factor V is constant, without loss of generality , we assume that $V=1$, then the two characteristic functions ϕ_1,ϕ_2 are given respectively by

$$\phi_1(u) = \begin{cases}\exp\Big\{iu\ln S_t+\frac{1}{2}\sigma^2 iu(1+iu)\tau+\lambda\tau\Big[\frac{p\eta_1}{\eta_1-1-iu}+\frac{q\eta_2}{\eta_2+1+iu}-\kappa(1+iu)-1\Big]\\ +\Big[\frac{-\sigma_r^2u^2+2iu\alpha_r\theta_r}{2\alpha_r^2}\tau+\frac{-iu\alpha_r\theta_r+\sigma_r^2u^2}{\alpha_r^2}B(\tau)-\frac{\sigma_r^2u^2}{4\alpha_r^2}B(2\tau)\Big]+iuB(\tau)r_t\Big\}, & \delta=0,\\ \exp\Big\{iu\ln S_t+\frac{1}{2}\sigma^2 iu(1+iu)\tau+\lambda\tau\Big[\frac{p\eta_1}{\eta_1-1-iu}+\frac{q\eta_2}{\eta_2+1+iu}-\kappa(1+iu)-1\Big]\\ +C_1+\frac{2ui(1-\mathrm{e}^{-\gamma_r\tau})}{2\gamma_r+(\alpha_r-\gamma_r)(1-\mathrm{e}^{-\gamma_r\tau})}r_t\Big\}, & \delta=0.5.\end{cases}$$

$$\phi_2(u) = \begin{cases}\exp\Big\{iu\ln S_t-\frac{1}{2}\sigma^2 iu(1-iu)\tau+\lambda\tau\Big[\frac{p\eta_1}{\eta_1-iu}+\frac{q\eta_2}{\eta_2+iu}-iu\kappa-1\Big]\\ +\Big[\frac{-\sigma_r^2u^2+2iu(\alpha_r\theta_r-\sigma_r^2)}{2\alpha_r^2}\tau+\frac{-iu(\alpha_r\theta_r-2\sigma_r^2)+\alpha_r\theta_r-\sigma_r^2+\sigma_r^2u^2}{\alpha_r^2}B(\tau)\\ -\frac{2iu\sigma_r^2+\sigma_r^2u^2}{4\alpha_r^2}B(2\tau)\Big]+iuB(\tau)r\Big\}, & \delta=0,\\ \exp\Big\{iu\ln S_t-\frac{1}{2}\sigma^2 iu(1-iu)\tau+\lambda\tau\Big[\frac{p\eta_1}{\eta_1-iu}+\frac{q\eta_2}{\eta_2+iu}-iu\kappa-1\Big]\\ +\Big[C_2-C_0\Big]+\Big[\frac{2(1-\mathrm{e}^{-\gamma\tau})}{2\gamma+(\alpha_r-\gamma)(1-\mathrm{e}^{-\gamma\tau})}-\frac{2(1-iu)(1-\mathrm{e}^{-\bar\gamma_r\tau})}{2\bar\gamma_r+(\alpha_r-\bar\gamma_r)(1-\mathrm{e}^{-\bar\gamma_r\tau})}\Big]r\Big\}, & \delta=0.5.\end{cases}$$

Corollary 3.7. If the interest rate r_t is constant, the two characteristic functions ϕ_1,ϕ_2 are then given respectively by

$$\begin{aligned}\phi_1(u) &= \Big[\frac{2\gamma_v}{2\gamma_v+[\alpha_v-\gamma_v-(1+iu)\rho\sigma\sigma_v](1-\mathrm{e}^{-\gamma_v\tau})}\Big]^{\frac{2\theta_v}{\sigma_v^2}}\exp\Big\{iu\ln S_t+\frac{\theta_v(\alpha_v-\gamma_v)\tau}{\sigma_v^2}\\ &\quad -\frac{(1+iu)\theta_v\sigma\rho\tau}{\sigma_v}+\lambda\tau\Big[\frac{p\eta_1}{\eta_1-1-iu}+\frac{q\eta_2}{\eta_2+1+iu}-\kappa(1+iu)-1\Big]+iur\tau+C_3V\Big\};\\ \phi_2(u) &= \Big[\frac{2\bar\gamma_v}{2\bar\gamma_v+[\alpha_v-\bar\gamma_v-iu\rho\sigma\sigma_v](1-\mathrm{e}^{-\bar\gamma_v\tau})}\Big]^{\frac{2\theta_v}{\sigma_v^2}}\exp\Big\{iu\ln S_t+\frac{\theta_v(\alpha_v-\bar\gamma_v)\tau}{\sigma_v^2}-\frac{iu\theta_v\sigma\rho\tau}{\sigma_v}\\ &\quad +\lambda\tau\Big[\frac{p\eta_1}{\eta_1-iu}+\frac{q\eta_2}{\eta_2+iu}-iu\kappa-1\Big]+iur\tau+C_4V\Big\}.\end{aligned}$$

Remark 4. Corollary 3.7 illuminates the SV_DexpJ model with jump risks. When $\lambda=0$,if $\delta=0$, the model is studied in [3]; if $\delta=0.5$, the model is similar to that in [5].

§4 Comparison of European options in different models

In this section, first we compare our model with BS model, DexpJ and other special cases of Corollary 3.6,3.7 in pricing European option and delta-hedging ratio with numerical experiment. Second, we illustrate implied volatility of four different models and analyze mainly the impact of ρ on implied volatility of SVSI_DexpJ model with CIR-MSR. Throughout this section, we take as parameters of our model $S = 100, \eta_1 = \eta_2 = 5, p = 0.4, \lambda = 1, \sigma = 0.2, r = 0.05, \theta_0.035$,$\alpha_r = 0.4, \sigma_r = 0.095, \alpha_v = 0.3, \theta_v = 0.6, \sigma_v = 0.1, \rho = -0.25, T = 0.5$. In Table 4.1, we compare option price in our model with that in other models. First, option price in BS model is the smallest. Second, the values of in-the-money (ITM)options with Vas-MSR are less than those with CIR-MSR, however, the values of out-the-money (OTM)options with Vas-MSR is greater than that with CIR-MSR. Finally, option prices in multi-factor economy(SVSI_DexpJ, SV_DexpJ, SI_DexpJ)are higher. In the same way, in Table 4.2 we make a comparison of the option deltas Δ_S in five different models. It shows that the option deltas Δ_S in BS model is greater than those in other models for ITM options. However, for OTM options, the option deltas in BS model is the lest, and overall the values of deltas for at-the-money(ATM)options do not change remarkably. Table 4.2 demonstrates the same tendency in Table 4.1 for the values of option deltas between Vas-MSR and CIR-MSR.

Table 4.1 Comparison of option prices for European Calls in 5 different models

K	90	95	100	105	110	115	120
Black-Scholes	13.4985	9.8727	6.8887	4.5817	2.9067	1.7616	1.0226
DexpJ	15.7203	12.4361	9.6858	7.4837	5.7897	4.5265	3.6025
SI_DexpJ(Vas)	15.5657	12.3985	9.7863	7.7289	6.1730	5.0329	4.2133
SI_DexpJ(CIR)	15.8328	12.5383	9.7744	7.5570	5.8480	4.5716	3.6368
SV_DexpJ	15.8361	12.5785	9.8418	7.6374	5.9282	4.6425	3.6939
SVSI_DexpJ(Vas)	15.6882	12.5452	9.9427	7.8794	6.3056	5.1416	4.2978
SVSI_DexpJ(CIR)	15.9476	12.6801	9.9301	7.7110	5.9872	4.6884	3.7292

Table 4.2 Comparison of Δ_S for European Calls in 5 different models

K	80	90	100	105	110	115	120
Black-Scholes	0.9660	0.8395	0.5977	0.4612	0.3349	0.2294	0.1488
DexpJ	0.9162	0.7930	0.5916	0.4817	0.3799	0.2932	0.2244
SI_DexpJ(Vas)	0.9164	0.7943	0.5949	0.4859	0.3845	0.2976	0.2283
SI_DexpJ(CIR)	0.9173	0.7956	0.5954	0.4857	0.3837	0.2965	0.2270
SV_DexpJ	0.9124	0.7886	0.5921	0.4853	0.3857	0.3000	0.2310
SVSI_DexpJ(Vas)	0.9127	0.7899	0.5953	0.4893	0.3901	0.3043	0.2348
SVSI_DexpJ(CIR)	0.9136	0.7910	0.5958	0.4891	0.3894	0.3032	0.2336

Figure 4.1 and Figure 4.2 illustrate the implied volatility of option. We price a call option with different exercise prices using the specified models in this paper with the same parameter-values as in Table 4.1, then we take those values as market prices of option and compute implied volatilities using BS formula. Figure 4.1 examines the implied volatility of four different models, including DexpJ, SV_DexpJ and SVSI_DexpJ models. First, it shows that the curve is not

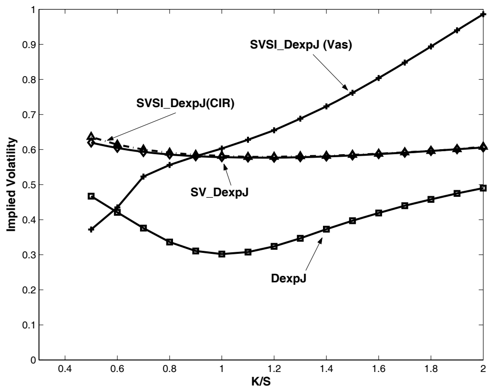

Fig 4.1 Implied volatility in 4 different models

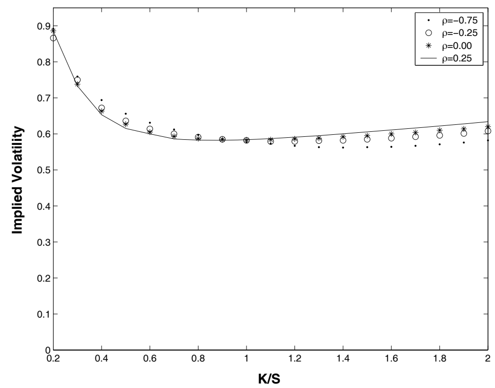

Fig 4.2 Implied volatility in ours with CIR-MSR

symmetric and resembles a "smile" pattern for DexpJ model because of the constant volatility in the DexpJ model. Second, the curve of the implied volatility is increasing with moneyness K/S for SVSI_DexpJ in Vas-MSR economy, which is possibly affected by Vasicek stochastic interest rate setup with negative effects. Finally, the implied volatilities for SV_DexpJ and SVSI_DexpJ(CIR) are almost constant, which should be valid for the real-world market and the best framework describing stock dynamics. Figure 4.2 examines how the implied volatility varies with correlation ρ in SVSI_DexpJ model with CIR-MSR. The monotonic downward sloping of the implied volatility with moneyness displayed in a "sneer" pattern implies that the market ITM options are undervalued by BS formula. Figure 4.2 shows that the smaller the value of correlation changes, the flatter the curve for the implied volatility is due to the negative correlation between stock return and volatility, which explains the asymmetric leptokurtic and "volatility smile" features.

§5 Conclusions

The double exponential jump-diffusion model with different market structure risks incorporates several important features of stock return. We derive the closed-form solution for European call option price in these models by using a Fourier inversion transform, P.D.E. and Feynman-Kac formula. Comparison and analysis of these models reveal that these MSR factors have a significant impact on option price, and show that the double exponential jump-diffusion model with CIR-MSR is suitable for modelling market variables. Not surprisingly, SVSI_DexpJ in CIR-MSR model will be useful for theoretical analysis and financial practice.

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