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An improved Viterbi algorithm for IF extraction of multicomponent signals

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Abstract Viterbi algorithm (VA) applied to time–frequency (TF) representation is a highly performed instantaneous frequency (IF) estimator for discrete-time signals, but it suffers from switch problem (SP) at the intersected points of multicomponents on TF plane. To suppress the SP in VA, an improved VA (IVA) presented in this paper assumes that the IF variation trends between two adjacent IF variation are not large, and then, a novel penalty function is introduced and added to the original VA. To verify the algorithm, the proposed algorithm applied to several multicomponent signals is firstly simulated; then, how parameter in the new penalty function influences the performance is analyzed. Comparison of the proposed algorithm with VA on signals in the background of noise is also made in the next. Simulations indicate that in contrast to the original VA, the proposed IVA can effectively suppress the SP caused by the intersected IFs and thus can achieve more accurate IFs for the multicomponent signals especially those with monotonous IFs.

Keywords Viterbi algorithm · Instantaneous frequency estimator · Multicomponent signals · Time–frequency analysis

1 Introduction

Instantaneous frequency (IF) is an important parameter for non-stationary signals in radar, sonar and other applica-

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tions, as it can reveal the time-varying characteristics of the Fourier spectrums in the non-stationary signals [1–3]. IF estimation for discrete-time signals has been attracted a lot of researches over the last decades, and various algorithms have been proposed [4–6], and most of methods are effective for the mono-component signals. IF estimate is commonly easier for mono-component signals than for multicomponent ones, therefore, a number of signal extraction algorithms based on null space pursuit [7,8], wavelet decomposition [9], empirical mode decomposition (EMD) [10–12], parameterized de-chirping [13] and blind source separation [14] can be implemented prior to the IF estimation. However, these extraction techniques are not always effective for the complicated multicomponent signal, and IF extraction for multicomponent signals is still a challenge. Due to the time-varying characteristics of Fourier spectrum in the multicomponent signals, IF estimation based on time–frequency (TF) analysis among other methods is a natural promising solution. After mapping one-dimensional (1-D) signal into two-dimensional (2-D) time–frequency representation (TFR) using TF transform, IFs are located at TF ridges, rather than TF points belonging to the noise and cross-terms. Therefore, most of the current methods are based on TF analysis. IF estimation based on TFR is generally influenced by the following factors: (a) noise; (b) other components; (c) time–frequency distribution used; (d) IF detection method. To suppress the noise and other components, TF filtering [15] and the component extraction algorithms presented above can be both applied. Corresponding to aforementioned (c), while TFR with constant window cannot obtain the optimal TF concentration because of the time-varying IFs, some researchers attribute development of TFRs with adaptive variable window widths [16–18].

Corresponding to aforementioned (d), other researchers focus on IF estimators for multicomponent signal from

TFR [19–23]. In [19–21], Hough transform applied to TFR is proposed to estimate IFs of multicomponent signals. However, this transform needs prior information about the class of signal's law and IFs must be mathematically depicted. The transform is inefficient for components with different laws. In [22], image processing techniques such as Markov random field model are also utilized to estimate the overlapped IFs.

In order to estimate IFs, one simple and direct approach is to detect peaks on the TF plane. However, for low signal-to-noise ratio (SNR) signals, the frequency fluctuation becomes large so that this IF estimates could far from true IFs. Thus, in [23], the continuous property was used by the IF detection on the TFR to obtain high-accurate estimates. Briefly, assume that the IF passes through as many TF points as possible of Wigner distribution with strong magnitudes and that the IF variations between two consecutive points are not too large, two penalty functions are defined, and Viterbi algorithm (VA) is introduced to estimate IF from TFR. VA is a most effective IF estimator for low SNR mono-component signals [23]. Furthermore, VA can be also potentially applied to estimate IFs of multicomponent signals when signals are separated well on the TF plane [24,25]. However, when IFs are overlapped on the TF plane, switch problem (SP) could arise, that is, one IF may switch to another IF at intersected TF points [23], which leads to wrong IF trajectory. To avoid this SP in the process of VA, this work attempts to propose an improved VA to extract overlapped IFs from TFR. Assuming that the IF variation trends between two adjacent IF variation are not intense, a novel penalty function is defined and added to the original VA. As a result, SP among different IFs can be largely suppressed and more accurate IFs can be acquired.

This paper is organized as follows: An improved VA scheme is depicted in the next section. Section 3 presents the validation and performance analysis of the proposed algorithm using a series of artificial multicomponent signals. Conclusions are drawn in the last section.

2 Algorithms

2.1 Viterbi algorithm

To introduce our proposed method, we first review the original VA. More detail can be found in reference [23].

After transforming a noisy signal into 2-D TFR, both signal and the noise are converted into TF points on the TF plane. The signal will concentrate on the IF ridge of TF points with large amplitude, and the IF of signal should be commonly smooth in a short time interval, while the noise will locate on the TF points randomly across the TF plane with small amplitude. Thus, the VA for IF estimation from TFR is pro-

posed based on two assumptions: (a) The estimated IF should pass through as many TF points as possible with strong magnitude. (b) The IF variation between two consecutive points is not extremely large. After defining two penalty functions corresponding to these two assumptions, the IF estimator can be written as the line minimizing the corresponding sum of the path penalty function [23]:

$$
\begin{aligned}
\hat{f}(n) &= \arg \min_{k(n) \in K} \left[\sum_{n=n_1}^{n_2-1} g(k(n), k(n+1)) \right. \\
&\quad \left. + \sum_{n=n_1}^{n_2} h\left(\text{TF}(n, k(n)) \right) \right] \\
&= \arg \min_{k(n) \in K} p(k(n); n_1, n_2),
\end{aligned}
\tag{1}
$$

where $\text{TF}(n, k)$ is a TF point at time bin n and frequency bin k, $h(x)$ is a non-increasing function corresponding to the first assumption (a) and $g(x, y)$ is a non-decreasing function with respect to the absolute difference between x and y corresponding to the second assumption (b), $p(k(n); n_1, n_2)$ is a sum of penalty functions $h(x)$ and $g(x, y)$, along the line $k(n)$, from time instant n_1 to time instant n_2, and K is the all paths between n_1 and n_2.

Assume a discrete TFR has M (Frequency) × N (Time) TF points, then $n_1 = 1, n_2 = N$, and the total path number K is equal to be M^N from n_1 to n_2. When one IF is estimated from TFR, the IF estimation in (1) is converted into searching a line $K(n)$ with minimal penalty from M^N paths.

For a considered time point n, the function $h(x)$ can be formed as follows. The TF values are first sorted into the non-increasing sequence [23]:

$$
\text{TF}(n, f_1) \geq \text{TF}(n, f_2) \geq \cdots \geq \text{TF}(n, f_j) \geq \cdots \text{TF}(n, f_M),
\tag{2}
$$

where $j = 1, 2, \ldots, M$, is the position within this sequence. Since larger magnitude of TF points corresponds to smaller penalty function, one simple $h(x)$ is defined as [23]:

$$
h\left(\text{TF}\left(n, f_j \right) \right) = j - 1.
\tag{3}
$$

In this way, at the considered time point n, $h(\text{TF}(n, f_1)) = 0$, $h(\text{TF}(n, f_2)) = 1, \ldots, h(\text{TF}(n, f_M)) = M - 1$, i.e., largest TF point results in smallest penalty, second largest TF points results in second smallest penalty, etc.

Meanwhile, a common linear form of $g(x, y)$ can be defined as [23]:

$$
g(x, y) = \begin{cases} 0, & |x - y| \leq \Delta, \\ c(|x - y| - \Delta), & |x - y| > \Delta. \end{cases}, \quad c, \Delta > 0
\tag{4}
$$

In this formula, the choice for Δ along c together determines the maximal expected value of the IF variation between consecutive points. In the realization, the selection of Δ is based on the frequency resolution of TF transform. Good results can be obtained by taking Δ and c corresponding to a few neighboring points when high-resolution TF transforms are used [23,25].

2.2 Proposed algorithm

VA performs well to estimate the mono-component signal in the low SNR; furthermore, it can be also potentially used to estimate well-separated IFs of multicomponent signal on the TF plane [25]. However, when VA is applied to the overlapped IFs, switch problem occasionally occurs. That is, one IF may switch to another IF at the intersected TF points between two components. To demonstrate the disadvantage, assume a multicomponent signal composed of one sinusoidal frequency modulation (SFM) and one linear frequency modulation (LFM) components as:

$$s(t) = \underbrace{\exp\left(j80\cos(\pi t + \pi/4)\right)}_{\text{SFM}} + \underbrace{\exp(j2t^2)}_{\text{LFM}}. \tag{5}$$

Assume the signal is sampled at the length of 256 points, with sample frequency 256 Hz, parameters Δ and c in (4) are set to be 2 and 8, respectively. In principle, all the types of TFRs can be selected for VA-based IF estimation. For multicomponent m-D signals, however, TFR with high TF resolution and good cross-term suppression should be preferred. It is reasonably concluded that higher performed TFR will result in more accurate VA-based IF estimation. While traditional short-time Fourier transform (STFT) lacks enough TF resolution and Wigner–Ville Distribution (WVD) suffers from serious cross-term interference, the filtered WVD, known as Cohen class, is one of the good candidates. In this paper, B-distribution (BD) is preferred for its high performance on TF resolution and crossing-term suppression [26]. After transforming the signal into TFR using BD, VA-BD is used to calculate two SFM IFs. The results are shown in Fig. 1a, b. It is shown in Fig. 1b that the measured IF has obviously switched from IF1 to IF2 at the region where two IFs are overlapped.

The switch problem can be explained as follows: According to (1), both IF1 and IF2 correspond to two paths with two minimal values across the BD. At the overlapped TF points, the magnitude penalty function $h(x)$ is almost the same for two components. However, when IF1 switches to IF2 at the overlapped points, the penalty function $g(x, y)$ along IF2 is smaller than the one along the IF1 itself. The penalty function $g(x, y)$ only assumes the IF variation between two consecutive points x and y is not extremely large. It cannot guarantee the continuous property of IF variation trends, i.e., when
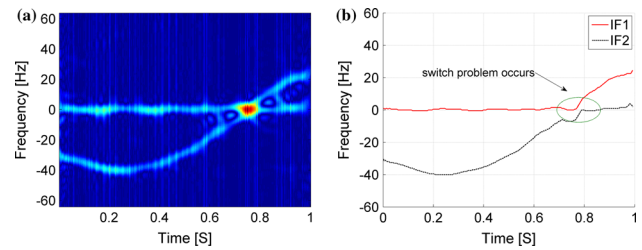

Fig. 1 VA-BD for two SFM multicomponent signals. **a** BD, **b** VA-based IF estimation. SP occurs where *arrow ticks*

the IF is calculated along the intersected region, the sign of two adjacent IF variation trends should be identical in common. Therefore, analysis above illuminates us present a third assumption to solve SP: The consecutive IF variation trends are not large, and then, a new penalty function is introduced as:

$$r(x, y, z) = \begin{cases} 0 & \text{if } (z-y)(y-x) > 0 \\ u\,|(z-y)-(y-x)| & \text{else} \end{cases} \tag{6}$$

where u is a parameter for penalty function $r(x, y, z)$, x, y, z are the time adjacent three IF points, which determines the suppressed level of SP in the IF estimation. Then, the function (1) is revised as:

$$\begin{aligned} \hat{f}(n) &= \arg\min_{k(n)\in K} \left[\sum_{n=n_1}^{n_2-1} g(k(n), k(n+1)) \right. \\ &\quad + \sum_{n=n_1}^{n_2-2} r(k(n), k(n+1), k(n+2)) \\ &\quad \left. + \sum_{n=n_1}^{n_2} h\left(\text{TF}(n, k(n))\right) \right] \\ &= \arg\min_{k(n)\in K} p(k(n); n_1, n_2), \end{aligned} \tag{7}$$

where the IF path is determined by three penalty functions. In this way, one IF estimate is also converted into searching a minimal path $k(n)$ from K paths determined by (7), but suppression of SP can be expected. Note that when $u = 0$, the improved VA is identical to VA. For simplicity, the new proposed algorithm is named as improved VA (IVA). Assume there are $L_{\max}$ components in a multicomponent signal, the IFs of multicomponent signal will be estimated by IVA-BD one by one. Note that the implementation of the proposed IVA is almost the same as VA, except that the penalty function $g(k(n), k(n+1))$ should be replaced as $g(k(n), k(n+1)) + r(k(n), k(n+1), k(n+2))$ when $n \geq 3$. It can be implemented by online realization, which has been explained in detail in [23,25]. Thus, implementation of the proposed method is summarized in the following textbox.

Initializing step:

  1. Setting IVA parameters (Δ, c, u), component order i = 0, component

    number L_{max} and masking bandwidth δ.

  2. Getting B-distribution $BD_i(n, k)$ of the considered signal.

  3. Establishing the penalty functions $h(BD_i(n, k))$, $g(x, y)$ and $r(x,y,z)$ as

    three sub-functions.

While i < L_{max}

  1. Estimating the IF $\hat{f}_i(n)$ from $BD_i(n, k)$ according to Eq.(7).

  2. Forming a new BD of signal by taking zero-values in the region around

    the determined IF by:

$$BD_{i+1}(n,k)=\begin{cases} 0, & k \in [\hat{f}_i(n)-\delta, \hat{f}_i(n)+\delta], \\ BD_i(n,k), & others, \end{cases}$$

  3. Updating $i \rightarrow i+1$.

End

As a comparison, the same signal (5) at SNR $= -2$ dB is evaluated using IVA. The parameters are set as: $\Delta = 2$, $c = 8$, $u = 12$, $\delta = 12$. The IVA-based IF estimation is shown in Fig. 2a. VA-based IF estimate with different Δ and c is presented in Fig. 2b–d. And the mean square error (MSE) and mean absolute error (MAE) of IFs measured by 200 independent realizations are also displayed in Table 1.
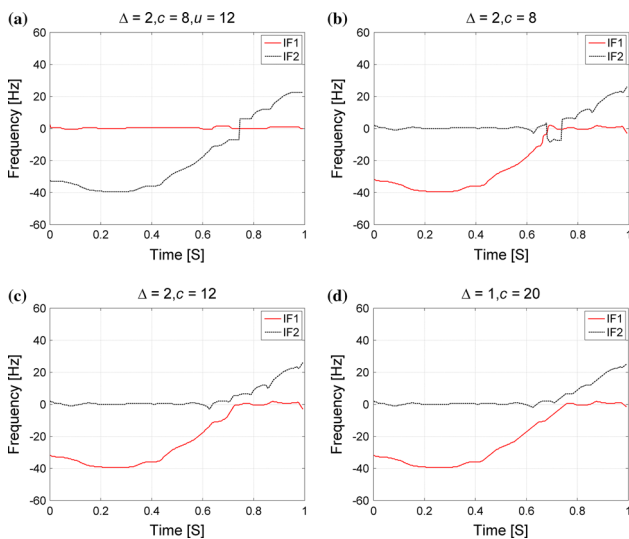

Fig. 2 **a** IVA-based IF estimation, **b–d** VA-based IF estimation with different parameters Δ and c

Table 1 MSE and MAE of IFs corresponding to Fig. 2

Components	IVA (a)	VA (b)	VA (c)	VA (d)
MSE				
SFM	4.57	6.92	6.08	5.68
LFM	6.48	8.29	7.67	7.32
MAE				
SFM	1.05	2.95	2.31	1.99
LFM	1.90	3.37	2.84	2.50

Note from Fig. 2b–d and Table 1 that smaller Δ and larger c in VA result in smoother IF estimation and less MSE and MAE, but SP always exists, meaning that wrong IFs are estimated, and VA cannot solve the SP by adjusting the VA parameters easily. However, as is shown in Fig. 2a, IVA succeeds to obtain IFs without SP by selecting proper parameters Δ, c and u, resulting in the smallest MSE and MAE for both two IFs. To validate the algorithm, more simulation on performance analysis will be implemented in the next section.

3 Simulation and performance analysis

In this section, a set of synthetic multicomponent signals embedded in the white Gaussian noise with independently real and imaginary parts are considered. The discrete signal is expressed as:

$$s(n\Delta t) = x(n\Delta t) + w(n\Delta t), \tag{8}$$

where $n = 1, 2, \ldots, N$, $N = 256$, sampling interval $\Delta t = 1/256$ s, $E(w(n\Delta t)) = 0$ and $var(w(n\Delta t)) = \sigma^2$. Assume that the amplitude of each component equals to 1, the SNR is defined as $10 \log_{10}(1/\sigma^2)$ dB. In the first part, IVA-based IF estimation for two multicomponent signals will be presented. And in the next part, performance analysis of the proposed algorithm on parameter selection of u is given. Meanwhile, the comparison of IVA with VA at different SNRs will be conducted.

3.1 Examples

3.1.1 Example 1

A signal with one LFM component and two SFM components, which is defined as:

$$x(t) = \underbrace{\exp\left(j \cdot (30\cos(2\pi t + \pi/6))\right)}_{SFM1}$$
$$+ \underbrace{\exp\left(j \cdot (25\cos(4\pi t + \pi/4))\right)}_{SFM2} + \underbrace{\exp(j \cdot (20t^2))}_{LFM}. \tag{9}$$

This signal is practical and usually modeled as the radar returned echo from the rigid target with micro-motions such as the helicopter with rotating blades [15,27]. Assume the signal is corrupted with white Gaussian noise with SNR $=$ 2 dB, the BD is displayed in Fig. 3, where three components are seriously overlapped on the TF plane.

Both IVA and VA are applied on BD to estimate three IFs. As is shown in VA, IFs are calculated one by one from BD. Therefore, for seriously TF overlapped signals, the latter IF
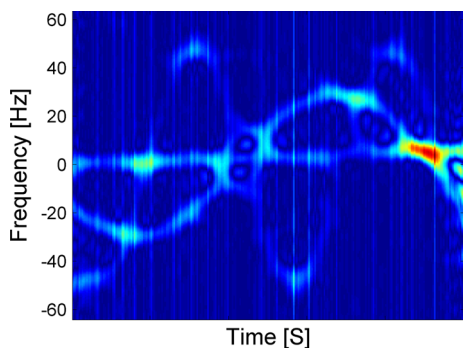
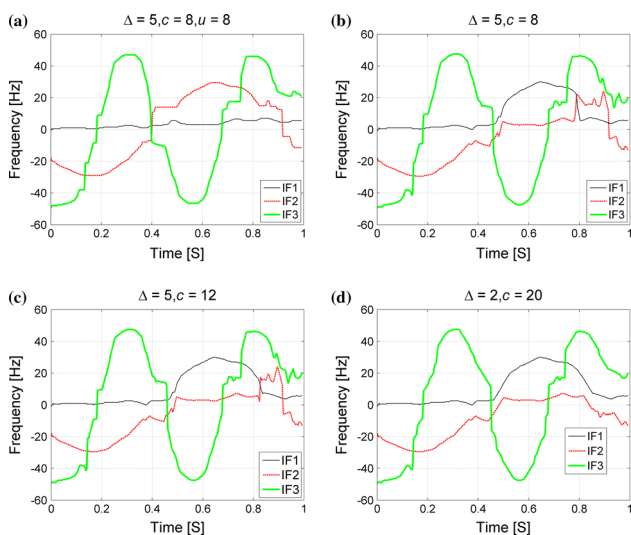**Fig. 3** BD of signal composed of one LFM and two SFM components at SNR = 2 dB

Fig. 4 IF estimation for the signal composed of one LFM and two SFM components. **a** IVA-based estimation, **b–d** VA-based estimation with different parameters (color figure online)

Table 2 MSE and MAE for three IFs

Components	IVA (a)	VA (b)	VA (c)	VA (d)
MSE				
SFM1	5.62	10.70	10.51	8.29
SFM2	15.28	9.46	10.21	20.49
LFM	5.33	10.33	10.04	7.90
MAE				
SFM1	2.99	6.64	6.63	5.07
SFM2	9.43	5.96	6.21	13.08
LFM	1.56	5.41	5.31	3.76

derives from TFR with TF points around former estimated IFs removed. As a result, the latter IF estimation is, the more TF points removed from TFR are. Therefore, the possible IF variation between two adjacent candidates of TF points, especially for those located in overlapped regions, may be large. So large $\Delta = 5$ is set. Comparison of the IVA-based IF estimation with VA-based IF estimation under various parameters is shown in Fig. 4. The MSE and MAE of three IFs are given in Table 2.

From Fig. 4, it is seen that compared with VA-BD, only IVA-BD can succeed to obtain all the three IFs without SP. Furthermore, for all the VAs, the LFM IF (red dashed line) and SFM1 (black solid line) IF always switch together. As is shown in Table 2, larger c and smaller Δ in VA result in more accurate IFs only for SFM1 and LFM components whose IFs are smoother. For SFM2 component with sharp IF, larger c and smaller Δ could lead to larger IF error. For IVA-based IF estimate, the most accurate SFM1 and LFM IFs can be measured as there is no SP; however, note that the

performance of IVA degrades for SFM2 component compared with VAs in Fig. 4b–c. The reason can be explained: This SFM IF has two peaks and one nadir, and the sign of IF variation reverses at these extremes. The IF estimates along the peaks or nadirs will be recognized as SP and suppressed by the proposed function (6). This example indicates the proposed IVA is more suitable to measure the multicomponent signals with monotonous and smooth IFs. To further validate the conclusion, another signal composed of three SFM components with sharp IFs will be considered.

3.1.2 Example 2

It has been verified from Example 1 that the selection of parameters Δ and c is related to the IF variation of the components in the considered signal. As a result, the performance of IVA will be different for the components in the same signal, depending on the IF variation. Also, when mapping the signal into TF plane using TFR, the TF concentration of the components will be also different; this further degrades the performance of the proposed algorithm. To alleviate from the influence of IF variation and focus on the analysis of parameter selection, we present another three-component signals with the same SFM but different time-shifting IFs:

$$
\begin{aligned}
x(t) = {} & \underbrace{\exp\left(j25\cos(2.4\pi t + \pi/3)\right)}_{\text{SFM1}} \\
& + \underbrace{\exp\left(j25\cos(2.4\pi (t - 0.3) + \pi/3)\right)}_{\text{SFM2}} \\
& + \underbrace{\exp\left(j25\cos(2.4\pi (t - 0.6) + \pi/3)\right)}_{\text{SFM3}}.
\end{aligned}
\tag{10}
$$

Assume SNR = 2 dB, the BD of the signal is given in Fig. 5. It is shown in Example 1 that the proposed VA cannot perform well for the sharp IF curve with peaks or nadirs. We applied VA and IVA with different parameters to the signal, respectively, and the results are displayed in Fig. 6. The MSEs and MAEs for three IFs are shown in Table 3.
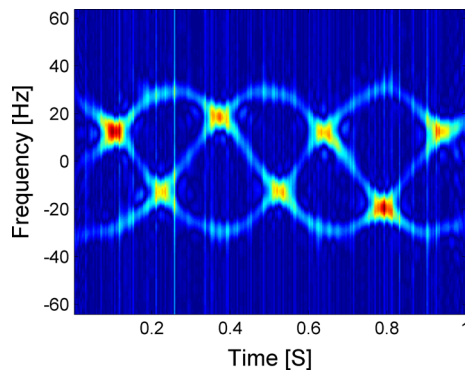
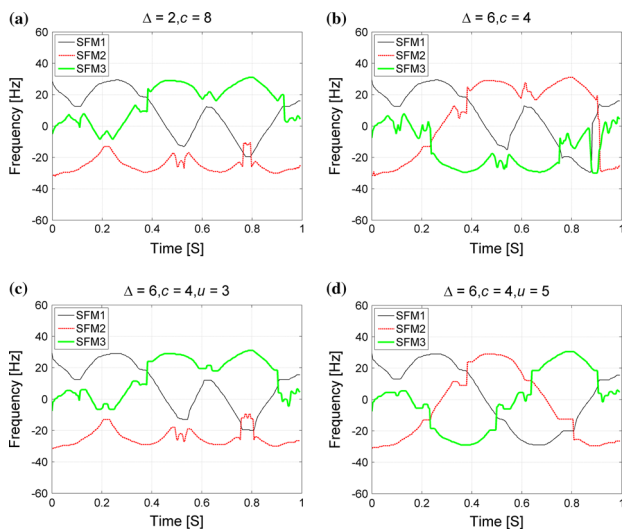**Fig. 5** BD of a signal composed of tree time-shifted IFs of SFM components

Fig. 6 IF estimation for signal composed of three SFM components. **a**, **b** VA-based estimation, **c**, **d** VA-based estimation

Table 3 MSEs and MAEs of estimated IFs corresponding to Fig. 6

Components	VA (a)	VA (b)	IVA (c)	IVA (d)
MSE				
SFM1	21.23	21.94	21.67	21.85
SFM2	20.77	22.53	21.01	21.98
SFM3	19.45	19.85	19.38	19.52
MAE				
SFM1	14.18	14.68	14.62	15.00
SFM2	13.56	14.42	13.90	14.82
SFM3	12.55	12.84	12.75	13.14

As is demonstrated in our previous work [25], smaller Δ and larger c may result in smoother IF; however, it may lead to the higher possibility of SP. Increasing the value of Δ and decreasing the value of c would result in less SP but lead to more IF fluctuation. The conclusion is validated in Fig. 6a, b, where IF estimate in Fig. 6a has smoother IFs but more SP; on the other hand, the estimate in Fig. 6b has more IF fluctuation but less SP. As the result, IF estimation by VA with $\Delta = 2$ and $c = 8$ is more accurate. However, no correct IF curve is successfully extracted.

As is shown in reference [25], larger Δ and smaller c should be selected for the seriously overlapped components. Therefore, set $\Delta = 6$, $c = 4$, IVAs with $u = 3, 5$ are applied to the BD to extract the IFs, the results are given in Fig. 6c, d, the simulation is also similar to previous Example 1, and larger u can result in better suppression of SP; however, IF estimate searching along the peaks and nadirs of IF curves is considered as SP, and this searching will be prevented by IVA. As is shown in Table 3, although IF curves are sometimes estimated successfully in Fig. 6d, the IF error is larger than this by VA, and larger u will lead to larger IF error.

3.2 Influence of parameter u on performance

In this part, we are going to investigate the performance of IVA under different u. To quantify the performance, the MSE between the estimated IF and theoretical IF is used. Then, set $\Delta = 5$, $c = 8$, and $u = 4, 8, 12, 16$ respectively, and MSEs of three components in Example 1 are calculated at SNRs from -10 to 5 dB. MSE at each SNR is calculated by the independent 200 process. The results for three components are shown in Fig. 7. As is shown in Fig. 4, the VA-based IFs of SFM1 and LFM switch together; therefore, accuracy of VA-based SFM1 and LFM in Fig. 7a, c is lowest. IVA can effectively avoid the problem, and when u or SNR becomes larger, the IF is more accurate. For the SFM2 estimation, since there is no SP, the accuracy of SFM2 IF will be almost the same for two algorithms when $u = 4$ in IVA, which is verified in Fig. 7b. However, when $u = 8, 12, 16$, IF error becomes larger and the performance of IVA is worse than VA. The reason is that SP of IVA for SFM2 also arises when u is too large.

In general, as is shown in Fig. 7a, c, larger u in IVA is, more SP is suppressed. Furthermore, larger u will result in less IF fluctuation. Therefore, more accurate IF can be obtained. However, the conclusion is more suitable for monotonous IF. For the non-monotonous IF, the sign of IF variation at the extremes will also reverse. Therefore, when u becomes too large for the sinusoidal IF, IF error increases. And more peaks or nadirs there are, more likely expansion of IF error occurs. As a result, the IVA-based sinusoidal IF1 with less peaks and nadirs and linear IF3 are more accurate than VA-based IFs under different parameter u at all SNRs from -10 to 5 dB. For the sinusoidal IF2 with larger oscillating frequency, however, IVA-based IF estimation accuracy is worse than VA for different u except when u is about 4.

To confirm the conclusion, a multicomponent noise-free signal composed of two SFM components is constructed as:
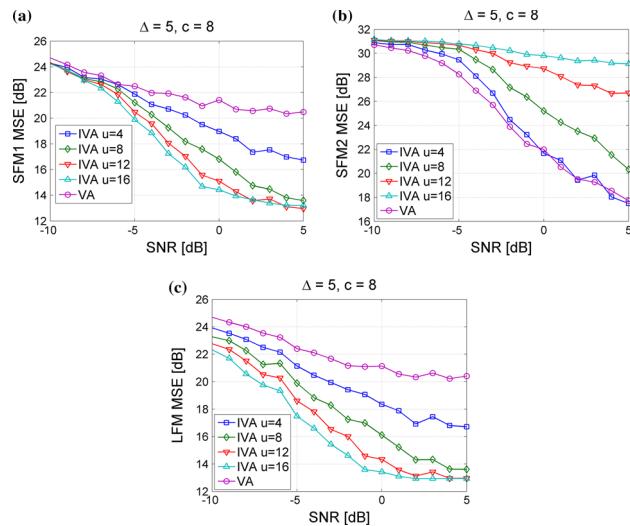
Fig. 7 IVA-based IF estimation for three components with different parameter u. **a** SFM1, **b** SFM2, **c** LFM

$$x(t) = \underbrace{\exp\left(ja_1/f_1\cos(2\pi f_1 t + \phi_1)\right)}_{\text{SFM1}}$$
$$+ \underbrace{\exp\left(ja_2/f_2\cos\left(2\pi f_2 t + \phi_2\right)\right)}_{\text{SFM2}}. \quad (11)$$

Set $a_1 = 40$, $f_1 = 0.5$ Hz, $\phi_1 = \pi/3$, $a_2 = 60$, $\phi_2 = -\pi/2$, vary f_2 from 1 to 5 Hz, and then, we investigate the IF estimation of SFM2 using the proposed IVA with $u \in (1, 12)$. Note that the performance of IVA is highly relevant to TFR, and when the oscillating frequency of SFM2 changes, the TF concentration and cross-term suppression of BD differ largely. Thus, to alleviate from the TFR influence on the performance, BD of each SFM component is first calculated respectively, and the IVA is then applied to sum of two BDs. In this process, window length of BD is also adjusted properly with f_2. Set $\Delta = 5$, $c = 8$, and MSE of each IF at different parameter u is calculated. The results are displayed in Fig. 8. The results are consistent with the previous analysis. When f_2 of SFM2 becomes larger, there are more peaks and nadirs in the SFM IF trajectory; therefore, larger parameter u leads to more IF error. For $f_2 = 1$, the IF curve is smooth, so the IF estimate is always accurate when u changes from 1 to 12. For $f_2 = 2$, IF is accurate when u changes from 1 to 8; For $f_2 = 3$, u should be set from 1 to 4. When $f_2 = 4$, the IF curve changes sharply, and more overlapped TF points, peaks and nadirs of IF curve arise, parameter u should be only 1. Till $f_2 = 5$, IVA fails to estimate the SFM2 IF with largest error.

To summarize this subsection, similar to the parameters in VA, the value of parameter u should be also set according to the IF characteristics in the considered signal. Generally, IVA is more suitable for the overlapped multicomponent signals with monotonous IFs, since larger parameter u could suppress the SP effectively. When IFs of the considered sig-
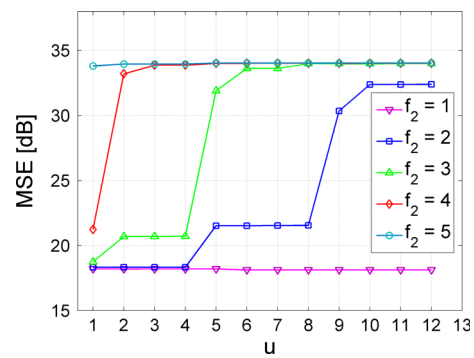

Fig. 8 MSE of IF estimate of SFM2 with different f_2 when u varies from 1 to 12 in IVA
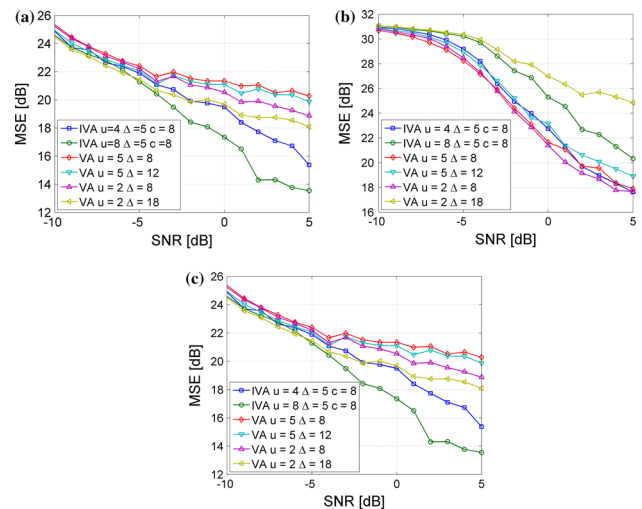

Fig. 9 Comparison of IVA with VA under different parameters for three components. **a** SFM1, **b** SFM2, **c** LFM

nal are non-monotonous, especially for the overlapped IFs with many peaks or nadirs, however, larger parameter u may probably lead to more SP, because the sign of IF variation also reverses at the peaks and nadirs in the IF curve and IF estimate along the extremes is considered as SP in mistake. Parameter u is recommended to be several values and should be adjusted depending on the IF characteristics in the signal.

3.3 Compared with VA with different c and Δ

To further verify the performance of IVA, more comparison of IVA with VA under different parameters will be made in this subsection. Signal at SNR from -10 to 5 dB in Example 1 is also used. Results are plotted in Fig. 9a–c. The conclusion is similar to that in Fig. 8. It is shown in Fig. 9a, c that when $u = 8$, $\Delta = 5$, $c = 8$, IVA can calculate the SFM1 and LFM components with the highest accuracy, but the accuracy for SFM2 is almost the lowest. When IVA parameters $u = 4$, $\Delta = 5$, $c = 8$, IVA can obtain the higher estimate for SFM2, but the IF accuracy for SFM1 and LFM becomes

lower. Furthermore, for SFM1 and LFM components, the accuracy is almost the same as that for VA with parameters $\Delta = 2, c = 18$ when SNR is below 1 dB. When SNR is larger than 1 dB, IVA estimate for SFM1 and LFM is better than those from all the VAs. When $\Delta = 2, c = 18$, the accuracy estimate is the lowest for SFM2 because of switch problem. Therefore, in order to obtain all three components with high accuracy, IVA with $u = 4, \Delta = 5, c = 8$ is a compromised choice.

4 Summary and conclusion

When VA is applied to estimate IFs of multicomponent signals from TFR, SP may occur at intersected points of several IFs; therefore, wrong IFs may be obtained. In order to suppress the SP in VA, this paper assumes that the sign of the two adjacent IF variations should not be reverse and a new penalty function is defined. To validate the IVA, two artificial multicomponent signals are simulated. Compared with VA, the smooth IFs are successfully extracted without SP. To quantify the performance of IVA, the MSE or MAE of each IF at various SNRs is also calculated for both VA and IVA. The results show that the proposed IVA can obtain more accurate IFs than VA for all the components.

Similar to VA, the performance of IVA is related to the several parameters in penalty functions [25]. In general, larger u, c and smaller Δ result in smoother IF. Larger u could suppress SP more effectively. However, for some signals especially those composed of non-monotonous or sharp IF components, the conclusion is not always correct and large u could lead to erroneous IFs.

The algorithm is more suitable for the multicomponent signals with overlapped monotonous IFs and cannot perform well for signals with step or sharp IFs. Similar to VA, the algorithm is also highly dependent on the selected TFR. Other highly performed TFR for multicomponent signals can be openly applied. All these works will be further investigated in the future.

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