SIViP (2017) 11:1537–1541
DOI 10.1007/s11760-017-1117-4

ORIGINAL PAPER

Estimation of sinusoidal frequency-modulated signal parameters in high-noise environment

Igor Djurović[1,2]

Received: 4 March 2017 / Revised: 20 April 2017 / Accepted: 13 May 2017 / Published online: 22 May 2017

Abstract In this paper, we consider the estimation of the sinusoidal frequency-modulated (FM) signal parameters in high-noise environments. For this purpose, we have combined the Viterbi algorithm for the estimation of the instantaneous frequency with the recently proposed technique for the parametric estimation of the FM signals based on the short-time Fourier transform. The proposed technique gives the accurate parameters estimation of the Cramer–Rao lower bound for signal-to-noise ratio of −2dB.

Keywords Sinusoidal FM signal · Parameter estimation · Short-time Fourier transform · Viterbi algorithm

1 Introduction

The sinusoidal frequency-modulated (FM) signals are common model of the micro-Doppler effect in the radar signal processing [1–7]. This effect is associated with rotating or vibrating parts of radar targets. Parametric estimation of these signals is important issue discussed in [8,9] with derived accuracy limits. In this paper, we are concentrated on the high-noise environments that are common in both military and civil applications where the micro-Doppler signal can be weak or disturbances strong. It is the reason for renewed interest in this signal model [10,11]. This is part of growing research interest in the parametric estimation of FM signals [12–14].

It should be noted that the sinusoidal FM model is just one of possible similar models. Other combinations representing signals with the phase that is equal to sum of the sinusoidal and polynomial function are commonly referred as the hybrid models [8,10]. However, here due to space limitations only the case of the sinusoidal FM signals will be considered, while the proposed extension can be applied to the hybrid and other models with modifications from [10].

Recently, we have proposed two-step procedure for the estimation of the sinusoidal FM signal parameters [10]. In the first step, the signal parameters are estimated from the IF obtained by the short-time Fourier transform (STFT)-based estimator [15,16]. In the second stage, these estimates are refined by the residual phase. The refinement procedure is performed several times in order to produce the mean square error (MSE) on the Cramer–Rao lower bound (CRLB) and to remove any residual bias from the estimates. The signal-to-noise ratio (SNR) threshold (position of the rapid departure of the MSE from the CRLB) is on SNR = 1dB. In this paper, we are going to consider is it possible to reduce this SNR threshold further. Instead of STFT, some other time-frequency representations that are robust to the noise influence can be used [17,18].

Firstly, we have proposed a three-stage procedure: rough stage based on the IF estimation; fine stage based on the residually signal phase; and nonlinear optimization using Nedler–Mead (NM) simplex algorithm [19]. The third stage in the procedure is introduced to avoid rerunning of the refinement stage that can also be prone to errors in high-noise environments. For extremely high-noise environment, each stage in this procedure can cause outliers, so it is important to additionally improve estimation results. Therefore, the IF has been also estimated by the application of the Viterbi esti-

✉ Igor Djurović
igordj@ac.me

1 Electrical Engineering Department, University of Montenegro, Cetinjski put bb, 81 000 Podgorica, Montenegro

2 Institute for Cutting Edge Information and Communication Technologies, Džordža Vašingtona 66/354, Podgorica, Montenegro

mation algorithm [20,21]. Finally, we have allowed that the criterion function selects any of the potential combinations of the parameter estimates (with or without Viterbi algorithm, with or without refinement, with or without the NM optimization). In this way, using different combinations we are avoiding outliers that can appear in any of the algorithms ingredients. Obtained results are significantly improved, and the SNR threshold has been reduced to just −2dB which is improvement of 3dB with respect to the algorithm from [10].

The manuscript is organized as follows. Signal model and the STFT are discussed in Sect. 2. The proposed algorithm is described in Sect. 3. Simulation results are given in Sect. 4 followed by the concluding remarks in Sect. 5.

2 Signal model and STFT

The considered signal model is the sinusoidal FM

$$f(n) = A\exp(i\phi(n)) = A\exp(ia\sin(bn+c)),\quad n\in[-N/2, N/2) \tag{1}$$

where N is number of available samples, corrupted by the white Gaussian noise $\nu(n)$ with variance σ^2. The SNR is defined as SNR$= 10\log_{10}[A^2/\sigma^2]$[dB]. The goal is to estimate signal parameters $\{a, b, c\}$ A from noisy mixture

$$x(n) = f(n) + \nu(n). \tag{2}$$

The IF of this signal is

$$\omega(n) = \phi'(n) = ab\cos(bn+c). \tag{3}$$

The amplitude can be estimated after estimating phase parameters $\{\hat{a}, \hat{b}, \hat{c}\}$ as

$$\hat{A} = \frac{1}{N}\left|\sum_n x(n)\exp(-j\hat{a}\sin(\hat{b}n+\hat{c}))\right| \tag{4}$$

so we are concentrated on estimation of the phase parameters $\{a, b, c\}$ only. The STFT will be used as important ingredient in the estimator already used due to its robustness to the noise [10,15,16] related to parametric estimation of the FM signals. The STFT is defined as

$$\mathrm{STFT}_h(n,\omega) = \sum_k w_h(k)x(n+k)\exp(-j\omega k) \tag{5}$$

where $w_h(k)$ is window function of the width h, $w_h(k)\neq 0$ for $k\in[-h/2, h/2)$. In this research, we are using only rectangular windows.

3 The proposed algorithm

3.1 IF estimation stage

The IF can be estimated from the STFT as

$$\hat{\omega}_h^{(0)}(n) = \arg\max_\omega |\mathrm{STFT}_h(n,\omega)|. \tag{6}$$

This IF estimator for considered signal is biased but robust to the noise influence. For high-noise environments, the robustness to the noise influence is more important than the bias influence. Under high-noise environment, we assume a noise with the standard deviation close or higher than the signal amplitude. Here, we are using superscript (0) to emphasize the STFT-position maxima-based IF estimator. The Fourier transform (FT) of the IF estimate is calculated

$$\hat{\Omega}_h^{(0)}(\omega) = FT\{\hat{\omega}_h^{(0)}(n)\}. \tag{7}$$

Position of $\hat{\Omega}_h^{(0)}(\omega)$ maxima corresponds to the estimate of parameter b. For more accurate results in [10] but for interpolation of $\hat{\Omega}_h^{(0)}(\omega)$, efficient Aboutanios–Mulgrew (AM) algorithm is applied [22]. Denote this estimate as $\hat{b}_h^{(0)}$. Amplitude of the FT on this frequency can be used to estimate parameter a,

$$\hat{a}_h^{(0)} = \frac{1}{\hat{b}_h^{(0)}}|\hat{\Omega}_h^{(0)}(\hat{b}_h^{(0)})|, \tag{8}$$

while parameter c, can be estimated from the phase of the FT as

$$\hat{c}_h^{(0)} = \measuredangle\hat{\Omega}_h^{(0)}(\hat{b}_h^{(0)}). \tag{9}$$

In the case of alternative models, this stage should be modified accordingly [10].

In order to achieve the accuracy improvement for high-noise influence, it is crucial to have the IF estimate that is robust to the noise influence. The Viterbi algorithm that is among the most accurate IF estimators for high noise environments can be used instead of the STFT representation maximum based estimator [20,21]. This IF estimation is path function $\hat{\omega}_h^{(1)}(n)$ (superscript (1) denotes the Viterbi algorithm application) minimizing the following optimization problem

$$\sum_n F\left(|\mathrm{STFT}_h\left(n,k(n)\right)|\right) + \sum_n G\left(|k(n)-k(n+1)|\right), \tag{10}$$

where $F(\cdot)$ is a nonincreasing function determined by sorting the STFT into nonincreasing order

$$|\text{STFT}_h(n,k_1)| \geq |\text{STFT}_h(n,k_2)| \geq \cdots \\ \geq |\text{STFT}_h(n,k_Q)|, \\ F\left(|\text{STFT}_h\left(n,k_l\right)|\right) = l-1. \tag{11}$$

Function $G(|x-y|)$ is determined as

$$G(|x-y|) = C(|x-y|-\Delta) \text{ for } |x-y| > \Delta \tag{12}$$

and zero elsewhere. We have selected $\Delta = 2$ frequency bins without penalization with function $G(\cdot)$, and $C = 10$. Path penalty function (10) is selected with two criterion: the IF estimate should pass through as strong as possible points of the time-frequency representation ($F(\cdot)$) and that path variations are small ($G(\cdot)$). If Δ is selected to be zero, the IF estimation would be over-smoothed while with large Δ the IF estimate would converge toward position of the time-frequency maxima that could be prone to the high noise errors. Small values of Δ about 2 are recommended in [20,23–25]. Details on the Viterbi algorithm IF estimator and its realization can be found in [20]. This IF estimates can be used to produce estimates of signal parameters in the same manner as in the previous technique $\hat{\omega}_h^{(1)}(n) \to \{\hat{a}_h^{(1)}, \hat{b}_h^{(1)}, \hat{c}_h^{(1)}\}$.

3.2 Residual phase refinement

Both the position of the maxima and the Viterbi algorithm IF estimates can be refined. This procedure can be described in the following manner [16]:

$$v(n) \\ = \text{unwrap}\left[\text{phase}\left[x(n)\exp\left[-i\hat{a}_h^{(j)}\sin\left(\hat{b}_h^{(j)}n+\hat{c}_h^{(j)}\right)\right]\right]\right] \\ \tilde{v}(n) = \frac{1}{2L+1}\sum_{k=n-L}^{n+L} v(n). \tag{13}$$

The phase estimate is now expressed as

$$\hat{\phi}(n) = \hat{a}_h^{(j)}\sin\left(\hat{b}_h^{(j)}n+\hat{c}_h^{(j)}\right) + \tilde{v}(n). \tag{14}$$

From the phase estimate, we reestimate phase parameters in similar manner as in the previous subsection as

$$\hat{\Xi}_h(\omega) = FT\{\hat{\phi}(n)\}, \tag{15}$$

$$\hat{b}_h^{(2+j)} = \arg\max_{\omega} |\hat{\Xi}_h(\omega)| \text{ (with the AM algorithm)}, \tag{16}$$

$$\hat{a}_h^{(2+j)} = \left|\hat{\Xi}_h\left(\hat{b}_h^{(2+j)}\right)\right| \tag{17}$$

$$\hat{c}_h^{(2+j)} = \measuredangle\hat{\Xi}_h\left(\hat{b}_h^{(2+j)}\right). \tag{18}$$

Here, adding 2 to the superscript emphasizes usage of the residual phase refinement. It should be noted that in this point we have four sets of estimators $\{\hat{a}_h^{(j)}, \hat{b}_h^{(j)}, \hat{c}_h^{(j)}\}$, $j \in [0,3]$, where odd j means that the Viterbi algorithm is used, while $j \geq 2$ means that the refinement is performed.

3.3 NM optimization

Additional 4 sets of estimates are obtained by application of the NM simplex algorithm [19]. For the NM optimization, the MATLAB fminsearch function is applied. These estimates are denoted as

$$\text{NM}\left\{\left\{\hat{a}_h^{(j)}, \hat{b}_h^{(j)}, \hat{c}_h^{(j)}\right\}\right\} = \left\{\hat{a}_h^{(4+j)}, \hat{b}_h^{(4+j)}, \hat{c}_h^{(4+j)}\right\}, \;\; j \in [0,3],$$

where the NM is the NM algorithm operator. Here, increasing the superscript for 4 means application of the NM algorithm.

3.4 Final estimate

In total, we have 8 sets of estimates $\{\hat{a}_h^{(j)}, \hat{b}_h^{(j)}, \hat{c}_h^{(j)}\}$, $j \in [0,7]$, calculated for various window widths $h \in H$ and the final estimates can be obtained as

$$\{\hat{a},\hat{b},\hat{c}\} = \left\{\hat{a}_{\hat{h}}^{\hat{j}}, \hat{b}_{\hat{h}}^{\hat{j}}, \hat{c}_{\hat{h}}^{\hat{j}}\right\}, \tag{19}$$

$$(\hat{j},\hat{h}) = \arg\max_{(j,h)} \\ \left|\sum_n x(n)\exp\left[-i\hat{a}_h^{(j)}\sin\left(\hat{b}_h^{(j)}n+\hat{c}_h^{(j)}\right)\right]\right|, \; j \in [0,7], h \in H. \tag{20}$$

Note that the same optimization function is inverse of those used in the NM optimization. In high-noise environments, all elements of the algorithm can be sensitive to outliers and they can appear in the IF estimates, refinement stage and the NM algorithm. However, multiple choices minimize possibility to be struck in the outlier and as it will be demonstrated in the next section, this setup brings improvement of 3dB in the SNR threshold with respect to technique from [10].

Calculation complexity of this procedure is influenced by the calculation of the STFT and the Viterbi algorithm, while other algorithm elements have negligible influence since they are evaluated with efficient procedures for one-dimensional signal.

4 Simulation study

The sinusoidal FM signal is considered with unit amplitude $A = 1$, phase parameters $a = 3$, $c = 0$, and b randomly selected in each trial according to the uniform distribution in the range $b \in [7\pi, 9\pi]$. Signal is considered in the interval $t \in [-1, 1)$ with the sampling interval $\Delta t = 1/128$ ($N =$

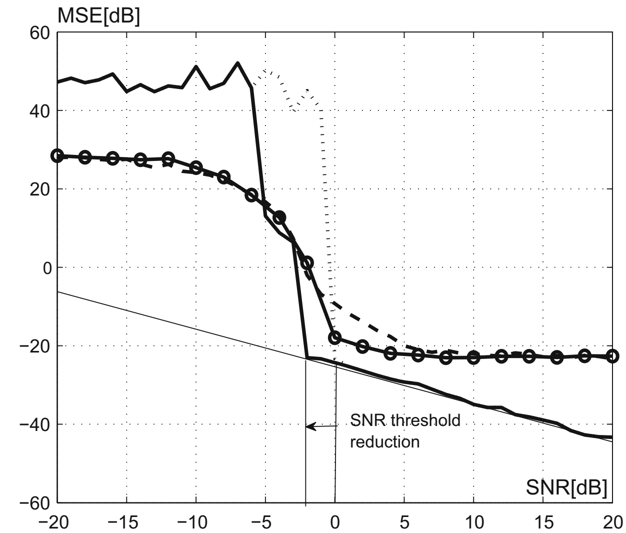

Fig. 1 MSE in estimation of parameter a. *Thick solid line*–proposed algorithm with ensemble of 8 estimates; *dotted line*—three-stage algorithm without application the Viterbi algorithm; *dashed line*—position of the STFT maxima with optimization of the window width; *line marker with circles*—refinement applied to the position of the STFT maxima with optimization of the window width; *thin solid line*—CRLB. SNR threshold reduction achieved with the proposed technique is depicted with *arrow*

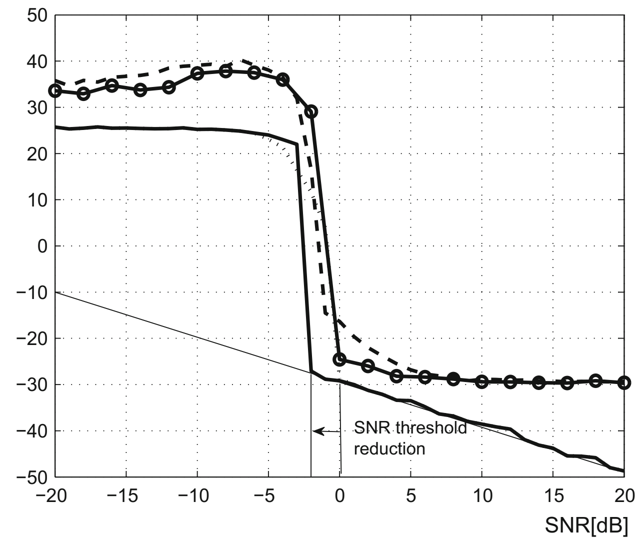

Fig. 2 MSE in estimation of parameter b. *Thick solid line*–proposed algorithm with ensemble of 8 estimates; *dotted line* —three-stage algorithm without application the Viterbi algorithm; *dashed line*—position of the STFT maxima with optimization of the window width; *line marker with circles*—refinement applied to the position of the STFT maxima with optimization of the window width; *thin solid line*—CRLB. SNR threshold reduction achieved with the proposed technique is depicted with *arrow*

256 samples in total). The same set of window widths is applied as in [10], i.e., $H = \{2r, r \in [1, 12]\}$. For each SNR, we have conducted 500 trials in the Monte Carlo simulation. The MSE in estimation of the signal parameters for phase parameters is given in Figs. 1–3 with the proposed method and for the position of the STFT maxima with optimized window width ($j = 0$), with refinement applied on the IF estimate from the position of the STFT maxima but without the NM algorithm ($j = 2$), combined algorithms with $j = 6$ (application of the phase refinement and the NM algorithm without the Viterbi algorithm) with optimization over set of different window widths. The obtained results are compared with the CRLB. In the presented way, we can see that the key ingredient in the reduction of the SNR threshold is the Viterbi algorithm since it reduces the SNR threshold to −2dB with respect to the other techniques with the SNR threshold about 0dB. The key algorithm element that is giving the MSE on the CRLB is the NM algorithm since without employing this algorithm we obtain the MSE significantly above the CRLB. Obtained results are better than with several reruns of the refinement procedure in [10] with the SNR threshold of 1dB. It means that the proposed algorithm can work for twice smaller the SNR than technique from [10] what is significant improvement (Figs. 1–3).

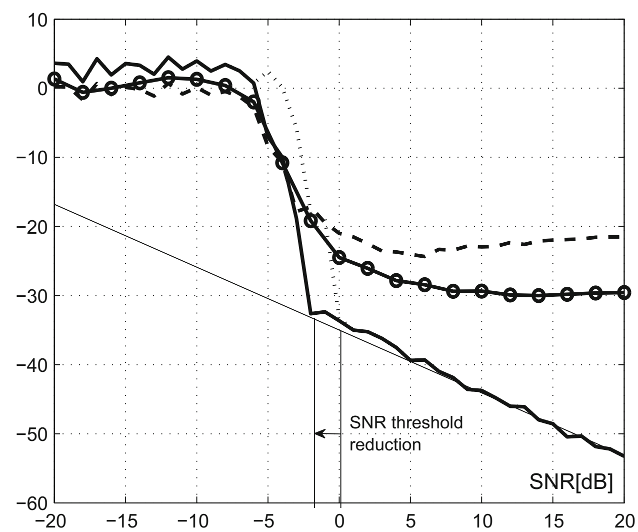

Fig. 3 MSE in estimation of parameter c. *Thick solid line*–proposed algorithm with ensemble of 8 estimates; *dotted line* —three-stage algorithm without application the Viterbi algorithm; *dashed line*—position of the STFT maxima with optimization of the window width; *line marker with circles*—refinement applied to the position of the STFT maxima with optimization of the window width; *thin solid line*—CRLB. SNR threshold reduction achieved with the proposed technique is depicted with *arrow*

5 Conclusion

We have proposed the sinusoidal FM parameters estimator for high-noise environments. Ensemble of 8 different sets of parameter estimates is formed with two different IF estimators (based on the position of the STFT maxima position and the Viterbi algorithm), with or without refinement in the phase, and with or without the NM optimization algorithm. The proposed algorithm improves estimation results with respect to the current state of the art for 3dB measured in the SNR threshold. This improvement is important for various radar micro-Doppler measurements that can be subject to high disturbance influence. Potential extension of this research is considering the multicomponent sinusoidal FM signals [12].

References

1. Chen, V.C., Li, F., Ho, S.-S., Wechsler, H.: Micro-Doppler effect in radar: phenomenon, model, and simulation study. IEEE Trans. Aerosp. Electron. Syst. **42**(1), 2–21 (2006)
2. Thayaparan, T., Abrol, S., Riseborough, E., Stanković, LJ., Lamothe, D., Duff, G.: Analysis of radar micro-Doppler signatures from experimental helicopter and human data. IET Radar Sonar Navig. **1**(4), 289–299 (2007)
3. Stanković, LJ., Thayaparan, T., Daković, M., Popović-Bugarin, V.: Micro-Doppler removal in the radar imaging analysis. IEEE Trans. Aerosp. Electron. Syst. **49**(2), 1234–1250 (2013)
4. Stanković, LJ., Thayaparan, T., Djurović, I.: Separation of target rigid body and micro-Doppler effects in ISAR imaging. IEEE Trans. Aerosp. Electron. **42**(4), 1496–1505 (2006)
5. Thayaparan, T., Stanković, LJ., Djurović, I.: Micro-Doppler human signature detection and its application to gait recognition and indoor imaging. J. Frankl. Inst. **345**(6), 700–722 (2008)
6. Arslan, M.T., Tofighi, M., Çetin, A.E.: Range resolution improvement in FM-based passive radars using deconvolution. Signal Image Video Process. **10**(8), 1481–1488 (2016)
7. Saeedi, J., Alavi, S.M.: Improved navigation-based motion compensation for LFMCW synthetic aperture radar imaging. Signal Image Video Process. **10**(2), 405–412 (2016)
8. Gini, F., Giannakis, G.B.: Hybrid FM-polynomial phase signal modeling: parameter estimation and Cramer-Rao bounds. IEEE Trans. Signal Process. **47**(2), 363–377 (1999)
9. Gini, F., Giannakis, G. B.: Parameter estimation of hybrid hyperbolic FM and polynomial phase signals using the multi-lag high-order ambiguity function. In: Asilomar Conference on Signals, Systems and Computers (1997). doi:10.1109/ACSSC.1997.680178
10. Djurović, I., Popović-Bugarin, V., Simeunović, M.: The STFT-based estimator of micro-Doppler parameters. IEEE Trans. Aerosp. Electron. Syst. doi:10.1109/TAES.2017.2669741
11. Wang, P., Orlik, P.V., Sadamoto, K., Tsujita, W., Gini, F.: Parameter estimation of hybrid sinusoidal FM-polynomial phase signal. IEEE Signal Process. Lett. **24**(1), 66–70 (2017)
12. Yang, P., Liu, Z., Jiang, W.-L.: Parameter estimation of multi-component chirp signals based on discrete chirp Fourier transform and population Monte Carlo. Signal Image Video Process. **9**(5), 1137–1149 (2015)
13. Djurović, I., Simeunović, M.: Combined HO-CPF and HO-WD PPS estimator. Signal Image Video Process. **9**(6), 1395–1400 (2015)
14. Djurović, I., Simeunović, M., Wang, P.: Cubic phase function: a simple solution for polynomial phase signal analysis. Signal Process. **135**, 48–66 (2017)
15. Djurović, I., Stanković, LJ.: STFT-based estimator of polynomial phase signals. Signal Process. **92**(11), 2769–2774 (2012)
16. Djurović, I., Stanković, LJ.: Quasi maximum likelihood estimator of polynomial phase signals. IET Signal Process. **13**(4), 347–359 (2014)
17. Mohammadi, M., Pouyan, A.A., Khan, N.A.: A highly adaptive directional time-frequency distribution. Signal Image Video Process. **10**(7), 1396–1377 (2016)
18. Ren, H., Ren, A., Li, Z.: A new strategy for the suppression of cross-terms in pseudo Wigner-Ville distribution. Signal Image Video Process. **10**(1), 139–144 (2016)
19. Pham, D.S., Zoubir, A.M.: Analysis of multicomponent polynomial phase signals. IEEE Trans. Signal Process. **55**(1), 56–65 (2007)
20. Djurović, I., Stanković, LJ.: An algorithm for the Wigner distribution based instantaneous frequency estimation in a high noise environment. Signal Process. **84**(3), 631–643 (2004)
21. Stanković, LJ., Djurović, I., Stanković, S., Simeunović, M., Daković, M.: Instantaneous frequency in time-frequency analysis: enhanced concepts and performance of estimation algorithms. Digit. Signal Process. **35**, 1–13 (2014)
22. Aboutanios, E., Mulgrew, B.: Iterative frequency estimation by interpolation on Fourier coefficients. IEEE Trans. Signal Process. **53**(4), 1237–1242 (2005)
23. Conru, C., Djurović, I., Ioana, C., Quinquis, A., Stanković, LJ.: Time–frequency detection using Gabor filter banks and Viterbi based grouping algorithm. In: Proceedings of IEEE ICASSP'2005, Philadelphija, USA, vol. 4, pp. 497–500 (2005)
24. Djurović, I.: Viterbi algorithm for chirp-rate and instantaneous frequency estimation. Signal Process. **91**(5), 1308–1314 (2011)
25. Stanković, LJ., Djurović, I., Ohsumi, A., Ijima, H.: Instantaneous frequency estimation by using Wigner distribution and Viterbi algorithm. In: Proceedings of ICASSP 2003, Hong Kong, China, vol. VI, pp. 121–124 (2003)