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ORIGINAL PAPER

Random-valued impulse noise removal using fuzzy weighted non-local means

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Abstract In this paper, we propose a fuzzy weighted non-local means filter for the removal of random-valued impulse noise. We introduce a new fuzzy weighting function, which can shut off the impulsive weight effectively, to the non-local means. According to the new weighting function, the more a pixel is corrupted, the less it is exploited to reconstruct image information. Experiments show that the performances of the new filter are surprisingly satisfactory in terms of both visual quality and quantitative measurement. Moreover, our filter also can be used to remove mixed Gaussian and random-valued impulse noise.

Keywords Fuzzy weight · Image restoration · Impulse noise · Mixed noise · Non-local means

1 Introduction

In many data acquisition, transmission and storage systems, noise is often inevitable as caused by various factors. For example, failures in sensors, readout circuits, A/D converters, or communication channels may introduce impulsive noise in digital images [1]; the thermal motion of electron in the photoelectric sensor may induce the white Gaussian noise. Therefore, image denoising is one of the most fundamental problems in image processing.

Many methods have been proposed for impulse noise removal in the literature. The median filter (MED) was once the most popular nonlinear filter for removing impulse noise [2]. But it tends to treat signal pixels and noisy pixels without distinction and results in destroying fine details and producing blotches in the restored images [3]. A solution to this problem is to devise an impulse detector to recognize noisy pixels from image signal pixels, and only the noise candidates are removed. For example, the genetic programming (GP) filter [4], the Luo filter [5], the directional weighted median (DWM) filter [6], and the contrast enhancement-based filter (CEF) [7] are such filters, which are recently proposed for the removal of random-valued impulse noise.

Many algorithms have been also proposed for Gaussian noise removal, such as [8–11]. The non-local means (NLM) algorithm [8] is one of the most efficient methods and has attracted a lot of attention from signal processing researchers. The NLM algorithm exploits the self-similarity or information redundancy within images and computes the estimated value of a pixel as a weighted average of all the similar pixels in the image. Pixel similarity is defined in NLM as the Euclidean distance between image patches. The NLM algorithm has two drawbacks. The one is computationally expensive, and many methods were proposed to accelerate it, see [12–17]. The other is that it is sensitive to impulse noise. It cannot adequately remove impulse noise because NLM interprets the noisy pixels as image structures.

In this paper, we present a new algorithm called the fuzzy weighted non-local means (FWNLM) filter for random-valued impulse noise removal. We provide an efficient fuzzy weighting function for the NLM algorithm to shut off the impulsive components. The new filter processes pixels in accordance with the rule: the more a pixel is corrupted, the less the pixel is exploited to reconstruct image information. Experiments show that the FWNLM filter has a surprisingly good denoising capability, as shown later.

The paper is organized as follows. In the next section, we briefly review the NLM algorithm. Then, in Sect. 3 we

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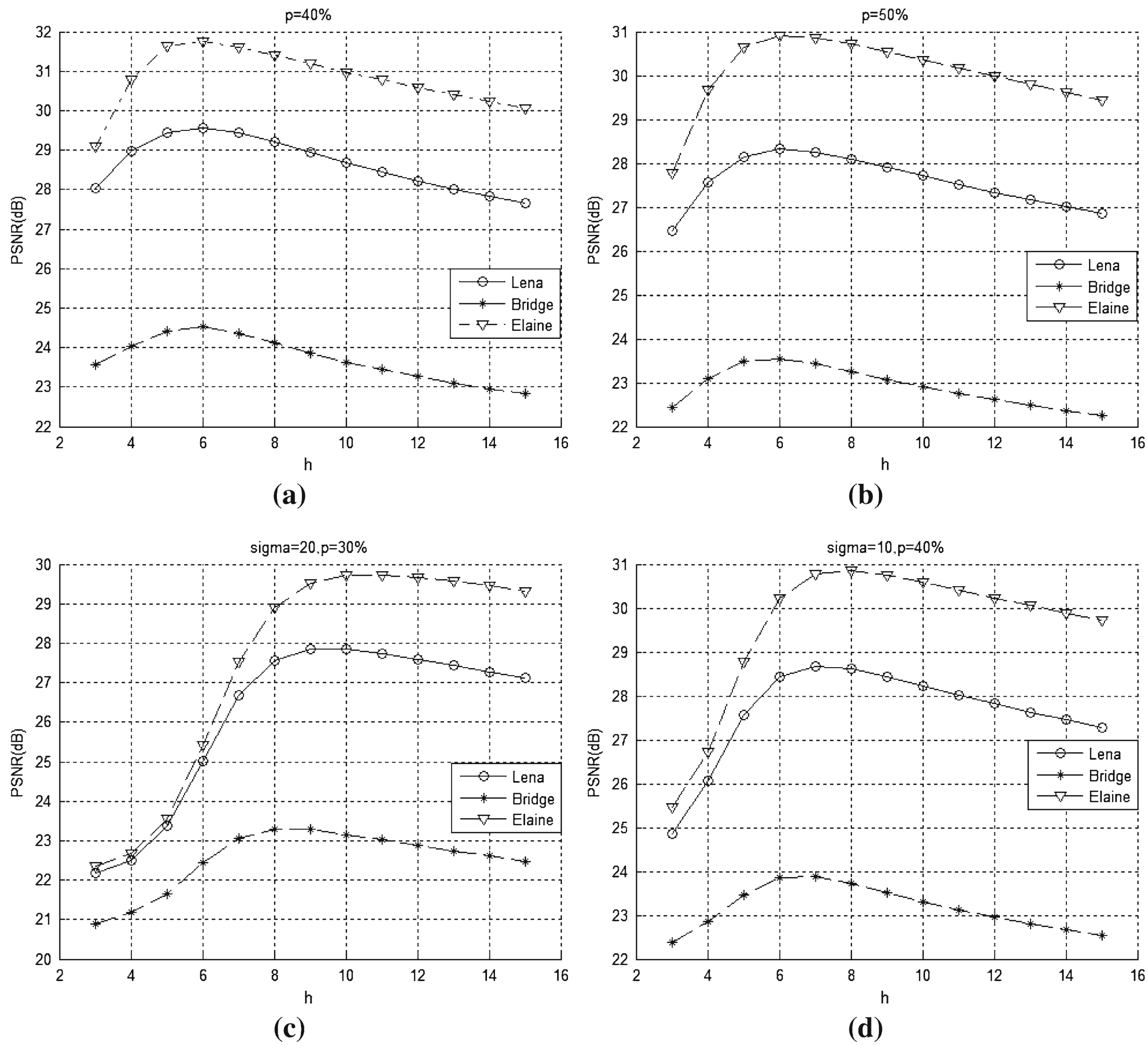

Fig. 1 The sensitivity of the parameter h at various noise levels. **a** Random-valued impulse noise, $p = 40\%$; **b** random-valued impulse noise, $p = 50\%$; **c** mixed noise, $\sigma = 20$, $p = 30\%$; **d** mixed noise, $\sigma = 10$, $p = 40\%$

Table 1 The suggested choice for the parameter h

Noise level	$p = 40\%$	$p = 50\%$	$p = 60\%$	$\sigma = 20$, $p = 30\%$	$\sigma = 10$, $p = 40\%$
Suggested h	5–7	5–7	6–8	8–12	6–9

describe our method in detail. Section 4 presents experimental results of the new filter. Finally, conclusions are given in Sect. 5.

2 Review of the NLM algorithm

Let us consider a noisy image $u : \Omega \subset \mathbf{R}^2 \rightarrow \mathbf{R}$. The gray-scale value at position $\mathbf{x} \equiv (x_1, x_2) \in \Omega$ is represented by $u(\mathbf{x})$. Each pixel of the filtered image is estimated by the NLM algorithm as a weighted average [8]:

$$\hat{u}(\mathbf{x}) = \frac{1}{\sum_{\mathbf{y}\in S} w(\mathbf{x}, \mathbf{y})} \sum_{\mathbf{y}\in S} w(\mathbf{x}, \mathbf{y}) u(\mathbf{y}), \tag{1}$$

where $S(S \subseteq \Omega)$ usually with a large size is a *searching window* centered at the pixel $\mathbf{x}$, and the weights $w(\mathbf{x}, \mathbf{y})$ depend on the pixel similarity between $\mathbf{x}$ and $\mathbf{y}$. The pixel similarity is calculated by comparing surrounding patches (here called *matching windows*) around $\mathbf{x}$ and $\mathbf{y}$. The similarity is defined by the expression:

$$w(\mathbf{x}, \mathbf{y}) = \exp\left(-\frac{\sum_{\mathbf{k}\in\Omega^m} g_a(\mathbf{k})|u(\mathbf{x}+\mathbf{k}) - u(\mathbf{y}+\mathbf{k})|^2}{h^2}\right), \tag{2}$$

where

$$\Omega^m \equiv \{\mathbf{k} = (k_1, k_2)| - m \leq k_1, k_2 \leq m\}, \tag{3}$$

and m denotes the size of the matching window, the weights $g_a(\mathbf{k})$ define a centered symmetric Gaussian kernel with the

Table 2 Results in PSNR after filtering images corrupted by impulse noise

Filters	Lena	Boat	Peppers	Elaine	Bridge
Random-valued impulse noise ($p = 40\%$)					
MED	22.94	23.45	23.53	24.79	21.20
NLM	18.91	19.47	19.42	20.96	18.06
SKR	26.44	26.45	26.60	28.34	23.04
GP	28.11	27.83	28.43	30.46	24.22
Luo	28.30	27.30	28.25	31.59	23.72
CEF	29.09	27.85	29.29	31.11	24.00
DWM	29.33	*28.26*	29.01	31.71	24.16
FWNLM	29.56	28.24	29.80	31.74	24.51
Random-valued impulse noise ($p = 50\%$)					
MED	19.73	20.53	20.57	21.63	19.06
NLM	17.37	18.03	18.02	19.44	16.97
SKR	22.84	23.65	23.71	25.25	21.17
GP	24.98	25.50	25.78	27.63	22.40
Luo	26.37	25.94	26.85	29.68	22.50
CEF	27.28	26.33	27.59	29.02	22.60
DWM	27.11	26.63	27.22	29.63	22.88
FWNLM	28.31	27.01	28.41	30.90	23.54
Random-valued impulse noise ($p = 60\%$)					
MED	17.12	18.08	17.89	19.12	17.18
NLM	16.05	17.00	16.77	18.23	16.04
SKR	19.47	20.62	20.43	22.03	19.13
GP	21.42	22.58	22.49	24.23	20.28
Luo	23.89	24.00	24.56	27.08	20.98
CEF	24.34	23.96	24.59	25.54	20.85
DWM	23.58	24.20	24.46	26.49	21.18
FWNLM	26.64	25.64	26.97	29.26	22.44

standard deviation a, and the scalar h is used to control smoothing.

The NLM algorithm performs impressively in Gaussian noise suppression. It averages the pixels with various weights in the entire searching window: Those pixels that are similar to the centered pixel in image structure get larger weights. Intuitively, the NLM algorithm matches local image structures rather than image intensities [14]. However, the NLM filter fails to remove impulse noise. As impulses damage image structures significantly, the algorithm tends to interpret the noisy pixels as image structures, and thus, the NLM algorithm performs not well in the presence of impulse noise, as shown later.

3 The proposed approach

3.1 The fuzzy weight

Following the initial idea of the NLM filter, we shut off the impulsive component by an impulsive weight and only use the impulse-free information to reconstruct the image. As the impulse noise is very difficult to detect precisely, we provide a fuzzy weight for each pixel. The fuzzy weight is calculated according to how impulse-like a pixel is. Here, we use the rank-ordered absolute differences (ROAD) statistic proposed in [18], to give a fuzzy index for each pixel. In this work, we calculate the ROAD as follows.

Let $\mathbf{x}$ be a pixel under consideration, assume that

$$d_{\mathbf{x},\mathbf{k}} = |u(\mathbf{x}) - u(\mathbf{x}+\mathbf{k})|, \quad \mathbf{k} \in \Omega^2, \tag{4}$$

and $0 \le u(\mathbf{x}) \le 255$. The notation Ω^2 indicates we use 5×5 window centered at $\mathbf{x}$. Next, we sort the twenty five $d_{\mathbf{x},\mathbf{k}}$ values in ascending order such that $r_1 \le r_2 \le \cdots \le r_{25}$, where r_i is the i^{th} smallest element. Then, the ROAD of $\mathbf{x}$ is defined by $\text{ROAD}(\mathbf{x}) = \sum_{i=1}^{13} r_i$.

The ROAD($\mathbf{x}$) provides us a simple but effective measure for detecting impulses: If the value is large, then the pixel is an impulse pixel; if the value is small, then the pixel is an uncorrupted pixel. Thus, we define the fuzzy weight for each pixel as

$$\lambda(\mathbf{x}) = \begin{cases} 0 & \text{ROAD}(\mathbf{x}) \ge T_1 \\ \frac{\text{ROAD}(\mathbf{x}) - T_1}{T_2 - T_1} & T_2 \le \text{ROAD}(\mathbf{x}) \le T_1 \\ 1 & \text{ROAD}(\mathbf{x}) \le T_2 \end{cases} \tag{5}$$

where T_1 and T_2 are two predefined parameters.

The weighting function $\lambda(\mathbf{x})$ ranges from 0 to 1 and indicates how much the information of a pixel is worth. The maximum value denotes the pixel is uncorrupted, and its information is fully useful for image reconstruction. On the contrary, the minimum value means the pixel is damaged and its information is totally useless.

3.2 The proposed fuzzy weighted non-local means filter

Here we introduce the fuzzy weighting function $\lambda(\mathbf{x})$ to the NLM algorithm to selectively pick pixels when calculating the pixel similarity. The proposed fuzzy weighted NLM (FWNLM) filter is

$$\hat{u}(\mathbf{x}) = \frac{1}{\sum_{\mathbf{y}\in S} \bar{w}(\mathbf{x},\mathbf{y})} \sum_{\mathbf{y}\in S} \bar{w}(\mathbf{x},\mathbf{y}) u(\mathbf{y}), \tag{6}$$

where the new weight of $\mathbf{y}$ with respect to the central pixel $\mathbf{x}$ is presented as (7).

$$\bar{w}(\mathbf{x},\mathbf{y}) = \lambda(\mathbf{y}) \exp\left(-\frac{\sum_{\mathbf{k}\in\Omega^m} g_a(\mathbf{k})\lambda(\mathbf{x}+\mathbf{k})\lambda(\mathbf{y}+\mathbf{k})|u(\mathbf{x}+\mathbf{k}) - u(\mathbf{y}+\mathbf{k})|^2}{h^2}\right). \tag{7}$$

In the new weighting function (7), the fuzzy weight $\lambda(\mathbf{y})$ tells the new filter how important the information of $\mathbf{y}$, the more the pixel is damaged ($\lambda(\mathbf{y})$ is near to zero), the more the

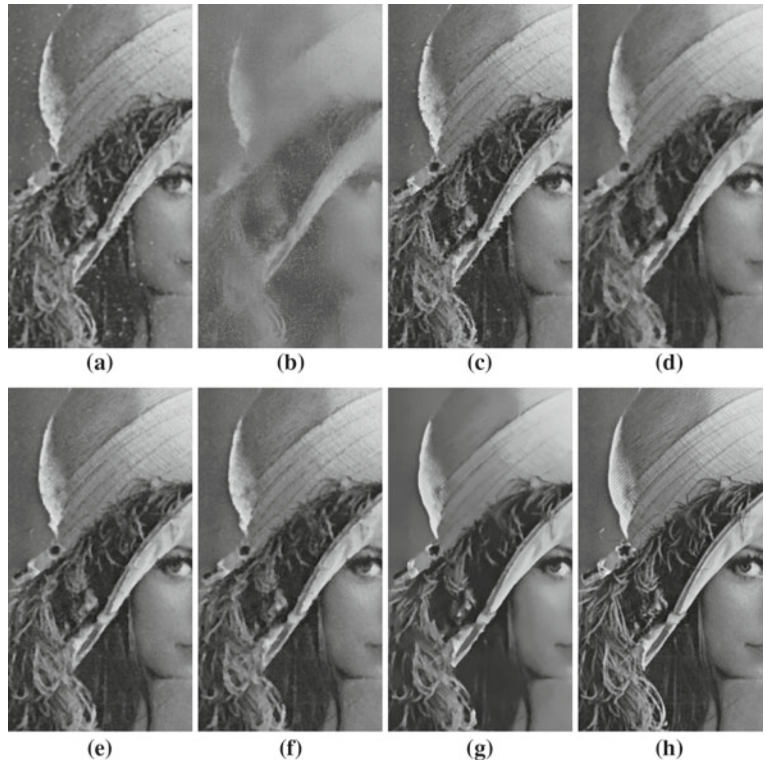

Fig. 2 Denoising of Lena image (part) corrupted with 40% random-valued impulse noise. **a** SKR, **b** NLM, **c** GP, **d** Luo, **e** DWM, **f** CEF, **g** FWNLM and **h** the noise-free image

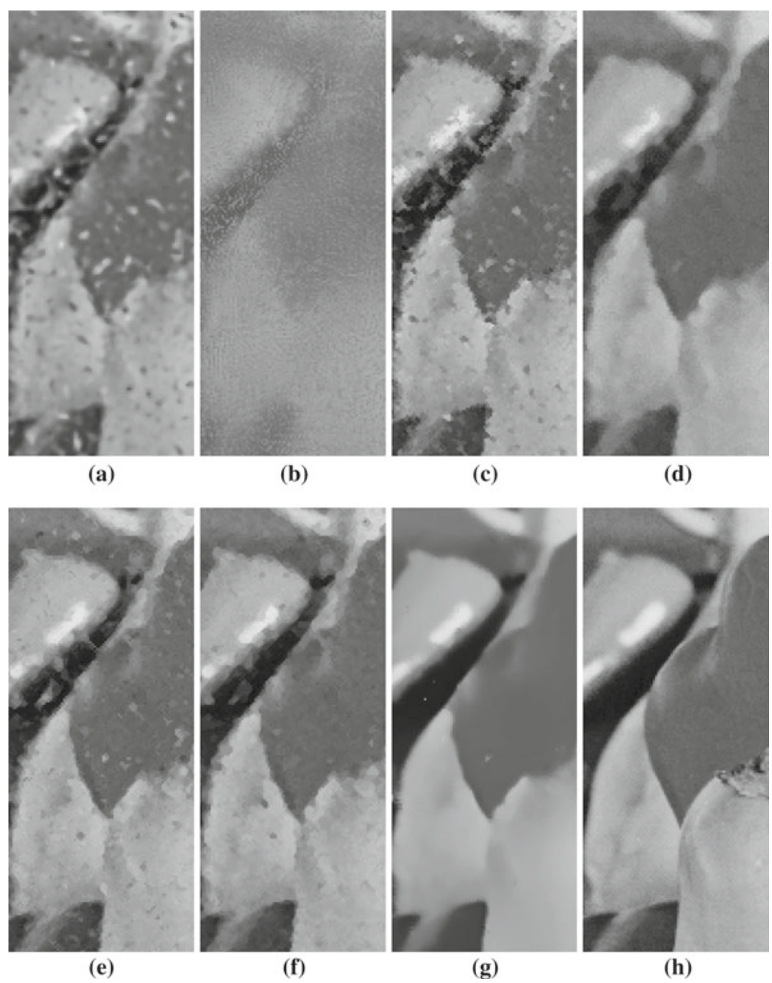

Fig. 3 Denoising of Peppers image (part) corrupted with 60% random-valued impulse noise. **a** SKR, **b** NLM, **c** GP, **d** Luo, **e** DWM, **f** CEF, **g** FWNLM and **h** the noise-free image

information of the pixel tends to be thrown away ($\bar{w}(\mathbf{x}, \mathbf{y})$ is close to zero). And the equation

$$\lambda(\mathbf{x}+\mathbf{k})\lambda(\mathbf{y}+\mathbf{k})|u(\mathbf{x}+\mathbf{k})-u(\mathbf{y}+\mathbf{k})|^2 \quad (8)$$

indicates that the more one of the pixels $\mathbf{x}+\mathbf{k}$ and $\mathbf{y}+\mathbf{k}$ is damaged, the less the distance between the two pixels is utilized. Therefore, the new weighting function is apt to discard impulse-like pixels when calculating the pixel similarity.

4 Simulations

In this section, we present experimental results to assess the performance of the proposed filter. Simulations were made on five 512×512 8-bit gray-scale standard images: Lena, Boat, Peppers, Elaine, and Bridge. They will be corrupted by random-valued impulse noise with very high noise levels—40, 50, and 60%. Restored results are quantitatively measured by the peak signal-to-noise ratio (PSNR); that is, if u^0 is the noise-free image of size $M \times N$, and $\hat{u}$ is a restored image of u^0, then the PSNR of $\hat{u}$ is calculated by

$$\mathrm{PSNR} = 10\log_{10}\frac{255^2}{\frac{1}{MN}\sum_{\mathbf{x}\in\Omega}\left(\hat{u}(\mathbf{x})-u^0(\mathbf{x})\right)^2}. \quad (9)$$

4.1 The selection of the parameters

For simplicity, we hold some parameters as constants in all the experiments. To reduce the burden of computation, we use a 21×21 window as the searching window as the authors of NLM suggested in [8], and the pixel similarity is calculated in a 9×9 matching window, that is, $m = 4$ in (3). We also fix the parameter $a = 2$ in Gaussian smooth kernel.

Through many experiments on the five images with 40, 50 and 60% random-valued impulse noise, we find that the good values of T_1 and T_2 are 380 and 120, respectively. In Fig. 1, we show the sensitivity of the parameter h. From the figure we can see that, for random-valued impulse noise, the h should increase slightly for better PSNR results when the noise level grows from 40 to 60%. But for mixed noise with the same impulse noise level, h should be larger. In Table 1, we list the suggested choice for the parameter h.

4.2 Experimental results

To evaluate the performance of the proposed filter[1], we provide some comparisons with the median (MED) filter (3×3), the NLM algorithm [8], the steering kernel regression (SKR) filter [11], the GP filter [4], the Luo filter [5], the DWM filter [6], and the CEF filter [7]. The parameters of these filters are chosen as their authors suggested (the GP filter has no parameter).

Table 3 Results in PSNR after filtering images corrupted by mixed Gaussian and random-valued impulse noise

Filters	Lena	Boat	Peppers	Elaine	Bridge
Mixed noise ($\sigma = 20$, $p = 30\%$)					
MED	23.48	23.53	23.76	24.35	21.74
NLM	20.59	20.96	21.19	22.58	19.21
SKR	25.50	25.28	25.55	26.45	22.90
GP	24.57	24.36	24.72	25.21	22.68
Luo	25.77	25.03	25.75	27.13	22.72
CEF	25.79	24.96	25.80	26.40	22.74
DWM	25.10	24.63	25.04	25.75	22.67
FWNLM	27.85	26.60	27.88	29.72	23.30
Mixed noise ($\sigma = 10$, $p = 40\%$)					
MED	22.24	22.68	23.07	23.92	21.01
NLM	18.88	19.35	19.54	20.75	18.07
SKR	25.19	25.38	25.75	26.97	22.62
GP	25.88	25.71	26.23	27.31	23.24
Luo	26.72	25.86	26.75	28.82	22.96
CEF	26.94	26.06	27.04	28.06	23.10
DWM	26.63	26.10	26.57	27.86	23.24
FWNLM	28.68	27.43	28.88	30.84	23.88

Table 2 presents numerical results for the five standard images corrupted by random-valued impulse noise with the high noise ratio $p = 40$, 50 and 60%. The parameter h of the proposed filter is chosen for better PSNR values as suggested in Table 1. It is easy to see that the proposed filter provides results with higher PSNR values almost in all cases. Especially, even when the noise level is very high (60%), the performances of our filter are still very satisfactory.

Subsequently, we show visual results restored by the different filters. Figure 2 shows the enlarged restoration results for a noisy Lena image corrupted by 40% random-valued impulse noise. We can see that the output of our filter is very clean and smooth. The visual comparison is further illustrated in Fig. 3. Even in the extremely high noise level—60%, our filter can deal well with it and obtain the best visual quality.

In addition, our filter can also perform well in the removal of mixed Gaussian and random-valued impulse noise. Table 3 lists the numerical results, and Fig. 4 shows the visual results for the seven filters. Again, from the table and figure, one can easily see that our filter yields the best results in terms of both the quantitative measurement and visual quality when removing mixed noise.

We end this section by considering the complexity of the FWNLM filter. Our method needs an extra calculation for the new weights $\lambda(\mathbf{x})$ at each pixel. Apart from this, it has extra multiplications in (7) when compared to the NLM filter's weighting equation (2). In Table 4, we show the computation

[1] The FWNLM filter is available at http://wudging.ys168.com/.

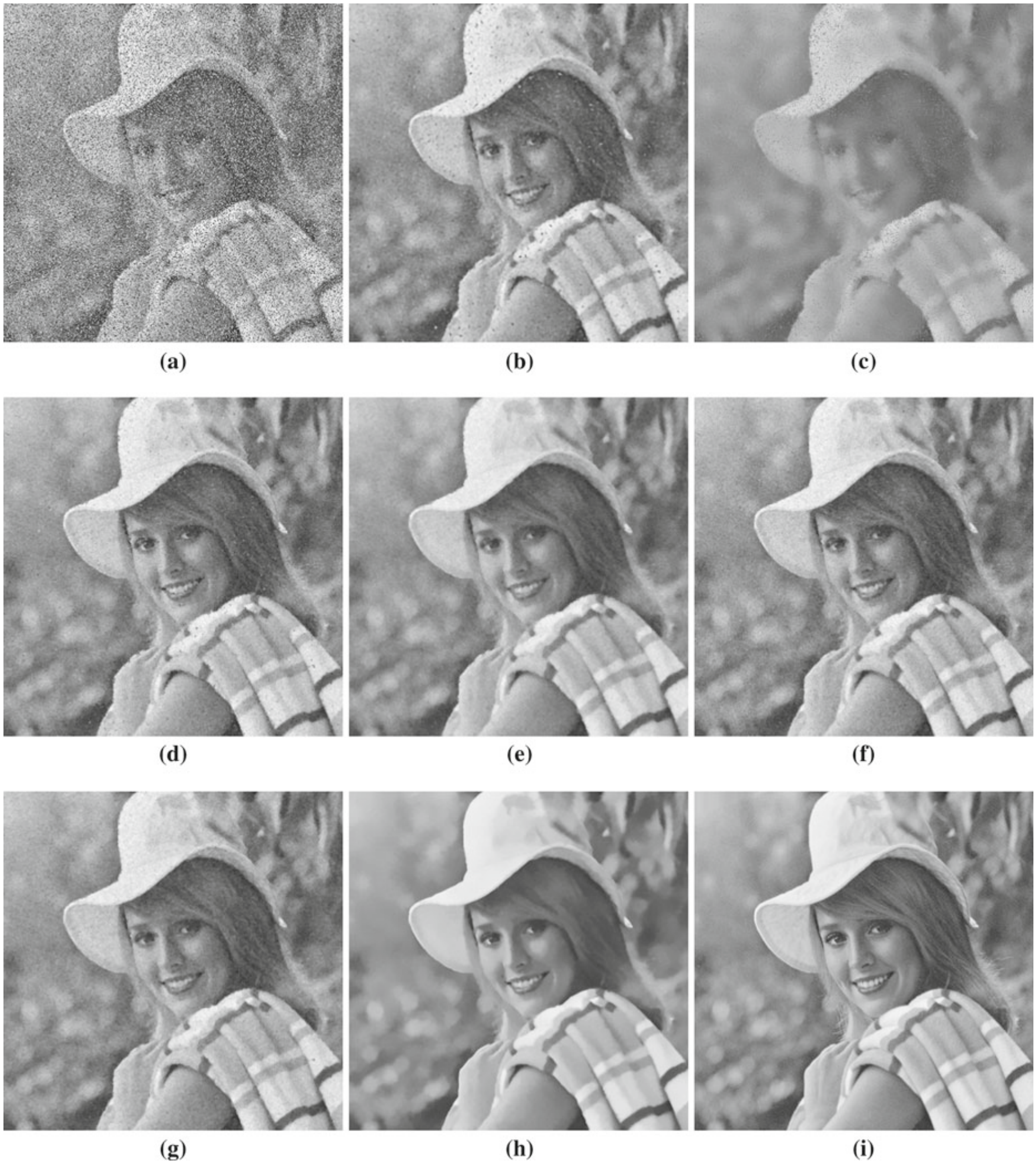

Fig. 4 Results of different filters in restoring mixed noise ($\sigma = 10$, $p = 40\%$). **a** Noisy image, **b** SKR, **c** NLM, **d** GP, **e** Luo, **f** DWM, **g** CEF, **h** FWNLM and **i** the noise-free image

time of the eight filters under the same PC equipped with a Pentium Dual-Core E5300 CPU and 2-GB RAM memory. The test picture is 512×512 Lena image. We collect the data of the FWNLM and NLM filter using the 21×21 searching window and the 9×9 matching windows. From the table, we see that the FWNLM filter is about 100–200 s slower than the NLM algorithm. However, as the calculation of the NLM and FWNLM filters is very alike, the acceleration methods, such as [12–17], also can be applied to our algorithm.

5 Conclusion

This paper presents a new filter for random-valued impulse noise removal. We introduce a new fuzzy weighting function to the non-local means algorithm to shut off the impulsive weight effectively. In contrast to the recently proposed techniques for the removal of random-valued impulse noise, the proposed filter has a distinct advantage in cleaning the noise. In particular, the proposed filter also performs

Table 4 Comparison of CPU time in seconds

	$p = 40\%$	$p = 50\%$	$p = 60\%$	$\sigma = 20$, $p = 30\%$	$\sigma = 10$, $p = 40\%$
MED	0.1	0.1	0.1	0.1	0.1
NLM	1,117.6	1,118.2	1,117.7	1,118.2	1,118.2
SKR	1,290.3	1,452.1	1,280.9	1,272.6	1,280.0
GP	2.7	2.7	2.7	2.8	2.7
Luo	31.0	31.0	31.0	31.1	31.4
CEF	8.1	12.3	12.6	12.5	12.3
DWM	18.8	18.8	18.9	18.8	18.9
FWNLM	1,235.9	1,269.5	1,269.1	1,269.1	1,270.0

surprisingly well in removing mixed Gaussian and random-valued impulse noise.

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