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Variable step-size Griffiths' algorithm for improved performance of feedforward/feedback active noise control

S. V. Narasimhan · S. Veena · H. Lokesha

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Abstract A new robust computationally efficient variable step-size LMS algorithm is proposed and it is applied for secondary path (SP) identification of feedforward and feedback active noise control (ANC) systems. The proposed variable step-size Griffiths' LMS (VGLMS) algorithm not only uses a step-size, but also the gradient itself, based on the cross-correlation between input and the desired signal. This makes the algorithm robust to both stationary and non-stationary observation noise and the additional computational load involved for this is marginal. Further, in terms of convergence speed and error, it is better than those by the Normalized LMS (NLMS) and the Zhang's method (Zhang in EURASIP J. Adv. Signal Process. 2008(529480):1–9, 2008). The convergence rate of the feedforward and feedback ANC systems with the VGLMS algorithm for SP identification is faster (by a factor of 2 and 3, respectively) compared with that using NLMS algorithm. For feedforward ANC, its convergence rate is faster (3 times) compared with Akhtar's algorithm (Akhtar in IEEE Trans Audio Speech Lang Process 14(2), 2006). Also, for higher main path lengths compared with SP, the proposed algorithm is computationally efficient compared with Akhtar's algorithm.

Keywords Active noise control · Griffiths' LMS algorithm · Variable step-size LMS algorithm · Secondary path (SP) identification

S. V. Narasimhan (✉) · S. Veena · H. Lokesha
Aerospace Electronics and Systems Division,
National Aerospace Laboratories, Bangalore 560 017, India
e-mail: narasim@nal.res.in

S. V. Narasimhan · S. Veena · H. Lokesha
Council of Scientific and Industrial Research (CSIR),
New Delhi, India

1 Introduction

The LMS adaptive algorithm due to its simplicity is widely used for many applications such as system identification, channel equalization, echo cancelation, noise removal, adaptive coding/compression and active noise control. In LMS algorithm, a larger step-size improves the convergence rate, but results in a larger misadjustment. Therefore to achieve both fast convergence rate and low misadjustment, a variable step-size LMS (VSS) algorithm is used. In VSS, to achieve faster convergence and a smaller misadjustment, the step-size is varied from a large to the small value [1,5,9–11,15]. In Kwong and Johnston [9], the step-size is made as a function of the error energy, however, in the presence of observation noise this will result in larger step-size. In Abolnasar and Mayyas [1], step-size is made proportional to the autocorrelation of the error and this is valid only for the white observation noise. The correlation LMS [11], uses cross-correlation between input and error and this suffers from negative step-size and adaptation stalling. In Okello's [10] VSS, the step-size is a function of the sum of the squared cross correlations between the error and the delayed inputs corresponding to the weights. This is free from the problems of Shan and Kailath [11], but may be sluggish for non-stationary signal. In the recent algorithm by Zhang [15], step-size is made proportional to the squared norm of the smoothed gradient vector. All these existing algorithms attempt to make the step-size robust to observation noise, but not the gradient for the weight vector adaptation.

In ANC, to account for the passage of the error through SP, for the LMS, original input (X) filtered by SP is used and this is known as filtered-X LMS (FXLMS) algorithm [7]. To ensure good performance of FXLMS algorithm, the SP estimate has to be accurate. For this, many approaches have been proposed [2,8,11,14]. The schemes in [8,14] use an adaptive noise canceler to reduce the effect of the observation noise, but at the cost of increased computations and memory. In Zhan et al. [14], the convergence speed of the SP has been improved by varying the gain for its excitation based on the residual noise energy. Here, the gain is affected by the observation noise and also as the excitation passes through the loudspeaker, it creates noise bursts, which are annoying. Therefore, a robust VSS is desirable to achieve a good SP estimate [2].

In this paper, a computationally efficient robust VSS is proposed and is applied to improve the performance of ANC. The proposed VSS estimates both the gradient and the step-size based on Griffiths' cross-correlation between the input and the error. Further, it is applied to improve the SP identification in ANC.

2 Proposed variable step-size Griffiths' LMS (VGLMS) algorithm

Griffiths' [4] has proposed a modification for the LMS algorithm that uses a gradient robust to observation noise and Okello [10] has proposed a VSS, in which the step-size adaptation is less affected by the observation noise. Here, Griffiths' and Okello's algorithms are combined to build robustness into both the gradient and the step-size.

2.1 Griffiths' algorithm (GLMS)

Consider a system $S(z)$ with impulse response $\boldsymbol{S}(n) = [s(0)\ s(1) \cdots s(M-1)]$ and white noise input $\boldsymbol{V}(n) = [v(n)\ v(n-1) \cdots v(n-M+1)]^{\mathrm{T}}$. The system output $d'(n)$ is corrupted by the observation noise $o(n)$ to give a desired signal $d(n) = d'(n) + o(n)$. The NLMS adaptation rule for this is given by,

$$\hat{\boldsymbol{S}}(n+1) = \hat{\boldsymbol{S}}(n) + \frac{\mu}{M E_v(n)} e(n) \boldsymbol{V}^{\mathrm{T}}(n) \quad (1)$$

where $e(n) = d(n) - y(n)$ and $y(n) = \hat{\boldsymbol{S}}^{\mathrm{T}}(n)\boldsymbol{V}(n)$.

If $\boldsymbol{G}(n) = E[d(n)\boldsymbol{V}^{\mathrm{T}}(n)] = E[\{d'(n) + o(n)\}\boldsymbol{V}^{\mathrm{T}}(n)]$, this represents the cross-correlation between the desired signal and the input. Further as $v(n)$ and $o(n)$ are independent, $E[o(n)\boldsymbol{V}^{\mathrm{T}}(n)] = 0$. Therefore, $\boldsymbol{G}(n) = E[d'(n)\boldsymbol{V}^{\mathrm{T}}(n)]$ and $E[e(n)\boldsymbol{V}^{\mathrm{T}}(n)] = \boldsymbol{G}(n) - E[y(n)\boldsymbol{V}^{\mathrm{T}}(n)]$.

As LMS algorithm uses instantaneous quantities and also as observation noise is not directly present in $y(n)\boldsymbol{V}^{\mathrm{T}}(n)$, LMS adaptation becomes [4],

$$\hat{\boldsymbol{S}}(n+1) = \hat{\boldsymbol{S}}(n) + \frac{\mu(n)}{M E_v(n)} [\boldsymbol{G}(n) - y(n)\boldsymbol{V}^{\mathrm{T}}(n)] \quad (2a)$$

where

$$E_v(n) = \beta_v E_v(n-1) + (1-\beta_v) v^2(n), \quad 0 < \beta_v < 1 \quad (2b)$$

The term $\boldsymbol{P}_G(n) = \boldsymbol{G}(n) - y(n)\boldsymbol{V}^{\mathrm{T}}(n)$, will be referred as *Griffiths' cross-correlation* and is free from the effect of observation noise component $o(n)$.

2.2 Okello's variable step-size algorithm

Okello [10] has proposed an algorithm that provides a step-size based on the cross-correlation $\boldsymbol{P}(n)$, between input and the error.

$$\boldsymbol{P}(n) = E[e(n)\boldsymbol{V}^{\mathrm{T}}(n)] = E[(d(n) - y(n))\boldsymbol{V}^{\mathrm{T}}(n)] \quad (3)$$

Since $E[o(n)\boldsymbol{V}^{\mathrm{T}}(n)] = 0$, $\boldsymbol{P}(n)$ is free from observation noise. In this, to avoid negative step-size, the variable step-size $\mu(n)$ is given by Okello et al. [10]

$$\mu(n) = \alpha\mu(n-1) + \sigma \boldsymbol{P}^{\mathrm{T}}(n)\boldsymbol{P}(n) \quad 0 < (\alpha, \sigma) < 1 \quad (4)$$

or

$$\mu(n) = \alpha\mu(n-1) + \sigma \sum_{i=0}^{L-1} P_i^2(n)$$

where $P_i(n) = \beta_{\mathrm{vs}} P_i(n-1) + (1-\beta_{\mathrm{vs}}) e(n) v(n-i+1)$, $0 < \beta_{\mathrm{vs}} < 1$.

It has been shown that $\mu(n) = \alpha\mu(n–1) + \sigma\boldsymbol{\theta}^{\mathrm{T}}(n)\boldsymbol{R}\boldsymbol{R}\boldsymbol{\theta}(n)$, where $\boldsymbol{R}(n) = E[\boldsymbol{V}(n)\boldsymbol{V}^{\mathrm{T}}(n)]$, is the autocorrelation matrix of $\boldsymbol{V}(n)$ and $\boldsymbol{\theta}(n) = E[\boldsymbol{S}^* - \hat{\boldsymbol{S}}(n)]$, $\boldsymbol{S}^*$ is the optimal weight vector. As convergence is reached, $\boldsymbol{\theta}(n)$ decreases resulting in a smaller step-size and hence smaller misadjustment. The expectation operation in $\boldsymbol{P}(n)$, reduces the effect of observation noise on step-size estimation. Further, $\boldsymbol{P}^{\mathrm{T}}(n)\boldsymbol{P}(n) \geq 0$ in Eq. 4, ensures positive step-size and the presence of $\alpha\mu(n-1)$ avoids stalling [1]. This being an NLMS algorithm, for stability the initial value for $\mu(n)$ is set close to unity [7] and the lower limit for $\mu(n)$ is fixed to prevent adaptation stalling and to provide minimum misadjustment tolerable.

2.3 Proposed VGLMS algorithm

It is proposed to combine the Griffiths' and Okello's algorithms to realize VGLMS algorithm. The only difference between the correlation term $\boldsymbol{P}(n) = E[d(n)\boldsymbol{V}^{\mathrm{T}}(n)] - E[y(n)\boldsymbol{V}^{\mathrm{T}}(n)]$ of Okello's algorithm and that of Griffiths' cross-correlation $\boldsymbol{P}_G(n) = E[d(n)\boldsymbol{V}^{\mathrm{T}}(n)] - y(n)\boldsymbol{V}^{\mathrm{T}}(n)$, is with the second term. The expectation operation for $y(n)\ \boldsymbol{V}^{\mathrm{T}}(n)$ can be removed due to use of instantaneous quantities in LMS algorithm and also as observation noise is not directly present in this term. Hence, the term $\boldsymbol{P}_G(n) = E[d(n)\boldsymbol{V}^{\mathrm{T}}(n)] - y(n)\boldsymbol{V}^{\mathrm{T}}(n)$ can be used for both step-size and gradient adaptation. It is important to note that as $y(n)$ is a time-varying quantity and averaging of this quantity in $\boldsymbol{P}(n)$ makes the adaptation process sluggish, justifying the use of $\boldsymbol{P}_G(n)$ over $\boldsymbol{P}(n)$ (illustrated in Sect. 3.2 for an example).

Therefore, the proposed adaptation rule for the individual coefficients is given by

$$\hat{s}_i(n+1) = \hat{s}_i(n) + \frac{\mu(n)}{ME_v(n)}[\hat{G}_i(n) - y(n)v(n-i)] \quad (5a)$$

and

$$\mu(n+1) = \alpha\mu(n) + \sigma \sum_{i=0}^{M-1} [\hat{G}_i(n) - y(n)v(n-i)]^2 \quad (5b)$$

$$\hat{G}_i(n+1) = \beta_g \hat{G}_i(n) + (1-\beta_g)d(n)v(n-i), \quad 0 < \beta_g < 1 \quad (5c)$$

Here, β_g decides the cutoff frequency, and hence, the degree of rejection of the observation noise. Equation 5a can be expressed as

$$\hat{s}_i(n+1) = \hat{s}_i(n) + \mu(n)\nabla(n) \quad (5d)$$

where $\nabla(n) = \frac{1}{ME_v(n)}[\hat{G}_i(n) - y(n)v(n-i)]$.

The second term of Eq. 5d is the product of $\mu(n)$ and the weight vector adaptation gradient $\nabla(n)$. The existing variable step-size algorithms have focused only on achieving noise free step-size $\mu(n)$ and not on the gradient $\nabla(n)$, though it is available. In the present algorithm, noise free $\mu(n)$ and $\nabla(n)$ are used.

In the proposed algorithm, a high σ value enables the step-size to track the changes in the error at a faster rate for non-stationary signal, whereas a small value of σ is acceptable for stationary signals. Noise-free $\boldsymbol{P}_G(n)$ used for both the gradient and step-size adaptation in VGLMS, significantly improves the convergence characteristics. Further, it does not require information regarding the observation noise power.

2.4 Zhang's method for comparison

The Zhang's algorithm [15] is the latest variable step-size algorithm and is is considered for comparison. In this, the step-size is made proportional to the squared norm of the smoothed gradient vector $\boldsymbol{P}(n)$ of Eq. 3, i.e., $\mu(n) = K \|\boldsymbol{P}(n)\|$ and for stationary and non-stationary observation noise, the proportionality constant K is computed separately. For stationary noise [15]:

$$K = P_{zs} = \frac{2J_{ex,VSS}(\infty)(1+\beta_{ex})}{(1-\beta_{ex})L^2 E_v E_o} \quad (6a)$$

$J_{ex,VSS}(\infty)$-excess error, selected as per application; L-length of the adaptive filter; E_v, E_o-power of the input and the observation noise, respectively. For non-stationary noise [15]:

$$K = \frac{P_{zn}}{L(E_v+E_e)^2}, \quad P_{zn} = \frac{2J_{ex,VSS,max}(\infty)(1+\beta_{ex})}{(1-\beta_{ex})E_v} \quad (6b)$$

$J_{ex,VSS,max}$, upper bound value of excess error.

This method uses smoothed gradient vector $\boldsymbol{P}(n)$ depending on the value of β_{vs} and suffers from slow in adaptation due to smoothing of $[y(n)v(n-i)]$. However, a low value of β_{vs}, enables fast adaptation, but it fails to suppress the noise in $[d(n)v(n-i)]$. Also, Zhang method requires different algorithms for stationary and non-stationary cases and noise power estimation unlike the VGLMS.

3 System identification simulation

For the simulation study, a system $S(z)$ of impulse response length 128 samples for an acoustic path is used. Here, the SNR is with reference to $d'(n)$ and $o(n)$, respectively. The performance of the algorithms is evaluated based on the following measures:

(a) The estimation error $\Delta S(n)$, deviation of the estimate $\hat{S}(z)$ from $S(z)$ is given by

$$\Delta S(n) = \|(\hat{\boldsymbol{S}}(n) - \boldsymbol{S})^2\| / \|\boldsymbol{S}^2\| = \sum_{k=0}^{M-1} (\hat{s}_k(n) - s_k)^2 \Big/ \sum_{k=0}^{M-1} s_k^2$$

Ensemble average of $\Delta S(n)$ expressed in dB,

$$\Delta S_{AV}(n)|_{dB} = 10\log_{10}\left\{\frac{1}{K}\sum_{i=1}^{K} \Delta S_i(n)\right\} \quad (7)$$

(b) The smoothed ensemble average square error (SEASE) expressed in dB, $\xi_{sd}(n)$

$$\xi_{sd}(n) = 10\log_{10}(\xi_S(n)) \quad (8)$$
$$\xi_S(n) = \lambda\xi_S(n-1) + (1-\lambda)\xi(n), \quad 0 < \lambda < 1$$
$$\xi(n) = \frac{1}{K}\sum_{i=1}^{K} e_i^2(n)$$

$e_i(n)$, error for ith realization; K, number of realizations. For all simulations, $K = 50$ and $\lambda = 0.99$ is considered.

3.1 Performance comparison of NLMS, Okello's and VGLMS algorithms

For this, the input $v(n)$ is a zero mean unit variance Gaussian white noise and the colored observation noise $o(n)$ is a sum of three sinusoids at 100, 200 and 300 Hz with additive Gaussian noise. The performance of various algorithms is evaluated for an SNR of −10 dB and using the parameters given in Table 1.

Table 1 Parameters for system identification algorithms at SNR = −10 dB

Parameter	μ	$\mu_{\max}, \mu_{\min}$	β_v	β_{vs}	β_g	α, σ
NLMS	1 and 0.005		0.99			
Okello		1, 0.005	0.99	0.9999		0.99, 0.99
VGLMS		1, 0.005	0.99		0.9	0.99, 0.0001

The NLMS algorithm with $\mu = 1$ is considered for faster convergence rate and $\mu = 0.005$ for minimum misadjustment and these are compared with VGLMS algorithm. The NLMS with $\mu = 0.005$ is able to achieve better $\Delta S_{\mathrm{AV}}(n)|_{\mathrm{dB}}$, but at the cost of convergence rate. The VGLMS has a faster convergence rate and lesser estimation error compared with NLMS algorithm (Fig. 1a). In Okello's algorithm due to the use of noisy gradient, the step-size is unable to reach $\mu_{\min} = 0.005$ and this makes the performance of Okello's algorithm poorer than VGLMS.

3.2 Performance comparison with $\boldsymbol{P}(n)$ and $\boldsymbol{P}_G(n)$ in VGLMS algorithm

For this, the input $v(n)$ is a zero mean unit variance Gaussian white noise and the observation noise is a colored noise (50 Hz fundamental and harmonics at 100, 150, 200, 250 Hz and additive random noise) and the SNR is 10 dB. The parameters used are given in Table 2. The indices $\xi_{\mathrm{sd}}(n)$ and $\mu_E(n)$ (the averaged step-size over 50 trials), indicate that the use of $\boldsymbol{P}(n)$ makes the convergence process sluggish (Fig. 1b and c) compared with using $\boldsymbol{P}_G(n)$ justifying the choice of $\boldsymbol{P}_G(n)$ for VGLMS algorithm.

Table 2 Parameters for using $\boldsymbol{P}(n)$ and $\boldsymbol{P}_G(n)$ at SNR = 10 dB

Parameter	$\mu_{\max}, \mu_{\min}$	β_v	β_{vs}	β_g	α, σ
$\boldsymbol{P}(n)$	1, 0.01	0.99	0.9999		0.99, 0.99
$\boldsymbol{P}_G(n)$	1, 0.01	0.99		0.9	0.99, 0.0001

3.3 Performance comparison of the VGLMS and Zhang's algorithms

The proposed VGLMS algorithm is compared with Zhang's algorithm [15] for both stationary and non-stationary noise fields. The parameters used for comparison are listed in Table 3. The input $v(n)$ is a Gaussian white noise (zero mean unit variance).

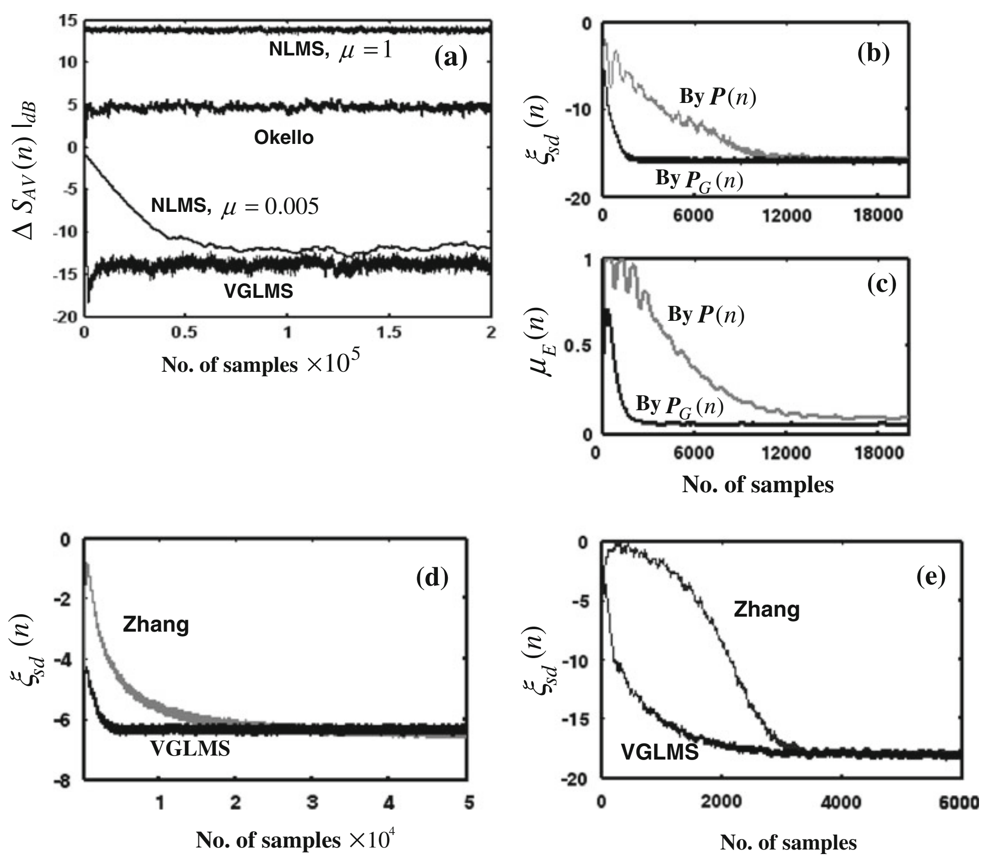

Fig. 1 **a** comparison of SEASE **b** $\xi_{\mathrm{sd}}(n)$ and **c** $\mu_E(n)$ using $\boldsymbol{P}(n)$ and $\boldsymbol{P}_G(n)$. Comparison of VGLMS and Zhang's algorithms **d** Stationary noise **e** non-stationary noise fields

Table 3 Parameters for comparison of VGLMS and Zhang's algorithm

Method	VGLMS method			Zhang's method		
Parameter	$\mu_{\max}, \mu_{\min}$	β_v, β_g	α, σ	$J_{\text{ex,VSS}}$	β_z	β_{ex}
Stationary noise	1, 0.01	0.99, 0.9	0.99, 0.0001	−35 dB	0.99	0.9999
Non-stationary noise	1, 0.01	0.99, 0.9	0.999, 0.99	−10 dB	0.9999	0.9995

Case 1 The stationary colored noise (50 Hz fundamental and harmonics at 100, 150, 200, 250 Hz plus an additive random noise) has 0 dB SNR. In the proposed method, the value of σ is kept very small (Table 3) to keep the variance of the step-size adaptation at a minimum. For the Zhang's method, the value of P_{zs} is computed from the parameters of Table 3. The results show that the VGLMS algorithm has five times faster convergence compared with the Zhang's method (Fig. 1d), as the gradient of the proposed method is free from observation noise.

Case 2 The non-stationary noise used is a chirp signal (fundamental frequency varying from 50 to 100 Hz and its harmonics at 100 and 150 Hz varying correspondingly plus a random noise at −30 dB). The SNR is kept at 10 dB. The value σ for VGLMS is 0.99 (higher compared with stationary case), as the step-size has to track changes at a faster rate to account for the non-stationarity. In Fig. 1e, $\xi_{\text{sd}}(n)$ shows that the convergence rate of the VGLMS algorithm is about two times faster than that of Zhang's. This shows the flexibility and superiority of the VGLMS for both the stationary and non-stationary cases.

4 Active noise control (ANC)

Feedforward ANC (FFANC): In FFANC, the antinoise built by adaptive filter $W(z)$ is given out by the loud speaker and this interacts with the existing (primary) noise field over a physical space resulting in a residual noise field. Effectively error $e(n)$ is available only after it passes through an acoustic path $S(z)$, the secondary path (SP) (Fig. 2).

$$e(n) = e_d(n) + e_v(n) = [d(n) - y(n)] * s(n) + v(n) * s(n) \quad (9)$$

'$*$' indicates convolution operation. The components $e_d(n)$ and $e_v(n)$ are due to residual noise field and SP excitation $v(n)$, respectively. Since during adaptation, $y(n)$ is changing, $e(n)$ becomes non-stationary. In ANC, $W(z)$ is adapted using filtered input LMS (FXLMS) algorithm [7],

$$w_i(n+1) = w_i(n) + 2\mu(n)e(n)x'(n-i), \quad i = 0.1, \ldots, L-1 \quad (10)$$

Here $x'(n) = \boldsymbol{S}(n)X(n)$, the filtered input and as $S(z)$ is not known, its estimate $\hat{S}(z)$ has to be found.

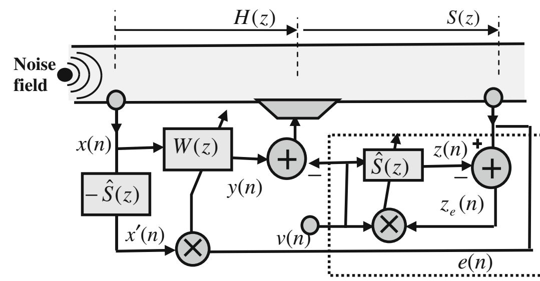

Fig. 2 Schematic of FFANC using FXLMS algorithm

Feedback ANC (FBANC) [7]: The FBANC is similar to FFANC except the reference signal is not available and it is derived from the only available signal, the error $e(n)$. Here $W(z)$ plays the role of a predictor and it is suitable for predictable noise fields. Further, the accuracy of the SP estimate not only determines the convergence of $W(z)$ but also the derived reference signal making FBANC doubly dependent on the SP estimate [7].

5 ANC with VGLMS algorithm for SP identification (VGLMS-SP)

5.1 On line Secondary path estimation

The performance of ANC depends on the accuracy of SP estimate. The estimation of $\hat{S}(z)$ is done on online basis using white noise $v(n)$ due to its merits [7]. $\hat{S}(z)$ is updated by NLMS algorithm using the step-size $\mu_s(n)$. The SP identification error is,

$$z_e(n) = e(n) - z(n) = e_d(n) + e_v(n) - v(n) * \hat{s}(n) \quad (11)$$

The identification has to be done in the presence of a strong observation noise $e_d(n)$. The level of $v(n)$ has to be below the primary noise level (by about 25 dB) as it passes through the loudspeaker and decides the noise floor level or the amount of attenuation achievable by ANC. Further, the identification should be fast as it is required in advance for filtering and generating the reference signal, calling for a larger adaptation step-size which results in a higher misadjustment. All these factors contribute to phase error and delay in the SP estimate, deteriorating the performance of ANC [3]. Therefore an accurate SP estimate is absolutely necessary for improving the performance of both FFANC and FBANC.

Table 4 Computations involved in implementing the algorithms

Equations considered for computation	No. of multiplications	No. of additions	No. of divisions
VGLMS-SP algorithm			
Cross-correlation, Eq. 5c	$2M + 1$	M	
Gradient (bracketed term of Eq. 5a)	M	M	
Weight vector adaptation, Eq. 5a	M	M	
Step-size adaptation, Eq. 5b	$M + 2$	M	
Normalization for step-size, Eqs. 2b,5a	4	1	1
	$5M + 7$	$4M + 1$	1
Akhtar's algorithm			
$\in (n)$ computation,			
$\in (n) = e(n) + S(n)[W(n)X(n)]^{\mathrm{T}} - W(n)[S(n)X(n)]^{\mathrm{T}}$	$2L + 2M$	$2L + 2M - 2$	
μ_s, $E_e(n)$, $E_{ze}(n)$, Eq. 12a, c, d	$2 + 3 + 3$	$2 + 1 + 1$	
$\rho(n)$, Eq. 12b			1
	$2L + 2M + 8$	$2L + 2M + 4$	1

To reduce the effect of errors in $\hat{S}(z)$, constrained/modified FXLMS algorithm has been proposed [6]. In literature, adaptive noise cancelers have been used to improve SP identification in the presence of colored non-stationary primary noise fields [8,12,13]. However, the introduction of additional adaptive filter increases the computational complexity. A gain scheduling procedure for the input for SP identification based on the error energy has also been proposed, but as it creates annoying intermittent high-noise level bursts [14], this is not desirable.

5.2 VGLMS algorithm for SP identification of ANC (VGLMS-SP)

It is proposed to use the VGLMS algorithm for improving the SP identification for ANC. $\boldsymbol{P}_G(n)$ of this algorithm enables better estimate of $S(z)$ even in the presence of a strong primary noise field, as its gradient and the step-size both have good noise immunity. The $\boldsymbol{P}_G(n)$ also enables faster adaptation, as only its noisy part is averaged. The choice of σ facilitates to effectively handle the non-stationary observation noise scenario of ANC. The use of better SP estimate provides a filtered input of good quality resulting in an improved ANC performance. Further, the VGLMS noise immunity enables the use of a lower excitation level which in turn reduces the noise floor level.

5.3 Akhtar's ANC algorithm for comparison

Akhtar has proposed a variable step-size algorithm that uses modified FXLMS (MFXLMS) algorithm [6] for the main path adaptation. The step-size $\mu_s(n)$ is computed as [2]

$$\mu_s(n) = \rho(n)\mu_{s(\min)} + (1 - \rho(n))\,\mu_{s(\max)} \tag{12a}$$

where

$$\rho(n) = \frac{E_e(n)}{E_{z_e}(n)}, \lim_{n\to\infty} \rho(n) \to 0 \tag{12b}$$

$$E_e(n+1) = \beta_p E_e(n) + (1-\beta_p)e^2(n) \tag{12c}$$

$$E_{z_e}(n+1) = \beta_p E_{z_e}(n) + (1-\beta_p)z_e^2(n), \quad 0 < \beta_p < 1 \tag{12d}$$

$\mu_{s(\min)}$ and $\mu_{s(\max)}$ are the lower and upper bounds of the step-size and are selected so that adaptation is neither too slow nor it becomes unstable.

This uses the fact that prior to convergence the observation noise $e_d(n)$ is high and use of a larger step-size will degrade $\hat{S}(z)$. Hence, this method uses a small value of step-size prior to convergence and is increased as the convergence progresses. For minimum misadjustment, $\mu_{s(\max)}$ has to be smaller and this limits $\mu_{s(\min)}$ which in turn limits the convergence rate of $\hat{S}(z)$. This is not desirable as the antinoise depends on the quality of the filtered input. Further this algorithm uses LMS instead of NLMS for SP identification, which may lead to divergence.

It is important to note that a larger step-size is desirable initially (prior to convergence) to have a faster $\hat{S}(z)$ convergence and a smaller step-size after convergence to get smaller misadjustment in $\hat{S}(z)$. The VGLMS-SP algorithm satisfies this requirement.

5.4 Computational considerations

Table 4 shows the computations involved in the SP identification of ANC using the VGLMS-SP and the Akhtar's algorithm. In VGLMS-SP algorithm, the use of $\mathbf{P}_G(n)$ for both step-size and gradient estimation reduces the computational load. In the Table 4, L and M are the main path and the SP

Table 5 Parameters for comparison of NLMS-SP and VGLMS-SP

Parameter	μ	μ_s	$\mu_{s(\max)}, \mu_{s(\min)}$	β_v	β_g	α, σ
NLMS-SP	0.01	0.3		0.99		
VGLMS-SP	0.01		0.3, 0.001	0.99	0.99	0.99, 0.99

impulse response lengths, respectively. In ANC, as $L \gg 2M$ is preferred to satisfy acoustical requirements, the computational load of VGLMS-SP is significantly low compared with Akhtar's algorithm. For $L < 2M$, their computational loads are comparable.

6 ANC simulation results

For this study, practical impulse responses for the mainpath (MP) and the SP, each of length 128 samples are used.

6.1 Comparison of FFANC with VGLMS-SP and NLMS-SP algorithms

For simulation, as the FXLMS algorithm is critical for a broadband noise field rather than for a narrowband noise, the former is considered. The input is a Gaussian white noise of zero mean unit variance. A filter length of 64 and 32 are selected for the MP and the SP, respectively. The uniform white noise injected for SP identification is 27 dB below the ambient noise level.

Normalized FXLMS algorithm is used for $W(z)$ adaptation. The VGLMS-SP and NLMS algorithms are used for online SP identification. The ANC algorithm using NLMS algorithm for SP is referred as **NLMS-SP** algorithm. The parameters used for the simulation is given in Table 5.

To obtain a faster $\hat{S}(z)$ estimate, the step-size for NLMS-SP algorithm is chosen equal to $\mu_{s(\max)}$ of VGLMS-SP algorithm. The convergence performance of the ANC error (Fig. 3a) shows that the VGLMS-SP has an improved convergence compared with the NLMS-SP algorithm and $\xi_{\text{sd}}(n)$ also supports this showing VGLMS-SP is twice faster than NLMS-SP algorithm (Fig. 3b). Initially, the step-size of the VGLMS-SP has a maximum value (Fig. 3c), making the SP identification faster. The good SP estimate improves the overall ANC performance resulting in a faster ANC convergence. Even in the presence of high observation noise prior to convergence, performance of VGLMS-SP algorithm is superior due to its built in noise immunity compared with that of NLMS-SP. Near to the convergence, both the algorithms have similar performance as the observation noise level is less. The performance measure $\Delta S_{\text{AV}}(n)|_{\text{dB}}$ for NLMS-SP and VGLMS-SP also justify the fact that SP identification of VGLMS-SP is better compared with NLMS-SP algorithm (Fig. 3d).

Table 6 Parameters for comparison of Akhtar's and VGLMS-SP algorithms for $L = 128$, $M = 64$

Parameter	μ	$\mu_{s(\max)}, \mu_{s(\min)}$	β_v	β_p	β_g	α, σ
Akhtar	0.01	0.01, 0.0005	0.999	0.9		
VGLMS-SP	0.01	0.01, 0.0005	0.999		0.999	0.9, 0.9

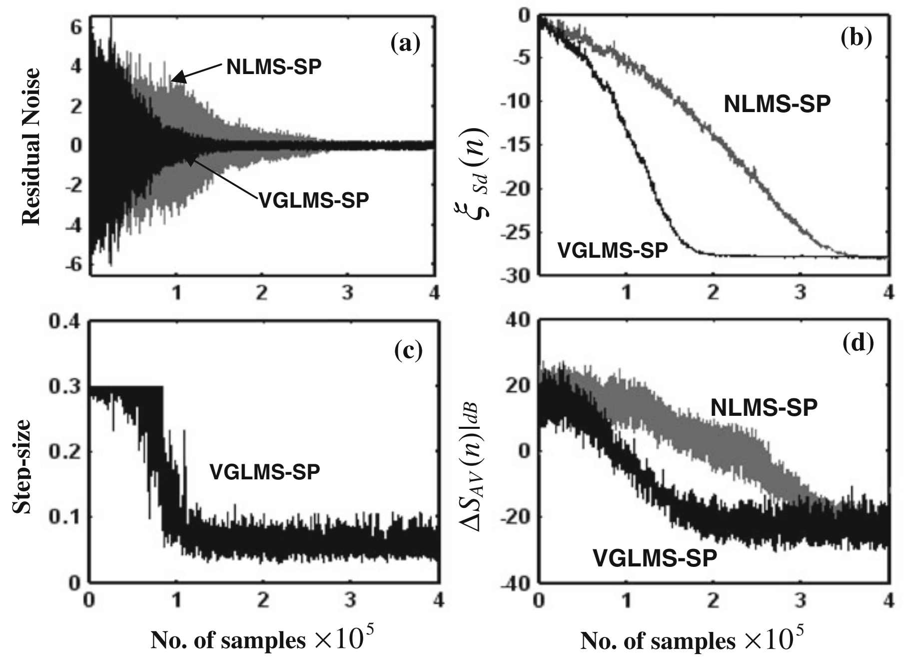

Fig. 3 **a** Residual noise, **b** $\xi_{\text{sd}}(n)$, **c** $\mu(n)$ of VGLMS-SP **d** $\Delta S_{\text{AV}}(n)|_{\text{dB}}$

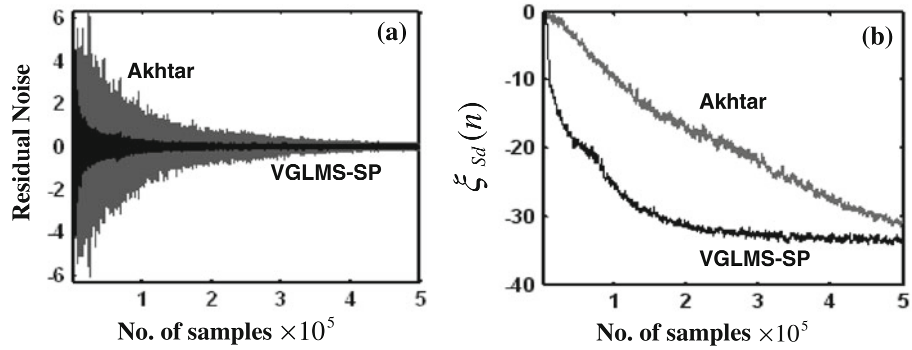

Fig. 4 Comparison of VGLMS-SP and Akhtar's algorithm **a** $e(n)$, **b** $\xi_{\mathrm{sd}}(n)$

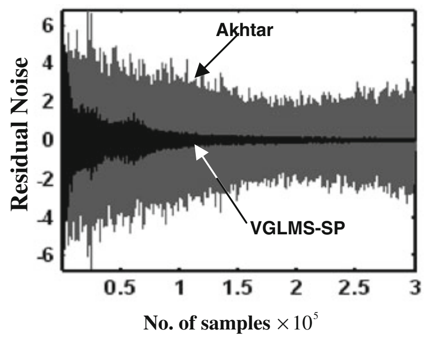

Fig. 5 Effect of reduced $v(n)$

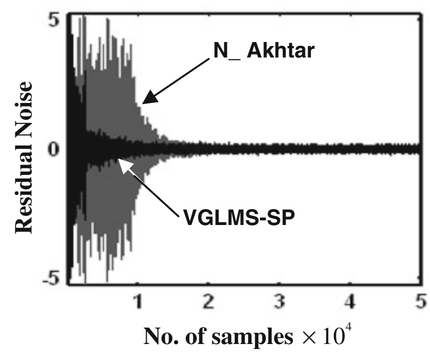

Fig. 6 Performance of N_Akhtar and VGLMS-SP algorithms

Table 7 Parameters for comparison of N_Akhtar with VGLMS-SP algorithms ($L = 64$, $M = 32$)

Parameter	μ	$\mu_{\max}, \mu_{\min}$	β_v	β_p	β_g	α, σ
N_Akhtar	0.0005	0.1, 0.01	0.99	0.99		
VGLMS-SP	0.05	0.1, 0.001	0.99		0.999	0.99, 0.99

Table 8 Parameters for comparison of NLMS-SP and VGLMS-SP

	μ	μ_s	$\mu_{\max}\mu_{\min}$	β_v	β_g	α, σ
NLMS-SP	0.001	0.0015		0.99		
VGLMS-SP	0.001		$1.5 \times 10^{-3}, 7 \times 10^{-5}$	0.99	0.99	0.9, 0.9

6.2 Comparison of VGLMS-SP and Akhtar's algorithms

The level of $v(n)$ injected for SP identification is kept at about 28 dB below the ambient noise level. The parameters selected are listed in Table 6.

For fair comparison, the maximum and minimum step sizes are divided by the SP length, i.e., $M = 64$, as input normalization is absent in the case of Akhtar's algorithm. In VGLMS-SP algorithm, a good SP estimate is obtained initially itself due to the use of higher SP step-size. This results in a faster convergence of the ANC error compared with that of Akhtar's algorithm (Fig. 4a). $\xi_{\mathrm{sd}}(n)$ (Fig. 4b) shows that VGLMS-SP algorithm is about three times faster than Akhtar's algorithm. It can be noted that in Akhtar's algorithm, although the SP step-size is scaled depending on the level of observation noise, it is not so with the gradient estimation. Hence, its performance is poorer compared with VGLMS-SP algorithm.

If the $v(n)$ level is further reduced to about −37 dB, the VGLMS-SP is able to converge where as the stability of Akhtar is affected (Fig. 5). This is due to the fact that the use of $\boldsymbol{P}_G(n)$ can handle a higher level of observation noise, as it removes its effect both on the step-size and the gradient.

Further, it is not advisable to use an un-normalized step-size, as changes in the input are not reflected on the step-size and this may lead to instability. For this study, the $v(n)$ is kept at the normal level of −28 dB. Therefore, the performance of the Akhtar's algorithm with input power normalization (N_Akhtar) for $W(z)$ is evaluated. N_Akhtar and VGLMS-SP algorithms are compared for their best performance using the parameters given in Table 7.

Since a faster SP of good quality is obtained in VGLMS-SP algorithm, it enables the use of higher main path step-size (0.05) resulting in faster convergence of ANC error (Fig. 6). However, for a step-size higher than 0.0005, N_Akhtar was found to diverge. In N_Akhtar, the use of MFXLMS algorithm [6] is expected to give a good performance, but

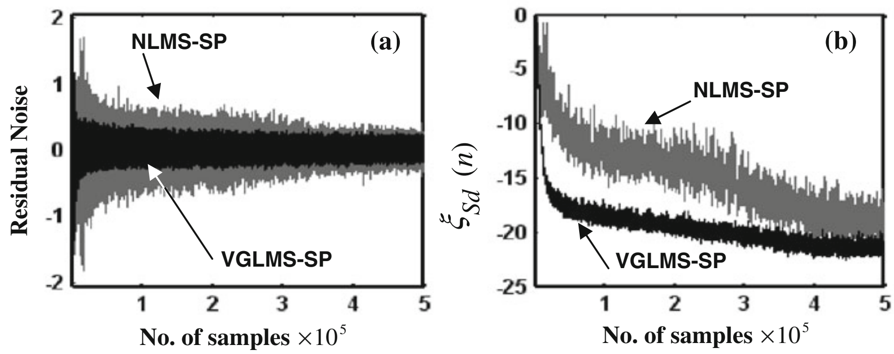

Fig. 7 Performance of VGLMS-SP based feedback ANC **a** $e(n)$, **b** $\xi_{\mathrm{sd}}(n)$

this is not so as the gradient used for SP adaptation is not free from observation noise leading to a poorer $\hat{S}(z)$.

6.3 Comparison of VGLMS and NLMS-SP algorithms for secondary path identification of Feedback ANC

The input noise is a narrowband noise with fundamental at 100 Hz and its harmonics at 200, 300 and 400 Hz, respectively. The filter lengths for the predictor and SP estimator were selected as 32. The parameters used are listed in Table 8.

The FBANC performance is doubly dependent on the accuracy of SP estimate. Even in the presence of narrowband observation noise, the proposed VGLMS-SP algorithm gives a good SP estimate. This leads to the better convergence rate of ANC error compared with NLMS-SP algorithm (Fig. 7a). Further, $\xi_{\mathrm{sd}}(n)$ shows that the convergence rate of VGLMS-SP is significantly faster (more than 3 times) compared with that of NLMS-SP algorithm (Fig. 7b).

7 Conclusion

A new computationally efficient robust variable step-size LMS (VGLMS) algorithm was proposed for system identification. The VGLMS uses Griffiths' cross-correlation to derive the step-size and gradient that are robust to observation noise. Further as same cross-correlation is used both for step-size and gradient, it is computationally efficient. The VGLMS is superior to recent Zhang's algorithm [15] as the latter uses only a noise free step-size, but its gradient is affected by observation noise and also it requires information about observation noise power.

FFANC and FBANC with VGLMS-SP estimation had an improvement in convergence rate by a factor of 2 and 3, respectively over NLMS-SP algorithm. When compared with Akhtar's algorithm, the convergence rate of the VGLMS-SP is three times faster as the former uses a gradient not free from observation noise. Further, for a higher main path length than SP length, the computational advantage of VGLMS-SP increases over that of Akhtar's algorithm.

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