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The ordered capacitated facility location problem

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Abstract In this paper, we analyze flexible models for capacitated discrete location problems with setup costs. We introduce a major extension with regards to standard models which consists of distinguishing three different points of view of a location problem in a logistics system. We develop mathematical programming formulations for these models using discrete ordered objective functions with some new features. We report on the computational behavior of these formulations tested on a randomly generated battery of instances.

Keywords Discrete location · Mixed integer programming · Strategic planning

Mathematics Subject Classification (2000) 90B80 · 90B06 · 90C11

1 Introduction

Location Analysis is one of the most active fields in Operations Research; in fact, many different models have been developed in the last decades to deal with differ-

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ent real world situations. A very important aspect of a location model is the correct choice of the objective function and in most classical location models the objective function is the main differentiator. The median objective is to minimize the sum of the distances from the clients to its server. The center objective is to minimize the maximum distance from a client to its server. The centdian objective is a convex combination of the median and center objectives; it aims to keep both the average cost behavior as well as the highest cost in balance. Despite the fact that all three objectives (and some more) are frequently encountered in the literature (see, for example, Drezner and Hamacher 2002), not much has been done in the direction of a unified framework for handling all of these objectives.

The need for location models that better fit different real world situations has made it necessary to develop new and flexible location models. To that end, Puerto and Fernández (1995) introduced a new type of objective function that generalizes the most popular objective functions mentioned above. This objective function, called ordered median function, applies a penalty to the distance from a client to its server, which depends on the position of that distance in the vector of all distances from the clients to the servers. For example, a different penalty might be applied to a client if the distance to the server is in the fifth position rather than in the second position. It is even possible to neglect some customers by assigning a zero penalty. This adds a "sorting" problem to the underlying facility location problem, making formulation and solution much more challenging.

In the last years, these flexible objective functions have attracted the attention of many researchers in the location-routing community, as well as in other fields (Berbeglia et al. 2007; Marín 2007; Plastria and Elosmani 2008; Tamir 2008). The papers (Puerto and Fernández 1995, 2000) study characterizations of the solution set for the general formulations. For the planar case with polyhedral gauges, Rodríguez-Chía et al. (2000) develops a polynomial time algorithm and Puerto et al. (1997) applies these models to semiobnoxious location problems. In network location problems, efforts are devoted to obtaining finite dominating sets and efficient algorithms to solve this kind of problems (Kalcsics et al. 2002, 2003; Nickel and Puerto 1999; Nickel et al. 2005; Puerto and Rodríguez-Chía 2005). The model that deals with the discrete location problem, namely the *Discrete Ordered Median Problem* (DOMP), has also been analyzed. Starting with a nonlinear formulation, several linearizations have been developed. Structural results as well as a specially tailored branch and bound procedure can be found in Boland et al. (2006). By using a different approach, following the principal idea of Elloumi et al. (2004), an improved branch and cut algorithm is developed in Marín et al. (2006, 2009). This way, it is now possible to optimally solve DOMPs with more than 100 clients in reasonable time. Moreover, a VNS-heuristic and an evolutionary approach can be found in Domínguez-Marín et al. (2005) and different genetic algorithms in Stanimirovic et al. (2007). Research in this area also led to a recent monograph; see Nickel and Puerto (2005).

In this paper, we introduce three major extensions of the basic DOMP. First, we consider capacitated models where the demands can be split. Second, we impose setup costs on building new facilities, thus giving rise to strategic models where the number of new supply facilities is not fixed a priori; rather it is chosen by the model using the setup costs. Therefore, the model is more suitable to cope with actual requirements from logistics. And third, we distinguish in the modeling phase which is the driving force of the logistics network. The objective function to be considered in these models has two main components, namely shipping plus setup costs.

The use of capacities in logistics problems is natural. The setup cost component of the objective function is modeled using the DOMP because it allows us to cover, among others, the following situations. In practice, if just one construction company is responsible for building all facilities, it sometimes grants a discount on the setup costs depending on the number of facilities they are commissioned to build. Another possible source of deductions is that the new facilities often have a similar design which can be re-used for consecutive facilities, with minor changes only. Therefore, we can assume that the setup costs decrease with each new facility being built. Hence, we pay the full setup costs for the first facility, but for each additional facility we get a certain percentage of reduction on the costs. Note that this discount is cumulative. However, the reduction applies in decreasing order of setup costs, i.e., we have to pay the most expensive facility completely, for the second most expensive we get the first reduction, and so on.

Finally, to distinguish the driving force of the logistics network, we introduce three different points of view of the planning process, which in turn result in three different ways of accounting for the shipping costs. Classical location models assume that shipping costs are paid by clients that receive the services. This interpretation will be called in the following the *client cost model*. The second point of view aims to introduce in the objective function the costs induced by the different workloads that occur at the distribution centers (see Zhou et al. 2002). Thus, under this interpretation we suppose that distribution centers pay the overall shipping costs (including workload induced costs). We call this situation the *supplier cost model*. The third point of view considers that a third party logistics provider is involved in the logistics system. Unlike the previous points of view, in this case the shipping costs are neither charged to the clients nor to the distribution centers. Instead, each transport relation is considered separately and has to be paid by the corresponding logistics provider. Moreover, each one of these costs may get discounts or "overcharges" (see Newman et al. 2005). We call this model *logistics provider cost model*.

Implementing the above differentiation on the shipping costs is naturally achieved using the ordered median objective function (see Nickel and Puerto 2005).

The rest of the paper is organized as follows: In Sect. 2, we introduce the model of the basic DOMP and illustrate the different solutions obtained for the ordered capacitated facility location problem. Afterwards, we provide mathematical formulations for the different points of view. Section 4 studies alternative formulations under the hypothesis that the modeling weights of the ordered median problem are given in non-decreasing order. In Sect. 5, we present a computational analysis to determine the limits of solving the problem with different formulations using standard MIP solvers. The paper ends with some conclusions.

2 The discrete ordered median problem

In this section, we introduce the basic discrete ordered median problem (DOMP). Let A denote the given set of M sites and identify these with the integers $1, \dots, M$, i.e.,

$A = \{1, \dots, M\}$. We assume, without loss of generality, that the set of candidate sites for new facilities is identical to the set of clients. Let $C = (c_{ij})_{i,j=1,\dots,M}$ be the given non-negative $M \times M$ cost matrix, where c_{ij} denotes the cost of satisfying the demand of client i from a facility located at site j. Let $N \le M$ be the number of facilities to be located. A solution to the location problem is given by a set of N sites; we use $X \subseteq A$ with $|X| = N$ to denote a solution. We assume that each client will be served by a facility located at a site which yields the cheapest cost of satisfying demand, i.e., given a solution X, we assume that each client i will be supplied from a site $j \in X$ such that

$$c_{ij} = c_i(X) := \min_{k \in X} c_{ik}.$$

The objective of the DOMP applies a linear cost, with coefficient $\lambda_i \ge 0$, to the ith cheapest cost of supplying a client, for each $i = 1, \dots, M$. So in order to calculate the objective, the costs of supplying clients, $c_1(X), \dots, c_M(X)$, must be sorted. We define σ_X to be a permutation on $\{1, \dots, M\}$ for which the inequalities

$$c_{\sigma_X(1)}(X) \le c_{\sigma_X(2)}(X) \le \cdots \le c_{\sigma_X(M)}(X)$$

hold. We call any such permutation a *valid permutation for* X. The Discrete Ordered Median Problem (DOMP) is defined as

$$\min_{X \subseteq A,\, |X| = N} \sum_{i=1}^{M} \lambda_i c_{\sigma_X(i)}(X).$$

Note that the linear representation of the DOMP is only pointwise defined, since $c_{\sigma_X(i)}(X)$ depends on X.

For different choices of λ we obtain different types of objective functions. To see that the DOMP objective generalizes well known location objectives, note that $\lambda = (1, 1, \dots, 1)$ makes the DOMP equivalent to the N-median problem; taking $\lambda = (0, 0, \dots, 0, 1)$ makes the DOMP equivalent to the N-center problem; taking $\lambda = (\alpha, \alpha, \dots, \alpha, 1)$ for $0 < \alpha < 1$ leads to the μ-centdian problem, a convex combination of the median and the center objective functions; and taking $\lambda = (0, \dots, 0, 1, \dots, 1)$, where the first $M - k$ entries are zero and the last k entries are one, leads to the k-centrum problem. Other objectives may also be of practical interest. One example is to take $\lambda = (0, \dots, 0, 1, \dots, 1, 0, \dots, 0)$, where the first k_1 and last k_2 entries are zero, the average of the "middle" part, the so-called $(k_1 + k_2)$-trimmed mean, which is a robust statistic, is minimized. Another example is to take $\lambda = (1, \dots, 1, 0, \dots, 0, 1, \dots, 1)$, where the first k_1 and the last k_2 entries are one: this leads to the problem of minimizing the sum of the k_1 smallest costs and the k_2 largest costs; the corresponding DOMP searches for a set of N facilities minimizing the average cost for the clients which are very close and very far away. A final example is to take $\lambda = (2, 0, \dots, 0, 1)$: this leads to the problem of minimizing the sum of the largest cost and the smallest cost (counted twice), with all other costs ignored. Clearly, classical location problems can easily be modeled under this common pattern. Moreover, new meaningful objective functions are easily derived, as shown above. An example presented by Nickel (2001) shows the great impact that the choice of the objective function has on the optimal location of the new facilities. Observe that the DOMP is NP-hard, as it is a generalization of the N-median problem (Kariv and Hakimi 1979).

Apart from the modeling aspect of the λ-weights, they also have neat economic interpretations. A possible interpretation of the correction factors λ_i could be the case of a company that wants to locate several factories to produce a specific item following a policy that

- Benefits the clients that support the largest transportation cost. Therefore, if the company wants to minimize the sum of the k-largest transportation costs of its clients, they can take $\lambda = (0, \ldots, 0, 1, \overset{k}{\ldots}, 1)$.
- Benefits the clients that support the major body of the transportation costs (excluding the smallest and largest ones which often distort the optimal solution, especially the latter ones). Therefore, if the company wants to minimize the trimmed mean transportation cost of its clients one can take $\lambda = (0, \overset{k_1}{\ldots}, 0, 1, \ldots, 1, 0, \overset{k_2}{\ldots}, 0)$.
- Reduces the maximal lead times for shipping products to the warehouses. Here, a suitable choice would be $\lambda = (0, \ldots, 0, 1)$.

Another interpretation could be the case of public services that are rather cheap for closest users and are also subsidized for remote ones. Therefore, $\wedge$-shaped (increasing–decreasing) correction factors would be suitable to model this kind of situation, e.g., $(0, 2, 4, 6, 5, 3, 1)$.

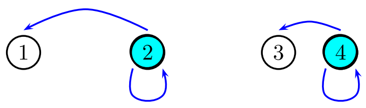
As described in the introduction, the goal of this paper is to develop a capacitated version with setup costs of the basic DOMP and introduce different models that depend on the different points of view described in the introduction. For these models, λ is the modeling vector of the shipping component of the cost, whereas $\mu = (\mu_1, \ldots, \mu_M) \geq 0$ is used to denote the modeling vector of the setup costs. Let a_k be the demand of client k, for all $k = 1, \ldots, M$ and b_j, f_j the capacity and the setup cost, respectively, of a supplier located at site j, for all $j = 1, \ldots, M$.

In the following example, we illustrate that the different points of view introduced above are not simply academic discussions. Indeed, even in these simple cases, results in quite different solutions.

Example 2.1 Let $A = \{1, 2, 3, 4\}$ be the set of clients located at $(0, 0)$, $(1, 0)$, $(3, 0)$, and $(3.5, 0)$ with the demand vector $a = (2.5, 1, 1.5, 2)$ and capacity vector $b = (3.5, 4.5, 4, 3.75)$. We assume that the transportation costs are proportional to distances but do not assume free self-service. That is, the transportation cost of supplying each site by itself is not zero and it is given by the vector $(0.75, 1.1, 0.77, 0.48)$. Thus, the unit transportation costs between sites are:

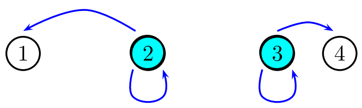
$$C = \begin{pmatrix} 0.75 & 1 & 3 & 3.5 \\ 1 & 1.1 & 2 & 2.5 \\ 3 & 2 & 0.77 & 0.5 \\ 3.5 & 2.5 & 0.5 & 0.48 \end{pmatrix}$$

and the setup cost vector $f = (2.5, 1.6, 2.3, 2.7)$. The goal is to compare the solutions obtained by the different models when we look for locating new supply facili-



Supplying scheme	Supplied amount
$2 \longrightarrow 1$	2.5
$2 \longrightarrow 2$	1
$4 \longrightarrow 3$	1.5
$4 \longrightarrow 4$	2

Fig. 1 Location and allocation pattern for the capacitated facility location problem



Supplying scheme	Supplied amount	λ assignment
$2 \longrightarrow 1$	2.5	λ_4
$2 \longrightarrow 2$	1	λ_3
$3 \longrightarrow 3$	1.5	λ_2
$3 \longrightarrow 4$	2	λ_1

Fig. 2 Location and allocation pattern for the client point of view

ties using the classical capacitated facility location approach ($\lambda = (1, \dots, 1)$) and the k-centrum objective ($\lambda = (0, \dots, 0, 1, \overset{k}{\dots}, 1)$) for the three different points of view with respect to the shipping costs. In all cases, we assume that the setup costs are discounted so that the most expensive facility is fully paid, and each new facility gets an accumulated 25% discount, i.e., $\mu = (0.25, 0.5, 0.75, 1)$. In the following, we present the different solutions obtained for these models:

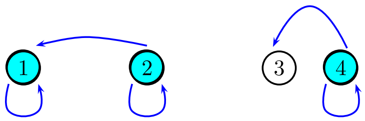
Capacitated facility location problem In an optimal solution the suppliers are located at the sites $\{2, 4\}$ with objective value 9.61 and a distribution pattern as shown in Fig. 1.

Client cost model For $\lambda = (0, 0, 1, 1)$ using the client cost model, it is optimal to locate new facilities at the sites $\{2, 3\}$ with objective value 7.155 and a distribution pattern as shown in Fig. 2.

As we consider the problem from the client point of view, we order the total transportation cost to supply each client (client cost). In this case, the transportation cost to supply client 1 is $2.5 \times 1 = 2.5$; client 2 is $1 \times 1.1 = 1.1$; client 3 is $1.5 \times 0.77 = 1.155$ and client 4 is $2 \times 0.5 = 1$. Thus, we assign λ_1 to the lowest client cost, that is, client 4; λ_2 is assigned to the second lowest client cost, i.e., client 2, and so on. Hence, the objective value is $0 \times 1 + 0 \times 1.1 + 1 \times 1.155 + 1 \times 2.5$, and the setup cost is $0.25 \times f_1 \times 0 + 0.5 \times f_4 \times 0 + 0.75 \times f_2 \times 1 + 1 \times f_3 \times 1$, this objective value is 7.155.

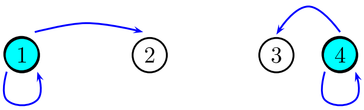
Supplier cost model For $\lambda = (0, 0, 1, 1)$, the optimal solution of this model requires to locate new facilities at the sites $\{1, 2, 4\}$ with objective value 8.63 and a distribution pattern as shown in Fig. 3.

As we consider the problem from the supplier point of view, we order the total transportation cost from each supplier (supplier cost). Note that there are three suppliers in this solution $\{1, 2, 4\}$. In this case, the transportation cost from supplier 1 is $2.06 \times 0.75 = 1.54$; supplier 2 is $0.44 \times 1 + 1 \times 1.1 = 1.54$ and sup-



Supplying scheme	Supplied amount	λ assignment
$1 \longrightarrow 1$	2.06	λ_2
$2 \longrightarrow 1$	0.44	λ_3
$2 \longrightarrow 2$	1	λ_3
$4 \longrightarrow 3$	1.5	λ_4
$4 \longrightarrow 4$	2	λ_4

Fig. 3 Location and allocation pattern for the supplier point of view.



Supplying scheme	Supplied amount	Supply cost	λ assignment
$1 \longrightarrow 1$	2.5	1.88	λ_{16}
$1 \longrightarrow 2$	1	1	λ_{15}
$4 \longrightarrow 3$	1	0.75	λ_{13}
$4 \longrightarrow 4$	2	0.96	λ_{14}

Fig. 4 Location and allocation pattern for the logistics provider point of view

plier 4 $1.5 \times 0.5 + 2 \times 0.48 = 1.71$. Hence, λ_1 is assigned to supplier 3, λ_2 to supplier 1, λ_3 to supplier 2 and λ_4 to supplier 4. Therefore, the objective value is $0 \times 0 + 0 \times 1.54 + 1 \times 1.54 + 1 \times 1.71 + 0.25 \times f_3 \times 0 + 0.5 \times f_2 + 0.75 \times f_1 + 1 \times f_4$, this value is 8.63.

Logistics provider cost model For $\lambda = (0, \dots, 0, 1, 1, 1, 1, 1, 1, 1)$, the optimal solution is to locate the suppliers at the sites $\{1, 4\}$ and a distribution pattern as shown in Fig. 4.

According to the values of λ and μ, the objective value is the sum of the seven largest transportation costs plus the discounted sum of the setup costs. Therefore, from the above table we obtain that the objective value is 9.16.

The above example shows that depending on the point of view used in the modeling phase we obtain different solutions that represent the different interests of the parties in a logistics problem.

3 The ordered capacitated facility location problem

In this section, we consider the ordered capacitated facility location problem (OCFL), an extension of the capacitated facility location problem (CFLP), see Drezner (1995). In this problem, the number of facilities to be located is not given in advance but it is part of the decision making process and, in addition, each supply facility has a setup cost to avoid free opening. This is an important difference to the capacitated DOMP where the number of new facilities was fixed a priori, see Kalcsics et al. (2008). The objective function of the OCFL takes into account the transportation and setup costs, where now both costs are sorted in non-decreasing order and multiplied by two vectors of correction factors λ and μ, respectively.

We are looking for a set of sites to locate supply facilities with enough capacity to cover the entire demand (we have assumed that the demands can be split). The goal is to minimize the λ-weighted ordered sum of the shipping costs to satisfy the demand of the clients and the μ-weighted ordered sum of the setup costs. The interpretation of the correction factors λ and μ is given in the introduction. Obviously, the combination of these two correction factors allows us to derive new decision policies.

In the following, we describe the different models. Each one of them uses its own set of variables while sharing the following family of variables:

$$y_{ij} = \begin{cases} 1 & \text{if a supplier is located at site } j \text{ and its setup cost } f_j \text{ is} \\ & \text{in the } i\text{th position of the setup cost vector,} \\ 0 & \text{otherwise.} \end{cases}$$

where $i, j = 1, \dots, M$.

3.1 Client cost model

In this section, we formulate the OCFL from a client point of view ($\mathsf{OCFL}_{\mathrm{CV}}$). The goal is to open a number of supply facilities with enough capacity to cover the demand so as to minimize the λ-weighted ordered sum of accumulated-per-client shipping costs plus the μ-weighted ordered sum of the setup costs. In order to formulate this model, we consider the following set of variables:

s_{ijk} = proportion of the demand a_k, covered by supplier j when the total transportation costs to cover the demand of client k are at the ith position of the client cost vector.

$$z_{ik} = \begin{cases} 1 & \text{if the overall transportation cost to cover client } k \text{ is at position } i, \\ 0 & \text{otherwise,} \end{cases}$$

for $i, j, k = 1, \dots, M$.

The client cost model, $\mathsf{OCFL}_{\mathrm{CV}}$, is now given as follows

$$(\mathsf{OCFL}_{\mathrm{CV}}) \quad \min \quad \sum_{i=1}^{M} \lambda_i \sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk}\, a_k\, c_{jk} + \sum_{i=1}^{M} \mu_i \sum_{j=1}^{M} y_{ij}\, f_j, \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{M} \sum_{j=1}^{M} s_{ijk} = 1, \quad \forall k = 1, \dots, M, \tag{2}$$

$$\sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk} = 1, \quad \forall i = 1, \dots, M, \tag{3}$$

$$\sum_{j=1}^{M} s_{ijk} = z_{ik}, \quad \forall i = 1, \ldots, M,\ k = 1, \ldots, M, \tag{4}$$

$$\sum_{j=1}^{M}\sum_{k=1}^{M} s_{ijk}\, a_k\, c_{jk} \le \sum_{j=1}^{M}\sum_{k=1}^{M} s_{i+1,jk}\, a_k c_{jk}, \quad \forall i = 1, \ldots, M-1, \tag{5}$$

$$\sum_{i=1}^{M}\sum_{k=1}^{M} s_{ijk}\, a_k \le b_j \sum_{i=1}^{M} y_{ij}, \quad \forall j = 1, \ldots, M, \tag{6}$$

$$\sum_{j=1}^{M} y_{ij}\, f_j \le \sum_{j=1}^{M} y_{i+1,j}\, f_j, \quad \forall i = 1, \ldots, M-1, \tag{7}$$

$$\sum_{i=1}^{M} y_{ij} \le 1, \quad \forall j = 1, \ldots, M, \tag{8}$$

$$\sum_{j=1}^{M} y_{ij} \le 1, \quad \forall i = 1, \ldots, M, \tag{9}$$

$$y_{ij} \in \{0, 1\}, \quad \forall i = 1, \ldots, M, j = 1, \ldots, M, \tag{10}$$

$$s_{ijk} \ge 0, \quad \forall i, j, k = 1, \ldots, M,$$

$$z_{ik} \in \{0, 1\}, \quad \forall i, k = 1, \ldots, M.$$

The objective function (1) is the λ-weighted ordered sum of the total shipping costs (the shipping cost for a client at position i is $\sum_{j=1}^{M}\sum_{k=1}^{M} s_{ijk}a_k c_{jk}$) plus the μ-weighted ordered sum of setup costs. Constraints (2) guarantee that the demand of all clients is covered. Constraints (3) and (4) ensure that total transportation cost to cover a client may be allocated to only one position in the ordered vector of total shipping costs. Constraints (5) guarantee the non-decreasing order of the entries of the shipping cost vector. Constraints (6) ensure that the total amount supplied by each server does not exceed its capacity. Constraints (7) guarantee the non-decreasing order of the entries of the setup cost vector. Constraints (8), (9) and (10) ensure that each setup cost may be allocated to at most one position in the vector of setup costs.

3.2 Supplier cost model

This section deals with the ordered capacitated facility location problem from the supplier point of view ($\mathsf{OCFL}_{\mathrm{SV}}$). As in the previous model, we are looking for a number of supply facilities with enough capacity to cover the entire demand of the clients. Unlike the previous model, we now consider the total transportation costs that accumulate at each supply facility (supplier cost). That is, the supplier cost of a facility is the sum of the transportation costs of shipments from that facility to the clients. The goal is to minimize the λ-weighted ordered sum of the supplier costs to deliver goods for covering the demand of all clients plus the μ-weighted ordered sum of the setup costs.

To formulate this model, we use the following sets of variables:

$s_{ijk} =$ proportion of the capacity of supplier j sent to customer k when the transportation cost of supplier j is the ith smallest value of the supplier cost vector.

$$z_{ij} = \begin{cases} 1 & \text{if the cost of supplier } j \text{ is in position } i, \\ 0 & \text{otherwise,} \end{cases}$$

for $i, j, k = 1, \ldots, M$.
Then, the model for the supplier point of view, OCFL_{SV}, is given as follows

$$(\mathsf{OCFL}_{SV}) \quad \min \quad \sum_{i=1}^{M} \lambda_i \sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk} b_j c_{jk} + \sum_{i=1}^{M} \mu_i \sum_{j=1}^{M} y_{ij} f_j, \tag{11}$$

$$\text{s.t.} \quad \sum_{i=1}^{M} \sum_{j=1}^{M} s_{ijk} b_j = a_k, \quad \forall k = 1, \ldots, M, \tag{12}$$

$$\sum_{k=1}^{M} s_{ijk} \le z_{ij}, \quad \forall i = 1, \ldots, M, \; j = 1, \ldots, M, \tag{13}$$

$$\sum_{j=1}^{M} z_{ij} \le 1, \quad \forall i = 1, \ldots, M, \tag{14}$$

$$\sum_{i=1}^{M} z_{ij} \le \sum_{i=1}^{M} y_{ij}, \quad \forall j = 1, \ldots, M, \tag{15}$$

$$\sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk} b_j c_{jk} \le \sum_{j=1}^{M} \sum_{k=1}^{M} s_{i+1,jk} b_j c_{jk}, \quad \forall i = 1, \ldots, M-1, \tag{16}$$

$$\sum_{j=1}^{M} y_{ij} f_j \le \sum_{j=1}^{M} y_{i+1,j} f_j, \quad \forall i = 1, \ldots, M-1,$$

$$\sum_{i=1}^{M} y_{ij} \le 1, \quad \forall j = 1, \ldots, M,$$

$$\sum_{j=1}^{M} y_{ij} \le 1, \quad \forall i = 1, \ldots, M,$$

$$y_{ij} \in \{0, 1\}, \quad \forall i = 1, \ldots, M, j = 1, \ldots, M,$$

$$s_{ijk} \geq 0, \quad \forall i, j, k = 1, \dots, M,$$
$$z_{ij} \in \{0, 1\}, \quad \forall i, j = 1, \dots, M.$$

The objective function (11) represents the λ-weighted ordered sum of total supplier costs (the supplier cost at position i is $\sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk} b_j c_{jk}$) plus the μ-weighted ordered sum of setup costs. Constraints (12) guarantee that the demand of all clients is covered. Constraints (13), (14), and (15) ensure that total transportation cost that accumulates at a supply facility may be allocated to at most one position in the vector of supplier costs. Constraints (16) guarantee the non-decreasing order of the entries in the cost vector. The remaining families of constraints have a similar interpretation as in the previous model.

3.3 Logistics provider cost model

The logistics provider cost model ($\mathsf{OCFL}_{\mathrm{LV}}$) accounts the overall cost of a certain solution by looking at all transportation links separately. The goal is to minimize the λ-weighted ordered sum of the transportation cost from each supplier to each client plus the μ-weighted ordered sum of setup costs. The λ-weights are correction factors that depend on the ordered sequence of the costs of the single transportation links. The reader may note that in this model $\lambda \in \mathbb{R}^{M^2}$ since we consider costs defined on each single transportation relation (link) and there are at most M^2 of these links.

To formulate the model, we use the following set of variables:

$s_{ijk} =$ proportion of demand a_k covered by supplier j when the transportation cost of that proportion of demand is the ith lowest value of the transportation links.

$$\delta_{ijk} = \begin{cases} 1 & \text{if the cost for supplying client } k \text{ from supplier } j \text{ is at position } i, \\ 0 & \text{otherwise,} \end{cases}$$

for $i = 1, \dots, M^2$, $j, k = 1, \dots, M$. The model for the logistics provider cost model, $\mathsf{OCFL}_{\mathrm{LV}}$, is given as follows

$$(\mathsf{OCFL}_{\mathrm{LV}}) \quad \min \quad \sum_{i=1}^{M^2} \lambda_i \sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk}\, a_k\, c_{jk} + \sum_{i=1}^{M} \mu_i \sum_{j=1}^{M} y_{ij}\, f_j, \tag{17}$$

$$\text{s.t.} \quad \sum_{i=1}^{M^2} \sum_{j=1}^{M} s_{ijk} = 1, \quad \forall k = 1, \dots, M, \tag{18}$$

$$s_{ijk} \leq \delta_{ijk}, \quad i = 1, \dots, M^2,\ j, k = 1, \dots, M, \tag{19}$$

$$\sum_{j=1}^{M} \sum_{k=1}^{M} \delta_{ijk} \leq 1, \quad i = 1, \dots, M^2, \tag{20}$$

$$\sum_{i=1}^{M^2} \delta_{ijk} \leq 1, \quad \forall j, k = 1, \dots, M, \tag{21}$$

$$\sum_{i=1}^{M^2} \delta_{ijk} \leq \sum_{i=1}^{M} y_{ij}, \quad \text{or} \quad \sum_{i=1}^{M^2} s_{ijk} \leq \sum_{i=1}^{M} y_{ij}, \quad \forall j, k = 1, \dots, M, \tag{22}$$

$$\sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk}\, a_k\, c_{jk} \leq \sum_{j=1}^{M} \sum_{k=1}^{M} s_{i+1,jk}\, a_k\, c_{jk}, \quad \forall i = 1, \dots, M^2 - 1, \tag{23}$$

$$\sum_{i=1}^{M^2} \sum_{k=1}^{M} s_{ijk}\, a_k \leq b_j \sum_{i=1}^{M} y_{ij}, \quad \forall j = 1, \dots, M,$$

$$\sum_{j=1}^{M} y_{ij}\, f_j \leq \sum_{j=1}^{M} y_{i+1,j}\, f_j, \quad \forall i = 1, \dots, M-1,$$

$$\sum_{i=1}^{M} y_{ij} \leq 1, \quad \forall j = 1, \dots, M,$$

$$\sum_{j=1}^{M} y_{ij} \leq 1, \quad \forall i = 1, \dots, M,$$

$$y_{ij} \in \{0, 1\}, \quad \forall i, j = 1, \dots, M,$$

$$s_{ijk} \geq 0, \quad \forall i = 1, \dots, M^2, \ \forall j, k = 1, \dots, M,$$

$$\delta_{ijk} \in \{0, 1\}, \quad i = 1, \dots, M^2, \ j, k = 1, \dots, M. \tag{24}$$

The objective function (17) represents the λ-weighted ordered sum of the transportation cost from each supplier to each client (the transportation cost from a supplier to a client at the ith position is $\sum_{j=1}^{M} \sum_{k=1}^{M} s_{ijk} a_k c_{jk}$) plus the μ-weighted ordered sum of setup costs. Constraints (18) guarantee that the demand of all clients is covered. Constraints (19), (20), (22), and (24) ensure that we can assign to a position i the transportation cost of at most one transportation link with origin at an open facility. In turn, constraints (21) guarantee that the cost of supplying a client from a supplier can appear in at most one position in the cost vector. Constraints (23) ensure the non-decreasing order of the entries of the sorted cost vector. The remaining families of constraints have a similar interpretation that in the previous model.

4 Some special models

In this section, we provide improved formulations for some particularly important cases where the modeling weights are in non-decreasing sequence. The reader may note that most classical models in the location analysis literature fall into this category, namely the median, center, centdian, k-centrum, etc. The rationale behind these formulations is based on the linear programming formulation that computes the minimum of the sum of the k-largest elements out of a set of n (see Ogryczak and Śliwiński 2003 and Ogryczak and Tamir 2003).

Again, we are looking for a set of sites to locate supply facilities with enough capacity to cover the entire demand. The goal is to minimize the λ-weighted ordered sum of the shipping costs to satisfy the demand of the clients and the μ-weighted ordered sum of the setup costs. Following the scheme of the previous sections, we propose different formulations that depend on the different points of view considered in the planning process. We will set $\lambda_0 = 0$ to simplify the notation.

We will now introduce improved formulations for the capacitated ordered median problem with setup cost (λ- and μ-weights given in non-decreasing sequence).

4.1 Client cost model

To state the improved model for the client point of view, $\mathsf{OCFL}^{\leq}_{\mathrm{CV}}$, we first define the following variables:

$$s_{jk} = \text{proportion of the demand of } a_k \text{ satisfied by supplier } j,$$

for $j, k = 1, \ldots, M$.
Then, the formulation is as follows:

$$(\mathsf{OCFL}^{\leq}_{\mathrm{CV}}) \quad \min \quad \sum_{i=1}^{M} (\lambda_{M-i+1} - \lambda_{M-i}) \left(i \cdot t_i + \sum_{k=1}^{M} d_{ki} \right) + \sum_{i=1}^{M} (\mu_{M-i+1} - \mu_{M-i}) \left(i \cdot r_i + \sum_{j=1}^{M} e_{ij} \right), \tag{25}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} s_{jk} = 1, \quad \forall k = 1, \ldots, M, \tag{26}$$

$$\sum_{k=1}^{M} s_{jk}\, a_k \leq b_j\, y_j, \quad \forall j = 1, \ldots, M, \tag{27}$$

$$d_{ki} \geq a_k \sum_{j=1}^{M} s_{jk}\, c_{jk} - t_i, \quad \forall i, k = 1, \ldots, M, \tag{28}$$

$$s_{jk} \geq 0, \quad \forall j, k = 1, \ldots, M, \tag{29}$$

$$d_{ki} \geq 0, \quad \forall i, k = 1, \ldots, M, \tag{30}$$

$$t_i \in \mathbb{R}, \quad \forall i = 1, \ldots, M, \tag{31}$$

$$e_{ij} \geq f_j\, y_j - r_i, \quad \forall i, j = 1, \ldots, M, \tag{32}$$

$$e_{ij} \geq 0, \quad \forall i, k = 1, \ldots, M, \tag{33}$$

$$r_i \in \mathbb{R}, \quad \forall i = 1, \ldots, M, \tag{34}$$

$$y_j \in \{0, 1\}, \quad \forall j = 1, \ldots, M. \tag{35}$$

According to Ogryczak and Tamir (2003), variables d_{ki} and t_i, together with the first part of objective function (25) and the constraints (26)–(31) are necessary to model the λ-weighted ordered sum of transportation costs ($\sum_{j=1}^{M} a_k s_{jk} c_{jk}$ for $k = 1, \ldots, M$). Moreover, variables e_{ij} and r_i with the second part of objective function (25) and the constraints (32), (33) and (34) are necessary to model the μ-weighted ordered sum of the setup costs ($f_j y_j$ for $j = 1, \ldots, M$). Therefore, the objective function (25) gives the λ-weighted ordered sum of the shipping costs plus the μ-weighted ordered sum of setup costs and altogether with constraints (26)–(35) model the improved version of the ordered capacitated client cost model.

4.2 Supplier cost model

To give the improved model for the supplier point of view, $\mathsf{OCFL}^{\leq}_{\mathrm{SV}}$, we first define the following variables:

$$s_{jk} = \text{proportion of the capacity of supplier } j \text{ sent to customer } k$$

for $j, k = 1, \ldots, M$.
Then, the formulation is the following:

$$(\mathsf{OCFL}^{\leq}_{\mathrm{SV}}) \quad \min \quad \sum_{i=1}^{M} (\lambda_{M-i+1} - \lambda_{M-i}) \left(i \cdot t_i + \sum_{j=1}^{M} d_{ji} \right) + \sum_{i=1}^{M} (\mu_{M-i+1} - \mu_{M-i}) \left(i \cdot r_i + \sum_{j=1}^{M} e_{ij} \right), \tag{36}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} s_{jk} b_j = a_k, \quad \forall k = 1, \ldots, M, \tag{37}$$

$$\sum_{k=1}^{M} s_{jk} \leq y_j, \quad \forall j = 1, \ldots, M, \tag{38}$$

$$d_{ji} \geq \sum_{k=1}^{M} s_{jk} b_j c_{jk} - t_i, \quad \forall i = 1, \ldots, M, \ j = 1, \ldots, M, \tag{39}$$

$$s_{jk} \geq 0, \quad \forall j = 1, \ldots, M, \ k = 1, \ldots, M, \tag{40}$$

$$d_{ki} \geq 0, \quad \forall i = 1, \ldots, M, \ k = 1, \ldots, M, \tag{41}$$

$$t_i \in \mathbb{R}, \quad \forall i = 1, \ldots, M, \tag{42}$$

$$e_{ij} \geq f_j y_j - r_i, \quad \forall i, j = 1, \ldots, M, \tag{43}$$

$$e_{ij} \geq 0, \quad \forall i, j = 1, \ldots, M, \tag{44}$$

$$r_i \in \mathbb{R}, \quad \forall i = 1, \ldots, M, \tag{45}$$

$$y_j \in \{0, 1\}, \quad \forall j = 1, \ldots, M. \tag{46}$$

The objective function (36) with constraints (39)–(45) gives the representation of the λ-weighted ordered sum of the elements of $\sum_{k=1}^{M} s_{jk}\, b_j\, c_{jk}$ for $j = 1, \ldots, M$, plus the μ-weighted ordered sum of the elements $f_j\, y_j$ for $j = 1, \ldots, M$. Constraints (37), (38) and (46) ensure that $\sum_{k=1}^{M} s_{jk}\, b_j\, c_{jk}$ for $j = 1, \ldots, M$, are the actual shipping costs and that capacity and demand requirements are satisfied.

4.3 Logistics provider cost model

To give the improved model for the logistics provider point of view, $\mathsf{OCFL}_{\mathrm{LV}}^{\leq}$, we first define the following variables:

$$s_{jk} = \text{proportion of the demand of } a_k \text{ covered by the supplier } j,$$

for $j, k = 1, \ldots, M$.
Then, the formulation is the following:

$$\begin{aligned}
(\mathsf{OCFL}_{\mathrm{LV}}^{\leq}) \quad \min \quad & \sum_{i=1}^{M^2} (\lambda_{M^2-i+1} - \lambda_{M^2-i}) \left(i \cdot t_i + \sum_{j=1}^{M} \sum_{k=1}^{M} d_{jki} \right) \\
& + \sum_{i=1}^{M} (\mu_{M-i+1} - \mu_{M-i}) \left(i \cdot r_i + \sum_{j=1}^{M} e_{ij} \right), \\
\text{s.t.} \quad & \sum_{j=1}^{M} s_{jk} = 1, \quad \forall k = 1, \ldots, M, \\
& \sum_{k=1}^{M} s_{jk}\, a_k \leq b_j\, y_j, \quad \forall j = 1, \ldots, M, \\
& d_{jki} \geq a_k\, s_{jk}\, c_{jk} - t_i, \quad \forall j, k = 1, \ldots, M,\ i = 1, \ldots, M^2, \qquad (47) \\
& s_{jk} \geq 0, \quad \forall j, k = 1, \ldots, M, \\
& d_{jki} \geq 0, \quad \forall j, k = 1, \ldots, M,\ i = 1, \ldots, M^2, \\
& t_i \in \mathbb{R}, \quad \forall i = 1, \ldots, M^2, \\
& e_{ij} \geq f_j\, y_j - r_i, \quad \forall i, j = 1, \ldots, M, \\
& e_{ij} \geq 0, \quad \forall i, j = 1, \ldots, M, \\
& r_i \in \mathbb{R}, \quad \forall i = 1, \ldots, M, \\
& y_j \in \{0, 1\}, \quad \forall j = 1, \ldots, M.
\end{aligned}$$

The objective function and constraints of this formulation are similar to the ones given in the two previous models. The only exception is that the first part of the objective function with constraints (47) models the λ-weighted ordered sum of the amounts $a_k s_{jk} c_{jk}$ for $j, k = 1, \dots, M$ which by the remaining constraints are the actual shipping costs of each transportation link in the system.

5 Computational results

In the following, we report some computational results for the proposed models to test the potential and limits of the different formulations when solving them with commercial solvers. For the tests, we randomly generated problem instances of varying size. The experiment was designed with the following parameters:

- Number of clients: Depending on the model, the values for M range from 6 to 70.
- Location of clients: The coordinates of the clients are uniformly distributed in the square $[0, 10] \times [0, 10]$.
- Demand of clients: The demands are uniformly distributed in the interval $[10, 20]$.
- Transportation cost: The costs are computed as $c_{ij} = r_i d(i, j)$ where r_i is a uniform random variable in the interval $[1, 5]$ (modeling the transportation cost rate) and $d(\cdot, \cdot)$ is the Euclidean distance.
- Capacity of suppliers: For each problem instance, we randomly choose a number N in the interval $[0.25M, 0.75M]$. The capacities of the facilities are then uniformly distributed in the interval $[1.1\frac{\sum_{i=1}^{M} a_i}{N}, 1.3\frac{\sum_{i=1}^{M} a_i}{N}]$.
- Setup costs of suppliers: The costs are chosen such that the number of established facilities in a solution is on average $\lceil M/b \rceil$.
- Modeling vector λ: We consider five different types of vectors
  L1 (*Median*): λ corresponding to the median problem, i.e., $\lambda = (1, \dots, 1)$.
  L2 (*Center*): λ corresponding to the center problem, i.e., $\lambda = (0, \dots, 0, 1)$.
  L3 (*kCentrum*): λ corresponding to the k-centrum model, i.e., $\lambda = (0, \dots, 0, 1, \overset{k}{\dots}, 1)$ where $k = \lceil 0.4M \rceil$.
  L4 (*TrimmedMean*): λ corresponding to the (k_1, k_2)-trimmed mean problem, i.e., $\lambda = (0, \overset{k_1}{\dots}, 0, 1, \dots, 1, 0, \overset{k_2}{\dots}, 0)$ where $k_1 = k_2 = \lceil 0.2M \rceil$.
  L5 ($\wedge$-*shaped*): λ corresponding to the $\wedge$-shaped problem with values $(0, 2, 4, \dots, M-3, M-1, M-2, M-4, \dots, 3, 1)$ if M is odd, and $(0, 2, 4, \dots, M-2, M-1, M-3, \dots, 3, 1)$ otherwise.
- Modeling vector μ: We consider two different types of vectors
  M1 (*Median*): μ corresponding to the median problem, i.e., $\mu = (1, \dots, 1)$.
  M2 (*Discount setup*): μ corresponding to a vector with increasing values, starting from 0.6 up to 1, i.e., $\mu = (0.6, \dots, 1)$.

For each number of clients M we randomly generated 10 instances. We solved the instances using CPLEX 9.1 where the maximal running time was limited to 2 hours. In what follows, we report the average and maximum running times in seconds for each model and the average number of new facilities in a solution.

In Table 1, we present results for the modeling vectors M1 (*Median*) and M2 (*Discounted*) for the setup costs. In the tables, 'M' denotes the number of clients and new

Table 1 Running times for all three points of view for the OCFL for M1 (upper table) and M2

	M	*Median*			*Center*			*kCentrum*			*Trimmed Mean*			*∧-shaped*		
		N	Avg	Max	N	Avg	Max	N	Avg	Max	N	Avg	Max	N	Avg	Max
Client	6	3.6	0.3	0.6	4.7	7.6	11.1	3.2	5.3	11.5	3.3	0.5	0.6	2.2	1.0	1.3
	7	3.2	0.7	1.1	3.3	101.2	226.7	2.7	44.4	79.2	2.7	4.4	7.9	2.8	2.7	4.8
	8	3.4	3.1	5.8	2.8	539.9	1114.0	3.0	259.1	562.8	2.7	20.7	42.0	2.8	12.5	32.8
	9	4.1	5.0	7.4	3.7	6072.1	7142.8	3.5	2705.1	7048.0	3.5	71.8	165.7	4.2	35.6	53.4
	10	4.3	24.0	63.9	3.3	7143.5	7156.7	3.8	5963.6	7172.5	3.4	304.5	537.2	4.6	111.2	153.1
Supplier	6	3.3	0.3	0.5	3.0	4.9	7.7	3.1	5.7	10.7	2.9	0.3	0.6	2.0	0.3	0.4
	7	3.0	0.6	1.3	3.1	18.4	27.7	3.2	20.6	50.3	2.8	0.7	1.4	2.2	0.4	0.6
	8	3.2	1.5	2.2	3.0	166.5	392.5	3.7	274.1	629.7	2.7	2.4	3.7	2.0	0.8	1.3
	9	3.9	4.4	6.8	3.7	443.7	789.9	4.3	1584.5	4683.7	4.1	15.7	73.0	3.2	3.7	6.1
	10	4.1	11.8	34.5	3.8	2223.0	7154.5	4.8	6153.7	7192.2	3.0	8.8	17.8	3.4	4.6	9.2
Logistics	6	2.2	2.6	4.2	2.0	7103.2	7145.9	2.0	7100.1	7140.8	2.0	7106.4	7144.3	2.2	16.2	31.4

	M	*Median*			*Center*			*kCentrum*			*Trimmed Mean*			*∧-shaped*		
		N	Avg	Max	N	Avg	Max	N	Avg	Max	N	Avg	Max	N	Avg	Max
Client	6	3.6	0.3	0.6	5.5	5.5	12.8	4.3	3.9	5.7	3.8	0.6	1.2	2.4	1.1	2.9
	7	3.5	1.0	1.3	4.5	96.6	211.0	3.0	49.8	129.9	2.8	3.4	5.3	2.8	2.7	5.6
	8	3.6	3.5	8.9	3.2	830.9	2374.1	3.0	406.2	860.8	3.0	20.5	38.8	2.6	12.1	25.5
	9	4.7	9.6	14.3	3.7	6568.7	7145.2	3.6	4045.8	7134.3	4.1	47.1	129.6	4.2	35.0	61.0
	10	5.0	31.1	62.6	4.2	7124.1	7158.5	4.8	6269.0	7139.8	3.6	235.8	651.5	4.8	190.0	498.6
Supplier	6	4.0	0.3	1.0	3.1	5.3	11.9	3.4	6.5	14.4	2.9	0.3	0.7	2.0	0.3	0.3
	7	3.4	0.8	1.3	3.2	23.4	37.3	3.6	33.7	72.8	2.8	0.7	1.8	2.2	0.4	0.6
	8	3.5	1.9	3.2	3.4	206.6	416.0	3.9	510.0	879.9	2.8	2.0	7.4	2.0	0.9	1.2
	9	4.3	7.0	15.9	3.9	990.7	2404.7	4.7	3661.0	7188.7	4.5	5.0	10.6	3.2	5.4	8.5
	10	4.6	13.9	61.0	4.0	2855.7	7103.4	5.3	5881.3	7192.6	3.0	9.8	20.3	3.4	7.5	11.6
Logistics	6	2.6	3.3	6.8	2.0	7115.3	7149.6	2.0	7111.9	7149.6	2.5	7128.7	7147.1	2.2	16.1	21.3

Table 2 Running times for all three points of view for the OCFL with modeling vector M1

	M	*Median*			*Center*			*kCentrum*		
View		N	Avg	Max	N	Avg	Max	N	Avg	Max
Client	30	6.1	0.3	0.7	6.0	13.0	50.7	6.0	11.2	35.5
	40	8.0	0.7	1.9	8.0	272.4	1227.4	8.0	51.7	133.0
	50	9.9	2.6	11.8	9.7	2339.3	7021.8	9.9	870.4	2407.9
	60	11.8	9.9	68.2	11.0	6930.1	7190.3	11.3	2497.6	7110.8
Supplier	10	3.6	0.0	0.0	4.0	0.2	0.3	3.3	0.1	0.2
	20	4.4	0.1	0.1	5.1	23.2	33.6	4.3	9.3	26.3
	30	5.3	0.2	0.3	5.2	359.3	756.8	5.0	158.4	461.3
	40	7.0	0.5	0.8	7.0	2673.6	6053.7	6.0	5375.0	7183.6
Logistics	6	2.2	2.8	3.9	2.0	7141.6	7188.3	2.0	7102.2	7168.1

facilities, 'N' denotes the average number of new facilities in a solution (over all instances of a given size), and 'Avg' and 'Max' the average and, respectively, maximal running times in seconds over all instances of a given size. Note that a maximal running time of close to 7200 seconds means that at least one of the instances could not be solved optimally within two hours; and with respect to the average running times, this means that (almost) none of the instances could be solved to optimality.

For the client point of view, the running times for the center and k-centrum problem are much higher than for the other three problem types. If we now compare the results for the client, supplier, and logistics provider models, we see that the approaches are not just different from a modeling point of view, but also from the computational side. The $\mathsf{OCFL}_{\mathrm{LV}}$ is the by far most difficult model. Here, already for the smallest problem size with six clients none of the instances of the center, k-centrum and trimmed mean could be solved to optimality (within two hours). In contrast to that, for the client and supplier point of view we could solve instances to optimality that are almost twice as large compared to $\mathsf{OCFL}_{\mathrm{LV}}$. Comparing the two different modeling vectors M1 and M2 for the setup costs, the second (M2) yields slightly more difficult models.

5.1 Improved formulations

In Sect. 4, we introduced alternative formulations for the OCFL for modeling vectors λ and μ that consist of non-decreasing values. The following results show that these models are much more efficient than the general ones, considerably increasing the size of the instances which can be solved to optimality (within two hours).

As we observe in Tables 2 and 3, the tractable problem sizes increase if we use the improved formulations. For M1, i.e., the median-type μ-modeling vector, we could optimally solve instances up to $M = 60$ for the client point of view and up to $M = 40$ for the supplier cost model. Compared to the general formulations, the tractable problem sizes increase by the factor six and four, respectively. However, the alternative formulations yield no improvement for the logistics provider point of view.

Table 3 Running times for all three points of view for the OCFL with modeling vector M2

View	M	*Median*			*Center*			*kCentrum*		
		N	Avg	Max	N	Avg	Max	N	Avg	Max
Client	20	4.2	0.7	0.9	4.0	10.2	18.3	4.0	5.5	8.0
	30	5.0	7.1	15.6	5.0	973.1	1524.0	5.0	159.7	340.1
	40	7.0	76.5	104.9	6.4	5920.0	7168.1	7.0	4722.7	6576.9
	50	8.6	614.2	1907.4	8.0	6366.3	7169.0	8.0	7154.4	7168.7
Supplier	10	3.7	0.1	0.1	4.0	0.5	0.7	3.3	0.4	0.6
	20	4.2	1.0	2.5	4.6	156.8	247.7	4.3	82.3	224.2
	30	5.0	6.3	12.4	5.0	4455.8	6743.5	5.0	3313.3	7168.9
	40	7.0	59.9	93.0	7.0	7167.7	7169.6	7.0	7165.2	7169.5
Logistics	6	2.4	2.2	2.7	2.0	7173.1	7182.3	2.0	7176.0	7187.4

For the μ-modeling vector M2 we obtain almost the same picture. The only exception is that the tractable problem sizes for the client cost model reduce to 50 clients. Moreover, as for the general formulations, using M2 leads to slightly more difficult problems.

6 Concluding remarks

This paper can be considered as a first building block in the analysis of capacitated strategic location problems with order requirements. Here by strategic we mean that the number of new facilities to be located is not fixed a priori but it is chosen by the model through setup costs. We have embedded this model within the framework of discrete ordered objective functions which in turn allows us to introduce different points of view to analyze the problem depending on the driving force in the logistics network. We have introduced different formulations for each one of these points of view and reported preliminary computational results showing the potential and limits of these formulations.

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