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An extension to rapid transit network design problem

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Abstract The rapid transit network design problem consists of the location of train alignments and stations, in a context where the demand makes its own decisions about the mode and route. The originality of this study is to incorporate in the model the line locations constraints with a bounded but variable number of lines, and lines with no predetermined origins and destinations. The computational experiments show the necessity of this extension to solve large networks, principally because of its computational advantage.

Keywords Underground train station and alignment location · Rapid transit network design

Mathematics Subject Classification (2000) 90B06 · 90C10 · 90C35

1 Introduction

Increasing mobility, longer journeys caused by the growth of cities and the traffic problems in city centers are some of the reasons why during the last few years new lines of rail transit systems (metro, light rail, etc.) have been constructed in some agglomerations, while in others existing lines and networks have been expanded. Due to the very high cost of constructing transit systems, it is important to pay close attention to their effectiveness in solving the urban traffic problem. A crucial part of the

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planning process is the underlying network design, which consists of two intertwined problems: determination of alignments and location of stations.

The above design decisions are considered at upper level and the user traffic behavior at lower level. At upper level the maximum coverage of demand using public network is the main aspect, taking line and budget constraints into account. At lower level the traffic demand decisions are taken into account in the transit network design alternatives, considering the traffic cost in private and public modes, based on the system offer (the network constructed) and, in general, the assumptions assumed in the modal traffic costs. The way of selecting and comparing these alternatives may be performed by considering that the demand chooses the path and mode.

The main efforts in this line of research have been aimed at determining a single alignment and the location of stations given it. Bruno et al. (2002) maximize the coverage of the demand by public network. Bruno et al. (1998) and Laporte et al. (2005) incorporate data of origin-destination matrix. The papers of Laporte et al. (2002) and Hamacher et al. (2001) deal with the problem of locating stations on a given alignment. García and Marín (2001, 2002) study the mode interchange and parking network design problems using Bilevel Programming. They consider the multimodal traffic assignment problem with combined modes at the lower level.

The reference of Laporte et al. (2006) extends the previous models by incorporating the station location problem, the alternative of several lines and defining the model using the maximum coverage of the public demand as an objective function and the budget constraints as side constraints. The contribution of this paper may be considered an extension of the above paper, where the lines are not initially given and they do not have fixed origins and destinations.

The paper is organized as follows. In the next section the previous network design model is described. Section 3 is dedicated to defining the proposed model extension. In Sect. 4 the computational experiments are studied, and in Sect. 5 the conclusions and further research are presented.

2 Rapid transit network design

The rapid transit network design (RTND) problem, described in Laporte et al. (2006), locates the stations that must belong to some of the lines of the network, and the authors study the problem of connecting them with a number of lines $L = \{l = 1, \dots, |L|\}$, each with origin o_l and destination d_l given, in competition with the private mode.

2.1 Data and notation

1. For key stations the set of potential locations is $N = \{i = 1, \dots, I\}$. From that it is defined the set E of feasible edges linking the key stations N. Therefore, we have an undirected graph $G(N, E)$ from which the rapid transit network is to be selected. For each node $i \in N$, we denote by $N(i)$ the set of nodes adjacent to i: $N(i) = \{j \in N : (i, j) \in E\} \cup \{j \in N : (j, i) \in E\}$.

2. The network is an expression of the location possibilities of the RTND. The set of possible links is a subset defined by $\{(i, j) : i < j, i, j \in N\}$ of the set of all bidirectional links. Note that (i, j) and (j, i) are identical because the links are assumed to be undirected edges.
3. The demand is given by the origin/destination pairs of nodes. The demand at pair w is given by the matrix: $g = (g_w)$, $\forall w = (p, q) \in W$, where W is the set of pair of demands. The demand is assumed known and of value 1, given that the model is of the incapacitated location type. The demand origin and destination nodes are taken from N.
4. The matrix $d = (d_{ij})$ of distances between pairs of nodes will be used to define the public cost. Note the values of matrix d could correspond to almost Euclidean distances because the system is designed to be underground.
5. In the service side, let c_{ij} and c_i be the costs of constructing an edge ij and a station at node i. These costs will depend on the available budget and other constraints: for the total cost of each line l the capacity values are $c^l_{\min}$, $c^l_{\max}$, and depending on the total network capacity are $c_{\min}$, $c_{\max}$.
6. The demand of the pair w takes decisions considering the generalized cost of satisfying it through the private and the public networks, which are u_w^{pri} and u_w^{pub}, respectively. Note that the later cost depends on the final topology of the public network and, therefore, on the edges that are selected, meanwhile u_w^{pri} are input data.

The variables are defined as follows:

$y_i^l = 1$, if line l is defined using the node i; $= 0$, otherwise.
$x_{ij}^l = 1$, if line l is defined using the edge ij; $= 0$, otherwise.
$f_{ij}^w = 1$, if the demand of the pair w uses edge ij; $= 0$, otherwise.
$p_w = 1$, if the pair w uses the public mode; $= 0$, otherwise.

Let us note that for the purpose of precisely defining variables f_{ij}^w edges can be substituted by two arcs, one in each direction. However, since the usual problem is aimed at selecting a set of nondirected links (given by nondirected variables x_{ij}^l) forming lines, and in order to avoid extra notation and constraints, nondirected notation will be maintained.

Thus, the RTND model can be stated in the following terms:

– Objective function:
  - Maximize the public trip covering $z_{\text{pub}} = \sum_{w \in W} g_w p_w$,
  - Minimize routing cost upper bound: $\bar{z}_r = \sum_{w \in W} (u^w_{\text{pri}}(1 - p_w) + u^w_{\text{pub}})$.

  The public trip covering is the main component of the objective function, but the routing cost upper bound term may be included for the demand routing be of minimum cost. The RTND maximizes the objective function defined by: $z = -\eta z_{\text{pub}} + (1 - \eta)\bar{z}_r$, where η is typically a number close to 1.
– Construction cost constraints: The line construction cost z_c^l and the total construction cost z_c are bounded:

$$z_c^l = \sum_{(i,j) \in E; i<j} c_{ij} x_{ij}^l + \sum_{i \in N} c_i y_i^l \in \left[c^l_{\min}, c^l_{\max}\right], \quad \forall l \in L; \tag{1}$$

$$z_c = \sum_{l\in L}\left(\sum_{(i,j)\in E; i<j} c_{ij}x^l_{ij} + \sum_{i\in N} c_i y^l_i\right) \in [c_{\min}, c_{\max}]. \quad (2)$$

– Line location constraints:

$$\sum_{j\in N(o_l)} x^l_{o_l j} = 1, \quad \forall l \in L, \quad (3)$$

$$\sum_{i\in N(d_l)} x^l_{i d_l} = 1, \quad \forall l \in L, \quad (4)$$

$$x^l_{ij} = x^l_{ji}, \quad \forall (i,j) \in E, i < j, \ \forall l \in L, \quad (5)$$

$$\sum_{j\in N(i)} x^l_{ij} = 2y^l_i, \quad \forall i \in N\backslash\{o_l, d_l\}, \ \forall l \in L, \quad (6)$$

$$y^l_{o_l} = y^l_{d_l} = 1, \quad \forall l \in L, \quad (7)$$

$$\sum_{i\in B}\sum_{j\in N} x^l_{ij} \le |B| - 1, \quad \forall B \subset N, |B| \ge 2, \ \forall l \in L. \quad (8)$$

– Routing demand constraints

$$\sum_{i\in N(k)} f^w_{ik} - \sum_{j\in N(k)} f^w_{kj} = \left\{\begin{array}{ll} -1, & \text{if } k = p : (p,q) = w \in W \\ 1, & \text{if } k = q : (p,q) = w \in W \\ 0, & \text{otherwise} \end{array}\right\},$$
$$\forall k \in N, \ \forall w \in W. \quad (9)$$

– Splitting demand constraints:

$$u^{\text{pub}}_w - \mu u^{\text{pri}}_w - M(1 - p_w) \le 0, \quad \forall w \in W, \quad (10)$$

where u^{pri}_w is data, μ is a congestion factor, $u^{\text{pub}}_w = \sum_{(i,j)\in E} d_{ij} f^w_{ij}$, and M is a big enough number.

– Location–allocation constraints

$$f^w_{ij} + p_w - 1 \le \sum_{l\in L} x^l_{ij}, \quad \forall (i,j) \in E, \ \forall w \in W. \quad (11)$$

Constraints (1) and (2) require lower and upper bounds on the cost of each line and on the overall network, respectively. Constraints (3) and (4) guarantee that each line starts and ends at its specified origin and destination. Constraints (5) mean that the edges are bidirectional. Constraints (6) require that each line has a path between the corresponding origin and destination. Constraints (7) ensure that all origins and destinations of the lines are constructed. Constraints (8) avoid the cycles. These last constraints are not explicitly considered but they are added when some cycle solution is obtained. Constraints (9) are the demand conservation at each node. Constraints (10) force demands to be assigned to the rapid transit mode if the associated public cost of using this network is less than or equal to the corresponding cost of the private mode, and the opposite. Constraints (11) guarantee that a demand is routed on an edge only if this edge belongs to the rapid transit network.

3 Extended rapid transit network design

The extension of RTND (ERTND) studies the following two topics:

- The number of lines is variable within a given bound.
- The lines do not have predetermined origins and destinations, all the network nodes are available for it.

Therefore, with the new formulation to obtain the optimum solution is not necessary to solve each of the RTND, which may be defined in terms of the possibilities of origins and destinations for each line, and of the number of line possibilities: one, two, up to $|L|$ lines.

In the ERTND the number of lines is a variable but the total number of lines is an upper fixed bound, the parameters o_l, d_l, $\forall l \in L$ are also variables that are not explicitly considered. ERTND is a new optimization model with a greater degree of freedom: the number of lines and their origin destination are variables within the bounds.

To define these changes, the line location constraints (3) to (8) in RTND are changed by the following location constraints:

$$x_{ij}^l \leq y_i^l, \quad \forall (i,j) \in E, i < j, \forall l \in L, \tag{12}$$

$$x_{ij}^l \leq y_j^l, \quad \forall (i,j) \in E, i < j, \forall l \in L, \tag{13}$$

$$x_{ij}^l = x_{ji}^l, \quad \forall (i,j) \in E, i < j, \forall l \in L, \tag{14}$$

$$\sum_{j \in N(i)} x_{ij}^l \leq 2, \quad \forall i \in N, \forall l \in L, \tag{15}$$

$$1 + \sum_{(i,j) \in E, i<j} x_{ij}^l = \sum_{i \in N} y_i^l, \quad \forall l \in L, \tag{16}$$

$$\sum_{i \in B} \sum_{j \in N} x_{ij}^l \leq |B| - 1, \quad \forall B \subset N, |B| \geq 2, \forall l \in L. \tag{17}$$

The constraints (12) and (13) ensure that the links are not located if their origin and destination nodes are not previously located. The constraints (14) change the undirected edge location variables to directed ones. The constraints (15) require that each node does not have more than two associated "edges". Constraints (16) require that the number of edges is one less than the number of nodes located at each line. The constraints (17) are the previous ones that require that the lines do not make cycles.

The above location constraints (16) need to be modified because they require that at least a link in each line must be located. To avoid this possibility, a new binary variable h is introduced to determine if there exists a link located for each line, so it must be verified that

$$h_l = \begin{cases} 1, & \text{if } \sum_{(i,j)\in E, i<j} x_{ij}^l \neq 0, \\ 0, & \text{if } \sum_{(i,j)\in E, i<j} x_{ij}^l = 0. \end{cases} \tag{18}$$

The constraints (16) and (18) are reformulated in the following:

$$h_l + \sum_{(i,j)\in E, i<j} x_{ij}^l = \sum_{i\in N} y_i^l, \quad \forall l \in L. \tag{19}$$

They are complemented by the following:

$$\begin{aligned} &\frac{1}{2} - \sum_{(i,j)\in E, i<j} x_{ij}^l + M h_l \geq 0, \quad \forall l \in L, \\ &\frac{1}{2} - \sum_{(i,j)\in E, i<j} x_{ij}^l + M(h_l - 1) \leq 0, \quad \forall l \in L, \end{aligned} \tag{20}$$

where M is a sufficiently large number, for example, M may be taken as the number of directed edges.

The ERTND is defined by the previous construction cost, routing, splitting demand and location–allocation constraints used to define RTND but changing the constraints (3) to (8) by the new location constraints defined by (12) to (15), (17), (19), and (20).

4 Computational experiments

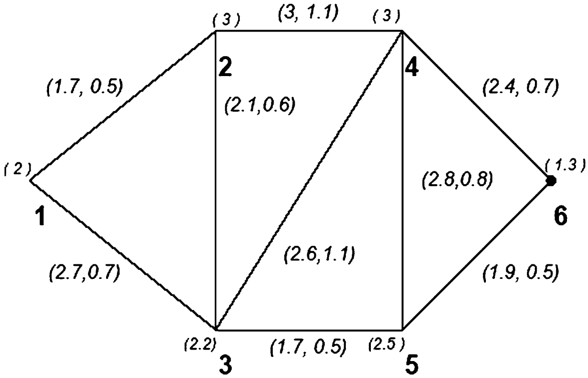
In the computational experiments it is shown that the proposed extension ERTND is more efficient than RTND, if the number of lines and their origin and destination is not fixed. These results have been obtained using Branch and Bound to solve two networks. It has been implemented with the help of Gams 21.6 which calls CPLEX 9.0.

Some experiments are defined by the network previously used by Laporte et al. (2006). This network is denoted by R1 and has 6 nodes and 9 edges. This network is given in Fig. 1. Each node has an associated construction cost c_i and each edge a pair (c_{ij}, d_{ij}) of weights: the construction cost and the distance.

The origin-destination demand g_w is given by the following matrix G:

$$G = \begin{pmatrix} - & 9 & 26 & 19 & 13 & 12 \\ 11 & - & 14 & 26 & 7 & 18 \\ 30 & 19 & - & 30 & 24 & 8 \\ 21 & 9 & 11 & - & 22 & 16 \\ 14 & 14 & 8 & 9 & - & 20 \\ 26 & 1 & 22 & 24 & 13 & - \end{pmatrix}.$$

Fig. 1 Test network R1



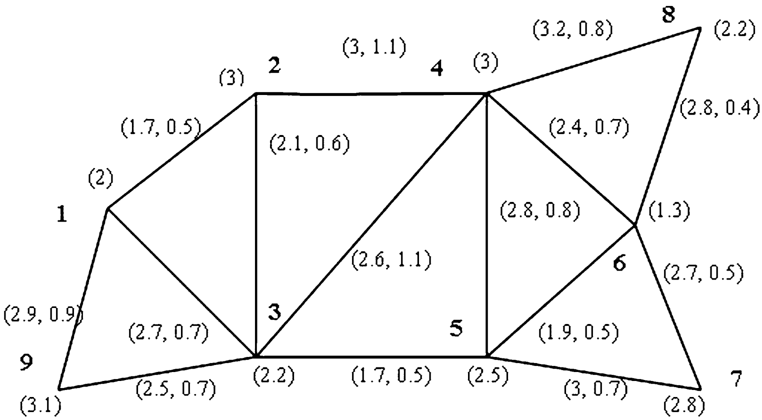


Fig. 2 Test network R2

The user private cost matrix U_w^{pri} is defined by the matrix U^{pri}:

$$U^{\text{pri}} = \begin{pmatrix} - & 1.6 & 0.8 & 2 & 2.6 & 2.5 \\ 2 & - & 0.9 & 1.2 & 1.5 & 2.5 \\ 1.5 & 1.4 & - & 1.3 & 0.9 & 2 \\ 1.9 & 2 & 1.9 & - & 1.8 & 2 \\ 3 & 1.5 & 2 & 2 & - & 1.5 \\ 2.1 & 2.7 & 2.2 & 1 & 1.5 & - \end{pmatrix}.$$

The second, larger network has also been defined and labeled R2. The network R2 has 9 nodes and 16 edges. The network R2 is equal to network R1 but only after deleting the nodes 7, 8, and 9, and their adjacent edges. The network R2 is ploted in Fig. 2.

As in R1, in R2 each node i has an associated construction cost c_i and each edge ij has a pair (c_{ij}, d_{ij}) of weights: the construction cost c_{ij} and the distance d_{ij} (which is also used to define the generalized cost to assign an edge to the public network).

In R2 the origin-destination demand g_w for each $w \in W$ is defined by the matrix:

$$G = \begin{pmatrix} - & 9 & 26 & 19 & 13 & 12 & 13 & 8 & 11 \\ 11 & - & 14 & 26 & 7 & 18 & 3 & 6 & 12 \\ 30 & 19 & - & 30 & 24 & 8 & 15 & 12 & 5 \\ 21 & 9 & 11 & - & 22 & 16 & 25 & 21 & 23 \\ 14 & 14 & 8 & 9 & - & 20 & 16 & 22 & 21 \\ 26 & 1 & 22 & 24 & 13 & - & 16 & 14 & 12 \\ 8 & 6 & 9 & 23 & 6 & 13 & - & 11 & 11 \\ 9 & 2 & 14 & 20 & 18 & 16 & 11 & - & 4 \\ 8 & 7 & 11 & 22 & 27 & 17 & 8 & 12 & - \end{pmatrix}.$$

The private cost u_w^{pri} for each $w \in W$ is defined by the matrix:

$$U^{\text{PRIV}} = \begin{pmatrix} - & 1.6 & 0.8 & 2 & 2.6 & 2.5 & 3 & 2.5 & 0.8 \\ 2 & - & 0.9 & 1.2 & 1.5 & 2.5 & 2.7 & 2.4 & 1.8 \\ 1.5 & 1.4 & - & 1.3 & 0.9 & 2 & 1.6 & 2.3 & 0.9 \\ 1.9 & 2 & 1.9 & - & 1.8 & 2 & 1.9 & 1.2 & 2 \\ 3 & 1.5 & 2 & 2 & - & 1.5 & 1.1 & 1.8 & 1.7 \\ 2.1 & 2.7 & 2.2 & 1 & 1.5 & - & 0.9 & 0.9 & 2.9 \\ 2.8 & 2.3 & 1.5 & 1.8 & 0.9 & 0.8 & - & 1.3 & 2.1 \\ 2.8 & 2.2 & 2 & 1.1 & 1.5 & 0.8 & 1.9 & - & 0.3 \\ 1 & 1.5 & 1.1 & 2.7 & 1.9 & 1.8 & 2.4 & 3 & - \end{pmatrix}.$$

To calculate the enormous computational difference between the use of RTND and ERTND, it may be observed that for a network with N nodes, for each ERTND, the following RTND items must be solved:

For the case with only one line, we will study $(\frac{N(N-1)}{2})$ RTND items; for 2 lines one must study $(\frac{N(N-1)}{2})(\frac{N(N-1)}{2} - 1)$ RTND items. For 3 lines we must study $(\frac{N(N-1)}{2})(\frac{N(N-1)}{2} - 1)(\frac{N(N-1)}{2} - 2)$ RTND items, and so on. For the network R1 with 6 nodes and for a simple case with 3 lines one must solve 2955 RTND items or one ERTND. This number is obtained as the sum of the cases for 1, 2, and 3 lines. For the network R2 with 9 nodes and in the case with 3 lines, the number of RTND items to solve is 42840 for only one ERTND model. The number of RTND items equivalent to one ERTND increases exponentially when the number of lines increases, and for large networks the optimal number of lines may be 4, 5, or a higher number.

The efficiency of ERTND is also evident in the following experiments. RTND and ERTND are run using the networks R1 and R2 for different congestion factors; the solution (optimal line and optimal objective function value) has been obtained with different $c_{\max}$. In all the experiments the maximum number of lines has been of 3 and the weight in the objective function is 1. The computational time is given in minutes.

Table 1 shows the optimal lines and their objective function values and the computational time to solve each item in network 1 with 30 demands and a budget equal to 24. In the first 9 rows the RTND results for 1, 2, and 3 lines for the congestion factors of 0.75, 1, and 1.5 are shown. In the following 3 rows the ERTND shows the results up to a maximum of 3 lines. To be able to compare the RTND computational

Table 1 Comparing RTND and ERTND for network R1 and a budget of 24

RTND	Congestion	Optimal lines	Obj.F.	C.Time
1 line	0.75	*L1*: 1–2–3–5–6–4	316	108.4
	1	*L1*: 1–2–3–5–6–4	365	179
	1.5	*L1*: 1–2–3–5–6–4	470	147.68
2 lines	0.75	*L1*: 1–3; *L2*: 2–3–5–6	248	474.57
	1	*L1*: 1–3; *L2*: 4–3–5–6	328	326.8
	1.5	*L1*: 1–3; *L2*: 3–5–6–4	368	223.52
3 lines	0.75	*L1*: 1–3 ; *L2*: 2–3; *L3*: 3–5–6	248	228.23
	1	*L1*: 1–3; *L2*: 2–3 ; *L3*: 3–5–6	309	127.11
	1.5	*L1*: 1–3; *L2*: 3–5–6; *L3*: 4–6	368	108.95
ERTND	**Congestion**	**Optimal lines**	**Obj.F.**	**C.Time**
Maximum	0.75	*L1*: 1–2–3–5–6–4	316	34.26
3 lines	1	*L1*: 1–2–3–5–6–4	365	142.45
	1.5	*L1*: 1–2–3–5–6–4	470	156.32

Table 2 Comparing RTND and ERTND for network R1 and 30 demands

$C_{\max}$	Congestion Factor	Optimal lines	Obj.F.	RTND C.Time	ERTND C.Time
24	0.75	*L1*: 1–2–3–5–6–4	316	811.2	34.26
24	1	*L1*: 1–2–3–5–6–4	365	632.91	142.45
24	1.5	*L1*: 1–2–3–5–6–4	470	480.15	156.32
32	0.75	*L1*: 4–3–5–6; *L2*: 1–3–2	330	662.87	9.21
32	1	*L1*: 2–3–5–6; *L2*: 1–3–4–6	470	1288.76	4.03
32	1.5	*L1*: 5–3–2–4–6; *L2*: 1–3	496	5196.85	3.46

times one must sum the computational time needed to solve each of the items (1, 2, and 3 lines). This simple example is evidence that the ERTND solves the items more efficiently.

In Tables 1 and 2, it may be observed that for a budget of 32 and for a congestion factor 0.75, RTND needs 811.2 minutes to obtain the optimal solution, meanwhile ERTND needs 34.26 minutes. With the congestion factor of 1.5, RTND uses 480.15 minutes and ERTND needs 156.32 minutes.

In the Table 2 the computational results for budgets of 24 and 32 have been compared, but the suboptimal solutions of each parametric RTND group solution have not been included. Logically, the computational time of RTND is the sum of the computational times of the Table 1 for a given congestion factor. Making the same experiment using a budget of 32, the efficiency of ERTND compared with RTND is greater, for example, for a congestion factor of 1.5, RTND uses 5196.85 minutes and ERTND needs only 3.46 minutes.

Table 3 Comparing RTND and ERTND for network R2 and 30 demands

C_{max}	Congestion Factor	Optimal lines	Obj.F.	RTND C.Time	ERTND C.Time
18	0.75	*L2*: 4–6–5; *L3*: 6–7	181	700.16	2.94
18	1	*L1*: 4–6; *L3*: 5–6–8	215	544.42	5.11
18	1.5	*L1:* 5–6–8; *L2*: 4–6	215	567.77	68.48
32	0.75	*L1*: 5–6; *L2*: 6–4–8 ; *L3*: 7–6–8	314	3147.97	5.36
32	1	*L1*: 3–4–6–8; *L2*: 9–3–5–6	349	5878.80	68.97
32	1.5	*L1*: 9–3–5–6–8; *L3*: 4–8	349	67278.69	14078.48

Table 4 Comparing RTND and ERTND for network R2 with 72 demands

C_{max}	Congestion Factor	Optimal lines	Obj.F.	RTND C.Time	ERTND C.Time
28	0.75	*L1*: 4–6–7; *L3*: 3–5–6–8	361	>240 hours	8 hours[a]
28	1	*L2*: 6–5–3–9; *L3*: 1–3–4	466	>240 hours	8 hours[b]
28	1.5	*L1*: 1–3–5–4–6–8	522	>240 hours	8 hours[c]
48	0.75	*L1*: 1–3–2; *L2*: 6–4–8; *L3*: 9–3–5–6–7	672	>240 hours	232.67
48	1	*L2*: 1–2–3–5–6–8; *L3*: 9–3–4–6–7	912	>240 hours	898.56
48	1.5	*L1*: 9–1–2–4–6–7; *L3*: 1–3–5–6–8	1035	>240 hours	165.23

[a]Relative gap 0.09

[b]Relative gap 0.26

[c]Relative gap 0.65

In the case of the network R2 with 30 demands, Table 3 shows the results for c_{max} equal to 18 and 32. It may be observed that to obtain the same optimal solution, the computational efficiency using ERTND is sometimes a hundred times better than using RTND.

The computational differences using both models are exponentially increased when the size of the problem increases. The need to use ERTND for large networks is clear. This effect has been proved in the Table 4. In the experiments with the network R2 and 72 demands, the difference computational time ranges from the hundreds of minutes using ERTND to more than 240 hours using RTND.

The relative gaps even for ERTND with a small network, for example R2 with 9 nodes and all the demands, show the difficulty in solving such a complex and large model. We are trying to define metaheuristic and decomposition methods to be able solve larger network problems.

5 Conclusions and further research

The rapid transit network design model has been extended to any line and any origin-destination for each line. This extended model includes all the network alternatives available on the previous model but studied simultaneously, so the choice of better alternatives is extremely more efficient.

The extended model of the Transit Network Design problem avoids the parametric analysis that was necessary using the previous model. For this reason, the computational time needed to obtain the same optimal solution is many times inferior in the extended model, depending on the system size.

Another improvement of the model includes a transshipment cost when the demand routes are defined for more than one line. The complexity of the Rapid Transit Network Design with transfers requires a study with a more careful methodology which is adequate to solve large networks. In this case the frequency of the lines is included in the model as a parameter for different topology alternatives, see García et al. (2006).

Another important extension includes multiperiod capacity expansion of the rapid transit network. In this case the network design decisions are temporarily taken considering the effect of dynamic construction cost, and temporary changes in the demand, see Marín and Jaramillo (2005).

Other approaches to characterize more computationally efficient line constraints have been studied by Marín and García (2007). They introduced a Logit mode distribution with advantages to simulate the user behavior and define the lines bounding the line intersections.

References

Bruno G, Ghiani G, Improta G (1998) A multi-modal approach to the location of a rapid transit line. Eur J Oper Res 104:321–332

Bruno G, Gendreau M, Laporte G (2002) A heuristic for the location of a rapid transit line. Comput Oper Res 29:1–12

García R, Marín A (2001) Urban multimodal interchange design methodology. In: Mathematical methods on optimization in transportation systems. Kluwer Academic, Dordrecht, pp 49–75

García R, Marín A (2002) Parking capacity and pricing in park'n ride trips: a continuous equilibrium network design problem. Ann Oper Res 116:153–178

García R, Garzón-Astolfi A, Marín A, Mesa JA, Ortega FA (2006) Analysis of the parameters of transfers in the rapid transit network design. Schloss Dagstuhl, 2006. Downloadable from http://drops.dagstuhl.de/opus/volltexte/2006/658

Hamacher H, Liebers A, Schobel A, Wagner D, Wagner F (2001) Locating new stops in a railway network. Electron Notes Theor Comput Sci 50

Laporte G, Mesa JA, Ortega FA (2002) Locating stations on rapid transit lines. Comput Oper Res 29:741–759

Laporte G, Mesa JA, Ortega FA (2005) Maximizing trip coverage in the location of a single rapid transit alignment. Ann Oper Res 136:49–63

Laporte G, Marín A, Mesa JA, Ortega FA (2006) An integrated methodology for the rapid transit network design. In: Garaets F, Kroon L, Schoebel A, Wagner D, Zarioliagis C (eds) Algorithmic methods for railway optimization. Lectures notes in computer science, vol 4359, accepted

Marín A, García R (2007) Location of the infrastructure in urban railway networks. Comput Oper Res, accepted

Marín A, Jaramillo P (2005) Urban rapid transit network capacity expansion. In: Advanced OR and AI methods in transportation. Publishing House of Poznan University of Technology, pp 218–225. ISBN: 83-7143-239-5