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An algorithm for a fuzzy transportation problem to select a new type of coal for a steel manufacturing unit

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Abstract Manufacturing of steel involves thermal energy intensive processes with coal as the major input. Energy generated is a direct function of ash content of coal and as such it weighs very high as regards the choice of coal. In this paper, we study a multiobjective transportation problem to introduce a new type of coal in a steel manufacturing unit in India. The use of new type of coal serves three non-prioritized objectives, viz. minimization of the total freight cost, the transportation time and the ratio of ash content to the production of hot metal. It has been observed from the past data that the supply and demand points have shown fluctuations around their estimated values because of changing economic conditions. To deal with uncertainties of supply and demand parameters, we transform the past data pertaining to the amount of supply of the ith supply point and the amount of demand of the jth demand point using level (λ, ρ) interval-valued fuzzy numbers. We use a linear ranking function to defuzzify the fuzzy transportation problem. A transportation algorithm is developed to find the non-dominated solutions for the defuzzified problem. The application of the algorithm is illustrated by numerical examples constructed from the data provided by the manufacturing unit.

Keywords Fuzzy transportation problem · Fuzzy numbers · Interval-valued fuzzy numbers · Non-dominated solutions

Mathematics Subject Classification (2000) 90C29 · 90C70 · 90C10 · 90C32

1 Introduction

In this paper, we consider a real life transportation problem relating to supply of coal to a multilocational steel manufacturing firm in India from the three suppliers,

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namely Vizag, Paradip, and Haldia collieries. The company has the option to use the following three types of coal:

(i) Imported coal
(ii) Prime coal
(iii) Medium coal

at its four main steel plants BSP, DSP, RSP, BSL located at different places in the country.

At the time the research was undertaken, all the four plants of the firm were primarily using domestically available coal, i.e. prime coal or medium coal. In either case, the production of the hot metal was low due to the heavy ash content. As a result, the company was finding it difficult to meet its monthly production targets of hot metal. A proposal for the use of imported coal is in the active consideration of the management because of its low ash content with a view of achieving three minimization objectives: (i) the freight cost involved in transporting required quantities from the three suppliers to the four steel plants; (ii) the transportation time; and (iii) the ratio of ash content to the production of hot metal. It is assumed that transportation from supply points to demand points is done in parallel, and the transportation time from the ith supply point to the jth demand point does not depend upon the volume of the shipment.

Based on the analysis of the decision making process, we have proposed the following transportation problem model:

$$\text{(P)} \quad \text{minimize} \left\{ \sum_i \sum_j c_{ij} x_{ij}, \operatorname{Max}_{\{(i,j)/x_{ij}>0\}} t_{ij}(x_{ij}), \frac{\sum_i \sum_j d_{ij} x_{ij}}{\sum_i \sum_j e_{ij} x_{ij}} \right\}$$

subject to

$$\sum_j x_{ij} = a_i, \quad i = 1, 2, \ldots, m, \tag{1}$$

$$\sum_i x_{ij} = b_j, \quad j = 1, 2, \ldots, n, \tag{2}$$

$$x_{ij} \geq 0, \quad i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n, \tag{3}$$

where

$$t_{ij}(x_{ij}) = \begin{cases} t_{ij} (\geq 0) & \text{if } x_{ij} > 0, \\ 0 & \text{if } x_{ij} = 0, \end{cases}$$

$a_i =$ supply available at the ith supply point,
$b_j =$ demand required at the jth demand point,
$c_{ij} =$ freight cost involved when one unit of product is transported from the ith supply point to the jth demand point,
$t_{ij} =$ transportation time from the ith supply point to the jth demand point,
$x_{ij} =$ total number of units transported from the ith supply point to the jth demand point,
$d_{ij} =$ ash content present in one unit of imported coal transported from the ith supply point to the jth demand point,
$e_{ij} =$ hot metal production using one unit of imported coal transported from the ith supply point to the jth demand point.

For consistency, we must have $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

The transportation problem (P) falls in the category of multiobjective transportation problems (MOTP). Intensive investigations on (MOTP) have been made by several researchers. Aneja and Nair (1979) presented a bicriteria transportation problem model. Lee and Moore (1973) studied the optimization of multiobjectives transportation problems. Diaz (1978, 1979) and Isermann (1979) proposed the procedures to generate all *non-dominated* solutions to the multiobjective linear transportation problem.

In particular, the transportation problem (P) generalizes the time–cost minimizing transportation problem studied in literature by Bhatia et al. (1976, 1979), Gupta (1977), Prasad et al. (1993), Li et al. (2001), Liu and Zhang (2005).

One of the objectives in the transportation problem (P) is the fractional function. Thus, the problem (P) also relates to the fractional transportation problem studied by Swarup (1966), Sharma and Swarup (1978), Chandra and Saxena (1987).

The works referred so far for solving the transportation problem assume that the parameters of the problem are exactly known. However, there are situations when the parameters may not be known in a precise manner. To deal with such situations, fuzzy set theory has been applied in literature to solve the transportation problem and to enrich and enhance the suggested solution methodologies. Bit et al. (1992) considered a k-objective transportation problem fuzzified by fuzzy numbers and used α-cut to obtain a transportation problem in the fuzzy sense expressed in linear programming form. Chanas and Kuchta (1996) used fuzzy numbers of the type L–L to fuzzify cost coefficients in the objective function and α-cut to express the objective function in the form of an interval. Hussien (1998) studied the complete set of α-possibly efficient solutions of multiobjective transportation problem with possibilistic coefficients of the objective functions. Li and Lai (2000) proposed a fuzzy compromise programming approach to a multiobjective linear transportation problem. For further references in this direction, we refer the reader to Chanas et al. (1993), Kikuchi (2000), Abd El-Wahed (2001), Ammar and Youness (2005), Chiang (2005).

We know that transportation plans are determined periodically for many companies, especially for large companies. Since the amount of supply at the ith supply point may not be exactly a_i each time, it could fluctuate a little. Hence, we fuzzify a_i to $\widetilde{a}_i$. Also, the demand at the jth demand point may not be exactly b_j each time, it could also fluctuate a little. Hence, we fuzzify b_j to $\widetilde{b}_j$. In this paper, we use (λ, ρ) *interval-valued* fuzzy numbers to formulate fuzzy transportation problem described as above. We use *signed distance ranking* to defuzzify the transportation problem. The signed distances of the (λ, ρ) *interval-valued* fuzzy numbers are measured from $\widetilde{0}_1$(y axis). The order properties of the signed ranking are the same as those of the real numbers, and hence this ranking is preferred as the defuzzification function in the present paper.

We are interested in developing a specialized method to find *non-dominated solutions* for the defuzzified transportation problem. The concept of nondominance is one of the basic concepts of multiobjective optimization. In the case of multiple objectives, there may not exist one solution which is best (global minimum or maximum) with respect to all objectives. In a typical multiobjective optimization problem, there exists a set of solutions which are superior to the rest of solutions in the search space when all objectives are considered but are inferior to other solutions in the space in one or more objectives. These solutions are known as *Pareto-optimal solutions* or *non-dominated solutions* (Chankong and Haimes 1983). The rest of the solutions are known as dominated solutions. Since none of the solutions in the non-dominated set is absolutely better than any other, anyone of them is an acceptable solution. The choice of one solution over the other requires a problem knowledge and a number of problem-related factors. Therefore, in multiobjective optimization problems, it may be useful to have a knowledge about alternative non-dominated solutions.

The rest of the paper is organized as follows. Section 2 contains basic definitions and preliminaries concerning (λ, ρ) *interval-valued* fuzzy numbers. In Sect. 3, we formulate fuzzy transportation problem using these fuzzy numbers. In Sect. 4, we develop a solution procedure for the defuzzified transportation problem based on multicriteria simplex method given by Yu and Zeleny (1975). The proposed solution method generalizes the method given in Gupta (1977) for the time–cost transportation problem. The solution procedure is summarized in the form of an algorithm in Sect. 5. Section 6 illustrates the application of the algorithm using numerical examples with practical data. In Sect. 7, we discuss the results and implementation of the solutions. Finally, some concluding remarks are made in Sect. 8.

2 Preliminaries

First, we introduce the concept of (λ, ρ) *interval-valued fuzzy numbers*. We state the following definitions and results from Chiang (2001) to be used in the formulation of the fuzzy transportation problem in the present paper.

Definition 1 If the membership function of the fuzzy set $\widetilde{a}_\alpha$ on $\mathbb{R} = (-\infty, \infty)$ is

$$\mu_{\widetilde{a}_\alpha}(x) = \begin{cases} \alpha, & x = a, \\ 0, & x \neq a, \end{cases} \tag{4}$$

then $\widetilde{a}_\alpha$ is called a *level α fuzzy point*.

Definition 2 If the membership function of the fuzzy set $\widetilde{A}$ on $\mathbb{R}$ is

$$\mu_{\widetilde{A}}(x) = \begin{cases} \lambda(x-a)/(b-a), & a \le x \le b, \\ \lambda(c-x)/(c-b), & b \le x \le c, \\ 0, & \text{otherwise}, \end{cases} \tag{5}$$

$a < b < c$, then $\widetilde{A}$ is called a *level λ fuzzy number*, $0 < \lambda \leq 1$. We denote $\widetilde{A} = (a, b, c; \lambda)$.

Let $F_N(\lambda)$ be the family of all level λ fuzzy numbers, i.e.

$$F_N(\lambda) = \left\{(a, b, c; \lambda) \mid \forall a < b < c, a, b, c \in \mathbb{R}\right\}, \quad 0 < \lambda \leq 1.$$

Definition 3 If the membership function of the fuzzy set $[a_\alpha, b_\alpha]$ on $\mathbb{R}$ is

$$\mu_{[a_\alpha, b_\alpha]}(x) = \begin{cases} \alpha, & a \leq x \leq b, \\ 0, & \text{otherwise}, \end{cases} \tag{6}$$

$a < b$, then $[a_\alpha, b_\alpha]$ is called a *level α fuzzy interval*, $0 < \alpha \leq 1$.

Definition 4 An interval-valued fuzzy set (*i-v* fuzzy set for short) $\widetilde{A}$ on $\mathbb{R}$ is given by $\widetilde{A} \triangleq \{(x, [\mu_{\widetilde{A}^L}(x), \mu_{\widetilde{A}^U}(x)])\}$, $x \in \mathbb{R}$, $\mu_{\widetilde{A}^L}(x)$, $\mu_{\widetilde{A}^U}(x) \in [0, 1]$, $\mu_{\widetilde{A}^L}(x) \leq \mu_{\widetilde{A}^U}(x)$, $\forall x \in \mathbb{R}$, and is denoted by $\widetilde{A} = [\widetilde{A}^L, \widetilde{A}^U]$.

This means that the grade of membership of x belongs to the interval $[\mu_{\widetilde{A}^L}(x), \mu_{\widetilde{A}^U}(x)]$, the least grade of membership at x is $\mu_{\widetilde{A}^L}(x)$ and the greatest grade of membership at x is $\mu_{\widetilde{A}^U}(x)$.

Interval-valued fuzzy sets were first used by Gorzafczany (1983) and have been applied to the fields of approximate inference, signal transmission and controller, etc.

Let

$$\mu_{\widetilde{A}^L}(x) = \begin{cases} \lambda(x-a)/(b-a), & a \leq x \leq b, \\ \lambda(c-x)/(c-b), & b \leq x \leq c, \\ 0, & \text{otherwise}. \end{cases} \tag{7}$$

Then $\widetilde{A}^L = (a, b, c; \lambda)$.

Let

$$\mu_{\widetilde{A}^U}(x) = \begin{cases} \rho(x-p)/(b-p), & p \leq x \leq b, \\ \rho(r-x)/(r-b), & b \leq x \leq r, \\ 0, & \text{otherwise}. \end{cases} \tag{8}$$

Then $\widetilde{A}^U = (p, b, r; \rho)$. Here $0 < \lambda \leq \rho \leq 1$, $p < a < b < c < r$.

We denote $\widetilde{A} = [(a, b, c; \lambda), (p, b, r; \rho)] = [\widetilde{A}^L, \widetilde{A}^U]$. $\widetilde{A}$ is called a level (λ, ρ) *i-v* fuzzy number (see Fig. 1). Let $F_{IV}(\lambda, \rho) = \{[(a, b, c; \lambda), (p, b, r; \rho)] | \forall\ p < a < b < c < r\}$, $0 < \lambda \leq \rho \leq 1$.

Remark 1 When $\lambda = 0$, $a = p$, $c = r$, $\widetilde{A} = [\widetilde{A}^L, \widetilde{A}^U]$ will reduce to level ρ fuzzy number $\widetilde{A}^U = (p, b, r; \rho)$.

Next, we consider the ranking of fuzzy numbers. Many ranking methods can be found in the fuzzy literature. Bass and Kwakernaak (1977) are among the pioneers in this area. Bortolan and Degani (1985), Wang and Kerre (1996) have reviewed different ordering methods. One can refer to Chen and Hwang (1992) for the description of

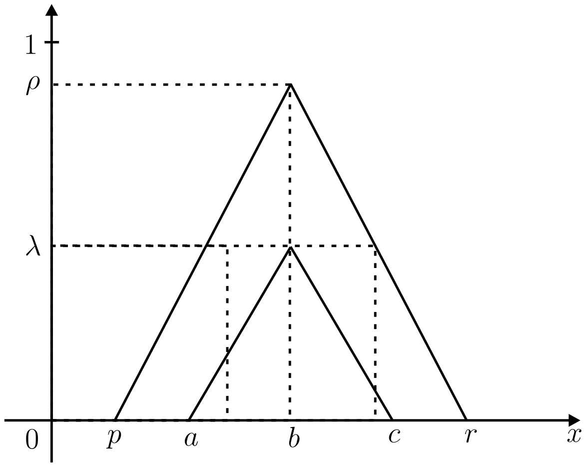


Fig. 1 Level (λ, ρ) *i-v* fuzzy number

various ordering methods. We consider the ranking of the level (λ, ρ) *i-v* fuzzy numbers using the concept of signed distance introduced by Yao and Wu (2000). First, we consider the signed distance on $\mathbb{R} = (-\infty, \infty)$.

Definition 5 Let $a \in \mathbb{R}$. We define the signed distance $d^*(a, 0) = a$. If $0 < a$, then a is to the right of 0, and the distance is $a = d^*(a, 0)$. If $a < 0$, then a is to the left of 0, and the distance is $-a = -d^*(a, 0)$.

We now consider the ranking of the level (λ, ρ) *i-v* fuzzy numbers.

Definition 6 Let $\widetilde{A} = [(a, b, c; \lambda), (p, b, r; \rho)] \in F_{IV}(\lambda, \rho)$. The signed distance of $\widetilde{A}$ from $\widetilde{0}_1$(y axis) is defined as follows. If $0 < \lambda \leq \rho \leq 1$, then

$$\begin{aligned} d_0(\widetilde{A}, \widetilde{0}_1) &= \frac{1}{\lambda}\int_0^{\lambda} \frac{1}{4}\left[a + c + p + r + (2b - a - c)\frac{\alpha}{\lambda} + (2b - p - r)\frac{\alpha}{\lambda}\right] d\alpha \\ &\quad + \frac{1}{\rho - \lambda}\int_{\lambda}^{\rho} \frac{1}{2}\left[p + r + (2b - p - r)\frac{\alpha}{\lambda}\right] d\alpha \\ &= \frac{1}{8}\left[6b + a + c + 4p + 4r + 3(2b - p - r)\frac{\lambda}{\rho}\right]. \end{aligned}$$

If $0 < \lambda = \rho \leq 1$, then

$$d_0(\widetilde{A}, \widetilde{0}_1) = \frac{1}{8}[12b + a + c + p + r]. \tag{9}$$

Remark 2 Let $\widetilde{A} = (a, b, c; \lambda) \in F_N(\lambda)$. The signed distance of $\widetilde{A}$ from $\widetilde{0}_1$(y axis) is

$$d_0(\widetilde{A}, \widetilde{0}_1) = \frac{1}{\lambda}\int_0^{\lambda} \frac{1}{2}\left[a + c + (2b - a - c)\frac{\alpha}{\lambda}\right] d\alpha = \frac{1}{4}(2b + a + c).$$

Definition 7 For each $0 < \lambda \le \rho \le 1$, let $\widetilde{A}$ and $\widetilde{B} \in F_{IV}(\lambda, \rho)$. The ranking of the level (λ, ρ) *i-v* fuzzy numbers in $F_{IV}(\lambda, \rho)$ is defined as

$$\begin{aligned} \widetilde{B} \prec \widetilde{A} \quad &\text{iff} \quad d_0(\widetilde{B}, \widetilde{0}_1) < d_0(\widetilde{A}, \widetilde{0}_1), \\ \widetilde{B} \approx \widetilde{A} \quad &\text{iff} \quad d_0(\widetilde{B}, \widetilde{0}_1) = d_0(\widetilde{A}, \widetilde{0}_1), \\ \widetilde{A} \prec \widetilde{B} \quad &\text{iff} \quad d_0(\widetilde{A}, \widetilde{0}_1) < d_0(\widetilde{B}, \widetilde{0}_1). \end{aligned}$$

Proposition 1 *$(F_{IV}(\lambda, \rho), \approx, \prec)$ satisfies the law of trichotomy.*

Proposition 2 *Let $\widetilde{A} = [(a_1, b_1, c_1; \lambda), (a_2, b_1, c_2; \rho)]$ and $\widetilde{B} = [(p_1, q_1, r_1; \lambda), (p_2, q_1, r_2; \rho)] \in F_{IV}(\lambda, \rho)$. Then*:

(1a) $\widetilde{A} \oplus \widetilde{B} = [(a_1 + p_1, b_1 + q_1, c_1 + r_1; \lambda), (a_2 + p_2, b_1 + q_1, c_2 + r_2; \rho)]$.
(2a) *If* $k > 0$, $\widetilde{k}_1(\cdot)\widetilde{A} = [(ka_1, kb_1, kc_1; \lambda), (ka_2, kb_1, kc_2; \rho)]$.
(3a) *If* $k < 0$, $\widetilde{k}_1(\cdot)\widetilde{A} = [(kc_1, kb_1, ka_1; \lambda), (kc_2, kb_1, ka_2; \rho)]$.
(4a) *If* $k = 0$, $\widetilde{k}_1(\cdot)\widetilde{A} = [(0, 0, 0; \lambda), (0, 0, 0; \rho)]$.

Proposition 3 *Let $\widetilde{A} = [(a_1, b_1, c_1; \lambda), (a_2, b_1, c_2; \rho)]$ and $\widetilde{B} = [(p_1, q_1, r_1; \lambda), (p_2, q_1, r_2; \rho)] \in F_{IV}(\lambda, \rho)$. Then*

$$d_0(\widetilde{A} \oplus \widetilde{B}, \widetilde{0}_1) = d_0(\widetilde{A}, \widetilde{0}_1) + d_0(\widetilde{B}, \widetilde{0}_1)$$

and

$$d_0(\widetilde{k}(\cdot)\widetilde{A}, \widetilde{0}_1) = kd_0(\widetilde{A}), \quad k > 0.$$

It follows that $d_0(\widetilde{D}, \widetilde{0}_1)$ is the signed distance of fuzzy set $\widetilde{D}$ from $\widetilde{0}_1(y$ axis) and $d_0(\widetilde{D}, \widetilde{E}) = d_0(\widetilde{D}, \widetilde{0}_1) - d_0(\widetilde{E}, \widetilde{0}_1)$ is the difference of the signed distances of two sets $\widetilde{D}, \widetilde{E}$ from $\widetilde{0}_1(y$ axis). The ordering relations $\prec, \approx$ on $F_{IV}(\lambda, \rho)$ satisfy the law of trichotomy. From Proposition 2 it follows that $(F_{IV}(\lambda, \rho), d_0, \prec, \approx, \oplus, \ominus, (\cdot))$ is an extension of $(R, d^*, <, =, +, -, \cdot)$. Hence, the ranking properties on $(R, d^*, <, =, +, -, \cdot)$ can be transferred to $(F_{IV}(\lambda, \rho), d_0, \prec, \approx, \oplus, \ominus, (\cdot))$. Moreover, it follows from Proposition 3 that the signed distance ranking is a linear ranking.

3 Problem formulation

Let T be a time period in which c_{ij}, t_{ij}, d_{ij} and e_{ij}, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, do not change in crisp transportation problem (P). In time period T, the degree of membership of a_i will not be in general 1. We may assume that the degree of membership of a_i falls within the interval $[\lambda, 1]$, $0 < \lambda < 1$. In other words, we let $\widetilde{a}_i$ be a *level* $(\lambda, 1)$ *i-v fuzzy number*,

$$\widetilde{a}_i = \big[(a_i - \alpha_{3i}, a_i, a_i + \alpha_{4i}; \lambda), (a_i - \alpha_{1i}, a_i, a_i + \alpha_{2i}; 1)\big], \tag{10}$$

where $0 < \alpha_{3i} < \alpha_{1i} < a_i$, $0 < \alpha_{4i} < \alpha_{2i} < a_i$, $i = 1, 2, \ldots, m$. Similarly, we fuzzify b_j to the *level* $(\lambda, 1)$ *i-v fuzzy number*

$$\widetilde{b}_j = \big[(b_j - \beta_{3j}, b_j, b_j + \beta_{4j}; \lambda), (b_j - \beta_{1j}, b_j, b_j + \beta_{2j}; 1)\big], \tag{11}$$

where $0 < \beta_{3j} < \beta_{1j} < b_j$, $0 < \beta_{4j} < \beta_{2j} < b_j$, $j = 1, 2, \ldots, n$. It may be noted that if $\alpha_{4i} = \alpha_{3i}$, $i = 1, 2, \ldots, m$, $\beta_{4j} = \beta_{3j}$, $j = 1, 2, \ldots, n$ and $\lambda = 0$ in (10) and (11), it reduces to the case of *level* 1 *fuzzy numbers* of $(a_i - \alpha_{1i}, a_i, a_i + \alpha_{2i}; 1)$, $(b_j - \beta_{1j}, b_j, b_j + \beta_{2j}; 1)$ or, more precisely, to the case of triangular fuzzy numbers.

Let $\widetilde{1}^* = [(1, 1, 1; \lambda), (1, 1, 1; 1)]$.

We obtain the following fuzzy transportation model:

$$\text{(P1)} \quad \text{minimize} \left\{ \sum_i \sum_j c_{ij} x_{ij}, \operatorname{Max}_{\{(i,j)/x_{ij}>0\}} t_{ij}(x_{ij}), \frac{\sum_i \sum_j d_{ij} x_{ij}}{\sum_i \sum_j e_{ij} x_{ij}} \right\} \tag{12}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \cdot \widetilde{1}^* \approx \widetilde{a}_i, \quad i = 1, 2, \ldots, m, \tag{13}$$

$$\sum_{i=1}^{m} x_{ij} \cdot \widetilde{1}^* \approx \widetilde{b}_j, \quad j = 1, 2, \ldots, n, \tag{14}$$

$$x_{ij} \geq 0, \qquad i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n. \tag{15}$$

For consistency, we must have

$$\sum_{i=1}^{m} \widetilde{a}_i \approx \sum_{j=1}^{n} \widetilde{b}_j, \tag{16}$$

i.e. $d_0(\sum_{i=1}^{m} \widetilde{a}_i, \widetilde{0}_1) = d_0(\sum_{j=1}^{n} \widetilde{b}_j, \widetilde{0}_1)$.

It may be noted that if $\widetilde{A} = \widetilde{B}$, then $\widetilde{A} \approx \widetilde{B}$. However, if $\widetilde{A} \approx \widetilde{B}$, it is not necessary that $\widetilde{A} = \widetilde{B}$. Therefore, "$\approx$" defined in ordering for fuzzy sets is not equal to "$=$" defined for fuzzy sets.

Using signed distance ranking of fuzzy numbers from Sect. 2 along with (10) and (11), we rewrite fuzzy transportation problem (P1) as

$$\text{(P2)} \quad \text{minimize} \left\{ \sum_i \sum_j c_{ij} x_{ij}, \operatorname{Max}_{\{(i,j)/x_{ij}>0\}} t_{ij}(x_{ij}), \frac{\sum_i \sum_j d_{ij} x_{ij}}{\sum_i \sum_j e_{ij} x_{ij}} \right\} \tag{17}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i + \frac{1}{16}\big[\alpha_{4i} - \alpha_{3i} + (4 - 3\lambda)(\alpha_{2i} - \alpha_{1i})\big], \quad i = 1, 2, \ldots, m, \tag{18}$$

$$\sum_{i=1}^{m} x_{ij} = b_j + \frac{1}{16}\big[\beta_{4j} - \beta_{3j} + (4 - 3\lambda)(\beta_{2j} - \beta_{1j})\big], \quad j = 1, 2, \ldots, n, \tag{19}$$

$$x_{ij} \geq 0, \quad i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n. \tag{20}$$

We can rewrite (18), (19) as

$$\sum_{j=1}^{n} x_{ij} = a_i^1, \quad i = 1, 2, \dots, m, \tag{21}$$

$$\sum_{i=1}^{m} x_{ij} = b_j^1, \quad j = 1, 2, \dots, n, \tag{22}$$

where

$$a_i^1 = a_i + \frac{1}{16}\big[\alpha_{4i} - \alpha_{3i} + (4 - 3\lambda)(\alpha_{2i} - \alpha_{1i})\big],$$

$$b_j^1 = b_j + \frac{1}{16}\big[\beta_{4j} - \beta_{3j} + (4 - 3\lambda)(\beta_{2j} - \beta_{1j})\big].$$

For consistency, we must have

$$\sum_{i=1}^{m}\big[\alpha_{4i} - \alpha_{3i} + (4 - 3\lambda)(\alpha_{2i} - \alpha_{1i})\big] = \sum_{j=1}^{n}\big[\beta_{4j} - \beta_{3j} + (4 - 3\lambda)(\beta_{2j} - \beta_{1j})\big]. \tag{23}$$

Remark 3 In the case study taken up for discussion in the present paper, decision making is required at the end of the steel manufacturing unit, an industrial user. It is imperative for an industrial unit to build a reliable supply chain to meet its demand. The unit has estimates of the demand for coal to be procured from the suppliers. Since we have sufficient information on the demands, in order to obtain a desired degree of satisfaction, we expect the three suppliers will match the demands as far as possible. For this reason, we use the same λ in *level* $(\lambda, 1)$ *i-v fuzzy number* for both the supplies as well as the demands.

Next, we develop a solution method to find non-dominated solutions for transportation problem (P2).

4 Solution methodology

Let $\bar{x}$ be any basic feasible solution for (P2) with basis $\bar{B}$. Define

$$\bar{d}_{ij} = \begin{cases} 1 & \text{if } t_{ij} = \bar{t}, \\ 0 & \text{if } t_{ij} < \bar{t}, \\ M & \text{if } t_{ij} > \bar{t}, \end{cases} \tag{24}$$

where $\bar{t} = \mathrm{Max}_{\{(i,j)/\bar{x}_{ij}>0\}} t_{ij}(\bar{x}_{ij})$ and M is an arbitrary large number. Consider the three problems:

$$
\begin{aligned}
(\mathrm{P}') \quad & \text{minimize } F_1 = \sum_i \sum_j c_{ij} x_{ij} \\
& \text{subject to } \sum_j x_{ij} = a_i^1, \quad i = 1, 2, \ldots, m, \\
& \sum_i x_{ij} = b_j^1, \quad j = 1, 2, \ldots, n, \\
& x_{ij} \geq 0 \qquad \text{for all } (i, j); \\
(\mathrm{P}'') \quad & \text{minimize } F_2 = \sum_i \sum_j \bar{d}_{ij} x_{ij} \\
& \text{subject to } \sum_j x_{ij} = a_i^1, \quad i = 1, 2, \ldots, m, \\
& \sum_i x_{ij} = b_j^1, \quad j = 1, 2, \ldots, n, \\
& x_{ij} \geq 0 \qquad \text{for all } (i, j); \\
(\mathrm{P}''') \quad & \text{minimize } F_3 = \frac{\sum_i \sum_j d_{ij} x_{ij}}{\sum_i \sum_j e_{ij} x_{ij}} \\
& \text{subject to } \sum_j x_{ij} = a_i^1, \quad i = 1, 2, \ldots, m, \\
& \sum_i x_{ij} = b_j^1, \quad j = 1, 2, \ldots, n, \\
& x_{ij} \geq 0 \qquad \text{for all } (i, j),
\end{aligned}
$$

where $\sum_i \sum_j e_{ij} x_{ij} > 0$ for all feasible solutions.

Clearly, $\bar{x}$ is a basic feasible solution for (P′), (P″) and (P‴). Let (u_i^1, v_j^1), (u_i^2, v_j^2), and $\{(u_i^3, v_j^3), (u_i^4, v_j^4)\}$ be the dual variables for problems (P′), (P″) and (P‴) determined using the following equations:

$$u_i^1 + v_j^1 = c_{ij} \quad \text{for } (i, j) \in \bar{B}, \tag{25}$$

$$u_i^2 + v_j^2 = \bar{d}_{ij} \quad \text{for } (i, j) \in \bar{B}, \tag{26}$$

$$u_i^3 + v_j^3 = d_{ij} \quad \text{for } (i, j) \in \bar{B}, \tag{27}$$

$$u_i^4 + v_j^4 = e_{ij} \quad \text{for } (i, j) \in \bar{B}. \tag{28}$$

Following Swarup (1966), we can rewrite F_3 as

$$F_3 = \frac{\sum_i \sum_j d_{ij}^1 x_{ij} + Z^1}{\sum_i \sum_j e_{ij}^1 x_{ij} + Z^2},$$

where

$$d_{ij}^1 = d_{ij} - u_i^3 - v_j^3,$$
$$e_{ij}^1 = e_{ij} - u_i^4 - v_j^4,$$
$$Z^1 = \sum_{i=1}^{m} a_i^1 u_i^3 + \sum_{j=1}^{n} b_j^1 v_j^3,$$
$$Z^2 = \sum_{i=1}^{m} a_i^1 u_i^4 + \sum_{j=1}^{n} b_j^1 v_j^4.$$

Using optimality conditions of the three problems described above, we have the following situations:

(i) $\bar{x}$ will dominate all other solutions x^* resulting from introducing the non-basic cell (i, j) for which $u_i^1 + v_j^1 - c_{ij} \le 0$, $u_i^2 + v_j^2 - \bar{d}_{ij} \le 0$ and $[Z^2(u_i^3 + v_j^3 - d_{ij}) - Z^1(u_i^4 + v_j^4 - e_{ij})] \le 0$ with one inequality strictly negative and $x_{ij}^* = \theta_{ij} > 0$; therefore, such a cell cannot be considered for introduction into the basis.
(ii) If there are cells (i, j) such that $u_i^1 + v_j^1 - c_{ij} \ge 0$, $u_i^2 + v_j^2 - \bar{d}_{ij} \ge 0$, and $[Z^2(u_i^3 + v_j^3 - d_{ij}) - Z^1(u_i^4 + v_j^4 - e_{ij})] \ge 0$ with one inequality strictly positive, then the new solution x^* obtained by introducing the cell (i, j) will dominate $\bar{x}$ if $x_{ij}^* = \theta_{ij} > 0$.
(iii) If there are cells (i, j) and (h, k) such that

$$\theta_{ij}\big(u_i^1 + v_j^1 - c_{ij}\big) \ge \theta_{hk}\big(u_h^1 + v_k^1 - c_{hk}\big),$$
$$\theta_{ij}\big(u_i^2 + v_j^2 - \bar{d}_{ij}\big) \ge \theta_{hk}\big(u_h^2 + v_k^2 - \bar{d}_{hk}\big),$$
$$\theta_{ij}\big[Z^2\big(u_i^3 + v_j^3 - d_{ij}\big) - Z^1\big(u_i^4 + v_j^4 - e_{ij}\big)\big]$$
$$\ge \theta_{hk}\big[Z^2\big(u_h^3 + v_k^3 - d_{hk}\big) - Z^1\big(u_h^4 + v_k^4 - e_{hk}\big)\big]$$

with strict inequality in at least one of the three inequalities, then the solution resulting from introducing the cell (h, k) is dominated by the solution resulting from introducing the cell (i, j).
(iv) $\bar{x}$ is a non-dominated solution if for all non-basic cells (i, j) either

$$u_i^1 + v_j^1 - c_{ij} < 0 \quad \text{or} \quad u_i^2 + v_j^2 - \bar{d}_{ij} < 0,$$
$$\text{or} \quad \big[Z^2\big(u_i^3 + v_j^3 - d_{ij}\big) - Z^1\big(u_i^4 + v_j^4 - e_{ij}\big)\big] < 0,$$

i.e. $\bar{x}$ is a unique optimal solution for at least one of the three problems (P′), (P″) and (P‴).

Thus the character of any basic feasible solution $\bar{x}$ may be determined from (ii) and (iv). But they do not cover all possible cases. We may have the possibility when there is at least one non-basic cell for which one of the quantities $u_i^1 + v_j^1 - c_{ij}$, $u_i^2 + v_j^2 - \bar{d}_{ij}$ and $[Z^2(u_i^3 + v_j^3 - d_{ij}) - Z^1(u_i^4 + v_j^4 - e_{ij})]$ is positive and the other two negative or two positive and one negative, it being assumed that the three quantities are non-positive for all the remaining non-basic cells. In these cases, to find whether $\bar{x}$ is non-dominated or dominated, we use the following *non-dominance criterion*.

We consider the following programming problem:

$$
\begin{aligned}
\text{(P3)} \quad & \text{minimize } -q_1 - q_2 - q_3 \\
& \text{subject to } \sum_j x_{ij} = a_i^1, \quad i = 1, 2, \dots, m, && (29) \\
& \sum_i x_{ij} = b_j^1, \quad j = 1, 2, \dots, n, && (30) \\
& x_{ij} \geq 0 \qquad \text{for all } (i, j), && (31) \\
& \sum_i \sum_j c_{ij} x_{ij} + q_1 = \sum_i \sum_j c_{ij} \bar{x}_{ij}, && (32) \\
& \sum_i \sum_j \bar{d}_{ij} x_{ij} + q_2 = \sum_i \sum_j \bar{d}_{ij} \bar{x}_{ij}, && (33) \\
& \frac{\sum_i \sum_j d_{ij}^1 x_{ij} + Z^1}{\sum_i \sum_j e_{ij}^1 x_{ij} + Z^2} + q_3 = \frac{\sum_i \sum_j d_{ij}^1 \bar{x}_{ij} + Z^1}{\sum_i \sum_j e_{ij}^1 \bar{x}_{ij} + Z^2}, && (34) \\
& q_1, q_2, q_3 \geq 0. && (35)
\end{aligned}
$$

From the construction of (32–34), it is trivial to conclude that $\bar{x}$ is a non-dominated solution if and only if problem (P3) has zero optimum and is dominated otherwise.

Now, consider the dual of problem (P3) as

$$
\begin{aligned}
\text{(P4)} \quad & \text{maximize } \Bigg\{ \sum_i u_i a_i^1 + \sum_j v_j b_j^1 + s_1 \sum_i \sum_j c_{ij} \bar{x}_{ij} + s_2 \sum_i \sum_j \bar{d}_{ij} \bar{x}_{ij} \\
& \qquad + s_3 \frac{\sum_i \sum_j d_{ij}^1 \bar{x}_{ij} + Z^1}{\sum_i \sum_j e_{ij}^1 \bar{x}_{ij} + Z^2} \Bigg\} \\
& \text{subject to} \quad u_i + v_j + s_1 c_{ij} + s_2 \bar{d}_{ij} + s_3 \frac{(Z^2 d_{ij}^1 - Z^1 e_{ij}^1)}{(e_{ij}^1 x_{ij} + Z^2)^2} \leq 0, && (36) \\
& s_1 \leq -1, && (37) \\
& s_2 \leq -1, && (38) \\
& s_3 \leq -1. && (39)
\end{aligned}
$$

Consider now problem (P3) without the additional constraints (32–35) and call it (P5). Let (x, q_1, q_2, q_3) be any basic feasible solution for (P3) with basis $\widehat{B}$. Following Klingman and Russell (1975), the basis $\widehat{B}$ could be initially partitioned as

$$\widehat{B}=\begin{bmatrix} B & B_A \\ c_B & c_A \\ \bar{d}_B & \bar{d}_A \\ \Delta_B & \Delta_A \end{bmatrix},$$

where $B[(m+n)\times(m+n-1)]$ is the basis for (P5). c_B, $\bar{d}_B$ and Δ_B are row vectors containing all those components c_{ij}, $\bar{d}_{ij}$ and $(Z^2d^1_{ij}-Z^1e^1_{ij})$, respectively, for which $(i,j)\in B$. Also, $B_A[(m+n)\times 3]$ is a *zero matrix*, $c_A=[1,0,0]$, $\bar{d}_A=[0,1,0]$, $\Delta_A=[0,0,1]$ if q_1,q_2 and q_3 are in the basis, otherwise they contain the appropriate columns.

Let $\widehat{B}^G$ denote the generalized left inverse of $\widehat{B}$. To determine the kth row of $\widehat{B}^G$, we need to solve the system of equations

$$\left(\xi^T,\eta^T,t_1,t_2,t_3\right)\begin{bmatrix} B & B_A \\ c_B & c_A \\ \bar{d}_B & \bar{d}_A \\ \Delta_B & \Delta_A \end{bmatrix}=e_k^T,$$

where ξ and η are vectors of dimension m and n, respectively, t_1, t_2, and t_3 are one-component vectors and e_k is the unit vector having a **1** in the kth component.

In order to determine the vector which enters the basis, consider the following system involving the dual variables $(U=(u_i), V=(v_j), s_1,s_2,s_3)$:

$$\left(U^T,V^T,s_1,s_2,s_3\right)\begin{bmatrix} B & B_A \\ c_B & c_A \\ \bar{d}_B & \bar{d}_A \\ \Delta_B & \Delta_A \end{bmatrix}=\left[0,\ E^T\right], \tag{40}$$

where

$$E^T=\begin{cases}(-1,-1,-1) & \text{if } q_1,q_2 \text{ and } q_3 \text{ are in the basis,}\\ (0,0,0) & \text{otherwise.}\end{cases}$$

Following Klingman and Russell (1975), the dual system in (40) can be transformed into the system

$$u_i+v_j+s_1c_{ij}+s_2\bar{d}_{ij}+s_3\Delta_{ij}=0 \quad \text{for } (i,j)\in B, \tag{41}$$

$$(s_1,s_2,s_3)H=E_H^T, \tag{42}$$

where $\Delta_{ij}=Z^2d^1_{ij}-Z^1e^1_{ij}$, $H[3\times 3]$ is a non-singular matrix such that H^{-1} occupies the last three components of the last three rows in $\widehat{B}^G$ and E_H^T is a 3-component row vector such that each component of E_H^T is a linear combination of the right-hand side of (40) where exactly one component of E^T is multiplied by one and other by zero.

Now, we can determine the dual variables for problem (P3) from (41) and (42) by finding (s_1, s_2, s_3), using (42) and then the remaining variables u_i's and v_j's using

$$u_i = -s_1 u_i^1 - s_2 u_i^2, \tag{43}$$

$$v_j = -s_1 v_j^1 - s_2 v_j^2. \tag{44}$$

The variables u_i and v_j in (43) and (44) satisfy (41). Therefore, once having known the dual variables, the non-basic cell (i, j) for which

$$\begin{aligned} u_i + v_j + s_1 c_{ij} + s_2 \bar{d}_{ij} + s_3 \Delta_{ij} = &-s_1\big(u_i^1 + v_j^1 - c_{ij}\big) - s_2\big(u_i^2 + v_j^2 - \bar{d}_{ij}\big) \\ &- s_3\big[Z^2\big(u_i^3 + v_j^3 - d_{ij}\big) - Z^1\big(u_i^4 + v_j^4 - e_{ij}\big)\big] \end{aligned}$$

is most positive may be determined, so that it qualifies for entry into the basis.

Let the cell chosen be (h, k). Now, we seek the representation Y of the entering cell (h, k). Let the column of the coefficient matrix associated with cell (h, k) be denoted by $\widehat{P}_{hk} = [P_{hk}, c_{hk}, \bar{d}_{hk}, \Delta_{hk}]^T$, where P_{hk} denotes the portion of $\widehat{P}_{hk}$ associated with problem (P5) (i.e. the first $m + n$ components). Let Y_{hk} denote the first $m + n$ components of Y. Using the previous basis partitioning, we may determine Y by solving the following system of equations:

$$\begin{bmatrix} B & B_A \\ c_B & c_A \\ \bar{d}_B & \bar{d}_A \\ \Delta_B & \Delta_A \end{bmatrix} \begin{bmatrix} Y_{hk} \\ Y_{A1} \\ Y_{A2} \\ Y_{A3} \end{bmatrix} = \begin{bmatrix} P_{hk} \\ c_{hk} \\ \bar{d}_{hk} \\ \Delta_{hk} \end{bmatrix}. \tag{45}$$

Given the entire generalized left inverse $\widehat{B}^G$, the revised simplex method may be used to solve the above system of equations. However, storing and updating the matrix $\widehat{B}^G[(m + n + 2) \times (m + n + 3)]$ is extremely expensive computationally. We propose to store and update only a small portion of $\widehat{B}^G$. Let $\widehat{B}_3^G$ denote the last three rows of $\widehat{B}^G$. Then

$$\begin{bmatrix} Y_{A1} \\ Y_{A2} \\ Y_{A3} \end{bmatrix} = \widehat{B}_3^G \begin{bmatrix} P_{hk} \\ c_{hk} \\ \bar{d}_{hk} \\ \Delta_{hk} \end{bmatrix}. \tag{46}$$

Proceeding as in Gupta (1977), we can obtain

$$\begin{aligned} Y_{A1} = &-a_{11}\big(u_h^1 + v_k^1 - c_{hk}\big) - a_{12}\big(u_h^2 + v_k^2 - \bar{d}_{hk}\big) \\ &- a_{13}\big[Z^2\big(u_h^3 + v_k^3 - d_{hk}\big) - Z^1\big(u_h^4 + v_k^4 - e_{hk}\big)\big], \end{aligned} \tag{47}$$

$$\begin{aligned} Y_{A2} = &-a_{21}\big(u_h^1 + v_k^1 - c_{hk}\big) - a_{22}\big(u_h^2 + v_k^2 - \bar{d}_{hk}\big) \\ &- a_{23}\big[Z^2\big(u_h^3 + v_k^3 - d_{hk}\big) - Z^1\big(u_h^4 + v_k^4 - e_{hk}\big)\big], \end{aligned} \tag{48}$$

$$\begin{aligned} Y_{A3} = &-a_{31}\big(u_h^1 + v_k^1 - c_{hk}\big) - a_{32}\big(u_h^2 + v_k^2 - \bar{d}_{hk}\big) \\ &- a_{33}\big[Z^2\big(u_h^3 + v_k^3 - d_{hk}\big) - Z^1\big(u_h^4 + v_k^4 - e_{hk}\big)\big], \end{aligned} \tag{49}$$

where $H^{-1} = (a_{ij})$.

In order to complete the determination of Y, we solve the following system:

$$Y_{hk} = B^G P_{hk} - B^G B_A \begin{bmatrix} Y_{A1} \\ Y_{A2} \\ Y_{A3} \end{bmatrix}, \tag{50}$$

where B^G is the generalized left inverse of the basis B. It should be noted that if q_1, q_2, and q_3 are in the basis B, then we need to calculate only the representation of P_{hk} as B_A is a zero matrix.

It can be observed from the nature of problem (P3) that $(\bar{x}, q_1 = 0, q_2 = 0, q_3 = 0)$ is the initial basic feasible solution with q_1, q_2 and q_3 in the basis. Therefore, initially

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = H^{-1}, \qquad E_H^T = [-1, -1, -1], \qquad B = \bar{B}.$$

Thus using (42), we get $s_1 = -1$, $s_2 = -1$, $s_3 = -1$.

Using these values, the entering cell may be obtained as explained above, and Y_A for this column can be determined from (47–49).

To check the non-dominance character of $\bar{x}$, under the non-degeneracy assumption, we proceed as follows. If $Y_A = [Y_{A1}, Y_{A2}, Y_{A3}]^T$ is non-positive, then $\widehat{P}_{hk}$ when entered into the basis will lead to a solution for which at least one of the quantities q_1,q_2, and q_3 is positive. Therefore, $\bar{x}$ will be a dominated solution. However, if at least one component of Y_A is positive, then the simplex iteration with one of the rows of H^{-1} as the pivot row can be carried out. The efficiency of the inverse compactification depends on having an updated version of H^{-1} and (s_1, s_2, s_3), therefore we need to update only H^{-1} and (s_1, s_2, s_3). Proceeding like this, we have the following two possibilities:

(i) Zero optimum for (P3) is obtained.
(ii) There exists a column $\widehat{P}_{hk}$ for which $-s_1(u_i^1 + v_j^1 - c_{ij}) - s_2(u_i^2 + v_j^2 - \bar{d}_{ij}) - s_3[Z^2(u_i^3 + v_j^3 - d_{ij}) - Z^1(u_i^4 + v_j^4 - e_{ij})]$ is positive and Y_A is non-positive.

In case (i), $\bar{x}$ is a non-dominated solution, whereas in case (ii) it is a dominated solution.

In the next section, we summarize the solution method described above in the form of an algorithm to determine non-dominated solutions.

5 The algorithm

Step 1 For some basic feasible solution $\bar{x}$, define $\bar{d} = (\bar{d}_{ij})$. Find the dual variables (u_i^1, v_j^1), (u_i^2, v_j^2) and $\{(u_i^3, v_j^3), (u_i^4, v_j^4)\}$. Check if $\bar{x}$ is a unique optimal solution for anyone of the three problems (P′), (P″) and (P‴). If so, then $\bar{x}$ is a non-dominated solution, and go to Step 4. Otherwise, go to Step 2.

Step 2 Choose a non-basic cell (i, j) for which

$$\left(u_i^1+v_j^1-c_{ij}\right)\geq 0, \qquad \left(u_i^2+v_j^2-\bar{d}_{ij}\right)\geq 0,$$
$$\left[Z^2\left(u_i^3+v_j^3-d_{ij}\right)-Z^1\left(u_i^4+v_j^4-e_{ij}\right)\right]\geq 0$$

with one of them strictly positive and $\theta_{ij}>0$. If such a cell exists, then $\bar{x}$ is a dominated solution. Introduce this cell (i,j) into the basis, and go to Step 1 if it leads to an unexplored basis, otherwise go to Step 4. On the other hand, if no such cell exists, then go to Step 3.

Step 3 Use the following non-dominance criterion (S) to check whether $\bar{x}$ is dominated or non-dominated:

(i) For any solution $\bar{x}$, choose $H^{-1}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$, $s_1=-1$, $s_2=-1$, $s_3=-1$, go to (ii).

(ii) Find the cell (i,j) for which $-s_1(u_i^1+v_j^1-c_{ij})-s_2(u_i^2+v_j^2-\bar{d}_{ij})-s_3[Z^2(u_i^3+v_j^3-d_{ij})-Z^1(u_i^4+v_j^4-e_{ij})]$ is most positive. If there is none, then $\bar{x}$ is a non-dominated solution, and go to Step 4, otherwise go to (iii).

(iii) Let (h,k) be the cell chosen at Step 3(ii). Find Y_{A1}, Y_{A2}, Y_{A3} using (47), (48) and (49).

Then we have the following cases:

(a) at least one of three quantities Y_{A1}, Y_{A2}, Y_{A3} is positive. Perform the simplex iteration with largest positive component taken as the pivot element. Update H^{-1} and (s_1,s_2,s_3) and go to Step 3(ii).

(b) (Y_{A1}, Y_{A2}, Y_{A3}) is non-positive and $\bar{x}$ is non-degenerate. In this case, $\bar{x}$ is a dominated solution, and go to Step 4.

(c) (Y_{A1}, Y_{A2}, Y_{A3}) is non-positive and $\bar{x}$ is degenerate. Let rth component of $\bar{x}$ be zero. Find $(Y_{hk})_r$. If $(Y_{hk})_r\leq 0$, then $\bar{x}$ is a dominated solution. Otherwise, perform the simplex iteration with rth component as the pivot element. Update H^{-1} and (s_1,s_2,s_3). Determine (u_i^1,v_j^1), (u_i^2,v_j^2) and $\{(u_i^3,v_j^3),(u_i^4,v_j^4)\}$ for the new basis for (P5), as $\bar{B}$ will no longer remain the basis for (P5). Go to Step 3(ii).

Step 4 Form the set R of those non-basic cells which lead to an unexplored solution such that it is not dominated by $\bar{x}$. If R is non-empty, then go to Step 5, otherwise terminate.

Step 5 Examine all cells in R to determine a cell that dominates all other solutions reachable from $\bar{x}$. Introduce this cell into the basis and go to Step 1.

6 Numerical example

We have collected real data for the transportation problem from the company, but we do not disclose units of data for confidentiality. The supplies of the three suppliers and demands of the four plants have been determined by planning department of

Table 1 c_{ij}

	BSP	DSP	RSP	BSL	a_i
Vizag	15	30	35	16	14
Paradip	9	10	12	4	18
Haldia	20	19	20	17	25
b_j	10	14	20	13	

Table 2 t_{ij}

	BSP	DSP	RSP	BSL
Vizag	8	14	15	9
Paradip	4	5	6	2
Haldia	11	10	11	9

Table 3 Ratio of ash content to the production of hot metal

	BSP		DSP		RSP		BSL	
Vizag	1	4	1	2	1	3	1	1
Paradip	1	1	2	3	1	2	1	3
Haldia	1	5	1	1	2	4	1	2

the company. It has been observed that supply and demand parameters have shown fluctuations in past. We use triangular fuzzy numbers in *Case* 1 to incorporate uncertainties in the data keeping the most likely estimate at the center and consider changes around the central values. In order to have a lower bound on degree of satisfaction, say 0.9, we need to tighten the left and right tolerances with respect to the central values in the triangular fuzzy numbers, achieved by using *level* (0.9, 1) *i-v fuzzy numbers* in *Case* 2.

We consider the following three cases:

Case 1: Let $\widetilde{a}_1 = (14-1, 14, 14+2; 1)$, $\widetilde{a}_2 = (18-2, 18, 18+1; 1)$,
$\widetilde{a}_3 = (25-1, 25, 25+3; 1)$, $\widetilde{b}_1 = (10-1, 10, 10+2; 1)$,
$\widetilde{b}_2 = (14-1, 14, 14+2; 1)$, $\widetilde{b}_3 = (20-2, 20, 20+3; 1)$ and
$\widetilde{b}_4 = (13-2, 13, 13+1; 1)$.

We obtain the following defuzzified transportation problem:

$$\text{minimize } F = \left\{ \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij}, \operatorname{Max}_{\{(i,j)/x_{ij}>0\}} t_{ij}(x_{ij}), \frac{\sum_{i=1}^{3}\sum_{j=1}^{4} d_{ij}x_{ij}}{\sum_{i=1}^{3}\sum_{j=1}^{4} e_{ij}x_{ij}} \right\}$$

$$\begin{aligned}
\text{subject to}\quad & x_{11}+x_{12}+x_{13}+x_{14}=14.25,\\
& x_{21}+x_{22}+x_{23}+x_{24}=17.75,\\
& x_{31}+x_{32}+x_{33}+x_{34}=25.5,\\
& x_{11}+x_{21}+x_{31}=10.25,\\
& x_{12}+x_{22}+x_{32}=14.25,\\
& x_{13}+x_{23}+x_{33}=20.25,\\
& x_{14}+x_{24}+x_{34}=12.75,\\
& x_{ij}\geq 0,\quad i=1,2,3,\ \ j=1,2,3,4.
\end{aligned}$$

Step 1 We obtain the first basic feasible solution x^0 given by $x^0_{11}=10.25, x^0_{12}=4, x^0_{22}=10.25, x^0_{24}=7.5, x^0_{33}=12.75, x^0_{34}=12.75$. According to the solution x^0, we have

$$F=\left(\sum_i\sum_j c_{ij}x^0_{ij}, t^0, \frac{\sum_i\sum_j d_{ij}x^0_{ij}}{\sum_i\sum_j e_{ij}x^0_{ij}}\right)=\left(938,\ 14,\ \frac{80.5}{171.25}=0.47\right). \tag{51}$$

Step 2 x^0 is a dominated solution because

$$u^1_2+v^1_4-c_{24}>0,\ u^2_2+v^2_4-\bar{d}_{24}=0 \text{ and } Z^2(u^3_2+v^3_4-d_{24})-Z^1(u^4_2+v^4_4-e_{24})>0.$$

Therefore, cell $(2,4)$ enter into the basis.

Further, following steps of the algorithm given in Sect. 5, we obtain different non-dominated solutions, which are listed below:

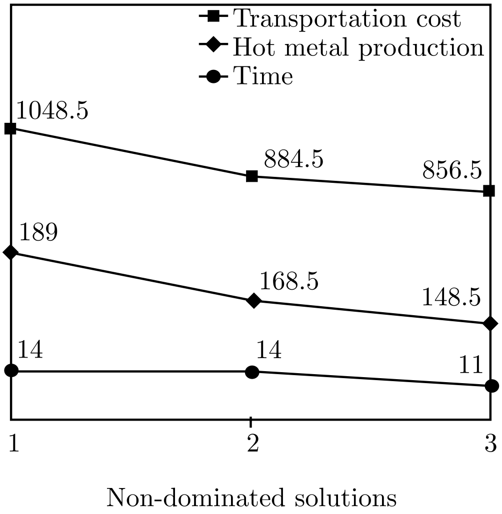
x^1: $x^1_{11}=0, x^1_{12}=14.25, x^1_{23}=5, x^1_{24}=12.75, x^1_{31}=10.25, x^1_{33}=15.25$ and $F=(1048.5,\ 14,\ \frac{72.75}{189}=0.3849)$;

x^2: $x^2_{11}=10.25, x^2_{12}=4, x^2_{23}=5, x^2_{24}=12.75, x^2_{32}=10.25, x^2_{33}=15.25$ with $F=(884.5,\ 14,\ \frac{72.75}{168.5}=0.4318)$;

x^3: $x^3_{11}=10.25, x^3_{14}=4, x^3_{23}=9, x^3_{24}=8.75, x^3_{32}=14.25, x^3_{33}=11.25$ with $F=(856.5,\ 11,\ \frac{68.75}{148.5}=0.4630)$.

Case 2: Let $\widetilde{a}^L_1=(14-3,14,14+1;0.9)$, $\widetilde{a}^L_2=(18-5,18,18+1;0.9)$,
$\widetilde{a}^L_3=(25-5,25,25+1;0.9)$, $\widetilde{b}^L_1=(10-1,10,10+1;0.9)$,
$\widetilde{b}^L_2=(14-3,14,14+1;0.9)$, $\widetilde{b}^L_3=(20-6,20,20+2;0.9)$,
$\widetilde{b}^L_4=(13-5,13,13+1;0.9)$, $\widetilde{a}^U_1=(14-4,14,14+8;1)$,
$\widetilde{a}^U_2=(18-6,18,18+8;1)$, $\widetilde{a}^U_3=(25-6,25,25+8;1)$,
$\widetilde{b}^U_1=(10-2,10,10+2;1)$, $\widetilde{b}^U_2=(14-4,14,14+8;1)$,
$\widetilde{b}^U_3=(20-7,20,20+9;1)$, $\widetilde{b}^U_4=(13-6,13,13+8;1)$.
We get $\widetilde{a}_i=[\widetilde{a}^L_i,\widetilde{a}^U_i]$, $i=1,2,3$ and $\widetilde{b}_j=[\widetilde{b}^L_j,\widetilde{b}^U_j]$, $j=1,2,3,4$.

Fig. 2 Graphical representation of Case 1



We obtain the following defuzzified transportation problem:

$$\text{minimize } F=\left\{\sum_{i=1}^{3}\sum_{j=1}^{4}c_{ij}x_{ij},\, \mathrm{Max}_{\{(i,j)/x_{ij}>0\}}\, t_{ij}(x_{ij}),\, \frac{\sum_{i=1}^{3}\sum_{j=1}^{4}d_{ij}x_{ij}}{\sum_{i=1}^{3}\sum_{j=1}^{4}e_{ij}x_{ij}}\right\}$$

subject to

$$x_{11}+x_{12}+x_{13}+x_{14}=14.2,$$
$$x_{21}+x_{22}+x_{23}+x_{24}=17.9125,$$
$$x_{31}+x_{32}+x_{33}+x_{34}=24.9125,$$
$$x_{11}+x_{21}+x_{31}=10,$$
$$x_{12}+x_{22}+x_{32}=14.2,$$
$$x_{13}+x_{23}+x_{33}=19.9125,$$
$$x_{14}+x_{24}+x_{34}=12.9125,$$
$$x_{ij}\geq 0,\quad i=1,2,3,\ \ j=1,2,3,4.$$

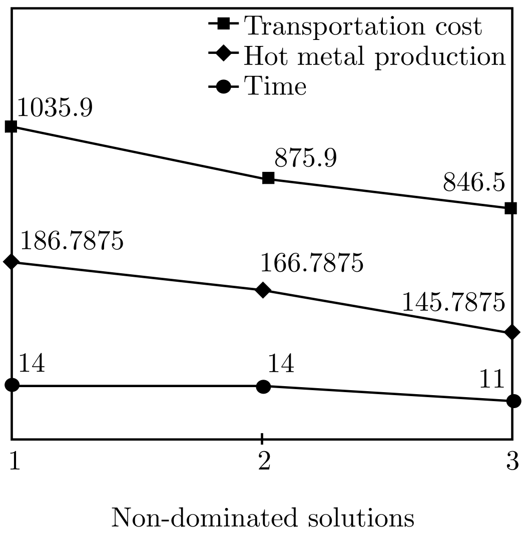
We have obtained different non-dominated solutions, which are listed below:

x^1: $x_{11}^1=0, x_{12}^1=14.2, x_{23}^1=5, x_{24}^1=12.9125, x_{31}^1=10, x_{33}^1=14.9125$ and $F=(1035.9, 14, \frac{71.9375}{186.7875}=0.3851)$;

x^2: $x_{11}^2=10, x_{12}^2=4.2, x_{23}^2=5, x_{24}^2=12.9125, x_{32}^2=10, x_{33}^2=14.9125$ and $F=(875.9, 14, \frac{71.9375}{166.7875}=0.4313)$;

x^3: $x_{11}^3=10, x_{14}^3=4.2, x_{23}^3=9.2, x_{24}^3=8.7125, x_{32}^3=14.2, x_{33}^3=10.7125$ and $F=(846.5, 11, \frac{67.7375}{145.7875}=0.4646)$.

Fig. 3 Graphical representation of Case 2



Case 3: We consider the following crisp transportation problem:

$$\text{minimize } F = \left\{ \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij}, \text{Max}_{\{(i,j)/x_{ij}>0\}}\, t_{ij}(x_{ij}), \frac{\sum_{i=1}^{3}\sum_{j=1}^{4} d_{ij}x_{ij}}{\sum_{i=1}^{3}\sum_{j=1}^{4} e_{ij}x_{ij}} \right\}$$

subject to

$$x_{11} + x_{12} + x_{13} + x_{14} = 14,$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 18,$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 25,$$
$$x_{11} + x_{21} + x_{31} = 10,$$
$$x_{12} + x_{22} + x_{32} = 14,$$
$$x_{13} + x_{23} + x_{33} = 20,$$
$$x_{14} + x_{24} + x_{34} = 13,$$
$$x_{ij} \geq 0, \quad i = 1, 2, 3, \ \ j = 1, 2, 3, 4.$$

We have obtained the following non-dominated solutions:

x^1: $x^1_{11} = 0, x^1_{12} = 14, x^1_{23} = 5, x^1_{24} = 13, x^1_{31} = 10, x^1_{33} = 15$ and $F = (1032, 14, \frac{72}{187} = 0.3850)$;

x^2: $x^2_{11} = 10, x^2_{12} = 4, x^2_{23} = 5, x^2_{24} = 13, x^2_{32} = 10, x^2_{33} = 15$ and $F = (872, 14, \frac{72}{167} = 0.4311)$;

x^3: $x^3_{11} = 10, x^3_{14} = 4, x^3_{23} = 9, x^3_{24} = 9, x^3_{32} = 14, x^3_{33} = 11$ and $F = (844, 11, \frac{68}{147} = 0.4626)$.

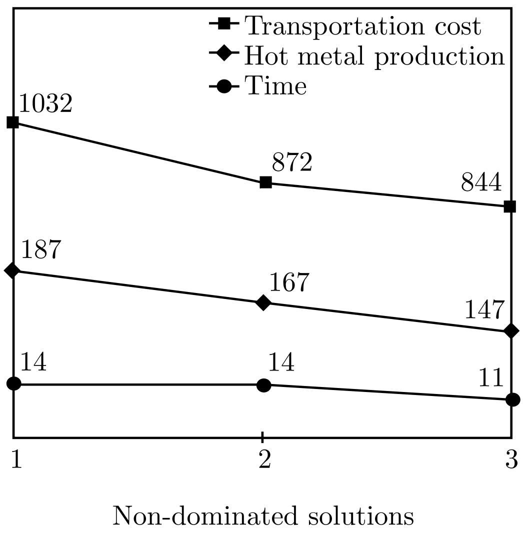


Fig. 4 Graphical representation of Case 3

Table 4

j	Case	Transportation cost (Z_j)	$r_{j,j-1} = \dfrac{Z_j - Z_{j-1}}{Z_{j-1}} \times 100\%$	Remark
1	Crisp Case	844		No statistical data in Case 1
2	Level 1 fuzzy numbers	856.5	$r_{21} = 1.48$	The fluctuation is significant for this fuzzy case
3	Level (0.9, 1) fuzzy numbers	846.5	$r_{31} = 0.296$ $r_{32} = -1.1675$	Combined by level 1 fuzzy number in Case 2 and level 0.9 fuzzy number

7 Discussion

As shown in Table 4, the changes in transportation cost are significant whenever fluctuation is large in the fuzzy case. The transportation cost is close to the crisp case whenever variation is small in the fuzzy case.

Table 5 presents the relative changes in the ratio of ash content to the production of hot metal corresponding to the two fuzzy cases with respect to the crisp case.

According to earlier transportation plans, it was reported that the unit transportation cost incurred was 868.5433. However, the company was not meeting the target production of hot metal set at 162. Our study shows that the company can meet target production of hot metal in all three cases with a small increase in unit transportation cost. The solutions thus arrived lead to some trade-off between hot metal production and unit transportation cost, which the decision makers have to attend to with reference to their value systems—some managers may value cost saving more than meeting the production targets and vice versa. Our approach, therefore, provides space for accommodating decision makers' value systems. Having said so, it should

Table 5

i	Case	Ratio of ash content to the hot metal production (R_i)	$r_{i,i-1} = \dfrac{R_i - R_{i-1}}{R_{i-1}} \times 100\%$
1	Crisp case	0.3850	
2	Level 1 fuzzy numbers	0.3849	$r_{21} = -0.0260$
3	Level (0.9, 1) fuzzy numbers	0.3851	$r_{31} = 0.0260$ $r_{32} = 0.0520$

be noted that these results are theoretical, and hence the actual impact of the implementation of these solutions may be affected by various factors impinging upon the working environment of the company. Nevertheless, for large companies, even small improvements in the current transportation plan may result in huge savings. In the present study, the decision makers have more than one solution to consider given the circumstances they find themselves in instead of having to contend with the optimal fuzzy compromise solution obtained using an aggregation operator, e.g. "min", "product".

8 Concluding remarks

(i) In this study, we have examined the decision making process followed on the use of a new variety of coal to meet the target production of hot metal in a large steel manufacturing unit in India. The numerical tests performed here seem promising. However, we do acknowledge that their actual implementation may be affected by some exogenous factors.

(ii) In practice, the availability and demand in manufacturing systems need not always be 100% achieved and at the same time they will not be 0%. However, the decision makers are interested in maximizing their degree of satisfaction. We have selected a range in which the degree of satisfaction lies between 0.9 and 1, represented conveniently by using (λ, ρ) *i-v* fuzzy sets. We have used linear ranking function to defuzzify the fuzzy transportation problem retaining the transportation structure of the original problem.

(iii) In crisp transportation problem (P), if a_i, $i = 1, 2, \ldots, m$, and b_j, $j = 1, 2, \ldots, n$, are unknown, then we can get the point estimates $\bar{a}_i = \frac{1}{N}\sum_{p=1}^{N} a_{ip}$ and $\bar{b}_j = \frac{1}{N}\sum_{p=1}^{N} b_{jp}$ of a_i and b_j, respectively, from the statistical data in the past. Let their variances be S_{ia}^2 and S_{ib}^2, respectively. Since the probabilities of the error between $\bar{a}_i$ and a_i and the error between $\bar{b}_j$ and b_j are unknown, we use a statistical confidence interval. We can set *i-v* fuzzy number corresponding to this interval. For any $i \in \{1, 2, \ldots, m\}$ and $j \in \{1, 2, \ldots, n\}$, corresponding to $(1-\alpha) \times 100\%$ and $(1-\beta) \times 100\%$ confidence intervals of a_i and b_j, we set level $1-\alpha$ fuzzy numbers: $\widetilde{a}_i^U, \widetilde{b}_j^U$ and level $1-\beta$ fuzzy numbers $\widetilde{a}_i^L, \widetilde{b}_j^L$. Then we obtain the following *level*

$(1-\beta, 1-\alpha)$ *i-v fuzzy numbers*: $\widetilde{a}_i = [\widetilde{a}_i^L, \widetilde{a}_i^U]$ and $\widetilde{b}_j = [\widetilde{b}_j^L, \widetilde{b}_j^U]$, where

$$\widetilde{a}_i^L = \big(\bar{a}_i - Z_i(\beta_1), \bar{a}_i, \bar{a}_i + Z_i(\beta_2); 1-\beta\big),$$
$$\widetilde{a}_i^U = \big(\bar{a}_i - Z_i(\alpha_1), \bar{a}_i, \bar{a}_i + Z_i(\alpha_2); 1-\alpha\big),$$
$$\widetilde{b}_j^L = \big(\bar{b}_j - Z_j(\beta_3), \bar{b}_j, \bar{b}_j + Z_j(\beta_4); 1-\beta\big),$$
$$\widetilde{b}_j^U = \big(\bar{b}_j - Z_j(\alpha_3), \bar{b}_j, \bar{b}_j + Z_j(\alpha_4); 1-\alpha\big),$$

$0<\alpha<\beta<1, 0<\alpha_k<\beta_k<1, k=1,2,3,4, \alpha_1+\alpha_2=\alpha, \alpha_3+\alpha_4=\alpha, \beta_1+\beta_2=\beta, \beta_3+\beta_4=\beta$. $t_{N-1}(r_k)$ is the r_k point of the t-distribution with degree $N-1$, i.e. $Z_i(r_k)=t_{N-1}(r_k)\frac{S_{ia}}{\sqrt{N}}$, $k=1,2$, and $Z_j(r_k)=t_{N-1}(r_k)\frac{S_{jb}}{\sqrt{N}}$, $k=3,4$, $r_k=\alpha_k$ or β_k. Thus, using statistical data, we can obtain a fuzzy transportation problem.

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