TEST (2024) 33:71–72
https://doi.org/10.1007/s11749-023-00918-2

DISCUSSION

Comments on: Shape-based functional data analysis by Wu, Huang and Srivastava

Lajos Horváth[1]

Received: 18 December 2023 / Accepted: 20 December 2023 / Published online: 9 February 2024

Just looking at the graphs of two functions, one could decide whether the functions have the same "shape". Of course, the opinions might differ. As a mathematical problem, it is hard since one needs a good definition of similar shapes. The authors provide a good summary of results in Hilbert spaces. Functional data analysis is a common practice to analyse functional observations. There are several possibilities how to use project the observations into a final dimensional space. The typical method is to use the covariance kernel, the long-run covariance kernel and there are further possibilities to choose an orthogonal bases. Horváth and Rice (2015) point out that the choice of the kernel used in the projection if a change in the mean is detected in functional analysis of variance problems. I agree with the authors that Hilbert space methods might not be the best to define "shape" of objects or similarity of objects. Curve fitting with a penalty term is usually better. The most popular method should be LASSO for functional data but since L_1 norm is used, now the observations in a Banach space. The other possibility is to use the space of continuous functions to try to define similarity of curves. We might think that two curves are similar of the number of the peaks is the same. The authors are correct that visibly different curves might have this property. Time-warping transformation might show similarity of curves. They suggest that elements of the set of all orientation-preserving diffeomorphisms of the support to itself are used as possible warping transformations. It is interesting that they define the shape of a function as a set. It is like using a wide brush on the functions. Due to the definition, it is hard or impossible to find a whole set. Hence, the mathematical theory based on Definition 1 is a challenging problem. Results from algebraic geometry might provide some tools. The present paper provides an excellent argument why the development of mathematical tools would be important and useful. In addition to the number peaks, the rates of the increase and decrease of the curves are important when someone want to say that the curves are similar. Hence the derivatives of the curves should be used in the fitting and/or in the penalty term. Sobolev spaces might be useful to develop theory.

✉ Lajos Horváth
horvath@math.utah.edu

[1] University of Utah, Salt Lake City, UT, USA

The paper provides a readable summary of the methods and challenges how shape analysis should be done. The examples show what can go wrong and when the suggested method works. I am sure that this review paper increases the interest in shape analysis from applies as well as from theoretical point of view.

Horváth, L. and Rice, G.: Testing equality of means when the observations are from functional time series. *Journal of Time Series Analysis* **36**(2015), 84–108.