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CORRECTION

Correction to: On the concept of *B*-statistical uniform integrability of weighted sums of random variables and the law of large numbers with mean convergence in the statistical sense

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Correction to: TEST 30, 83–102 (2021) https://doi.org/10.1007/s11749-020-00706-2

Professor Lê Văn Thành (Vinh University, Nghe An, Viet Nam) has so kindly called to the authors' attention an error in their original article. On line 6 of page 89, the authors incorrectly asserted in the proof of (i) in the necessity half of Theorem 1 that

$$\left\{ n \in \mathbb{N} : \sum_{k=1}^{\infty} |a_{nk}| E|X_k| I_{\{|X_k| \le a\}} > a \right\} = \varnothing.$$

The argument for establishing (i) is corrected as follows.

By (2), we can choose M such that

$$\sup_{n \in \mathbb{N}} {}_{\mathrm{st}_B} \sum_{k=1}^{\infty} |a_{nk}| < M < \infty. \tag{*}$$

The original article can be found online at https://doi.org/10.1007/s11749-020-00706-2.

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Note that with $a > 0$ as in (4),

$$\left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} |a_{nk}| E|X_k| > Ma + \frac{\varepsilon}{2}\right\} \tag{**}$$
$$\subset \left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} |a_{nk}| E|X_k| I_{\{|X_k| \le a\}} > Ma\right\}$$
$$\bigcup \left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} |a_{nk}| E|X_k| I_{\{|X_k| > a\}} > \frac{\varepsilon}{2}\right\}$$
$$\subset \left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} |a_{nk}| > M\right\}$$
$$\bigcup \left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} |a_{nk}| E|X_k| I_{\{|X_k| > a\}} > \frac{\varepsilon}{2}\right\}.$$

Combining (*), (4), and (**), we obtain that

$$0 \le \delta_B\left(\left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} |a_{nk}| E|X_k| > Ma + \frac{\varepsilon}{2}\right\}\right)$$
$$\le \delta_B\left(\left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} |a_{nk}| > M\right\}\right)$$
$$+ \delta_B\left(\left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} |a_{nk}| E|X_k| I_{\{|X_k| > a\}} > \frac{\varepsilon}{2}\right\}\right)$$
$$= 0 + 0 = 0$$

and so

$$\delta_B\left(\left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} |a_{nk}| E|X_k| > Ma + \frac{\varepsilon}{2}\right\}\right) = 0.$$

Thus, the real number $Ma + \frac{\varepsilon}{2}$ is a B-statistical upper bound of the sequence $\left\{\sum_{k=1}^{\infty} |a_{nk}| E|X_k| : n \in \mathbb{N}\right\}$. Hence (i) holds.

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