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Workspace of Translation 3-UPU Parallel Manipulators

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Abstract To determine workspace and relationship between the workspace and geometry of parallel manipulator is important for optimum design of parallel manipulators. In this paper, the workspace and the relationship between the workspace and the geometry of 3-UPU parallel manipulators with pure translation are investigated. Geometric and non-geometric constraints are defined and taken account of in determining the workspace of the translation 3-UPU manipulators. A direct average condition number is used as the global performance index of the workspace. This research shows that there exists an optimal value of the direct average condition number favorable for a good design of parallel mechanisms. The results presented in this paper are useful for the optimum design of 3-UPU parallel manipulators.

Key words 3-UPU parallel manipulator, workspace, translation.

1 Introduction

Many recent researches are focused on parallel manipulators due to its advantages of better load-carrying capacity, better stiffness, and better precision compared to a serial manipulator. Hunt[1] is among those who first proposed to use the parallel manipulator as a robotic mechanism. However these advantages are achieved at the expense of a reduced workspace, difficulty in mechanical design, and more complex kinematics and control algorithms. So it is important to obtain an applicable workspace of the parallel manipulator with optimum geometry design.

A parallel manipulator consists of a moving platform and a fixed base connected by more than one limbs. There are many kinds of parallel manipulators, in which the parallel manipulator with 3 degrees of freedom (DOF), termed a tripod, is important.

The basic topologic structure of a tripod mechanism consists of a moving platform and a fixed base connected by three limbs. Each limb has a single actuator. There are many kinds of tripod configurations[2-6]. The moving platform can move with pure translation[2,3,6], pure spherical rotation[4,5] or in hybrid motion of translation and spherical rotation, with respect to a fixed base.

Gregorio[2-4], Karouia[5] and Tsai[7] presented several 3-UPU mechanisms, as shown in Fig. 1. The 3-UPU mechanism features interconnection between platform and base by three serial kinematical chains of type UPU, where U stands for universal joint, and P for the prismatic pair which is actuated independently. When certain assembly or geometric conditions are satisfied, 3-UPU mechanisms can realize pure translation[2,3] or pure spherical rotation[4,5].

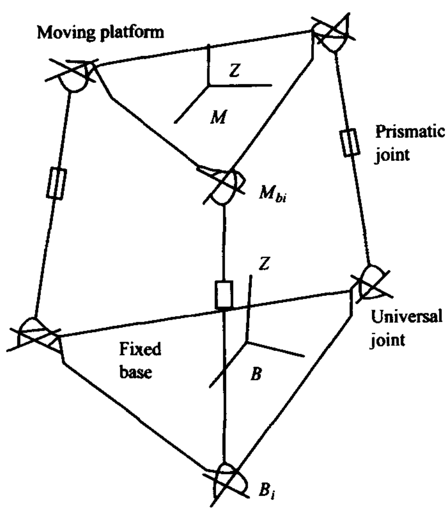

Fig.1 The sketch of 3-UPU parallel manipulators

The 3-UPU translation parallel manipulator is made of two equilateral triangular plates connected at cor-

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ners with three identical limbs. By properly orienting the axes of universal joints, the moving platform can have only translation motion. The mobility and singularity of this manipulator have been studied in Refs. [2,3]. Direct kinematical analyses have been studied in Ref. [7].

The workspace of parallel manipulators has been studied extensively[8-11]. For the 3-UPU manipulators, Tsai[9] and Badescu[11] studied the isotropy and the workspace. In Tsai's work, no angle constraints were considered for the universal joints. In Badescu's work, however, the shape of workspace was not properly described.

In this paper, workspace analysis of translation 3-UPU parallel manipulators with geometric and non-geometric constraints is performed. The workspace is parameterized using two geometric parameters, the minimum link length and the radius difference between the two circum-circles of the fixed base and the moving platform of 3-UPU manipulators. The workspace volumes corresponding to different combinations of varied constraints and the two parameters are calculated. A global performance index, the average condition number, is applied to evaluate the workspace of different geometries. Based on these results, an optimal design of 3-UPU parallel manipulators for a given application is obtained.

2 Geometric and Non-geometric Constrains

A base coordinate system $\{B; x, y, z\}$, which is a reference frame, is placed at the fixed base center B with its Z-axis perpendicular to the base plane. Similarly, the moving coordinate system $\{M; x, y, z\}$ is fixed to moving platform at the platform center M, shown in Fig.1. The corners of the two equilateral triangles are denoted B_1 through B_3 and M_{b1} through M_{b3}, respectively.

2.1 Geometric constraints

For the translation 3-UPU parallel manipulators, limb length limitations and universal joint angle constraints are viewed as geometric constraints.

Since the moving platform is assumed to be parallel to the fixed base for the sake of simplicity, the transformation unit quaternion of frame $\{B\}$ with respect to frame $\{M\}$ equals 1, *i.e.* $q = 1$. The link $\boldsymbol{l}_i$ for $i = 1,2,3$ can be expressed as:

$$\boldsymbol{l}_i = \boldsymbol{BM} + \boldsymbol{MM}_{bi} - \boldsymbol{BB}_i, \tag{1}$$

Let $\boldsymbol{D}_i = \boldsymbol{MM}_{bi} - \boldsymbol{BB}_i$, $\boldsymbol{P}_i = \boldsymbol{BM}$, Eq. (1) can be written as:

$$\boldsymbol{l}_i = \boldsymbol{P} + \boldsymbol{D}_i. \tag{2}$$

Since the fixed base and moving platform are two equilateral triangles, the norm of $\boldsymbol{D}_i$ is really the radius difference of two circum-circles of the two triangles, *i.e.*, $D = \| \boldsymbol{D}_i \| = R_B - R_M$, where R_B and R_M are radius of the fixed base and moving platform respectively. From Eq. (2), it is clear that configurations of the parallel manipulator is fully determined by two parameters, the link length and the radius difference. So its workspace is a function of the two geometric parameters.

Thus, the link length, denoted by l_i, are given by

$$l_i = \| \boldsymbol{P} + \boldsymbol{D}_i \|. \tag{3}$$

The link length constraints can be described as:

$$l_{min} \leqslant l_i \leqslant l_{max}. \tag{4}$$

The angle, θ_i, between the link and the Z-axis of frame $\{B\}$ is used to be the angle limitation of universal joints. Denoted unit vectors $\boldsymbol{u}_i = \boldsymbol{l}_i / l_i$, its value can be computed as:

$$\theta_i = \arccos(\boldsymbol{u}_i \cdot \boldsymbol{k}), \tag{5}$$

where $\boldsymbol{k}$ is the unit vector along the Z-axis. The angle limitations are expressed as:

$$\theta_i \leqslant \theta_{max}. \tag{6}$$

2.2 Non-geometric constraints

The Jacobian matrix of a parallel manipulator is defined as the matrix representing the transformation mapping of the actuated joint rates into the moving platform velocities. This transformation in the 3-UPU manipulators is written as:

$$\boldsymbol{J}\dot{l} = \dot{\boldsymbol{P}} \text{ or } \dot{l} = \boldsymbol{J}^{-1}\dot{\boldsymbol{P}}, \tag{7}$$

where $\dot{l} = [\dot{l}_1 \ \dot{l}_2 \ \dot{l}_3]^T$ is the rate of link length change, $\dot{\boldsymbol{P}}$ is velocity vector of the moving platform. For the 3-UPU parallel manipulators, the Jacobian matrix, $\boldsymbol{J}$, is of the following form:

$$\boldsymbol{J}^{-1} = \{\boldsymbol{u}_i\}. \tag{8}$$

For the controllable motion of parallel manipulator, the Jacobian matrix should be non-singular:

$$\det(\boldsymbol{J}) \neq 0. \tag{9}$$

Since Jacobian matrix of force is the transpose of matrix of kinematics, condition (9), which is satisfied for the motion, is also satisfied for force or moment controllable state.

2.3 Workspace definition

Because the 3-UPU parallel manipulators are pure translation, its workspace is composed of only reachable points of the moving platform. The set of reachable points of the center of the moving platform is viewed as a 3-UPU manipulators workspace.

Research on workspace can be classified into two parts. One is to search the workspace of a given mechanism. The other is to evaluate the performance of the obtained workspace. For the evaluation of the workspace, Badescu[11] and others[9] have presented many performance indices, wherein the condition number of Jacobian matrix plays an important role in workspace evaluation. The condition number of matrix is defined as:

$$k = \| \boldsymbol{J} \| \cdot \| \boldsymbol{J}^{-1} \|, \tag{10}$$

where $\| \cdot \|$ denotes the norm 2 of a matrix.

The condition number of a matrix is always no less than 1. In a linear system, the relative accuracy between input and output is determined with the condition number. For a large condition number, a small relative error in the input will produce a large relative error in the output, and vice versa. Therefore the solution reliability of a linear system is low with a large condition number. A linear system with a small condition number is of a homologous configuration. Especially when the condition number equals 1, the matrix is called isotropy matrix, and the configuration of the linear system is called isotropy. It is obvious that, for either serial or parallel manipulators, condition numbers in the workspace of manipulators are different from position to position. Hence it is difficult to evaluate the performance of workspace of a parallel manipulator with only one index. In this paper, it is the direct average condition number, but not the inverse average in Ref. [9], that used to be the global performance index of the translation 3-UPU parallel manipulators. It is defined as follows:

$$\eta = \frac{A}{B}, \tag{11}$$

where

$$A = \int_W k \mathrm{d}W, \quad B = \int_W \mathrm{d}W, \tag{12}$$

in which W is the workspace of the parallel manipulator, k is the condition number at a particular point of W, and the denominator B is the volume of the workspace. It is evident that a better behaving manipulator corresponds to a smaller average condition number.

3 Algorithm for Determining the Workspace

Because the 3-UPU parallel manipulators have only translation, its workspace is defined as the set of reachable locations of the center of moving platform in frame $\{B\}$. Thus the workspace can be described as set intersection of several sets as follows:

$$W = W(1) \cap W(2) \cap W(3), \tag{13}$$

where $W(1)$ is the set satisfying condition $W(4)$, $W(2)$ is the set satisfying condition (6), and W(3) is the set satisfying condition (9).

For a given geometry and limitations of link length and universal joint angle, the workspace can be searched as follows where the position vector $\boldsymbol{P}$ is denoted (X, Y, Z):

(1) Set $Z = 0$. The maximum Z can be obtained as: $z_{\max} = \sqrt{l_{\max}^2 - D^2}$, where D is defined in Section 2.1.

(2) Determine the maximum square parallel to the xy-plane defined by Z and the maximum link length $l_{\max}$. The lateral of the square can be calculated as: $x_{\max} = D + \sqrt{l_{\max}^2 - Z^2}$.

(3) For a given interval, δ, of X and Y, search the available vector $\boldsymbol{P}$, which is an element in the workspace $\boldsymbol{W}$ and meets conditions (4, 6, 9) within $[-x_{\max}, x_{\max}]$. Increase the number of available vectors by 1 and the workspace volume by δ^3.

(4) Increase Z by the given interval, δ, repeat steps (2) and (3), and keep searching until $Z = z_{\max}$.

In the virtue of Eqs. (11) and (12) and the above algorithm, the average condition number can be obtained as follows:

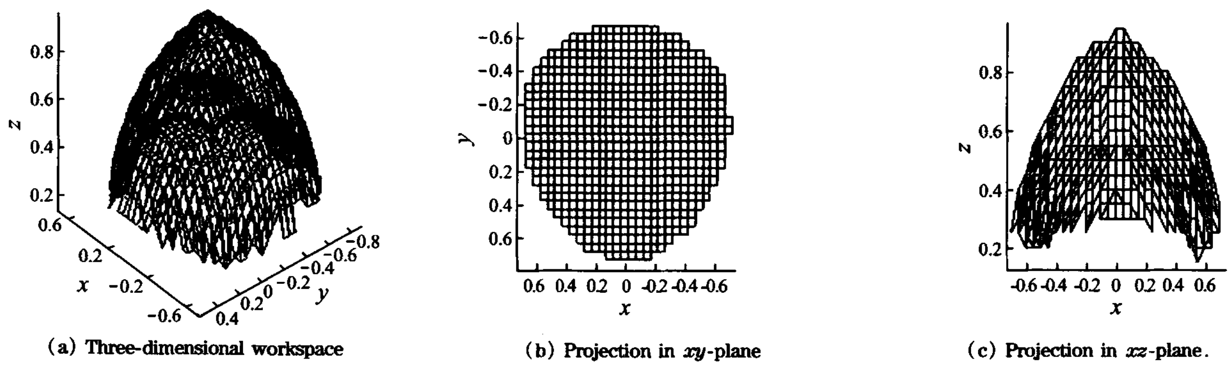
$$\eta = (\sum_W k)/N, \tag{14}$$

where N is the total number of available vectors or the total number of points in the workspace.

4 Results

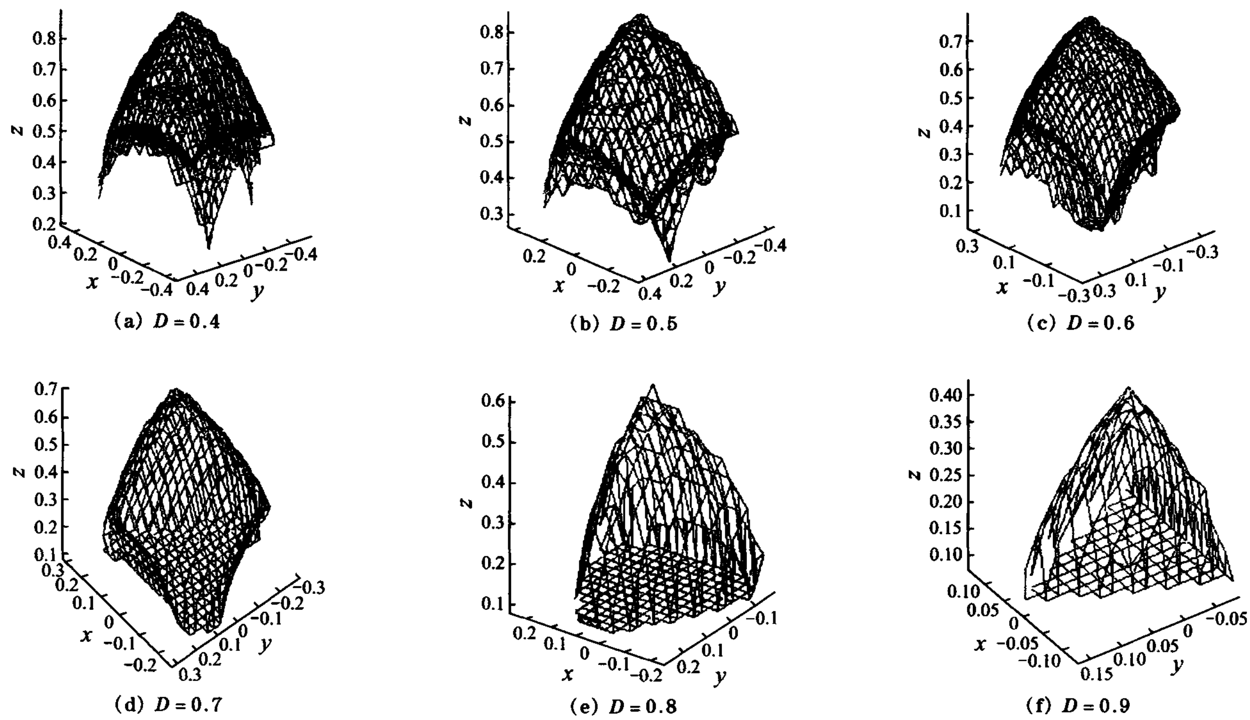
A normalized method is used to search the workspace of the 3-UPU parallel manipulators. Let $l_{max} = 1$, the radius difference D and the minimum link length l_{min} are normalized based on l_{max}. Thus the link stroke is $s = l_{max} - l_{min}$. With D varying within [0.27, 0.93], l_{min} in [0.54, 0.84] and angle constraints in [50°, 100°], the workspace is calculated based on various combinations of D and l_{min}, and the angle constraints in this paper. These results are shown from Fig. 2 to 6.

Figure 2 shows the workspace of the parallel manipulator for $l_{min} = 0.54$ and $D = 0.27$ with the angle constraint $\theta_{max} = 85°$. Fig.2 (a), (b) and (c) present a 3-dimensional shape of the workspace, its projection on the *xy*-pane and *xz*-plane, respectively. From Fig.2 (a), it can clearly be seen that the shape of the workspace is like a parabolic cylinder with a cavity composed of three similar smaller parabolic cylinders. The shape of the workspace in Fig.2 is symmetric in space due to the symmetric configuration of the translation 3-UPU parallel manipulators. The shape and volume of the workspace will be changed when the geometric constraints are changed, as shown in Figs.2 and 3.



(a) Three-dimensional workspace (b) Projection in *xy*-plane (c) Projection in *xz*-plane.

Fig.2 Workspace volume for $l_{min} = 0.54$, $D = 0.27$, $\theta_{max} = 85°$, $\delta = 0.05$



(a) $D = 0.4$ (b) $D = 0.5$ (c) $D = 0.6$

(d) $D = 0.7$ (e) $D = 0.8$ (f) $D = 0.9$

Fig.3 Workspace volume for $l_{min} = 0.54$, $\theta_{max} = 85°$, $\delta = 0.05$

Figure 4 shows the volume of the constrained workspace as a function of the radius difference D and of the minimun link length l_{min} with angle constraint $\theta_{max} = 80°$. From this plot, it is apparent that the workspace volume will increase as the radius difference D decreases and the minimum link length decreases or the stroke of the prismatic, joint increases.

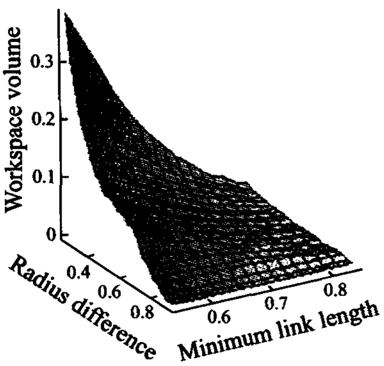


Fig.4 Workspace volume

Figure 5 shows the workspace volume varied as a function of radius difference D and angle constraints for $l_{min} = 0.54$. It can be seen that the workspace volume increases as the constraint angle increases.

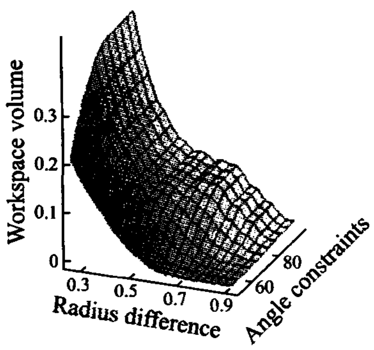


Fig.5 Workspace volume

Figure 6 presents the average condition number as a function of radius difference D and of the angle constraints for $l_{min} = 0.54$. It is shown that the average condition number is affected a lot by the radius difference D. The angle constraints and the minimum link length only impose restricts on the workspace volume. And there is an interval at about $D = 0.6$ where the average condition number is minimal and corresponds to better configurations of the parallel manipulators. This is different compared to the one presented in Ref.[11] where there was no optimal values of average condition number in the relationship between the average inverse condition number and the two design parameters. On considering that an optimal value of the average condition number is an important criteria for getting a good design of parallel manipulators, it is favored to apply a direct average condition number other than the average inverse condition number to be the global performance index of the workspace for the translational 3-UPU manipulators. From the result shown in Fig.6, it is apparent that the translational 3-UPU with the radius difference D at about 0.6 is of better configurations.

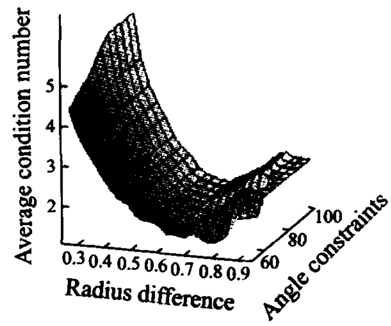


Fig.6 Average condition number

5 Conclusion

In this paper, the workspace of the transltion 3-UPU parallel manipulators is investigated under consideration of prismatic, universal joint constraints and non-singularity conditions. The workspace is parameterized using two design parameters, which are the minimum length of prismatic joints and the radius difference of two circum-circles of the fixed base and the moving platform. The workspace is calculated as a function of the minimum link length, the radius difference and the universal angle constraints. A global performance index of workspace, the direct average condition number, is presented and calculated. And the results show that there exists an optimal value of average condition number. The results given in this paper are useful for optimal design of the translational 3-UPU parallel manipulators in accordance with requirements of practical applications.

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