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QUALITY ASSURANCE

Performance measurement for offline inspections under variable interactions and inspection errors in low-volume production

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Abstract
The assessment of the performance of inspection strategies is a crucial element in the design phase of product quality inspections of manufacturing companies. The aspects that inspection designers need to consider include: (1) the typology of quality inspection, (2) the inspection variables involved, (3) the potential interaction between variables and (4) the presence of inspection errors. In particular, low-volume inspection design is critical due to the lack of historical data and the inadequacy of traditional statistical approaches. By considering these issues, this paper proposes a novel approach to support inspection designers in the prediction of offline quality inspection performance. The development of a probabilistic model based on the analysis of the possible variable interactions and inspection errors and the definition of some performance measures may successfully help designers in the early design stages of inspection process planning. The approach is supported by a practical application in the Additive Manufacturing field.

Keywords Quality control · Offline inspection · Inspection performance · Variable interaction · Inspection errors · Additive manufacturing

List of symbols

X_i	Input variable ($i=1,\ldots,m$)
Y_j	Output variable ($j=1,\ldots,n$)
p_{Y_j}	Probability of occurrence of a defective output variable Y_j
α_{Y_j}	Probability of erroneously classifying the output variable Y_j as defective (type I inspection error)
β_{Y_j}	Probability of erroneously not classifying the output variable Y_j as defective (type II inspection error)
LSL_j	Lower specification limit of Y_j
USL_j	Upper specification limit of Y_j
$VAR\left(Y_j\right)$	Variance of Y_j
$\boldsymbol{X}=\left[x_1,\ldots,x_m\right]^{\mathrm{T}}$	Vector of the m input variables
$\boldsymbol{A}=\left[a_0,a_1,\ldots,a_m\right]^{\mathrm{T}}$	Vector of the mathematical model coefficients
$\boldsymbol{K}=[\boldsymbol{X}\,\boldsymbol{A}]^{\mathrm{T}}$	Vector of size $2\,m+1$ of the input variables and the coefficients of the mathematical
$cov(\boldsymbol{K})$	Variance–covariance matrix of $\boldsymbol{K}$
$\left[\frac{\partial Y_j}{\partial \boldsymbol{K}}\right]$	Vector of the partial derivatives of Y_j with respect to each component of $\boldsymbol{K}$
HB	Brinell hardness in the scale HBW 2.5/62.5
P	Laser power (W)
v	Scan speed (mm/s)
h_d	Hatching distance (mm)
$p_{Y_j}^{X_i}$	Probability of occurrence of the defective-output variable Y_j due to the input variable X_i
$p_{Y_1\cap Y_2\cap\cdots\cap Y_k}^{X_i}$	Probability that the input variable X_i causes k defective-output variables, with $k \le n$.
$p_{Y_j}^{X_1\cap X_2\cap\cdots\cap X_s}$	Probability that s input variables cause the defective-output variable Y_j, with $s \le m$.
W_j	Bernoulli random variable related to the output variable Y_j

The order of family name and given name corrected for all author's.

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W_P	Bernoulli random variable related to the product
D_{Y_j}	Mean number of real defective-output undetected for the jth output-variable
D_{tot}	Inspection effectiveness measure without considering variable interactions
D'_{tot}	Inspection effectiveness measure under variable interactions
D^*_{tot}	Inspection effectiveness measure under variable interactions, derived by assuming independence between output variables
$\boldsymbol{I}$	Vector of model inputs related to the inspection effectiveness measure D'_{tot}
$cov(\boldsymbol{I})$	Variance–covariance matrix of model inputs
RP	Recycled powder
LT	Layer thickness
PO	Porosity
MP	Mechanical properties
DA	Dimensional accuracy

1 Introduction

Nowadays, in order to fight the competition and maintain their market position, manufacturing companies are increasingly interested in quality performance evaluation tools as well as quality monitoring and control systems [1]. In particular, choosing effective quality inspections is a key factor within organizations to meet customer needs and maintain the competitive advantage in the marketplace [2, 3]. For years, manufacturing companies have exploited traditional approaches to design quality inspections [4, 5]. Nowadays, the increasing complexity and customization of products require more sophisticated, flexible and therefore expensive quality control strategies [6–8].

There are several aspects that inspection designers have to consider during the inspection process planning, including (1) the typology of production to be inspected, and (2) the kind of quality control to be performed. In particular, regarding production typology, the design of quality-inspections for low-volume productions is a remarkable issue because of the inadequacy of traditional techniques, e.g., cost–benefit models, simulations, optimization models [3, 9, 10]. This production typology is characterized by a low production rate and often by a high level of complexity and customization [11]. As far as quality control is concerned, inspections can be performed in-process or offline [12]. Production units are inspected during the production process in the case of in-process inspections, also referred to as online or in-line inspections in the scientific literature [13–16]. Conversely, in offline inspections, the finished products are inspected after the manufacturing process is completed [13, 17].

Inspection design of low-volume productions is attracting increasing interest from researchers and practitioners. Regarding in-process inspections, some studies have proposed methods to design an economical in-process control procedure, supporting the choice of the best sampling strategy for low-volume productions [18, 19]. Another line of research has focused on the development of suitable defect prediction models for low-volume manufacturing processes and their use to plan quality inspection strategies [20–27]. Also with regard to offline inspections, some studies aimed to develop probabilistic models for predicting defects and define adequate performance indicators outlining the overall effectiveness and affordability of alternative offline inspection strategies [26, 27]. Despite this general interest, previous studies concerning offline inspections were based on the hypothesis of no interaction between process and inspection variables. This assumption, which could be true in some cases, can be particularly strong, especially in complex contexts such as Additive Manufacturing processes. This paper aims to extend previous studies in the field of offline inspection design by proposing a quantitative method for assessing offline inspection effectiveness considering: (1) possible interactions between process and inspection variables, in terms of cause-and-effect relationships, and (2) potential inspection errors. More in detail, the method was developed to address the following research question (RQ):

> RQ: "How to quantify offline inspection effectiveness when the interactions between process and inspection variables and the inspection errors may not be neglected?".

The proposed approach, by providing some performance measures of offline inspections, can offer adequate support to inspection designers of low-volume productions during the early stages of inspection process planning. In detail, the proposed probabilistic model and the related performance measures can be adopted to support the decision-making process in the early design phases on the most effective inspection strategy, meant as the combination of inspection methods on quality characteristics. Indeed, especially in the case of low-volume productions, which are typically characterized by high levels of customization and complexity, the choice of the most appropriate inspection is a non-trivial problem for two main reasons:

(i) the variety of products that can be produced with the same technology makes it difficult to standardize and adopt a unique inspection strategy;

(ii) amongst all the different possible inspections, there may be several eligible and suitable strategies for the considered low-volume production.

In this regard, using adequate inspection performance measures to quantify the effectiveness of alternative inspection strategies from the early inspection design phases is of paramount importance and contributes to achieving zero-defect manufacturing goals.

The remainder of the paper is structured as follows. A problem statement that arises from a real application case in the Additive Manufacturing field is presented in Sect. 2. In Sect. 3, the manufacturing process and the inspection process variables are described and integrated into an overall probabilistic model. Furthermore, a self-adaptive approach is proposed to estimate model probabilities. Section 4 discusses the approach adopted for predicting inspection performance in terms of effectiveness, including possible variables interactions and inspection errors. Practical examples to illustrate the proposed method applied to the real case presented in Sect. 2 are the subject of Sect. 5. Finally, Sect. 6 proposes closing remarks, research limitations, and future developments.

2 Problem statement in a real case application

Consider a part produced by the selective laser melting (SLM) technique, which is a promising additive manufacturing process that fully melts a metal material into a solid three-dimensional part. The part is manufactured layer by layer by consolidating metal powder particles using a focused laser beam that selectively scans the surface of the powder bed [28]. In this process, several input variables can affect the quality of the finished product, including continuous variables, such as laser power, scan speed and hatching distance, and discrete variables, e.g., the use of virgin/recycled powder and the layer thickness [29–33]. These input variables can affect a variety of quality characteristics of products, which will be called from now on output variables, including surface roughness, macro-hardness, porosity, tensile strength and dimensional accuracy [34, 35].

A first problem is to determine which is the probability of occurrence of defects related to the selected output variables. In order to solve this issue, the relationships between input and output variables can be exploited to obtain the probabilities of occurrence of defective-output variables, as will be discussed in Sect. 3.

In order to check the conformity of a product with respect to the output variables, offline inspections can be performed, such as dimensional verifications, visual checks, comparison with reference exemplars, mechanical tests. Two inspection errors can be associated with each inspection activity, namely detecting a defect when it is not present (type I error) and not detecting a defect when it is actually present (type II error). Despite inspection designers try to minimize such inspection errors through sophisticated (manual and/or automatic) quality monitoring techniques, they can never be eliminated. The combination of inspection methods to perform quality controls on output variables defines an inspection strategy. Thus, a second problem is measuring and predicting the performances of alternative quality inspection strategies that can be performed on a product. A first preliminary methodology to solve this problem was proposed in the study of Verna et al. [26]. However, this approach does not consider any interactions between output variables and inspection errors. In real situations, on the contrary, there is often a dependence between the occurrence of defective-output variables and/or inspection errors. Accordingly, Sect. 4.1 proposes an approach able to also consider variables interactions.

3 Process and inspection modelling

Consider a manufacturing process in ideal settings condition with m input variables that influence the final quality of a single product, evaluated by measuring n quality characteristics, i.e., output variables. In addition, each output variable can be inspected using a specific offline inspection method, which can be subject to inspection errors.

In the proposed model, schematized in Fig. 1, Xi refers to the input variable, where the index i ranges between 1 and m, being m the total number of input variables. Y_j identifies the output variable, where j is in the range from 1 to n, being n the total number of output variables. Besides, the following probabilities can be associated with each jth output variable:

- p_{Y_j}: probability of occurrence of a defective output variable Y_j in nominal operating conditions;

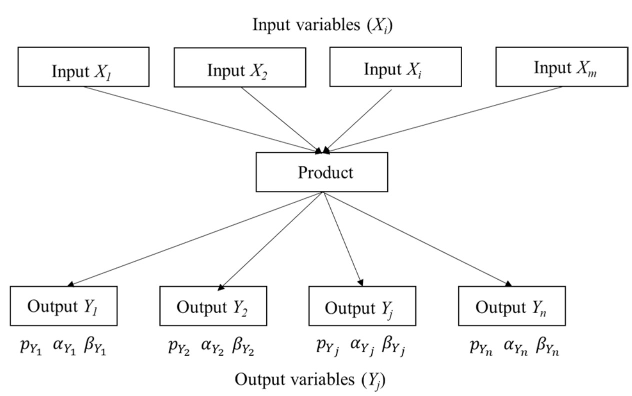


Fig. 1 Representation of a production process with m input variables and n output variables with related probabilities (adapted from [26])

- α_{Y_j}: probability of erroneously classifying the output variable Y_j as defective (i.e., type I inspection error);
- β_{Y_j}: probability of erroneously not classifying the output variable Y_j as defective (i.e., type II inspection error).

The probability p_{Y_j} concerns the quality of a process and it is strictly related to the intrinsic propensity to generate defects. The inspection errors α_{Y_j} and β_{Y_j} depend on the quality of the jth output variable inspection activity. They are strongly related to factors such as the technical skills and experience of the inspectors, the type of inspection performed, the time allowed for inspection, the work environment, and other work- and inspection-related factors [13, 17, 36, 37]. In practical applications, the probabilities p_{Y_j}, α_{Y_j} and β_{Y_j} may be a priori estimated using adequate probabilistic models, empirical methods (historical data, previous experience on similar processes, process knowledge, etc.) or simulations [20, 22, 23, 38, 39]. In the next Sect. 3.2, a self-adaptive approach will be presented to estimate such probabilities.

3.1 Defective-output probability p_{Y_j}

As schematized in Fig. 1, the underlying assumption of the model is the relationship between input and output variables. Therefore, if a defective output occurs, it may be caused by some input variables and their interactions. As a consequence, the probabilities of occurrence of defective output can be obtained by exploiting the relationships between input and output variables. Such relationships can be derived by implementing methods proposed in the scientific literature. For instance, Eger et al. [7] propose a data-driven analysis tool to identify the correlations between process variables in multistage production systems. This approach allows deriving the dependencies between variables in highly connected processes [7].

Process input variables can be continuous or discrete. Section 3.1.1 reminds how to estimate the probabilities of occurrence of defective-output variables for continuous variables. Section 3.1.2 proposes a novel methodology for discrete variables.

3.1.1 Continuous input variables

When dealing with continuous input variables, a methodology to estimate the probabilities of occurrence of defective-output variables was proposed in a previous study of Verna et al. [26]. Specifically, probabilities of occurrence of defective-output can be obtained using a linear mathematical model relating input and output variables by composing the uncertainties of the input variables and the coefficients of the mathematical model through the law of composition of variances [4, 40].

In detail, defined the vector of the m input variables as $\boldsymbol{X} = \left[x_1, \ldots, x_m\right]^{\mathrm{T}}$, the variability of each input variable contributes to the variability of the related Y_j output variable, along with the contribution of the coefficients of the mathematical model, $\boldsymbol{A} = \left[a_0, a_1, \ldots, a_m\right]^{\mathrm{T}}$, as shown in Eq. (1) [26]:

$$VAR\left(Y_{\mathrm{j}}\right) \approx \left[\frac{\partial Y_{\mathrm{j}}}{\partial \boldsymbol{K}}\right]^{\mathrm{T}} \cdot cov(\boldsymbol{K}) \cdot \left[\frac{\partial Y_{\mathrm{j}}}{\partial \boldsymbol{K}}\right] \quad (j = 1, \ldots, n) \tag{1}$$

where $\boldsymbol{K}$ is the vector of size $2\,m+1$ of the input variables and the coefficients of the mathematical model, defined as $\boldsymbol{K} = [\boldsymbol{X}\,\boldsymbol{A}]^{\mathrm{T}}$, $cov(\boldsymbol{K})$ is the variance–covariance matrix [41] and $\left[\frac{\partial Y_{\mathrm{j}}}{\partial \boldsymbol{K}}\right]$ is the vector of the partial derivatives of Y_j with respect to each component of $\boldsymbol{K}$.

At this point, if the probability distribution of each output variable Y_j is known, the probability p_{Y_j}, representing the probability that Y_j falls outside the specification limits, can be estimated by computing the area of the distribution outside the two specification limits, respectively LSL_j and USL_j, as follows:

$$p_{Y_j} = 1 - P\left(LSL_j \leq Y_j \leq USL_j\right) \tag{2}$$

To clarify this methodology, a simple example is provided. In a previous study, it was found from planned experimentation that Brinell hardness in the scale HBW 2.5/62.5 (HB) of parts produced by SLM process can be expressed as a function of process parameters, i.e., laser power (P), scan speed (v) and hatching distance (h_d), as follows [27]:

$$HB = \beta_0 + \beta_1 \cdot P + \beta_2 \cdot v + \beta_3 \cdot h_d + \beta_4 \cdot v^2 + \beta_5 \cdot v \cdot h_d \tag{3}$$

where the mean value and standard deviation of the parameters $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5$ are provided in Table 1.

The variance of HB can be obtained by composing the variance of the mathematical model parameters, reported in Table 1, and the standard uncertainty of process variables,

Table 1 Mean value and standard deviation of the model parameters of Eq. (3) [27]

Parameter	Mean value	Standard deviation
β_0 [HB]	$-5.12\cdot10^{1}$	$3.57\cdot10^{1}$
β_1 [HB/W]	$-1.42\cdot10^{-1}$	$7.16\cdot10^{-2}$
β_2 [HB/(mm/s)]	$2.19\cdot10^{-1}$	$3.28\cdot10^{-2}$
β_3 [HB/mm]	$4.85\cdot10^{2}$	$1.10\cdot10^{2}$
β_4 [HB/(mm/s)^2]	$-5.46\cdot10^{-5}$	$1.16\cdot10^{-5}$
β_5 [HB/(mm^2/s)]	$-2.69\cdot10^{-1}$	$8.22\cdot10^{-2}$

Table 2 Standard uncertainty of process variables

Process variable	Standard uncertainty
P (W)	$2.89 \cdot 10^{-2}$
v (mm/s)	$2.89 \cdot 10^{-2}$
h_d (mm)	$2.89 \cdot 10^{-3}$

evaluated as the resolution of the AM machine under the assumption of uniform distribution, see Table 2 [27].

Accordingly, the variance of HB can be obtained as follows:

$$VAR(\mathrm{HB}) \approx \left[\frac{\partial HB}{\partial \boldsymbol{K}}\right]^{\mathrm{T}} \cdot cov(\boldsymbol{K}) \cdot \left[\frac{\partial HB}{\partial \boldsymbol{K}}\right] = 4.62\ \mathrm{HB}^2 \quad (4)$$

where $\boldsymbol{K} = \left[P, v, h_d, v \cdot v, v \cdot h_d, \beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5\right]^{\mathrm{T}}$ and $cov(\boldsymbol{K})$ includes respectively their variances and covariances.

Finally, under the hypothesis of normal distribution, the probability of hardness-defect, p_{HB}, was obtained by Eq. (2). In detail, given the nominal value of hardness (122.45 HB), the variance shown in Eq. (4), and the lower specification limit ($LSL = 114$ HB), the resulting probability is the following [27]:

$$p_{HB} = P(HB \leq LSL) = 0.55\% \quad (5)$$

It has to be specified that, in this case, technological requirements only impose a lower, and not an upper, specification limit.

3.1.2 Discrete input variables

The probability of occurrence of the jth defective-output, p_{Y_j}, can be derived from the probabilities of occurrence of defects caused by the input variables. Accordingly, each ith input variable is associated with a probability p_{X_i}, i.e., the probability of occurrence of defects in the final part due to the input variable X_i.

The relation between input and output variables is represented through the probability $p^{X_i}_{Y_j}$, i.e., the probability of occurrence of the defective-output variable Y_j due to the input variable X_i. Besides, each input variable may be a source of more defective-output variables. In this situation, the probability that the input variable X_i causes k defective-output variables is denoted as $p^{X_i}_{Y_1 \cap Y_2 \cap \cdots \cap Y_k}$, with $k \leq n$. Similarly, each defective-output variable may be caused by more input variables. In such a case, the probability that s input variables cause the defective-output variable Y_j is identified with the probability $p^{X_1 \cap X_2 \cap \cdots \cap X_s}_{Y_j}$, with $s \leq m$.

Consider an exemplifying process with 3 input variables and 4 output variables, as shown in Fig. 2.

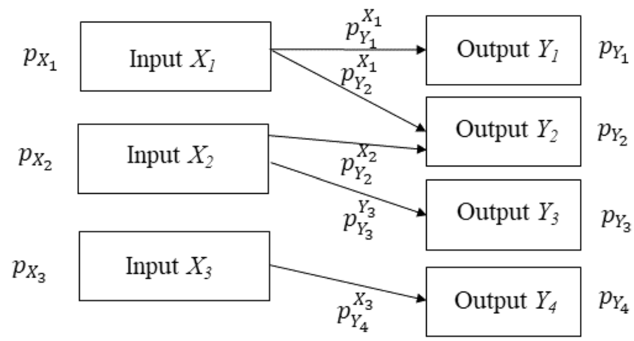


Fig. 2 Representation of an exemplifying process with 3 input variables and 4 output variables

In this specific example, the probabilities of occurrence of defects in the product due to the input variables, p_{X_i} ($i = 1,2,3$), are:

$$p_{X_1} = p^{X_1}_{Y_1} + p^{X_1}_{Y_2} - p^{X_1}_{Y_1 \cap Y_2} \quad (6a)$$

$$p_{X_2} = p^{X_2}_{Y_2} + p^{X_2}_{Y_3} - p^{X_2}_{Y_2 \cap Y_3} \quad (6b)$$

$$p_{X_3} = p^{X_3}_{Y_4} \quad (6c)$$

More in general, p_{X_i} can be calculated, for each $i \in \{1, 2, \ldots, m\}$, as follows:

$$p_{X_i} = \sum_{j=1}^{k} p^{X_i}_{Y_j} - \sum_{j_1 < j_2} p^{Xi}_{Y_{j_1} \cap Y_{j_2}} + \cdots + (-1)^{r+1} \cdot \sum_{j_1 < j_2 < \cdots < j_r} p^{X_i}_{Y_{j_1} \cap Y_{j_2} \cap \cdots \cap Y_{j_r}} + \cdots + (-1)^{k+1} \cdot p^{Xi}_{Y_1 \cap Y_2 \cap \cdots \cap Y_k} \quad (7)$$

where each sum $\sum_{j_1 < j_2 < \cdots < j_r}$ is calculated for all the $\binom{k}{r}$ possible subsets of r elements of the set $\{1, 2, \ldots, k\}$, and k is the total number of defective-output variables caused by the input variable X_i, with $k \leq n$.

At this point, the probabilities of occurrence of defective-output variables of the example illustrated in Fig. 2, p_{Y_j} ($j = 1,2,3,4$), can be derived as follows:

$$p_{Y_1} = p^{X_1}_{Y_1} \quad (8a)$$

$$p_{Y_2} = p^{X_1}_{Y_2} + p^{X_2}_{Y_2} - p^{X_1 \cap X_2}_{Y_2} \quad (8b)$$

$$p_{Y_3} = p^{X_2}_{Y_3} \quad (8c)$$

$$p_{Y_4} = p_{Y_4}^{X_3} \tag{8d}$$

where the probability $p_{Y_2}^{X_1 \cap X_2}$ in Eq. (8b) can be calculated, according to the definition of conditional probability [4], as follows:

$$p_{Y_2}^{X_1 \cap X_2} = \begin{cases} p_{Y_2}^{X_1} \cdot p_{Y_2}^{X_2} \text{ if the occurrence of } X_1 \text{ and that of } X_2 \text{ are independent} \\ p_{Y_2}^{X_2|X_1} \cdot p_{Y_2}^{X_1} \text{ if the occurrence of} X_1 \text{ and that of } X_2 \text{ are dependent} \\ \quad \left(\text{the occurrence of } X_1 \text{ is the conditioning event}\right) \\ p_{Y_2}^{X_1|X_2} \cdot p_{Y_2}^{X_2} \text{ if the occurrence of } X_1 \text{ and that of} X_2 \text{ are dependent} \\ \quad \left(\text{the occurrence of } X_2 \text{ is the conditioning event}\right) \end{cases} \tag{9}$$

In Eq. (9), $p_{Y_2}^{X_1|X_2}$ is the conditional probability that the defective-output variable Y_2 caused by X_1 occurs, given that the defective-output variable Y_2 caused by X_2 has occurred (or vice versa for $p_{Y_2}^{X_2|X_1}$).

More in general, p_{Y_j} can be calculated, for each $j \in \{1, 2, \dots, n\}$, as follows:

$$p_{Y_j} = \sum_{i=1}^{s} p_{Y_j}^{X_i} - \sum_{i_1 < i_2} p_{Y_j}^{X_{i_1} \cap X_{i_2}} + \dots + (-1)^{r+1} \cdot \sum_{i_1 < i_2 < \dots < i_r} p_{Y_j}^{X_{i_1} \cap X_{i_2} \cap \dots \cap X_{i_r}} + \dots + (-1)^{s+1} \cdot p_{Y_j}^{X_1 \cap X_2 \cap \dots \cap X_s} \tag{10}$$

where each sum $\sum_{j_1 < j_2 < \dots < j_r}$ is calculated for all the $\binom{s}{r}$ possible subsets of r elements of the set $\{1, 2, \dots, s\}$, and s is the total number of input variables that cause the defective-output variable Y_j jointly, with $s \le m$. The generic probability $p_{Y_j}^{X_{i_1} \cap X_{i_2} \cap \dots \cap X_{i_r}}$, expressed in Eq. (10), can be derived by exploiting the definition of conditional probability [4] according to the logic-causal criteria between input variables. However, when independence between input variables can be assumed, i.e., when only controlled independent inputs of the process affecting the quality of the finished product are considered, it can be expressed as shown in Eq. (11).

$$p_{Y_j}^{X_{i_1} \cap X_{i_2} \cap \dots \cap X_{i_r}} = p_{Y_j}^{X_{i_1}} \cdot p_{Y_j}^{X_{i_2}} \cdot \dots \cdot p_{Y_j}^{X_{i_r}} \quad j \in \{1, 2, \dots, n\} \tag{11}$$

3.2 Self-adaptive approach to estimate defect and inspection error probabilities

The estimation of the probability of occurrence of the defective-output variables, both continuous and discrete, and the probability of inspection errors—see Sect. 3, is a key point of the proposed probabilistic model. Since the approach proposed in this study is mainly beneficial for low-volume productions, where few historical data are available, the estimation of such probabilities may not be straightforward. Therefore, in order to estimate the above probabilities, the adoption of a self-adaptive approach is suggested. In particular, the probabilities of defective-output variables presented in Sect. 3.1 may be estimated in the design stages of inspections by Eqs. (2) and (10) and using, as a first approximation, historical data relevant to similar products of the same manufacturing process—with slightly different characteristics. Then, as new experimental data becomes available, the prediction models described in Sects. 3.1.1 and 3.1.2 can be updated accordingly to improve estimates accuracy.

A similar approach can be applied to the estimation of inspection errors. As abovementioned, inspection errors, α_{Y_j} and β_{Y_j}, are affected by a plurality of factors, including operators/inspectors' experience and technical skills of operators/inspectors, the typology of inspection performed (manual, automatic or a mixture of both), the time allowed for inspection, the work environment, and other work- and inspection-related factors. Owing to this large number of factors that can lead to inspection errors, it is challenging to estimate the corresponding probabilities. In the scientific literature, some papers treat inspection errors only from a theoretical point of view [13, 38, 42, 43]; instead, others estimate them by adopting approaches based on prior knowledge of the inspection process [11, 12, 20, 23, 26]. As the inspection errors are mostly related to the measuring procedure (instrument, operator and working conditions), empirical data relevant to different products of similar manufacturing processes can be used as a first approximation to estimate them, especially in the case of new productions or in the design stages of inspections. Indeed, most of the controls performed in a company are common to different typologies of products, as for electromechanical products [23]. Thereafter, a self-adaptive approach can be implemented, which involves updating and refining the estimates with new data acquired as production progresses.

Such an auto-adaptive approach allows for up-to-date and accurate estimates of model (process and inspection) probabilities. Clearly, the more data used and the greater the periodicity of the self-adaptation, the greater the model's accuracy and the resulting prediction of the performance measures described in the next Sect. 4.

4 Performance assessment of inspection strategies

According to the process and inspection modelling proposed in Sect. 3 and the tree diagram shown in Fig. 3, for each jth output variable ($j = 1,\ldots,n$) the following probabilities can be obtained [26]:

$$\mathrm{P}(\text{classify the output variable } \mathrm{Y_j} \text{ as defective}) = \mathrm{p_{Y_j}} \cdot \left(1 - \beta_{\mathrm{Y_j}}\right) + \left(1 - \mathrm{p_{Y_j}}\right) \cdot \alpha_{\mathrm{Y_j}} \tag{12}$$

$$\mathrm{P}(\text{classify the output variable } \mathrm{Y_j} \text{ as conforming}) = \mathrm{p_{Y_j}} \cdot \beta_{\mathrm{Y_j}} + \left(1 - \mathrm{p_{Y_j}}\right) \cdot \left(1 - \alpha_{\mathrm{Y_j}}\right) \tag{13}$$

As stated in Eq. (12), an output variable can be classified as defective when it is actually defective, with a probability $p_{Y_j} \cdot \left(1 - \beta_{Y_j}\right)$, or when it is conforming (false positive), with a probability $\left(1 - p_{Y_j}\right) \cdot \alpha_{Y_j}$. On the other hand, an output variable Y_j can be classified as conforming when there is an inspection error (false negative), with a probability $p_{Y_j} \cdot \beta_{Y_j}$, or when there is the real absence of any defect, with a probability $\left(1 - p_{Y_j}\right) \cdot \left(1 - \alpha_{Y_j}\right)$, as shown in Eq. (13).

Then, n Bernoulli random variables (W_j) are defined as follows:

- $W_j = 0$, when either (1) the truly defective output variable Y_j is detected as such or (2) the output variable Y_j is not defective;
- $W_j = 1$, the truly defective output variable Y_j is not detected as such (false negative).

Since defects that are not detected by inspections are the objective of this study, the following probability can be obtained according to Eq. (13) ($j = 1,\ldots,n$):

$$P\left(\mathrm{W_j} = 1\right) = \mathrm{p_{Y_j}} \cdot \beta_{\mathrm{Y_j}} \tag{14}$$

as the term $\left(1 - p_{Y_j}\right) \cdot \left(1 - \alpha_{Y_j}\right)$ represents the probability of classifying conforming outputs as conforming.

Therefore, the mean number of real defective-output undetected for the jth output-variable is:

$$D_{Y_j} = E\left(W_j\right) = p_{Y_j} \cdot \beta_{Y_j} \tag{15}$$

When considering the overall inspection strategy, the mean total number of defective-output variables which are erroneously not detected can be defined as:

$$D_{tot} = \sum_{j=1}^{n} E\left(W_j\right) = \sum_{j=1}^{n} p_{Y_j} \cdot \beta_{Y_j} \tag{16}$$

In first approximation, D_{tot} can be considered a reasonable estimate of the inspection effectiveness as it provides a measure of the overall effectiveness of the inspection strategy performed on the product. It has to be pointed out that Eq. (16) is obtained under the hypothesis of no interaction between inspection errors and defect probabilities of different output variables. As a consequence, the two output variables can be considered decoupled.

For each output variable Y_j, a total cost related to the inspection, including costs for the inspection activity, defects removal and undetected defects, can also be considered, as described in the study of Verna et al. [26]. However, a detailed cost analysis will be the object of future developments of this research.

4.1 Interaction between model variables

As mentioned above, Eq. (16) is obtained under the assumption of no interaction between defects and inspections errors of different output variables. This allows to decouple the corresponding output variables and, therefore, to consider the related events as mutually exclusive, i.e., disjoint events.

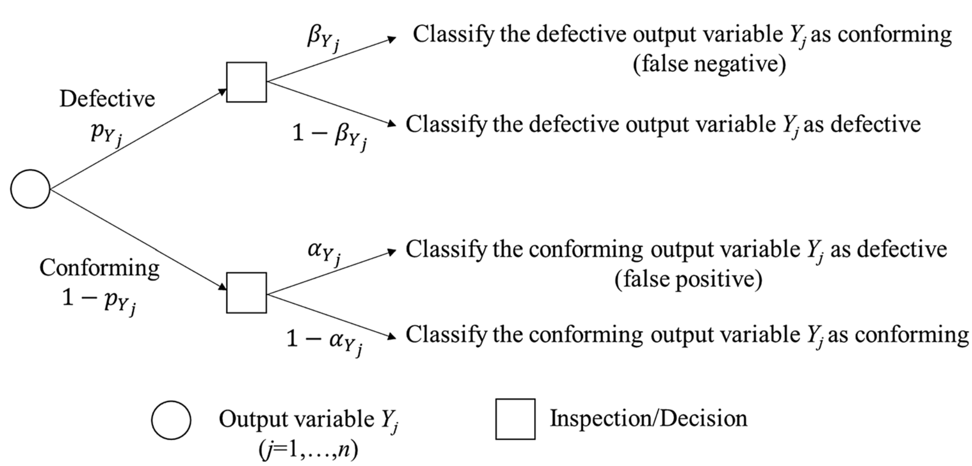


Fig. 3 Tree diagram of the inspection process of each jth output variable

However, in practical situations, different defective-output variables can occur jointly, requiring the proposed model and performance measures to be refined.

It is worth noting that possible interactions between variables are intended in this study as cause-and-effect relationships and not merely as correlations. Indeed, a correlation is a statistical measure of the relationship between two or more variables that, however, does not provide information about the cause-and-effect relationship of the data [7]. Besides, it has to be clarified the distinction between the concept of variables interaction and that of independence. Interaction may arise when the effect of one causal variable on an outcome depends on the state of a second causal variable (i.e., when effects of the two causes are not additive) [44]. On the other hand, two events are independent if the occurrence of one does not affect the probability of occurrence of the other. Similarly, two random variables are independent if the realization of one does not affect the probability distribution of the other [41]. Accordingly, in a scenario of variable interactions, there can be situations of either dependence or independence between events or variables.

A summary reporting the assumptions (dependence/independence) introduced in this study in the modeling of the interaction between variables is provided in Table 3.

Table 3 Summary of the assumptions (dependence/independence) introduced in the modeling of interaction between variables

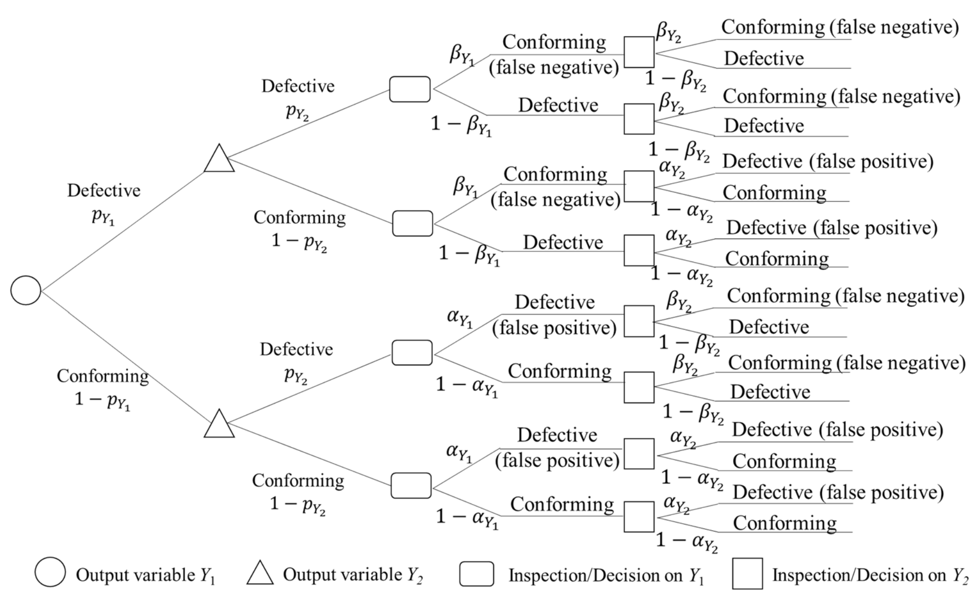
	Occurrence of defects	Inspection errors	Occurrence of defects and inspection errors
Figure 4	Independence	Independence	Independence
Figure 5	Dependence	Independence	Independence
Figure 6	Dependence	Independence	Dependence

Consider, for example, two output variables denoted by Y_1 and Y_2 that are inspected on the final product. In the case of interaction between defects and inspections errors of Y_1 and Y_2, there are 16 different possibilities in such an inspection process, including some cases of misclassifications and other of correct classifications. This scenario is depicted in Fig. 4.

It has to be highlighted that the events represented in Fig. 4, both related to the occurrence of defects and inspection errors, are considered independent. For instance, the occurrence of the defective-output variable Y_2 is independent of the occurrence of the defective output-variable Y_1. Besides, inspections on Y_1 and Y_2 are performed separately, as it happens in most practical cases, and the corresponding inspection errors do not depend on the typology of the defect. Accordingly, as shown in Fig. 4, the type I and type II inspection errors are the same in all the paths of the graphical model. In graphical terms, this situation is indicated by the absence of any direct arrow between the nodes of the events in the tree diagram.

However, in real situations, the assumption of independence between the defective-output variables can be an oversimplification. In general, probabilities are context sensitive. For instance, the probability of occurrence of the defective-output variable Y_2 can be conditioned on the occurrence of the other defective-output variable Y_1, or vice versa. Referring to the application case described in Sect. 2, consider as output variables mechanical properties (MP) and porosity (PO). Suppose that the probabilities of occurrence of defects are $p_{PO} = 2\%$ and $p_{MP} = 2.98\%$. If the occurrence of MP and that of PO are independent, then the probability that the two defective-output variables occur jointly, $p_{MP \cap PO}$, will be $p_{MP \cap PO} = p_{MP} \cdot p_{PO} = 0.06\%$. On the other hand, in case of dependence between the occurrences of the defects, and supposing that the occurrence of MP is conditioned to the

Fig. 4 Tree diagram of the inspection process of 2 output variables in case of independence between the occurrence of defects, inspection errors and both



occurrence of PO (i.e. $p_{MP|PO} = 80\%$), then $p_{MP\cap PO} = p_{MP|PO} \cdot p_{PO} = 1.6\%$. Thus, in this case, assuming independence between MP and PO would result in underestimating the joint probability $p_{MP\cap PO}$.

In such a case, i.e., when there is a dependence between the occurrence of defective-output variables, the scenario is depicted in Fig. 5. The four possible combinations of defects in such a scenario are: Event (A): Y_1 defective and Y_2 defective; Event (B): Y_1 deffective and Y_2 conforming; Event (C):Y_1 conforming and Y_2 defective; Event (D): Y_1 conforming and Y_2 conforming. The probabilities associated with each event are reported in Fig. 5. Specifically, the probability that the two defective-output variables occur jointly, $p_{Y_1\cap Y_2}$, can be obtained, according to the definition of conditional probability [4], as follows:

$$p_{Y_1\cap Y_2} = \begin{cases} p_{Y_2} \cdot p_{Y_1} \text{ if the occurrence of } Y_1 \text{ and that of } Y_2 \text{ are independent} \\ p_{Y_2|Y_1} \cdot p_{Y_1} \text{ if the occurrence of } Y_1 \text{ and that of } Y_2 \text{ are dependent} \\ \qquad \text{(the occurrence of } Y_1 \text{ is the conditioning event)} \\ p_{Y_1|Y_2} \cdot p_{Y_2} \text{ if the occurrence of } Y_1 \text{ and that of } Y_2 \text{ are dependent} \\ \qquad \text{(the occurrence of } Y_2 \text{ is the conditioning event)} \end{cases} \tag{17}$$

In light of this, according to the structure of the problem and the directionality of the cause-and-effect relationship between the output variables, in the graphical model depicted in Fig. 5, $p_{Y_1\cap Y_2}$ should be replaced by the probabilities reported in Eq. (17). It should be noted that, when the occurrence of Y_1 and that of Y_2 are independent, the diagram in Fig. 5 can lead back to the diagram in Fig. 4.

As far as inspection errors are concerned, their probability could also be related to the occurrence of the defective-output variables, i.e., to the four different events (A), (B), (C) and (D). In this case, simple probabilities should be replaced by conditional probabilities, as shown in Fig. 6. In detail, four different inspection errors can occur when inspecting Y_1 ($\beta_{Y_1|A}$, $\beta_{Y_1|B}$, $\alpha_{Y_1|C}$ and $\alpha_{Y_1|D}$), and other four when inspecting Y_2 ($\beta_{Y_2|A}$, $\beta_{Y_2|C}$, $\alpha_{Y_2|B}$, $\alpha_{Y_2|D}$). It has to be noted that, for Y_1, the errors $\beta_{Y_1|C}$ and $\beta_{Y_1|D}$ are not considered because in the events (C) and (D) the output Y_1 is conforming. Accordingly, we are not interested in evaluating type II errors for those scenarios. Similarly, type I errors $\alpha_{Y_1|A}$ and $\alpha_{Y_1|B}$ related to Y_1 are not of interest in events (A) and (B), respectively, in which Y_1 is defective. The same reasoning can be applied to Y_2, for which inspection errors $\beta_{Y_2|B}$, $\beta_{Y_2|D}$, $\alpha_{Y_2|A}$ and $\alpha_{Y_2|C}$ are not regarded.

In practical applications, inspection errors are not mainly related to the part to be inspected and its defects. Instead, they depend closely on factors such as the measuring device and procedure, the inspector abilities, and other work- and inspection-related factors [45, 46]. For that reason, as a first approximation, the model and performance measure proposed in this study rely on the independence between inspection errors, and between inspection errors and the occurrence of defects, as depicted in Fig. 5. Such a hypothesis helps obtain a generalization of the performance measure with n output variables, which will be described in the next section.

In order to generalize the proposed model to n output variables inspected, the possible combinations in which the

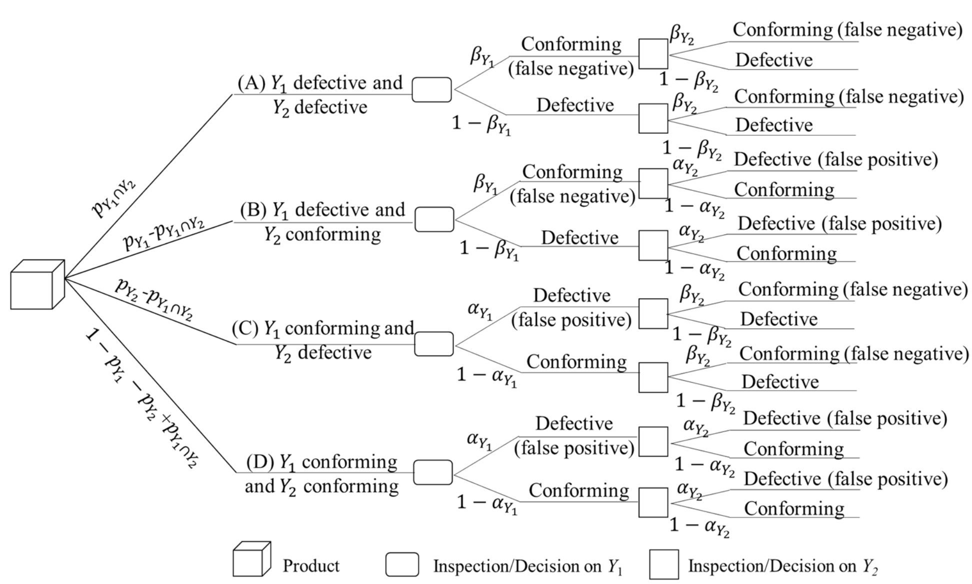


Fig. 5 Tree diagram of the inspection process of 2 output dependent variables in case of independence between inspection errors, and between inspection errors and the occurrence of defects

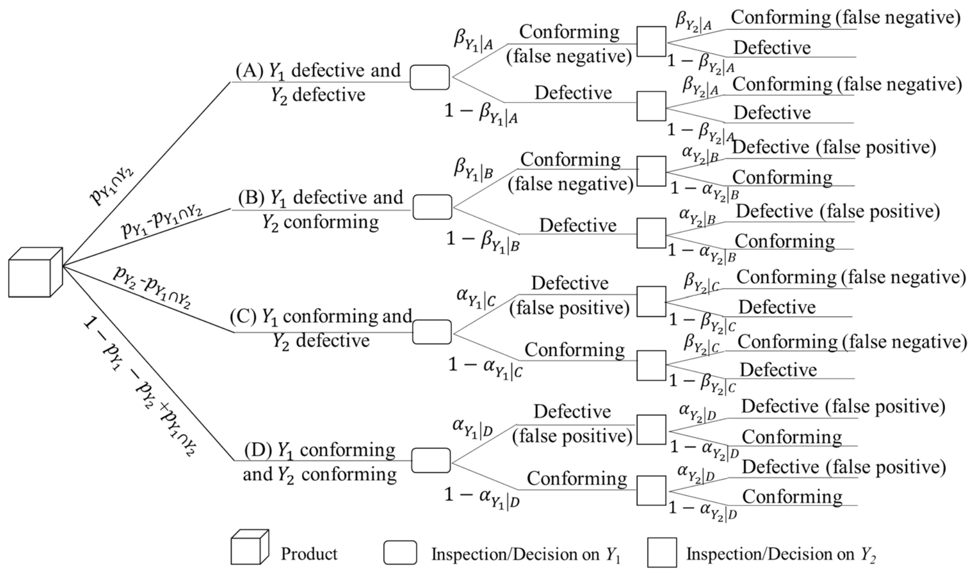


Fig. 6 Tree diagram of the inspection process of 2 output dependent variables in case of independence between inspection errors, and dependence between inspection errors and the occurrence of defects

defects can occur are 2^n, each one associated with 2^n possible combinations of inspection errors, resulting in a total of 2^{2n} combinations (i.e., all possible branches of the tree diagram).

4.1.1 Inspection effectiveness

Again, a Bernoulli random variable related to the product (W_P) can be defined as follows:

- $W_P = 0$, when either (1) a truly defective output variable is classified as defective or (2) an output variable is not defective;
- $W_P = 1$, a truly defective output variable is not classified as defective.

According to the graphical models of Figs. 4, 5 and 6, $P(W_P = 0)$ can be obtained by multiplying the probabilities on the paths where conforming (both false positive and truly conforming) and truly defective output variables are encountered. On the other hand, $P(W_P = 1)$ can be derived by multiplying the probabilities on the paths where false negative output variables are encountered. In the specific case of independence between inspection errors and the related defective-output variables (see Fig. 5), the following two relationships are obtained, given that the two probabilities are complementary:

$$\mathrm{P}(\mathrm{W_P} = 0) = 1 - \mathrm{p}_{\mathrm{Y}_1} \cdot \beta_{\mathrm{Y}_1} - \mathrm{p}_{\mathrm{Y}_2} \cdot \beta_{\mathrm{Y}_2} + \mathrm{p}_{\mathrm{Y}_1 \cap \mathrm{Y}_2} \cdot \beta_{\mathrm{Y}_1} \cdot \beta_{\mathrm{Y}_2} \tag{18}$$

$$\begin{aligned} P(W_P = 1) = & p_{Y_1 \cap Y_2} \cdot \left[\beta_{Y_1} + \left(1 - \beta_{Y_1}\right) \cdot \beta_{Y_2}\right] + \left(p_{Y_1} - p_{Y_1 \cap Y_2}\right) \cdot \beta_{Y_1} \\ & + \left(p_{Y_2} - p_{Y_1 \cap Y_2}\right) \cdot \alpha_{Y_1} \cdot \beta_{Y_2} + \left(p_{Y_2} - p_{Y_1 \cap Y_2}\right) \\ & \cdot (1 - \alpha_{Y_1}) \cdot \beta_{Y_2} = p_{Y_1} \cdot \beta_{Y_1} + p_{Y_2} \cdot \beta_{Y_2} - p_{Y_1 \cap Y_2} \cdot \beta_{Y_1} \cdot \beta_{Y_2} \end{aligned} \tag{19}$$

Therefore, according to Eqs. (18) and (19), the mean total number of defective-output variables which are erroneously not detected in the inspection process for the two variables Y_1 and Y_2 can be defined as:

$$D'_{tot} = E\left(W_P\right) = p_{Y_1} \cdot \beta_{Y_1} + p_{Y_2} \cdot \beta_{Y_2} - p_{Y_1 \cap Y_2} \cdot \beta_{Y_1} \cdot \beta_{Y_2} \tag{20}$$

Thus, if the inspection process is examined in its totality and, therefore, the two output variables are not decoupled, Eq. (20) differs from Eq. (16) for the component $p_{Y_1 \cap Y_2} \cdot \beta_{Y_1} \cdot \beta_{Y_2}$, which represents the mean total number of undetected defects of Y_1 and Y_2 when they occur jointly in the product.

More in general, if there are n output variables to be inspected on the product, by exploiting the total probability theorem [47], the inspection effectiveness indicator becomes:

$$\begin{aligned} D'_{tot} = & \sum_{j=1}^{n} \left(p_{Y_j} \cdot \beta_{Y_j}\right) - \sum_{j_1 < j_2} \left[\left(p_{Y_{j_1} \cap Y_{j_2}}\right) \cdot \left(\beta_{Y_{j_1}} \cdot \beta_{Y_{j_2}}\right)\right] \\ & + \cdots + (-1)^{t+1} \cdot \sum_{j_1 < j_2 < \cdots < j_t} \left[\left(p_{Y_{j_1} \cap Y_{j_2} \cap \cdots \cap Y_{j_t}}\right) \cdot \left(\beta_{Y_{j_1}} \cdot \beta_{Y_{j_2}} \cdot \cdots \cdot \beta_{Y_{j_t}}\right)\right] \\ & + \cdots + (-1)^{n+1} \cdot \left[\left(p_{Y_1 \cap Y_2 \cap \cdots \cap Y_n}\right) \cdot \left(\beta_{Y_1} \cdot \beta_{Y_2} \cdot \cdots \cdot \beta_{Y_n}\right)\right] = \sum_{j=1}^{n} (-1)^{j+1} \\ & \cdot \sum_{1 \le k_1 < \cdots < k_j \le n} \left[\left(p_{\bigcap_{q=1}^{j} Y_{k_q}}\right) \cdot \left(\prod_{q=1}^{j} \beta_{Y_{k_q}}\right)\right] \end{aligned} \tag{21}$$

where each sum $\sum_{j_1 < j_2 < \cdots < j_t}$ is calculated for all the $\binom{n}{t}$ possible subsets of t elements of the set $\{1, 2, \ldots, n\}$. Thus, D'_{tot} is obtained by summing the probabilities of occurrence of defects multiplied by the related type II errors, minus the probabilities associated with defects appearing in even numbers, also multiplied by the related type II errors, and by summing again the probabilities associated with defects appearing in odd numbers, also multiplied by the related type II errors. Although Eq. (21) is formulated for the case of independence between inspection errors and the related defective-output variables, it can be considered a reasonable approximation of the indicator of inspection effectiveness when n defective-output variables can occur jointly.

4.2 Variability evaluation of performance measures

The reliability of the performance measure of inspection effectiveness can be assessed by providing a quantitative evaluation of the variability of the estimate. The approach that can be used to this aim is the method based on the law of composition of variances [4, 20]. According to this approach, the variability affecting all the model inputs, i.e., probabilities of occurrence of defects and inspection errors, can be combined and propagated to obtain the variability of the performance measure D'_{tot}. In detail, known the equation model relating model inputs and the performance measure, see Eq. (21), the variability, expressed in terms of variance (VAR), of the inspection effectiveness measure may be defined as follows:

$$VAR\left(D'_{tot}\right) = \left[\frac{\partial D'_{tot}}{\partial \boldsymbol{I}}\right]^{\mathrm{T}} \cdot cov(\boldsymbol{I}) \cdot \left[\frac{\partial D'_{tot}}{\partial \boldsymbol{I}}\right] \tag{22}$$

where $\boldsymbol{I}$ is the vector of model inputs and $cov(\boldsymbol{I})$ is the variance–covariance matrix of model inputs.

It has to be remarked that Eq. (22), in case of absence of variable interactions, i.e., when considering Eq. (16) instead of Eq. (21), leads to the simplified model given in Eq. (23):

$$\begin{aligned} VAR\left(D_{tot}\right) &= \sum_{j=1}^{n}\left[\left(\frac{\partial D_{tot}}{\partial p_{Y_j}}\right)^2 \cdot VAR\left(p_{Y_j}\right) + \left(\frac{\partial D_{tot}}{\partial \beta_{Y_j}}\right)^2 \cdot VAR\left(\beta_{Y_j}\right)\right] \\ &= \sum_{j=1}^{n}\left[\beta_{Yj}^2 \cdot VAR\left(p_{Y_j}\right) + p_{Y_j}^2 \cdot VAR\left(\beta_{Y_j}\right)\right] \end{aligned} \tag{23}$$

5 Case study application

Referring to the case study described in Sect. 2, consider a part produced by SLM for which the probabilities of occurrence of defective-output variables are evaluated by the two discrete variables: recycled powder (RP) and layer thickness (LT). The use of recycled powder may be considered a Boolean variable (use or not of the recycled powder). The second variable, the layer thickness, is primarily chosen based on the particle size and cannot be thinner than the largest particle in the powder [48]. Besides, in AM machines, the layer thickness can typically assume discrete values in the permissible range. For instance, in the EOSINT® M 270 metal sintering system [49], the layer thickness can vary from 20 µm to 100 µm (20 µm, 30 µm, 40 µm, 50 µm, 60 µm, 70 µm, 80 µm, 90 µm and 100 µm) depending on the material. Extensive studies in the scientific literature have shown the effect of recycled powder and layer thickness on porosity (PO), mechanical properties (MP) and dimensional accuracy (DA) of components produced with SLM technique. In particular, some authors found empirically that the use of recycled powder may affect porosity and mechanical properties, e.g., tensile strength [31, 32, 50], while layer thickness on dimensional accuracy as well as mechanical properties [51–53]. Although recycled powder and layer thickness may also affect other output variables, e.g., surface roughness [54], this example is restricted to analyzing porosity, mechanical properties and dimensional accuracy (Fig. 7). However, the proposed approach can be extended to further output variables.

Assume that the probabilities of occurrence of defects in the product due to RP and LT, p_{RP} and p_{LT}, are respectively 2% and 3%. In detail, RP can cause PO, MP and joint PO and MP defects with, respectively, probabilities p_{PO}^{RP}, p_{MP}^{RP} and $p_{PO\cap MP}^{RP}$ (see Eq. (24a)). On the other hand, LT can cause MP, DA and joint MP and DA defects with probabilities p_{MP}^{LT}, p_{DA}^{LT}, and $p_{MP\cap DA}^{LT}$, respectively (see Eq. (24b)).

$$p_{RP} = p_{PO}^{RP} + p_{MP}^{RP} - p_{PO\cap MP}^{RP} = 2\% \tag{24a}$$

$$p_{LT} = p_{MP}^{LT} + p_{DA}^{LT} - p_{MP\cap DA}^{LT} = 3\% \tag{24b}$$

Probabilities in Eqs. (24a) and (24b) can be estimated with real data based on literature data and/or previous direct manufacturing experience gained in producing the same (or similar) parts via SLM. Alternatively, if such data are not easily available, preliminary experimental campaigns

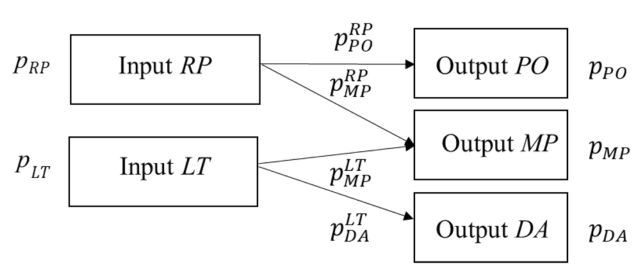


Fig. 7 Schematic of the SLM process with 2 input variables (RP and LT) and 3 output variables (PO, MP, DA) with the related probabilities. RP recycled powder, LT layer thickness, PO porosity, MP mechanical properties, DA dimensional accuracy

could be conducted. In detail, the following steps should be followed.

Step 1) Parts manufacturing
To evaluate the effect of *RP*, a number N_p of parts should be manufactured in the same build, in optimal working conditions, by using recycled powder. It should be clarified that the term "build" and "job" indicate, in industry and the literature, the stack of parts produced via SLM in one single process run [55]. On the other hand, to evaluate the effect of *LT*, a number N_q of parts should be manufactured in the same build, by using optimal parameters settings and a fixed layer thickness.
Step 2) Inspections
Appropriate quality controls should be performed to evaluate the defectiveness in terms of *PO* and *MP* for the N_p parts of the first campaign, and *MP* and *DA* for the N_q parts of the second campaign. A part is recorded as defective if the considered quality characteristic value is out of a specification range.
Step 3) Probability estimation
The probabilities of occurrence of defects should be estimated by using the classical definition of probability, i.e., number of defective parts over total number of produced parts. For instance, if $N_p = 50$ and 1 part is signaled as defective in terms of porosity, then $p_{PO}^{RP} = 1/50 = 2\%$. It has to be clarified that, when estimating p_{PO}^{RP}, all the parts that present porosity-defects should be accounted, even those with mechanical properties-defects. Similarly, in p_{MP}^{RP}, all parts with mechanical properties-defects should be included. On the other hand, $p_{PO \cap MP}^{RP}$ is obtained considering only the parts with both defects of *PO* and *MP*. The same method should be applied for evaluating p_{MP}^{LT}, p_{DA}^{LT} and $p_{MP \cap DA}^{LT}$. For instance, if $N_q = 100$ and 3 parts are signaled as defective in terms of *DA*, then $p_{DA}^{LT} = 3/100 = 3\%$.

Then, the probabilities of occurrence of defective-output variables can be derived, according to Eqs. (10) and (11):

$$p_{PO} = p_{PO}^{RP} = 2\% \tag{25a}$$

$$p_{MP} = p_{MP}^{RP} + p_{MP}^{LT} - p_{MP}^{RP \cap LT} = 1\% + 2\% - (1\% \cdot 2\%) = 2.98\% \tag{25b}$$

$$p_{DA} = p_{DA}^{LT} = 3\% \tag{25c}$$

Now, combining the type II inspection errors of each output variable (see Table 4) with the related defect probabilities, the indicator of effectiveness may be derived. Such inspection errors may be estimated by the use of prediction models and/or empirical methods—based on historical data, previous experience and process knowledge [22, 23, 26]—or by adopting the self-adaptive approach described

Table 4 Inspection errors related to porosity *PO*, mechanical properties *MP* and dimensional accuracy *DA*, based on historical data related to 100 parts

Output variable Y_j	β_{Y_j} (%)
PO	7.0
MP	5.0
DA	5.0

in Sect. 3.2. In this case study, inspection errors were firstly experimentally estimated as the fraction of false negative parts out of the total number of inspected parts, based on historical data related to 100 similar parts manufactured by the SLM process. Such values are listed in Table 4. For instance, β_{PO} was 7.0% as 7 parts were classified as non-defective (when actually defective) out of a total of 100 inspected parts.

When the interaction between variables is not considered, the effectiveness indicator can be derived by exploiting Eq. (16):

$$D_{tot} = p_{PO} \cdot \beta_{PO} + p_{MP} \cdot \beta_{MP} + p_{DA} \cdot \beta_{DA} = 4.39 \cdot 10^{-3} \tag{26}$$

As mentioned in Sect. 4.1, interactions between variables can be commonplace in a complex contest such as AM processes. Thus, when considering the interaction, the indicator of effectiveness should be evaluated according to Eq. (21):

$$\begin{aligned} D'_{tot} = {} & p_{PO} \cdot \beta_{PO} + p_{MP} \cdot \beta_{MP} + p_{DA} \cdot \beta_{DA} \\ & - \left(p_{MP \cap PO} \cdot \beta_{MP} \cdot \beta_{PO}\right) \\ & - \left(p_{DA \cap PO} \cdot \beta_{DA} \cdot \beta_{PO}\right) \\ & - \left(p_{DA \cap MP} \cdot \beta_{DA} \cdot \beta_{MP}\right) \\ & + \left(p_{MP \cap DA \cap PO} \cdot \beta_{MP} \cdot \beta_{DA} \cdot \beta_{PO}\right) \end{aligned} \tag{27}$$

A first preliminary estimate of the probabilities that defects can occur jointly, i.e., $p_{MP \cap PO}$, $p_{DA \cap PO}$, $p_{DA \cap MP}$ and $p_{MP \cap DA \cap PO}$, can be derived by assuming independence between output variables. As a consequence, Eq. (27) may be re-written as:

$$\begin{aligned} D^{*}_{tot} = {} & p_{PO} \cdot \beta_{PO} + p_{MP} \cdot \beta_{MP} + p_{DA} \cdot \beta_{DA} \\ & - \left(p_{MP} \cdot p_{PO} \cdot \beta_{MP} \cdot \beta_{PO}\right) \\ & - \left(p_{DA} \cdot p_{PO} \cdot \beta_{DA} \cdot \beta_{PO}\right) \\ & - \left(p_{DA} \cdot p_{MP} \cdot \beta_{DA} \cdot \beta_{MP}\right) \\ & + \left(p_{MP} \cdot p_{DA} \cdot p_{PO} \cdot \beta_{MP} \cdot \beta_{DA} \cdot \beta_{PO}\right) \\ & = 4.38 \cdot 10^{-3} \end{aligned} \tag{28}$$

where $p_{MP} \cdot p_{PO} = 0.06\%$, $p_{DA} \cdot p_{PO} = 0.06\%$, $p_{DA} \cdot p_{MP} = 0.09\%$ and $p_{MP} \cdot p_{DA} \cdot p_{PO} = 0.006\%$.

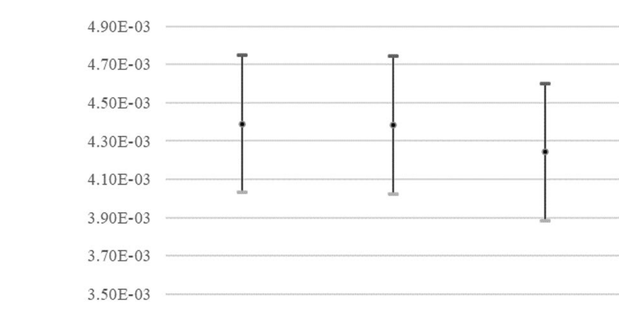
It can be shown that, in the assumption of independence between output variables, the following relationship holds: $D_{tot} > D^{*}_{tot}$, being all defect probabilities and inspection errors values ranged between 0 and 1. In light of the relationship existing between D_{tot} and D^{*}_{tot}, and also considering that the defect probabilities and inspection errors are typically low values in nominal working conditions, the difference between the two indicators is typically negligible. Thus, in conservative terms, the performance measure D_{tot} can represent a reasonable overestimation of the inspection effectiveness in the case of independence between defective output variables.

In order to verify if D_{tot} can also be a reasonable approximation for evaluating D'_{tot} in the assumption of dependence of the occurrence of defects (see Eq. (27)) joint probabilities should be estimated experimentally. In this latter case, literature data or previous similar manufacturing experience may be used. Alternatively, a specific experimentation should be conducted to estimate the joint probabilities ($p_{MP \cap PO}$, $p_{DA \cap PO}$, $p_{DA \cap MP}$ and $p_{MP \cap DA \cap PO}$). For instance, suppose we experimentally obtain the following values (by quantifying the number of defective parts in which joint defects occurs over the total number of produced parts): $p_{MP \cap PO} = 1.6\%$, $p_{DA \cap PO} = 1.3\%$, $p_{DA \cap MP} = 1.8\%$ and $p_{MP \cap DA \cap PO} = 0.06\%$. In this case, the assumption of independence between variables is not valid. Indeed, joint probabilities are not equal to the product of single probabilities. Instead, they are derived from the related conditional probabilities (i.e., $p_{MP|PO} = \frac{p_{MP \cap PO}}{p_{PO}} = 80\%$, $p_{DA|PO} = \frac{p_{DA \cap PO}}{p_{PO}} = 65\%$, $p_{DA|MP} = \frac{p_{DA \cap MP}}{p_{MP}} = 3\%$, $p_{(MP \cap DA)|PO} = \frac{p_{MP \cap DA \cap PO}}{p_{PO}} = 60\%$).

Now, by applying Eq. (27) in case of dependence between variables, the following value is obtained:

$$D'_{tot} = 4.25 \cdot 10^{-3} \tag{29}$$

The estimates of inspection effectiveness obtained by Eq. (26), (28) and (29) should be complemented with their estimated variabilities. As a first approximation, the standard deviation of each model input (i.e., probabilities of occurrence of defects and inspection errors) are assumed to be 5% of the relevant value of the input itself. Then, the variances related to inspection effectiveness measures D'_{tot} and D^*_{tot} are calculated by applying Eq. (22), and for D_{tot} by implementing Eq. (23). The 95% Confidence Intervals (CI) are finally obtained from the variability of the performance measures, as shown in Fig. 8.



	D_{tot}	D^*_{tot}	D'_{tot}
Upper limit 95% CI	4.75E-03	4.74E-03	4.60E-03
Lower limit 95% CI	4.03E-03	4.02E-03	3.88E-03
Mean value	4.39E-03	4.38E-03	4.24E-03

Fig. 8 Graphical comparison of the 95% confidence intervals (CI) of the inspection effectiveness measures D_{tot}, D^*_{tot} and D'_{tot}

As can be noted, the dependence between the occurrences of defects results in a slight decrease (about 3.5%) in the mean number of undetected defective-output variables with respect to D_{tot} and D^*_{tot} (see respectively Eqs. (26) and (28)). However, in all three cases (i.e., D_{tot}, D^*_{tot} and D'_{tot}), given a production of 1000 components, there are nearly 5 defective-output variables that are erroneously not identified. Moreover, as represented in Fig. 8, the three confidence intervals overlap, thereby highlighting no systematic difference between the three performance measures. As a result, the indicator D_{tot} can represent a preliminary conservative estimation of inspection effectiveness also in case of dependence between output variables.

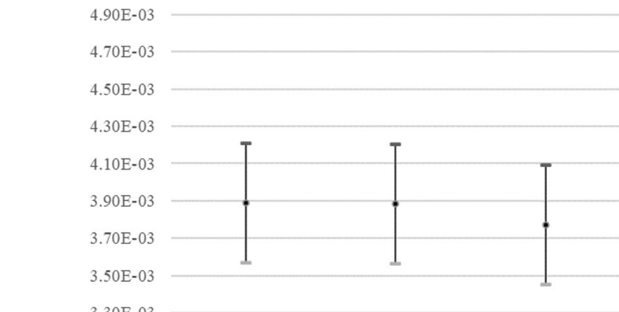
As mentioned in Sect. 3.2, the estimates of model probabilities can be gradually refined using a self-adaptive approach. Consider, as an example, that a new job of 30 parts is produced and a 100% inspection is performed. This quality control enables the refinement of the inspection error estimates shown in Table 4. In detail, regarding *PO*, 2 parts were classified as non-defective when actually defective, whereas 1 false negative part was identified for *MP* and 0 for *DA*. Taking PO as an example, 2 false negative parts were added to the previous 7 parts (historical data, cf. Table 4) out of a total of 130 inspected parts (100 previously inspected and 30 related to the new produced job), resulting in $\beta_{PO} = \frac{(2+7)}{130} = 6.9\%$. The probabilities estimates listed in Table 4 were accordingly updated, as shown in Table 5.

Using such new estimates of inspection errors, the performance measures derived by Eqs. (26), (27) and (28) can be refined to improve the accuracy of the prediction, as represented in Fig. 9.

It is worth noting that the empirical validation of the proposed approach and performance measure is a delicate issue. Since the mean number of undetected defects is, generally, very low (as in this case which is of the order of 10^{-3}) and, typically, AM productions involved some tens per build, a real data collection cannot be easily completed in a short time. For instance, referring to the proposed case study, 1000 parts should be produced to observe about 4 or 5 defective-output variables which are not detected. However, as a first

Table 5 Inspection errors related to porosity *PO*, mechanical properties *MP* and dimensional accuracy *DA*, updated according to the self-adaptive approach (cf. Sect. 3.2) after a new produced job

Output variable Y_j	β_{Y_j} (%)
PO	6.9
MP	4.6
DA	3.8



	D_{tot}	D^{*}_{tot}	D'_{tot}
Upper limit 95% CI	4.21E-03	4.20E-03	4.09E-03
Lower limit 95% CI	3.57E-03	3.57E-03	3.46E-03
Mean value	3.89E-03	3.89E-03	3.77E-03

Fig. 9 Graphical comparison of the 95% confidence intervals (CI) of the inspection effectiveness measures D_{tot}, D^{*}_{tot} and D'_{tot}, obtained after the refinement of inspection error estimates using the self-adaptive approach (cf. Sect. 3.2)

approximation, data relevant to different parts produced by SLM may be put together, considering similar geometries, similar materials, similar AM systems and the same application field (e.g., aerospace and automotive). At least a thousand parts should be collected to experimentally count the average number of defects undetected and then compare it with the estimated measures of inspection effectiveness. This can represent a preliminary validation procedure of the proposed methodology. In the long term, the real data collection may be completed for a more refined estimation of inspection effectiveness performance. Through the use of the performance measures, inspection designers can quantify the effectiveness of alternative inspection strategies and, as a result, implement changes and improvements to the inspection methods adopted with the goal of achieving zero defects.

6 Conclusion

For manufacturing companies, planning effective inspection strategies has always been a key factor in gaining competitive advantage. Several are the aspects that designers need to consider when designing quality inspections, including the typology of production as well as the typology of controls. To date, the assessment of performances of offline inspections in low-volume productions is still critical due to the complexity of the process, resulting in (1) possible interactions between process and inspection variables and (2) potential inspection errors. By considering these issues, this paper attempted to answer the following research question (RQ): "How to quantify offline inspection effectiveness when the interactions between process and inspection variables and the inspection errors may not be neglected?". To address this question, a general methodology is proposed throughout the manuscript to evaluate and predict, from the early stages of inspection design, the offline inspection effectiveness under variables interactions and inspection errors. The method is based on a probabilistic model for defect prediction based on the relationships between process variables and output variables inspected on the final product (i.e., quality characteristics). From the early design phases of inspection planning, model probabilities can be estimated using a self-adaptive approach that allows for up-to-date and accurate predictions. This method initially requires the use of available historical data, also related to productions similar to the one considered, and then includes experimental data that are progressively collected to enhance the accuracy of estimates. Moreover, an effectiveness performance indicator is proposed, together with a method for evaluating its variability, to assist designers in the early design stages of inspection planning. An excerpt of application of the method to a real case study in the field of Additive Manufacturing processes is proposed. The findings reported in this study revealed that evaluating inspection effectiveness by considering or not the interaction amongst output variables leads to comparable results. This is because low-volume productions under nominal working conditions are considered, where the probability of occurrence of defects and that of inspection errors is typically low. From an operational perspective, neglecting the interaction between output variables means slightly overestimating the number of defects not detected by the inspection strategy. However, this can be considered a reasonable approach in most real cases, also given the limited number of parts produced. As a result, inspection designers can, as a first approximation, avoid estimating the joint probabilities of occurrence of defects and still obtain reasonable estimations of inspection effectiveness. The proposed approach can be applied to (1) evaluate the performance of alternative inspection strategies in terms of effectiveness, (2) select the most appropriate according to the manufacturer requirements, and (3) stimulate the improvement of each inspection methods adopted in the inspection strategies with the goal of achieving zero-defects.

Some limitations of this study have to be highlighted. First of all, the proposed model and related performance measure require the estimation of some not-so-easily-quantifiable probabilities. Thorough knowledge of the process, the operator/inspector experience and preliminary experimental tests can help overcome this issue. Secondly, the validation of the method would require a long time given the low production volume, as in the case of AM processes. However, a preliminary validation can be performed by collecting real data of similar parts produced with the same technology, characterized e.g., by similar geometries, materials and application fields. Future research steps will include implementing the proposed methodology to sheet metal production, which allows scalability and taking into account errors that are mutually dependent. Besides, the authors are planning to extend this methodology to evaluate the overall inspection costs and include it within a broader costs' assessment related to the entire product life cycle.

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