

# Parametric equations for notch stress concentration factors of rib–deck welds under bending loading

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**ABSTRACT** The effective notch stress approach for evaluating the fatigue strength of rib–deck welds requires notch stress concentration factors obtained from complex finite element analysis. To improve the efficiency of the approach, the notch stress concentration factors for three typical fatigue-cracking modes (i.e., root–toe, root–deck, and toe–deck cracking modes) were thoroughly investigated in this study. First, we developed a model for investigating the effective notch stress in rib–deck welds. Then, we performed a parametric analysis to investigate the effects of multiple geometric parameters of a rib–deck weld on the notch stress concentration factors. On this basis, the multiple linear stepwise regression analysis was performed to obtain the optimal regression functions for predicting the notch stress concentration factors. Finally, we employed the proposed formulas in a case study. The notch stress concentration factors estimated from the developed formulas show agree well with the finite element analysis results. The results of the case study demonstrate the feasibility and reliability of the proposed formulas. It also shows that the fatigue design curve of FAT225 seems to be conservative for evaluating the fatigue strength of rib–deck welds.

**KEYWORDS** notch stress concentration factor, rib–deck weld, parametric analysis, regression analysis, parametric equation

## 1 Introduction

Rib–deck welds are one of the most fatigable details in orthotropic steel decks (OSDs). Fatigue assessment of rib–deck welds has raised great concern recently [1–6]. Several stress-based approaches, such as the nominal stress, hot-spot stress, and effective notch stress approaches, could be employed in the fatigue assessment of rib–deck welds [7]. Among them, the nominal stress approach is the simplest. In this approach, fatigue assessment is performed using specific S–N curves corresponding to the structural details. However, the structure-related stress concentration is not included in the approach, which could lead to inaccurate evaluation results. The hot-spot stress method has higher accuracy than the nominal stress method because it considers the effect of the structural configuration (i.e., the structure-related stress concentration). However, the notch stress induced by weld beads cannot be determined by the hot-spot stress approach due to the limitation of stress extrapolation, as shown in Fig. 1. Additionally, the

hot-spot stress approach is not applicable to the weld roots of rib–deck welds where most fatigue cracks initiate [7]. Alternatively, the effective notch stress approach, which has an increased precision by considering the nonlinear peak induced by the presence of the weld, could precisely account for the notch stress. Therefore, the effective notch stress approach has a broad application prospect in structural engineering.

The concept of the effective notch stress approach was conceived by Radaj [8] when he derived a fictitious rounding of a notch stress model of welds based on Neuber's assumptions [9]. Köttgen et al. [10] further developed the effective notch stress approach, as embedded in the IIW recommendations [7]. To employ the effective notch stress approach, a stress concentration factor ( $K_t$ ) is required.  $K_t$  could be obtained by replacing the actual weld contour with a fictitious round notch. For most structural components with plate thickness greater than 5 mm, an effective notch radius of  $r = 1$  mm is usually recommended [7]. In this case,  $K_t$  could also be intrinsically interpreted as the effective notch stress concentration factor ( $K_f$ ).  $K_f$  is key in the application of

the effective notch stress approach. Once  $K_f$  is obtained, the effective notch stress ( $\sigma_{en}$ ) could be determined by multiplying the nominal stress ( $\sigma_n$ ) by  $K_f$ .

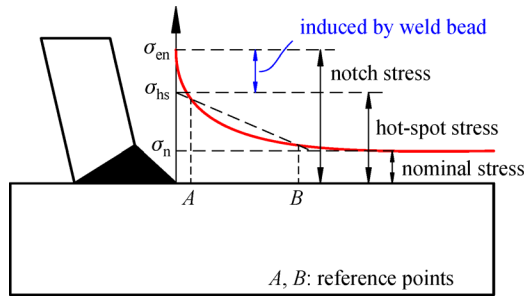


Fig. 1 Fatigue stress at a weld.

$K_f$  could be obtained by numerical simulations, such as finite element analysis (FEA). This process requires the development of a finite element model introduced by an effective notch at weld toe or root and sensitivity analysis of the meshing size to obtain reliable and numerically good-quality results [11]. Such requirement prevents many engineers from employing the effective notch stress approach in fatigue assessment of welded joints. As an alternative to the complex and time-consuming numerical simulations, empirical formulas obtained from regression analysis are a better choice for determining  $K_f$ . If such formulas provide high-accuracy results, their application would save time, effort, and cost. For this reason, several efforts have been made to develop empirical formulas for  $K_f$  in simple connections, such as welded cruciform joints [12–14], T-butt welded connections [15,16], and butt welds [17]. Recently, studies on the application of the effective notch stress approach on rib–deck welds have been reported [6,18–20]. However, only a few studies have focused on the empirical formula for  $K_f$ . Therefore, there is a need to develop the empirical formulas for  $K_f$  in rib–deck welds to enhance the efficient use of the effective notch stress approach.

In this study, finite element models (FEMs) that simulate rib–deck welded structures were developed and validated. Parametric analysis was performed to investigate the effect of weld geometric parameters on  $K_f$ . On this basis, regression analysis was performed to obtain the empirical formula for  $K_f$ . Finally, a case study was presented to

investigate the application of the proposed formulas in the fatigue assessment of rib–deck welded specimens.

## 2 Finite element modeling

### 2.1 Model

FEA was performed to obtain  $K_f$  in the weld toe and root of a rib–deck weld. Figure 2 shows the schematic diagram of the model. The model has a transverse width of  $3 \times W_d$  and a longitudinal length of  $L_d$ . In this study,  $W_d$  and  $L_d$  were set to 300 and 100 mm, respectively. The height of the U-rib was 240 mm. Since this study was focused on local parameters (i.e.,  $K_f$  at the weld toe and root), the dimensions were large enough for the effect of boundary conditions to be neglected, according to Saint Venant's principle [21]. The geometric parameters of the rib–deck welded joint are depicted in Fig. 2, showing the deck thickness ( $t_d$ ), rib thickness ( $t_r$ ), weld penetration rate ( $1 - t_p/t_r$ ), weld leg length in the deck ( $l_{w,d}$ ), weld leg length in the rib ( $l_{w,r}$ ), and the angle between the deck and rib ( $\theta$ ). The geometric parameters of the rib–deck weld were assigned different values for the parametric analysis, as discussed in Section 4.

To obtain  $K_f$  at the weld toe and root, a round notch with a radius of 1 mm was introduced according to the IIW recommendations [7]. All models were simulated by SOLID186-type elements in ANSYS. This element is defined by 20 nodes having three degrees of freedom each; translations in the nodal  $x$ ,  $y$ , and  $z$  directions. The element supports plasticity, hyper-elasticity, creep, stress-stiffening, large deflection, and large strain capabilities. To obtain  $K_f$ , an elastic analysis was performed by setting the Young modulus ( $E$ ) and Poisson ratio ( $\nu$ ) of the steel material to 206 GPa and 0.3, respectively. For the boundary conditions, both ends of the deck plate in the transverse direction were fixed. That is, for the nodes at the two ends of the deck plate, displacements along  $x$ ,  $y$ , and  $z$  directions were fixed. Surface loads of magnitude 1.0 MPa were applied at both sides of the U-rib,  $W_d/2$  away from the end of the deck plate. In this case, the objective rib–deck weld was subjected to pure bending loading. The developed FEMs built are shown in Fig. 3. The meshing of the FEMs is detailed in Section 2.2.

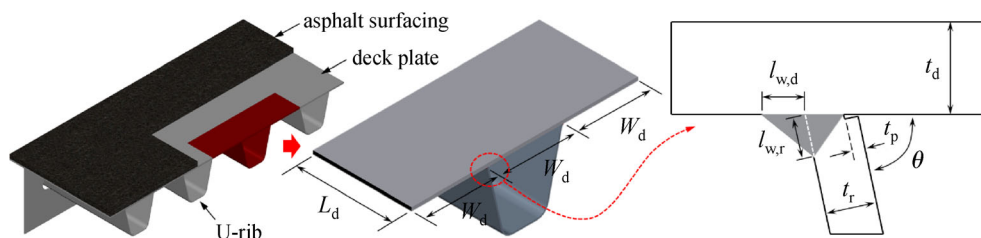


Fig. 2 Schematic of the model.

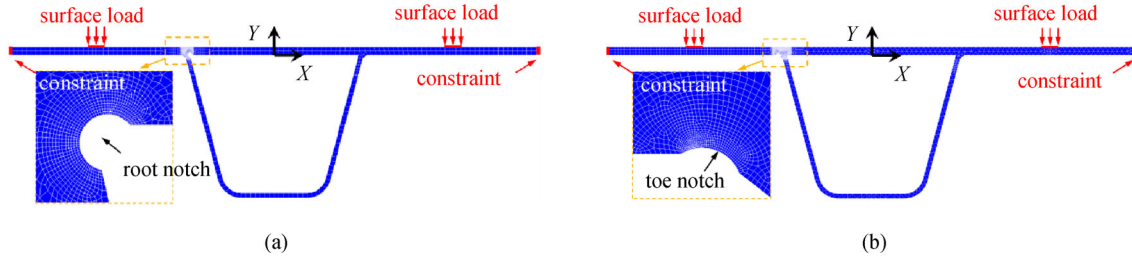


Fig. 3 Finite element models with weld root or toe notch under bending loading. (a) Weld root notch; (b) weld toe notch.

Steel is usually a strain-hardening material with a specific constitutive relationship. However, to obtain  $K_f$  at the introduced notch, an elastic analysis is considered, as recommended by IIW [7]. Thus, in this study, the steel material was simply assigned a linear elastic constitutive relation.

2.2 Effect of meshing size

Effective notch stress is sensitive to the size of the element around the notch. For a 20-node SOLID186-type element-simulated model, the number of elements in  $360^\circ$  arc is recommended to be at least 24 [7]. Thus, the number of elements in  $282^\circ$  arc (i.e., corresponding to  $\theta$  of  $78^\circ$ ) should be at least 18.8. To further determine the suitable number of elements for rib–deck welds, the effective notch stresses in the weld root of a fully penetrated rib–deck weld were extracted for sensitivity analysis. The number of elements at the notch edge was set to 5, 10, 20, 30, and 40, respectively. To improve the calculation accuracy and decrease the computational cost, the meshing transition region was arranged near the notch, as illustrated in Ref. [19]. The coarse meshing size was set to 6 mm.

Considering that the stress distribution at the middle part of the FEM was less affected by the boundary conditions, the stresses thereat were extracted for investigation. The maximum stress perpendicular to the weld ( $\sigma_x$ ), the first principal stress ( $\sigma_1$ ), and von Mises equivalent stress ( $\sigma_s$ ) around the notch are plotted in Fig. 4. The stresses first decrease and then remain stable as the meshing size increases. When the number of elements at the notch edge exceeds 20, the stresses become constant with the meshing size. This indicates that the number of elements must not be less than 20 in a  $282^\circ$  arc (i.e., 26 in  $360^\circ$  arc). Therefore, the number of elements in  $360^\circ$  arcs was taken as 30 in this study. The meshing size was precise enough for determining  $K_f$ .

2.3 Definition of the notch stress concentration factor

To determine  $K_f$ , it is essential to obtain the nominal stress at the weld toe or root first, as expressed in Eq. (1). Figure 5 shows the distribution of the stress component  $\sigma_x$ . There is a severe stress concentration at both the weld toe and root.

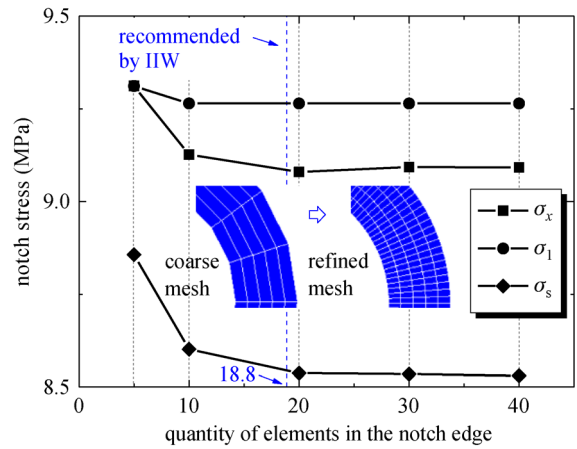


Fig. 4 Effect of meshing size on the effective notch stress.

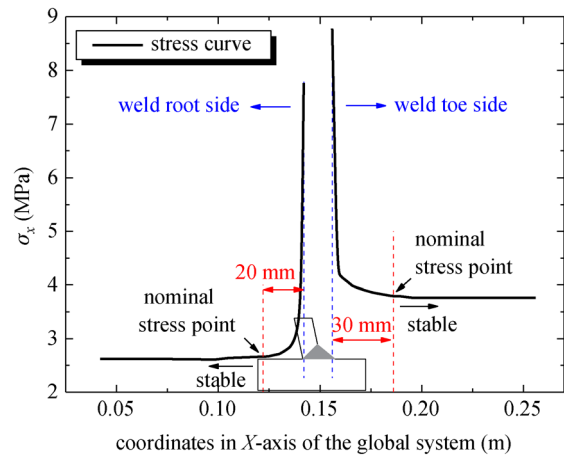


Fig. 5 Determination of the nominal stress points.

The stress decreases sharply first and then remains stable as the extraction point moves away from the weld toe or root. For rib–deck welds, the extracting point 20 mm away from the weld root or 30 mm from the weld toe could be regarded as the nominal stress point.

Furthermore, effective notch stress could be determined by different stresses ( $\sigma_x$ ,  $\sigma_1$ , or  $\sigma_s$ ), which could affect the calculated  $K_f$ . To investigate such effects, taking the fully

penetrated rib–deck weld as an example, the values of  $K_f$  determined by different stresses are plotted in Fig. 6.  $K_f$  corresponding to  $\sigma_1$  is close to that corresponding to  $\sigma_s$ . It reveals that  $K_f$  could be determined by either  $\sigma_1$  or  $\sigma_s$ . In this study,  $\sigma_s$  was used to determine  $K_f$ .

$$K_f = \sigma_{en}/\sigma_n. \quad (1)$$

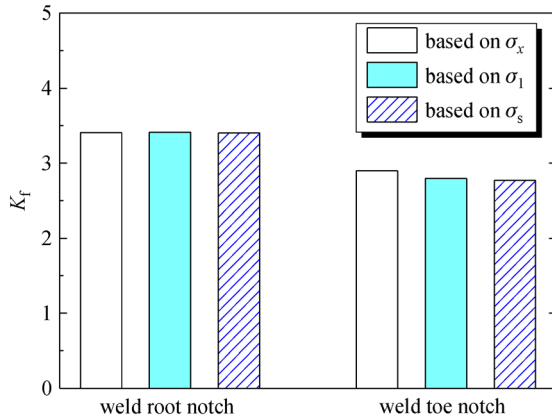


Fig. 6  $K_f$  based on different stresses.

### 3 Validation of the model

As mentioned earlier, a few empirical formulas for the  $K_f$  of rib–deck welds have been reported. Hence, we referred to the empirical formula for the  $K_f$  of a cruciform joint proposed by Oswald et al. [11] to validate the accuracy of the model for determining  $K_f$ . The estimation results were compared to those of the FEA using the same method discussed in Section 2.

Considering the full penetrated cruciform joint (Fig. 7),  $K_f$  at the weld toe could be predicted using Eq. (2). For comparison, a FEM for simulating cruciform joints was also developed using the same method discussed in Section 2 (Fig. 8). A round notch with a radius of 1 mm was set at the weld toe. Symmetric boundary conditions were employed to improve the computing accuracy, as marked in Fig. 8. A 0.5 MPa tensile load was then applied to the FEM for elastic analysis.

$$K_f = \sum_{k=1}^{10} c_k \cdot f_k(\alpha, t_1, y), \quad (2)$$

where  $\alpha$  is the flank angle,  $t_1$  the plate thickness,  $y = l_1/t_1$ ,  $l_1$  the weld leg lengths,  $f_k$  the variables,  $c_k$  the coefficients, and subscript  $k$  the sequence number of each term. Since there are 10 terms in Eq. (2),  $k$  ranges from 1 to 10. The values of  $c_k$  are listed in Table 1. Both estimated and numerical results are plotted in Fig. 9. The numerical results are consistent with the estimated ones (i.e., calculated by Eq. (2)). The maximum error of the estimated results relative to the numerical results is within 1%, demonstrating the feasibility and reliability of the model in determining  $K_f$ .

## 4 Parametric analysis

### 4.1 Objective stress concentration positions

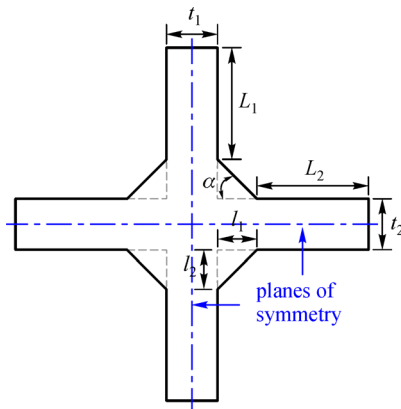
Extensive studies have been conducted to investigate the fatigue performance of rib–deck welds [1–3,5,6,22,23]. Through experimental investigations, three typical fatigue cracks are observed in rib–deck welds: (1) the root–toe crack (Fig. 10(a)); (2) the root–deck crack (Fig. 10(b)); and (3) the toe–deck crack (Fig. 10(c)). In a previous study [19], the feasibility of predicting the crack initiation position through the effective notch stress approach was preliminarily validated. In this study, FEMs for simulating the tested specimens in Refs. [2,3] were developed. The corresponding stress nephogram around the notch is depicted in Fig. 10. The crack initiation positions determined by FEA are similar to those determined by experiments. For example, considering 15% penetrated specimens (Fig. 10(a)), the stress concentration position at the notch edge (i.e., CP1) is consistent with the initiation position of the root–toe crack. Hence, in this study,  $K_f$  in all possible crack initiation positions (i.e., CP1 for root–toe crack, CP2 for root–deck crack, and CP3 for toe–deck crack) are investigated.

### 4.2 Parametric analysis

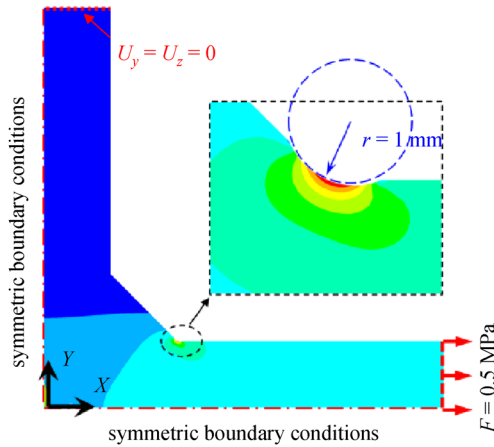
As shown in Fig. 2, the geometry of a rib–deck weld is determined using deck thickness ( $t_d$ ), rib thickness ( $t_r$ ), weld penetration rate ( $1 - t_p/t_r$ ), weld leg length in the deck ( $l_{w,d}$ ), weld leg length in the rib ( $l_{w,r}$ ), and angle between the deck and rib ( $\theta$ ). We investigated multiple geometric parameters using the developed FEMs. The relative deck thickness ( $t_d/t_r$ ) corresponding to a rib thickness of 6 mm

Table 1 Coefficients of the regression formula for  $K_f$  of full penetrated cruciform joints

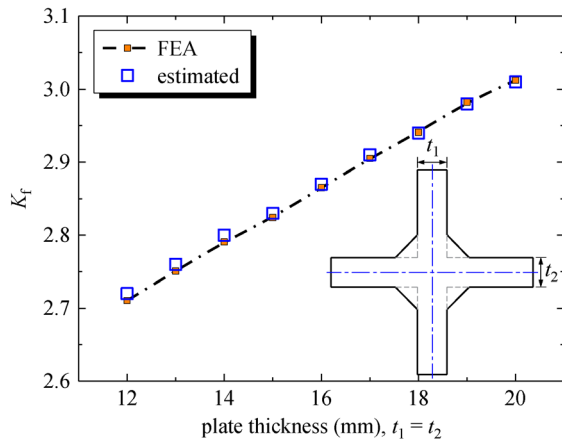
$k$	$f_k$	$c_k$	$k$	$f_k$	$c_k$	$k$	$f_k$	$c_k$
1	1	0.07171	5	$\alpha^2$	- 0.00072	9	$t_1 y$	0.00432
2	$\alpha$	0.07442	6	$t_1^2$	- 0.00026	10	$\alpha y$	0.00892
3	$t_1$	0.02698	7	$y^2$	- 0.16891	-	-	-
4	$y$	- 0.06121	8	$t_1 \alpha$	0.00049	-	-	-



**Fig. 7** Geometry of a full penetrated cruciform joint. Herein,  $t_1 = t_2 = 20$  mm,  $L_1 = L_2 = 40$  mm,  $l_1 = l_2 = 10$  mm,  $\alpha = 45^\circ$ .



**Fig. 8** Nephogram of the first principle stress, including the boundary conditions.



**Fig. 9** Comparison of the FEA and estimated results.  $L_1 = L_2 = 40$  mm,  $l_1 = l_2 = 10$  mm,  $\alpha = 45^\circ$ .

was set to 1.8, 2.0, and 2.5 to investigate the effect of deck thickness on  $K_f$ . The rib thickness ( $t_r$ ) was set to 6, 8, and 10 mm to investigate the effect of rib thickness. The weld penetration rate ( $1 - t_p/t_r$ ) was increased from 0 to 0.8 at a 0.1 interval. The relative weld leg length in the deck ( $l_{w,d}/t_r$ ) and that in the rib ( $l_{w,r}/t_r$ ) was increased from 0.8 to 1.2 at a 0.2 interval. Furthermore,  $\theta$  was increased from  $70^\circ$  to  $80^\circ$  at a  $5^\circ$  interval.  $K_f$  in CP1, CP2, and CP3 were determined using Eq. (1), and the results are plotted in Figs. 11–13, respectively. In light of the purpose of this study, other quantitative results are not presented; however, they are available with the authors if required.

Figure 11 shows the variation of  $K_f$  in CP1. It decreases with an increase in the weld penetration rate ( $1 - t_p/t_r$ ) but increases with the rib thickness (Fig. 11(a)). It decreases with an increase in  $l_{w,d}/t_r$  and  $l_{w,r}/t_r$ ,  $\theta$ , and  $t_d/t_r$ . It varies more with  $l_{w,r}/t_r$  and  $t_d/t_r$  (Figs. 11(c) and 11(e)) and less with the other geometric parameters (Figs. 11(a), 11(b), and 11(d)). This shows that increasing  $t_d$  or  $l_{w,r}$  is more beneficial than increasing  $l_{w,d}$ ,  $t_r$  and  $\theta$  for reducing the stress concentration in CP1.

Figure 12 shows the variation of  $K_f$  in CP2. In general,  $K_f$  in CP2 increases with the weld penetration rate ( $1 - t_p/t_r$ ). It decreases with an increase in  $t_r$  at small weld penetration rates (less than 40%) and remains unchanged at large weld penetration rates. As shown in Figs. 12(b) and 12(d),  $K_f$  in CP2 increases with  $l_{w,d}/t_r$  and  $\theta$ , but it remains constant at  $l_{w,r}/t_r$  less than 1.0, as shown in Fig. 12(c). At  $l_{w,r}/t_r$  greater than 1.0, a slight decrease in  $K_f$  is observed.  $K_f$  decreases with an increase in  $t_d/t_r$ , as shown in Fig. 12(e).

Figure 13 shows the variation of  $K_f$  in CP3, i.e., in the weld toe.  $K_f$  is independent of the weld penetration rate ( $1 - t_p/t_r$ ). It is also considered independent of  $\theta$  and  $t_d/t_r$ , as shown in Figs. 13(d) and 13(e), respectively. As shown in Figs. 13(a)–13(c),  $K_f$  increases with  $t_r$  and  $l_{w,r}/t_r$  but decreases with increase in  $l_{w,d}/t_r$ . However, the effect of  $l_{w,d}/t_r$  and  $l_{w,r}/t_r$  is not obvious.

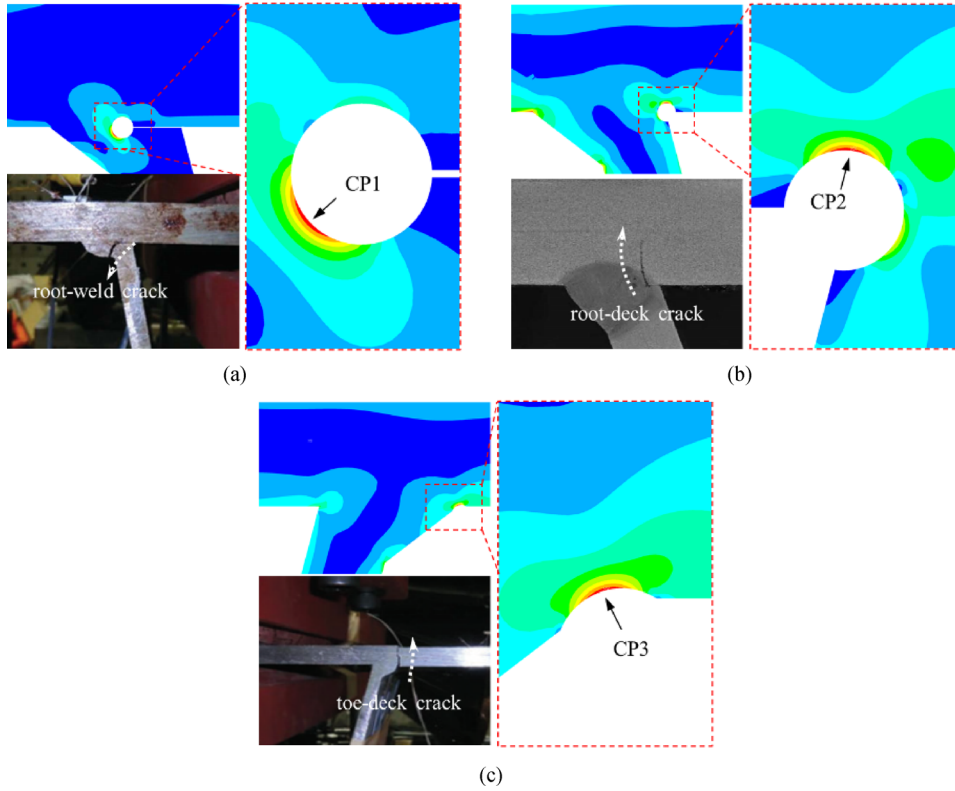
The observations from Figs. 11–13 reveal the significant effect of the geometric parameters of rib–deck welds on  $K_f$ . To develop empirical formulas for  $K_f$ , the coupled effects of different parameters should be reasonably considered.

## 5 Regression analysis

### 5.1 Regression formula for effective notch stress concentration factor

Based on the parametric analysis results, multi-parameter regression analysis was performed to develop the formulas for  $K_f$  in rib–deck welds (i.e., CP1, CP2, and CP3). The parametric analysis results show that the relationship between the geometric parameters and  $K_f$  is fitted with the linear fitting curve. In addition, since six variables may be related to  $K_f$ , it is essential to find the best-fitting





**Fig. 10** Notch stress concentration positions obtained in this study. (a) Concentration position related to root-toe crack (CP1); (b) concentration position related to root-deck crack (CP2); (c) concentration position related to toe-deck crack (CP3).

combination of the variables to develop the formulas. Hence, we adopted multiple linear stepwise regression using a software named Statistical Product and Service Solutions (SPSS) developed by IBM. The linear regression functions with quadratic order and coupling terms are fitted using the values of  $K_f$  calculated from FEA, as expressed by Eq. (3).

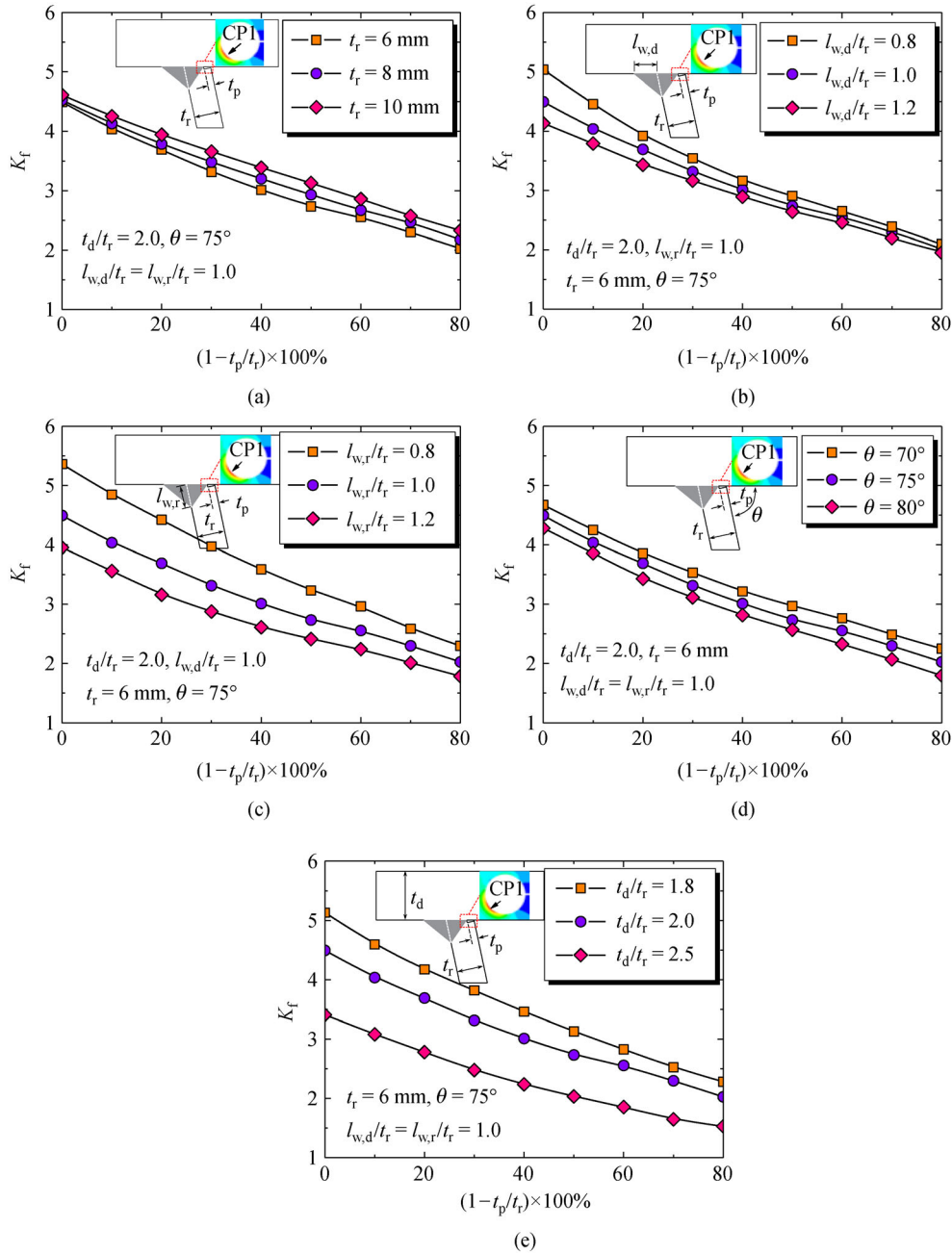
$$K_f^{CPi} = \sum_{k=1}^{28} c_k^{CPi} \cdot f_k(X_1, X_2, X_3, X_4, X_5, X_6), \quad (3)$$

where  $i = 1, 2$ , and  $3$ , e.g.,  $K_f^{CPi}$  represents  $K_f$  in CP1;  $X_1 = 1 - t_p/t_r$ ;  $X_2 = t_r/10$ ;  $X_3 = l_{w,d}/t_r$ ;  $X_4 = l_{w,r}/t_r$ ;  $X_5 = 2\theta/180^\circ$ ;  $X_6 = t_d/t_r$ ;  $c_k^{CPi}$  are the coefficients obtained by the regression analysis. Since six normalized variables ( $X_1, X_2, X_3, X_4, X_5$ , and  $X_6$ ), their squares, each combination of two variables, and an additional constant term (i.e.,  $c_k^{CPi}$ ) are used, a total of 28 terms are involved in the regression functions. Among the 28 terms, some may be dominant; however, the effect of a part of the terms on  $K_f$  could be neglected, as illustrated in the parametric analysis. To simplify the regression functions, the variables were automatically reduced during the regression process [11]. That is, only the variables that meet the conditions of significance and importance filter are considered in the regression functions.

The detailed stepwise regression analysis results are given in Appendix A. The coefficients of the regression formula for  $K_f^{CP1}$ ,  $K_f^{CP2}$ , and  $K_f^{CP3}$  are listed in Table 2. The coefficients could be related to their influence on the dependent variable (i.e.,  $K_f$ ). Table 2 shows that some of the coefficients are set to zero, indicating that the influence of the corresponding terms on  $K_f$  could be neglected. Additionally, the absolute value of the coefficient quantifies the influence of the corresponding term, whereas the negative sign only represents the variation trend of  $K_f$ . Taking the coefficient of  $X_1$  (i.e.,  $1 - t_p/t_r$ ) for instance, the value of  $c_k^{CP1}$  is  $-12.861$ . The negative sign indicates that  $K_f^{CP1}$  decreases as  $X_1$  increases, and the absolute value indicates the considerable effect of  $X_1$  on  $K_f^{CP1}$  that are consistent with the results plotted in Fig. 11.

Figure 14 compares the values of  $K_f^{CP1}$ ,  $K_f^{CP2}$ , and  $K_f^{CP3}$  calculated by Eq. (3) and the values obtained by FEA. All the correlation coefficients ( $R^2$ ) are close to 1.0, indicating that the proposed empirical formulas could provide good predictions of  $K_f^{CP1}$ ,  $K_f^{CP2}$ , and  $K_f^{CP3}$ .

$K_f$  could also be predicted using the Deep Neural Network (DNN) method, as reported in [11,23]. Compared with the proposed regression formulas, the DNN method can handle all load cases and give all results once. Meanwhile, uncertainties could be automatically



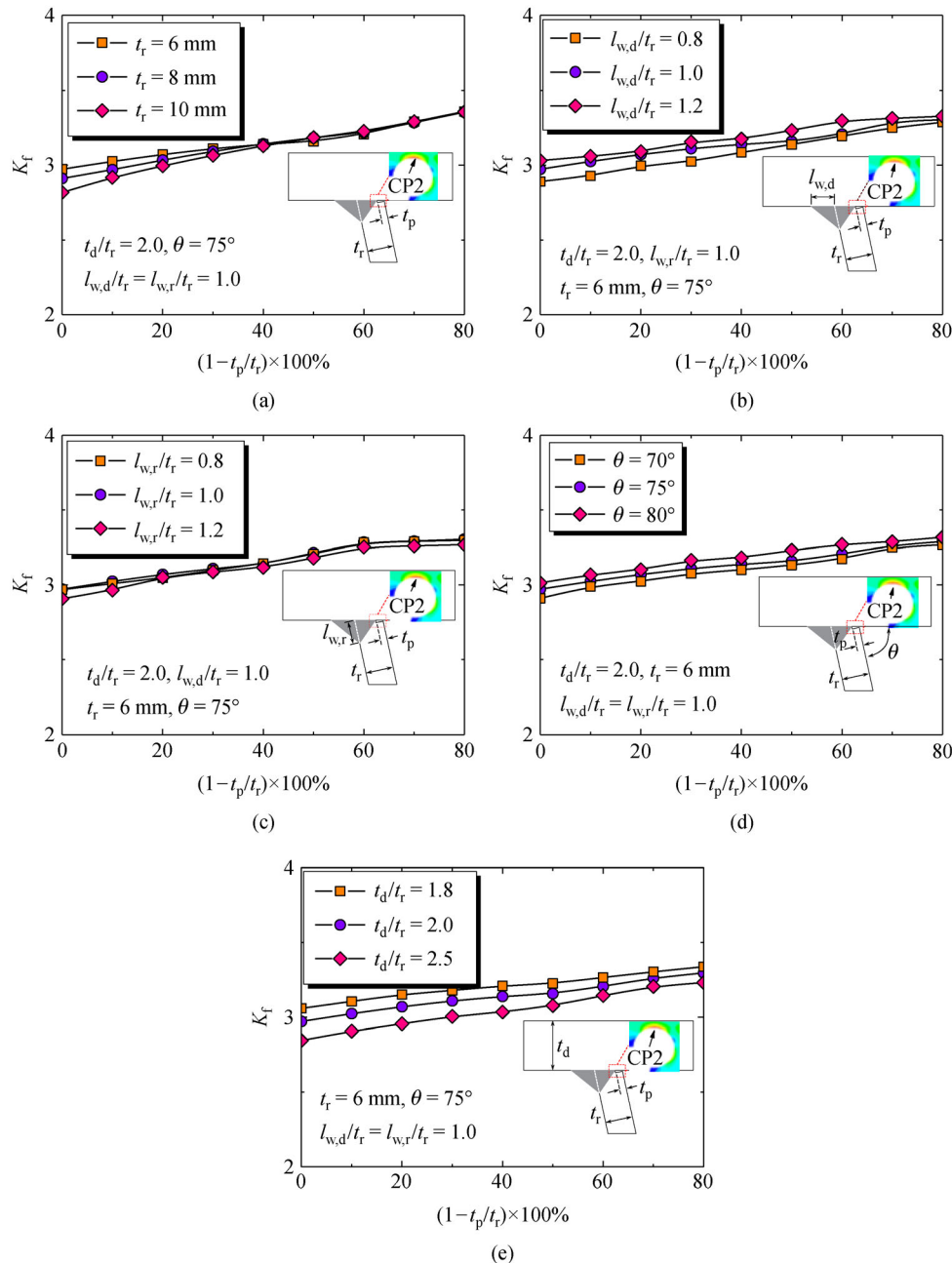
**Fig. 11** Variation of effective notch stress concentration factor ( $K_f$ ) at CPI of weld root. (a) Effect of the weld penetration rate ( $1 - t_p/t_r$ ) and rib thickness ( $t_r$ ); (b) effect of the weld penetration rate ( $1 - t_p/t_r$ ) and relative weld leg length in the deck ( $l_{w,d}/t_r$ ); (c) effect of the weld penetration rate ( $1 - t_p/t_r$ ) and relative weld leg length in the rib ( $l_{w,r}/t_r$ ); (d) effect of the weld penetration rate ( $1 - t_p/t_r$ ) and the angle between the deck and the rib ( $\theta$ ); (e) effect of the weld penetration rate ( $1 - t_p/t_r$ ) and relative deck thickness ( $t_d/t_r$ ).

accounted for due to the advantage of the DNN method. Further work is needed to employ the DNN method in predicting  $K_f$ .

## 5.2 Validation of the proposed formula

To further validate the reliability of the proposed formulas

for predicting  $K_f$ , we adopted additional data for  $l_{w,d}/t_r = 0.6$ ,  $l_{w,r}/t_r = 1.5$ ,  $t_r = 7$  mm, and  $\theta = 60^\circ$ , which were not considered in the regression analysis. The results are plotted in Fig. 15. We observed that the estimated values of  $K_f$  are in good agreement with the numerical results obtained by FEA. This demonstrates that the proposed formulas could be adopted for determining  $K_f$  in rib–deck welds.



**Fig. 12** Variation of  $K_f$  at CP2 of weld root. (a) Effect of the weld penetration rate  $(1 - t_p/t_r)$  and  $t_r$ ; (b) effect of the weld penetration rate  $(1 - t_p/t_r)$  and  $l_{w,d}/t_r$ ; (c) effect of the weld penetration rate  $(1 - t_p/t_r)$  and  $l_{w,r}/t_r$ ; (d) effect of the weld penetration rate  $(1 - t_p/t_r)$  and  $\theta$ ; (e) effect of the weld penetration rate  $(1 - t_p/t_r)$  and  $t_d/t_r$ .

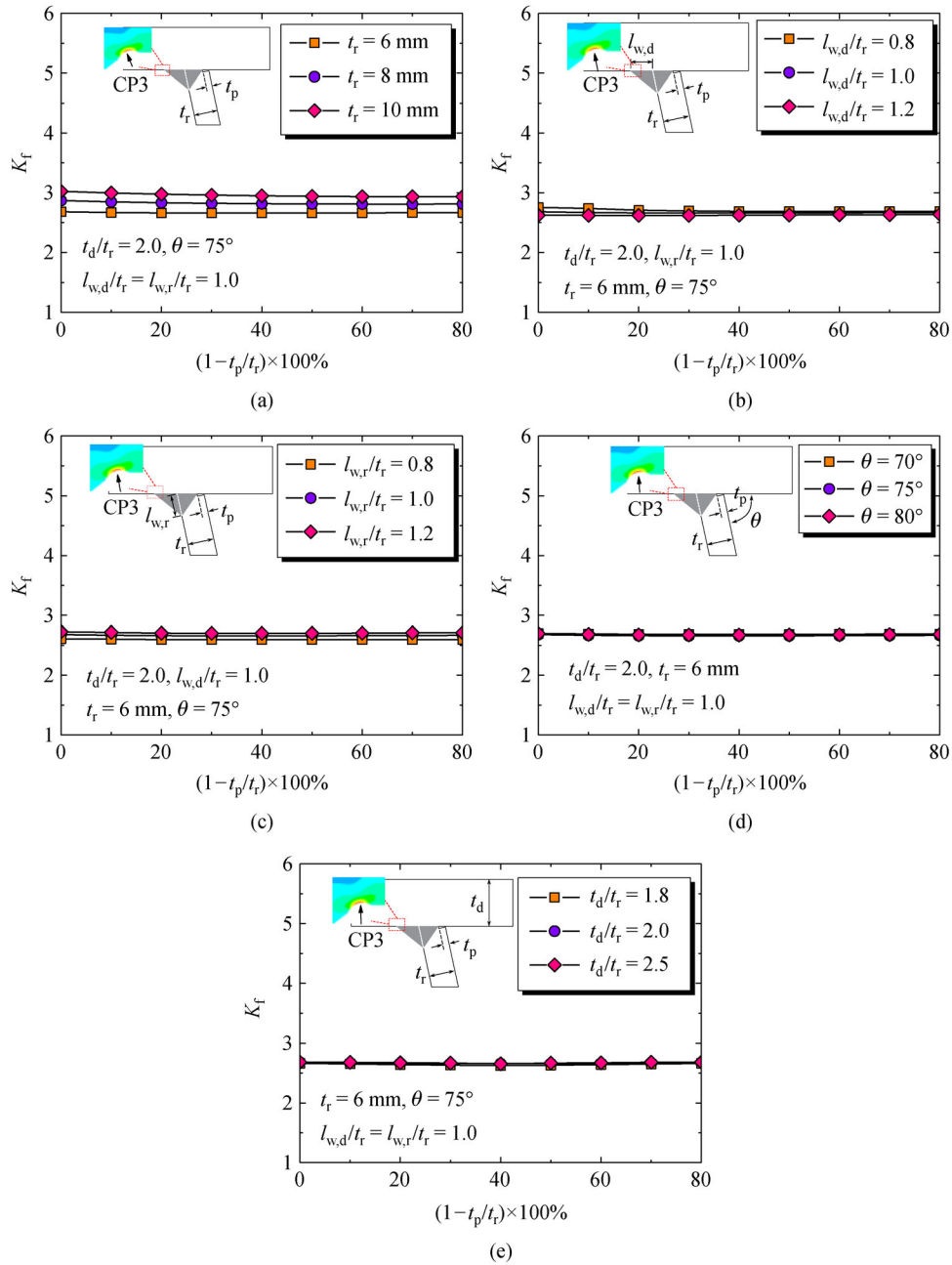
## 6 Case study

Once the crack type is determined,  $K_f$  at the crack initiation position (i.e., the objective stress concentration position) could be quickly determined using Eq. (3). Then, the nominal stress-based fatigue data could be converted to the notch stress system, subsequently evaluated using the notch stress S–N curves. In this study, one example is presented to show the application of the proposed formulas for  $K_f$ .

### 6.1 Review of the fatigue test

Fatigue testing data of nine full-scale rib–deck welded specimens were considered [24]. The nine specimens were divided into three equal groups according to  $t_d$  (i.e., 14, 16, and 18 mm). All specimens had the same  $t_r$  of 8 mm and a penetration rate of 80%. The geometric parameters of the rib–deck weld are plotted in Fig. 16(a). For other geometric sizes of the specimen, please refer to Ref. [24]. Figure 16(b) shows the experimental diagram of the





**Fig. 13** Variation of  $K_f$  at CP3 of weld toe. (a) Effect of the weld penetration rate  $(1 - t_p/t_r)$  and  $t_r$ ; (b) effect of the weld penetration rate  $(1 - t_p/t_r)$  and  $l_{w,d}/t_r$ ; (c) effect of the weld penetration rate  $(1 - t_p/t_r)$  and  $l_{w,r}/t_r$ ; (d) effect of the weld penetration rate  $(1 - t_p/t_r)$  and  $\theta$ ; (e) effect of the weld penetration rate  $(1 - t_p/t_r)$  and  $t_d/t_r$ .

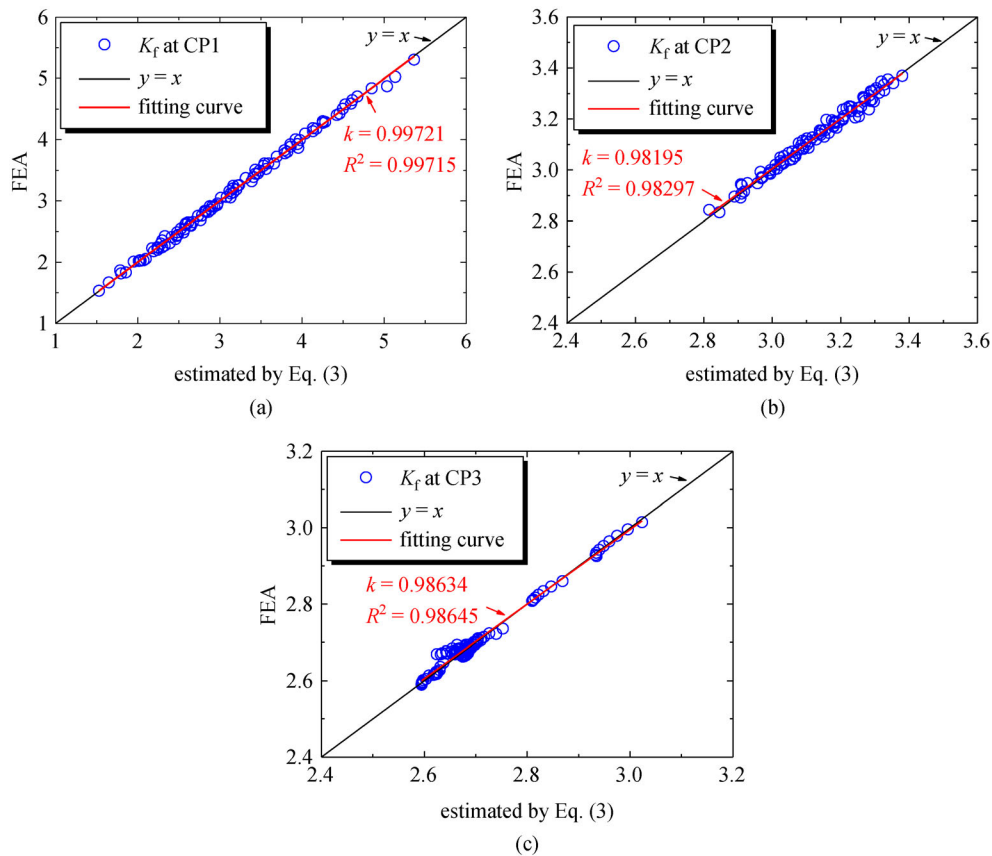
rib–deck welded specimens. The loading force was equally distributed by the distribution beam on the deck plate surface. Thus, the bending moment between the two loading positions was consistent. When a fatigue crack penetrated the deck plate, the specimen was considered failed, and the experiment was terminated. The definition of failure here is similar to that proposed by IIW [7]. Hence, the S–N curves recommended by IIW could be employed in evaluating fatigue testing data.

## 6.2 Application of the proposed formula

In the fatigue test, all the fatigue cracks initiated at the weld toe and propagated along the thickness of the deck plate, as shown in Fig. 17. Hence,  $K_f$  in the weld toe (i.e.,  $K_f$  in CP3) needs to be determined. We developed FEMs to simulate 14, 16, and 18 mm thick specimens using the same modeling method discussed in Section 2 to determine the values of  $K_f$ . Furthermore, the proposed formula (Eq. (3)),

**Table 2** Coefficients for the regression formula of  $K_f^{CP1}$ ,  $K_f^{CP2}$ , and  $K_f^{CP3}$ 

$k$	$f_k$	$c_k^{CP1}$	$c_k^{CP2}$	$c_k^{CP3}$	$k$	$f_k$	$c_k^{CP1}$	$c_k^{CP2}$	$c_k^{CP3}$
1	1	11.167	3.824	2.367	15	$X_1X_3$	2.159	-0.216	0.077
2	$X_1$	-12.861	0.000	0.000	16	$X_1X_4$	2.960	0.000	0.000
3	$X_2$	0.000	0.000	0.000	17	$X_1X_5$	0.000	0.000	0.000
4	$X_3$	0.000	0.000	0.000	18	$X_1X_6$	1.543	0.182	0.000
5	$X_4$	0.000	0.000	0.000	19	$X_2X_3$	0.000	0.000	0.000
6	$X_5$	0.000	0.000	0.000	20	$X_2X_4$	0.000	-0.320	0.873
7	$X_6$	0.000	-0.801	0.000	21	$X_2X_5$	0.000	0.000	0.000
8	$X_1^2$	1.363	-0.085	0.114	22	$X_2X_6$	0.177	0.000	0.000
9	$X_2^2$	0.000	0.000	0.000	23	$X_3X_4$	0.000	0.791	-0.259
10	$X_3^2$	0.000	-0.229	0.000	24	$X_3X_5$	0.000	0.000	0.000
11	$X_4^2$	1.989	-0.334	0.000	25	$X_3X_6$	-0.919	0.000	0.024
12	$X_5^2$	-2.245	0.353	0.000	26	$X_4X_5$	0.000	0.000	0.000
13	$X_6^2$	0.526	0.117	0.000	27	$X_4X_6$	-3.785	0.000	0.000
14	$X_1X_2$	0.792	0.578	-0.295	28	$X_5X_6$	0.000	0.000	0.000

**Fig. 14** Comparison of FEA and estimated results. (a)  $K_f$  at CP1; (b)  $K_f$  at CP2; (c)  $K_f$  at CP3.

as well as the coefficients listed in Table 2, was also adopted to predict the values of  $K_f$ . The FEA and estimated results are compared in Table 3, and they are in good agreement, with a maximum discrepancy of 2.91%.

### 6.3 Fatigue data in the notch stress system

The estimated values of  $K_f$  were applied, and then, the nominal stress-based fatigue data were converted to the

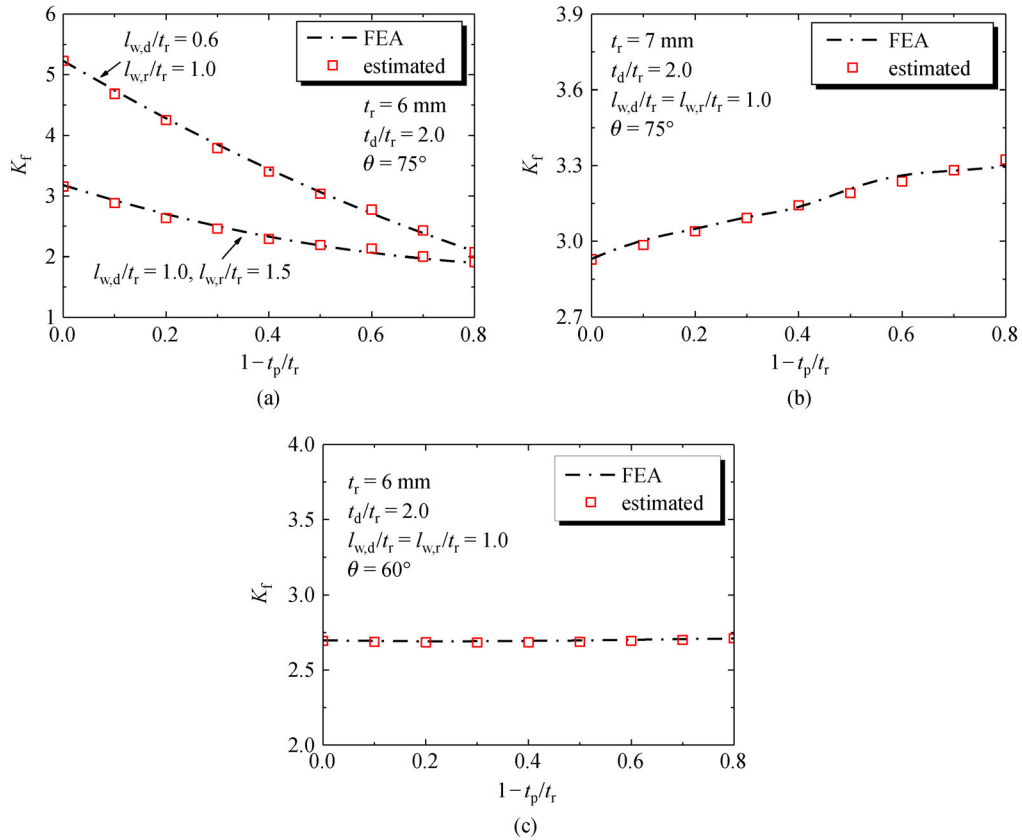


Fig. 15 Validation of the proposed formula using additional data. (a)  $K_f$  at CP1; (b)  $K_f$  at CP2; (c)  $K_f$  at CP3.

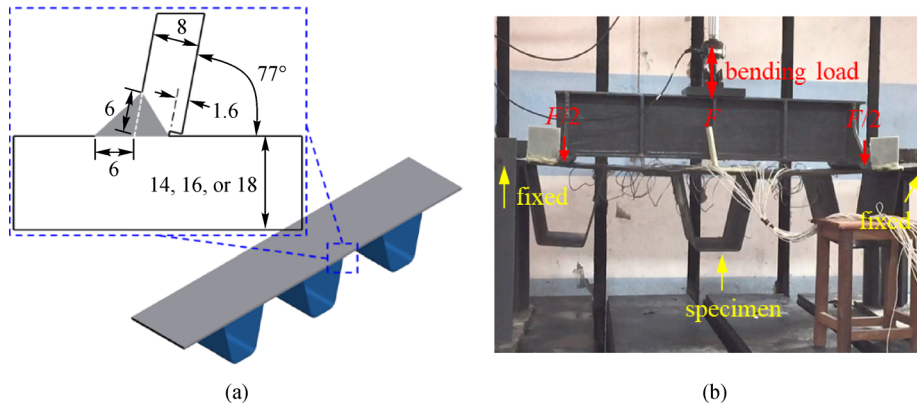
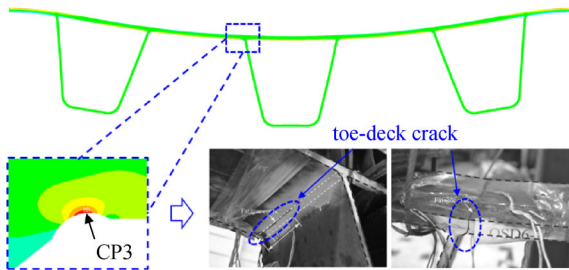


Fig. 16 Illustration of the fatigue test. (a) Geometry of full and 80% penetrated specimens; (b) bending loading mode.

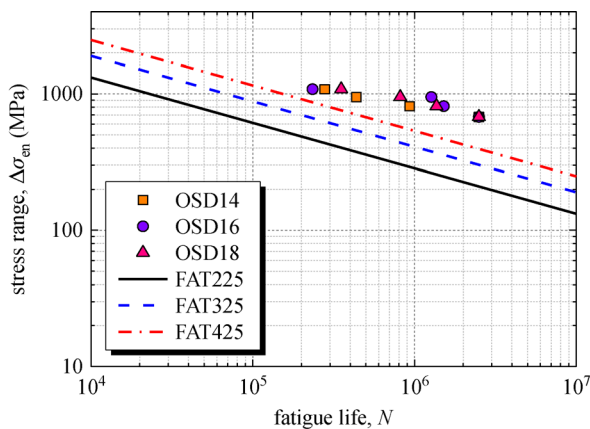
Table 3 Comparison of FEA and estimated values of  $K_f$

deck plate thickness (mm)	FEA, $K_{f1}$	estimated, $K_{f2}$	$(K_{f1} - K_{f2}) / K_{f2} \times 100\%$
14	2.681	2.707	-0.96%
16	2.759	2.711	1.77%
18	2.795	2.716	2.91%



**Fig. 17** Notch stress concentration and fatigue cracking of OSD series.

effective notch stress-based fatigue data, as plotted in Fig. 18. The FAT class S–N curves are also included in the figure. All data points fall above the fatigue design curve of FAT225 in the IIW fatigue design recommendations [7]. The data points could also satisfy the S–N curves of FAT325 and FAT425. It implies that the fatigue design curve of FAT225 in the IIW fatigue design recommendations is conservative for the fatigue design of rib–deck welds possibly subjected to toe–deck cracking.



**Fig. 18** Fatigue data in the notch stress system.

## 7 Conclusions

We performed a parametric analysis to investigate the effect of geometric parameters on the effective notch stress concentration factors ( $K_f$ ) in rib–deck welds. We also performed regression analysis to develop empirical formulas for predicting  $K_f$  in rib–deck welds. A case study is presented to show the application of the proposed formulas. The following conclusions can be drawn.

1) We propose a modeling method for investigating the effective notch stress in rib–deck welds. The number of elements in the 360° arc should be no less than 26. The nominal stress point close to the weld toe or root should be 30 or 20 mm away from the weld toe or root, respectively.

2) Based on the parametric analysis results, we propose empirical formulas for predicting  $K_f$  in rib–deck welds.

The proposed formulas cover  $K_f$  for three types of fatigue-cracking modes. The correlation coefficients between the FEA and estimated results are close to 1.0, showing good agreement between the results.

3) A case study was conducted to exemplify the application of the proposed formulas. A maximum discrepancy of 2.91% between the FEA and estimated values of  $K_f$  confirms the reliability of the proposed formulas. FAT225 recommended by IIW seems conservative for evaluating the fatigue data of rib–deck welds. Further research is needed to develop the optimal notch stress-based S–N curves for rib–deck welds.

## Appendix

### Appendix A

The detailed stepwise regression analysis results of  $K_f^{CP1}$ ,  $K_f^{CP2}$ , and  $K_f^{CP3}$  are listed in Tables A1, A2, and A3, respectively. To develop the optimal regression formula for  $K_f^{CP1}$ , 16 trial regression models were adopted. Similarly, 13 trial regression models were adopted for  $K_f^{CP2}$  and six models for  $K_f^{CP3}$ . The more independent variables the regression equation contains, the larger the correlation coefficients ( $R^2$ ), the smaller the standard error of estimate, and the better the simulation effect. In this study, the final correlation coefficients ( $R^2$ ) for  $K_f^{CP1}$ ,  $K_f^{CP2}$ , and  $K_f^{CP3}$  were 0.997, 0.981, and 0.986, respectively.

**Table A1** Stepwise regression analysis results of  $K_f^{CP1}$

model	predictor variables	$R^2$	standard error of estimate
1	$X_0, X_1X_6$	0.832	0.35968972
2	$X_0, X_1X_6, X_4X_6$	0.930	0.23125343
3	$X_0, X_1X_6, X_4X_6, X_1X_5$	0.946	0.20322842
4	$X_0, X_4X_6, X_1X_5$	0.947	0.20242335
5	$X_0, X_4X_6, X_1X_5, X_3X_6$	0.961	0.17420706
6	$X_0, X_4X_6, X_1X_5, X_3X_6, X_1^2$	0.970	0.15196302
7	$X_0, X_4X_6, X_1X_5, X_3X_6, X_1^2, X_1X_2$	0.977	0.13208737
8	$X_0, X_4X_6, X_1X_5, X_3X_6, X_1^2, X_1X_2, X_1X_4$	0.980	0.12270214
9	$X_0, X_4X_6, X_1X_5, X_3X_6, X_1^2, X_1X_2, X_1X_4, X_1X_6$	0.985	0.10805856
10	$X_0, X_4X_6, X_1X_5, X_3X_6, X_1^2, X_1X_2, X_1X_4, X_1X_6, X_1$	0.989	0.09292036
11	$X_0, X_4X_6, X_1X_5, X_3X_6, X_1^2, X_1X_2, X_1X_4, X_1X_6, X_1, X_5^2$	0.992	0.07950012
12	$X_0, X_4X_6, X_3X_6, X_1^2, X_1X_2, X_1X_4, X_1X_6, X_1, X_5^2$	0.992	0.07909066
13	$X_0, X_4X_6, X_3X_6, X_1^2, X_1X_2, X_1X_4, X_1X_6, X_1, X_5^2, X_1X_3$	0.994	0.06530485

(Continued)

model	predictor variables	$R^2$	standard error of estimate
14	$X_0, X_4X_6, X_3X_6, X_1^2, X_1X_2, X_1X_4, X_1X_6, X_1, X_5^2, X_1X_3, X_2X_6$	0.995	0.06159826
15	$X_0, X_4X_6, X_3X_6, X_1^2, X_1X_2, X_1X_4, X_1X_6, X_1, X_5^2, X_1X_3, X_2X_6, X_6^2$	0.995	0.05886461
16	$X_0, X_4X_6, X_3X_6, X_1^2, X_1X_2, X_1X_4, X_1X_6, X_1, X_5^2, X_1X_3, X_2X_6, X_6^2, X_4^2$	0.997	0.04973409

**Table A2** Stepwise regression analysis results of  $K_f^{CP2}$

model	predictor variables	$R^2$	standard error of estimate
1	$X_0, X_1X_3$	0.856	0.04981720
2	$X_0, X_1X_3, X_6$	0.920	0.03700029
3	$X_0, X_1X_3, X_6, X_5^2$	0.931	0.03432015
4	$X_0, X_1X_3, X_6, X_5^2, X_2X_4$	0.939	0.03247062
5	$X_0, X_1X_3, X_6, X_5^2, X_2X_4, X_1X_2$	0.953	0.02850526
6	$X_0, X_1X_3, X_6, X_5^2, X_2X_4, X_1X_2, X_3X_4$	0.969	0.02319788
7	$X_0, X_6, X_5^2, X_2X_4, X_1X_2, X_3X_4$	0.969	0.02321962
8	$X_0, X_6, X_5^2, X_2X_4, X_1X_2, X_3X_4, X_4^2$	0.971	0.02241161
9	$X_0, X_6, X_5^2, X_2X_4, X_1X_2, X_3X_4, X_4^2, X_1X_6$	0.974	0.02118114
10	$X_0, X_6, X_5^2, X_2X_4, X_1X_2, X_3X_4, X_4^2, X_1X_6, X_1X_3$	0.977	0.01988373
11	$X_0, X_6, X_5^2, X_2X_4, X_1X_2, X_3X_4, X_4^2, X_1X_6, X_1X_3, X_6^2$	0.979	0.01901173
12	$X_0, X_6, X_5^2, X_2X_4, X_1X_2, X_3X_4, X_4^2, X_1X_6, X_1X_3, X_6^2, X_1^2$	0.980	0.01836800
13	$X_0, X_6, X_5^2, X_2X_4, X_1X_2, X_3X_4, X_4^2, X_1X_6, X_1X_3, X_6^2, X_1^2, X_3^2$	0.981	0.01806100

**Table A3** Stepwise regression analysis results of  $K_f^{CP3}$

model	predictor variables	$R^2$	standard error of estimate
1	$X_0, X_2X_4$	0.918	0.03051650
2	$X_0, X_2X_4, X_3X_4$	0.967	0.01940469
3	$X_0, X_2X_4, X_3X_4, X_1X_2$	0.975	0.01685447
4	$X_0, X_2X_4, X_3X_4, X_1X_2, X_1^2$	0.982	0.01423178
5	$X_0, X_2X_4, X_3X_4, X_1X_2, X_1^2, X_3X_6$	0.984	0.01355348
6	$X_0, X_2X_4, X_3X_4, X_1X_2, X_1^2, X_3X_6, X_1X_3$	0.986	0.01291298

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