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RESEARCH ARTICLE

Multi-objective optimal design of braced frames using hybrid genetic and ant colony optimization

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ABSTRACT In this article, multi-objective optimization of braced frames is investigated using a novel hybrid algorithm. Initially, the applied evolutionary algorithms, ant colony optimization (ACO) and genetic algorithm (GA) are reviewed, followed by developing the hybrid method. A dynamic hybridization of GA and ACO is proposed as a novel hybrid method which does not appear in the literature for optimal design of steel braced frames. Not only the cross section of the beams, columns and braces are considered to be the design variables, but also the topologies of the braces are taken into account as additional design variables. The hybrid algorithm explores the whole design space for optimum solutions. Weight and maximum displacement of the structure are employed as the objective functions for multi-objective optimal design. Subsequently, using the weighted sum method (WSM), the two objective problem are converted to a single objective optimization problem and the proposed hybrid genetic ant colony algorithm (HGAC) is developed for optimal design. Assuming different combination for weight coefficients, a trade-off between the two objectives are obtained in the numerical example section. To make the final decision easier for designers, related constraint is applied to obtain practical topologies. The achieved results show the capability of HGAC to find optimal topologies and sections for the elements.

KEYWORDS multi-objective, hybrid algorithm, ant colony, genetic algorithm, displacement, weighted sum method, steel braced frames

1 Introduction

The aim of optimal design is to obtain the best solution based on the objective function in the given condition. Structural optimization is a complex, not simply solved, problem. If the layout of the structure is assumed, under the loading and boundary conditions, the structural responses could be obtained using the finite element analysis. The purpose of the optimization, however, is to attain an optimal structural system under the constraints. Since, the structural optimization problems usually include nonlinear and implicit functions, the analytical methods are unable to solve these practical problems. Therefore, numerical methods remain to be the only option in solving practical real optimization problems.

Three types of optimal designs in structural optimization include topology, shape and size optimization. The aim of topology optimization is to determine the optimal connection and relation of structural elements, which define the existence or non-existence of the elements. Shape optimization is the process of looking for an optimum geometry for the structure, and optimal sizing is focused on the determination of optimal cross sections for the structural elements. The proposed hybrid genetic ant colony algorithm (HGAC) in this paper is to find optimal topology and sizing for steel braced frames.

During the recent decades, there has been a tendency to develop heuristic methods to solve discrete optimization problems, which have been inspired from the nature. Although in these methods, there is no guarantee for feasibility and optimality of the solutions, however, they are able to obtain near optimal solutions, which are not possible to attain by conventional gradient-based methods. The developed heuristic methods for optimal design of structures consist of genetic algorithm (GA) [1,2], evolutionary programming (EP) [3], simulated annealing

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(SA) [4], evolution strategies (ESs) [5], ant colonies (AC) [6,7], particle swarm optimization (PSO) [8], tabu search (TS) [9], and cellular automata (CA) [10]. Elbeltagi et al. [11] presented a comparative study on five evolutionary heuristic optimization algorithms in 2005. GAs, among the heuristic algorithms, are most famous for structural optimal design and have a great potential in producing optimal solutions for practical problems [12,13].

In 2005, Espinoza et al. demonstrated that hybrid GAs (HGAs) with local search algorithms required significantly fewer function evaluations to achieve convergence when compared with the simple GA [14]. As stated in the literature, GA has strong ability to exploit using crossover and mutation operators, while ant colony (ACO) is more famous to explore providing diverse population [15,16]. Chan and Wong [17] proposed a hybrid algorithm based on GA and optimality criteria (OC) for structural topology and size optimization of tall steel frames. Chen et al. proposed a co-evolutionary model based on dynamic combination of GA and ACO [18]. Kicinger and Arciszewski [19] reported that HGAs have formed a rapidly growing research area with great potential in solving structural design problems. They studied empirical analysis of memetic algorithms for conceptual design of tall buildings. Kareem et al. reported a strong work by utilizing computational fluid dynamics to optimize tall building design [20]. Spence and Kareem [21] investigated data-enabled design and optimization for tall buildings. Performance-based topology optimization for wind-sensitive tall buildings has been studied and successive results reported [22].

Practical optimal topology for reinforced concrete moment resisting frames (RCMRFs) is reported in the literature [23]. Similar studies are reported in the literature for steel moment resisting frames with and without bracings [24,25]. Kaveh et al. worked on plastic analyses of frames using genetic algorithm and ant colony algorithm [26]. Babaei et al. studied multi-objective optimization of RCMRFs using a non-dominated sorting genetic algorithm II (NSGA-II) [27]. To solve quadratic assignment problems (QAP), Xu et al. [28] proposed a novel adaptive GA-ACO-local search hybrid algorithm, which is a synergy between the main algorithms. More recently, optimization of tall buildings with outrigger belt trusses were investigated and optimum location and number for trusses were proposed based on two objective functions [29].

The following section defines the optimal design problem formulation, to review the application of GA and ACO algorithms, and to develop the proposed novel hybrid algorithm. Section 3 includes case studies to illustrate the capability of the HGAC method for topology and size optimization of steel braced frames. Significant conclusions are drawn in Section 4.

2 Formulation and optimization method

A minimization problem obtains the design variables so that the objective function is minimized by satisfying all the defined constraints. In general, constrained minimization problems are formulated as follows:

$$\begin{aligned} &\text{minimize :} \quad f(x), \\ &\text{subject to :} \quad g_m(x) \leqslant 0 \quad (m = 1,2,...M), \\ &x_i^L \leqslant x_i \leqslant x_i^U \quad (n = 1,2,...,N), \end{aligned} \tag{1}$$

where $f(x)$ is objective function; $x = \{x_1, x_2, ..., x_N\}$ is the design variable vector of N members; and M is the number of constraints.

In multi-objective minimization problems, the design variables are obtained based on the different combinations of the objective functions. Typically, a multi-objective minimization problem could be defined as follows:

$$\begin{aligned} &\text{minimize :} \quad F(x) = (f_1(x), f_2(x), ..., f_n(x)), \\ &\text{subject to :} \quad g_m(x) \leqslant 0 \quad (m = 1,2,...M), \\ &x_i^L \leqslant x_i \leqslant x_i^U \quad (n = 1,2,...,N), \end{aligned} \tag{2}$$

where $f_1(x), f_2(x), ..., f_n(x)$ are objective functions; n is the number of objective functions; $x = \{x_1, x_2, ..., x_N\}$ is the design variable vector of N members; and M is the number of constraints.

There are many different methods to convert a multi-objective optimization problem into a single objective optimization problem [30], the most popular one of which is the weighted sum method (WSM) [31]. In this method, the main objective function is obtained by a linear combination of all weighted objective functions as follows:

$$F(x) = \sum_{i=1}^{M} w_i f_i(x_1, x_2, ..., x_n), \tag{3}$$

where w is a nonzero weight vector for objective functions, defined so that $\sum_{i=1}^{M} w_i = 1$. If all the weights are positive, then the minimum value of the main objective function is Pareto front [32].

The objective function for multi-objective optimal design of steel braced frames using weighted sum method could be written as:

$$\begin{aligned} f(x) &= w_1 f_1(x)/f_{1\min}(x) + w_2 f_2(x)/f_{2\min}(x) \\ &= w_1 \cdot \sum_{i=1}^{n} \rho_i . l_i . x_i / f_{1\min}(x) + w_2 \cdot_{roof} / f_{2\min}(x). \end{aligned} \tag{4}$$

The stress constraints are as:

$$|\frac{\sigma_i}{\sigma_i^a}| \leqslant 1; \quad i = 1 \text{ to } n. \tag{5}$$

In addition, the displacements constraints are:

$$\delta_i \leqslant \delta_{\max}; \quad i = 1 \text{ to } m, \tag{6}$$

where $f_1(x)$ and $f_2(x)$ are the total weight and displacement objective function of the structure, respectively; $f_{1\min}(x)$ and $f_{2\min}(x)$ are the minimum weight and displacement objective functions obtained by assigning the weakest and the strongest cross sections to the elements, respectively;σ_i and σ_i^a are the existing and allowable bending stresses, respectively; δ_i and $\delta_{\max}$ are the story and allowable deflections; and ρ_i is the density of the structural elements.

The penalized objective function is as follows:

$$\begin{aligned} f(x) &= w_1 \cdot f_1(x) \cdot (1 + C_w)^{P_w} / f_{1\min}(x) \\ &\quad + w_2 \cdot f_2(x) \cdot (1 + C_d)^{P_d} / f_{2\min}(x) \\ &= w_1 \cdot \Big(\sum_{i=1}^{m} \gamma_i \cdot L_i \cdot A_i\Big) \cdot (1 + C_w)^{P_w} / f_{1\min}(x) \\ &\quad + w_2 \cdot \Delta_{roof} \cdot (1 + C_d)^{P_d} / f_{2\min}(x), \end{aligned} \tag{7}$$

where C_w, C_d and p_w, p_d are the penalty value and exponent for weight and displacement objective functions, respectively.

Assuming $w_1 = \alpha$ and $w_2 = 1-\alpha$, (since $\sum_{i=1}^{M} w_i = 1$) , the objective function may be written as:

$$\begin{aligned} f(x) &= \alpha \cdot f_1(x) \cdot (1 + C_w)^{P_w} / f_{1\min}(x) \\ &\quad + (1-\alpha) \cdot f_2(x) \cdot (1 + C_d)^{P_d} / f_{2\min}(x) \\ &= \alpha \cdot \Big(\sum_{i=1}^{m} \gamma_i \cdot L_i \cdot A_i\Big) \cdot (1 + C_w)^{P_w} / f_{1\min}(x) \\ &\quad + (1-\alpha) \cdot \Delta_{roof} \cdot (1 + C_d)^{P_d} / f_2(x). \end{aligned} \tag{8}$$

Since the two objective functions are not in the same scale, they must be normalized properly. Therefore, the normalized compound objective function could be written as follows:

$$\begin{aligned} f(x) &= \alpha \cdot F_1(x) \cdot (1 + C_w)^{p_w} \\ &\quad + (1-\alpha) \cdot F_2(x) \cdot (1 + C_d)^{p_d}, \end{aligned} \tag{9}$$

where $F_1(x)$ and $F_2(x)$ are the normalized total weight and displacement objective functions of the structure, respectively.

Genetic algorithm is one of the stochastic search methods, which is inspired by the nature. GAs have many applications in solving optimization problems. In the nature, better generations are achieved by combination of better chromosomes. Sometimes, during the combinations, mutation in chromosomes occurs, which may result in a better chromosome. There are, usually, some operators for selection, combination and mutation of chromosomes, however, GAs utilizes four operators of fitness, selection, crossover and mutation to solve the problems.

GAs, initially, create a number of solutions, renowned as first generation/population, stochastically or systematically. Each solution is named a chromosome. Subsequently, new generations are created, using selection, crossover and mutation operators. Finally, the old population is combined and replaced by some or all of the new population.

Ant system-rank (AS-rank) algorithm is the most famous method of ant colony algorithms, introduced by Bullnheimer et al. [33]. In this algorithm, each ant trails a pheromone base on its rank and the best ant contains the largest value of pheromone, similar to elite ant system (EAS). The pheromone updating formula is can be written as:

$$\tau_i^j = (1-\rho) \cdot \tau_i^j + \sum_{r=1}^{w-1} (w-1) F(s^r) + wF(s^{bs}), \tag{10}$$

where τ_i^j is the intensity of trial; ρ is the evaporation constant between 0 and 1; $F(s^r)$ is the trial change for the ranked ants; $wF(s^{bs})$ is the extra trial for best, elite, solution.

Since better performance of GA and ACO is represented in the literature [33], they are employed as the main algorithms of this research. Many different combinations of these methods are developed [34]. In this paper, a dynamic hybridization of GA and ACO is developed and applied to solve multi-objective optimal design of steel braced frames, which has not applied in the literature for steel braced frames optimization. The phenomenon of long time convergence, precocity, and stagnation sometimes emerge from GA and ACO. The status of execution process in each algorithm is different. In the other words, in the executing procedure stage, best chromosome, in GA, is different from best ant. Therefore, a hybrid algorithm is presented to run ACO and then switches to GA by evaluating the running state.

According to the code requirements and engineering experiences, in a frame structure the elements of lower levels are stronger than the upper levels. Although it depends on the loading condition and the topology of the structure, but in a building frame structure, beam, column or even brace sections in the first floor need to be the heaviest, while the sections at the top level of the structure need to be the lightest profiles. In this article, sorting of the element sections based on their section number is proposed and applied in the developed algorithm, as a new operator. Many tested examples and their results have shown the effectiveness of this new operator, as illustrated in experimental section.

The stagnation and slow convergence phenomenon emerges in GA and ACO. To overcome this phenomenon during the optimization process, the following definition is applied in the hybrid algorithm:

$$Convergence(i) = |f_{avg}(i) - f_{avg}(i-1)|, \tag{11}$$

$$f_{avg}(i) = \frac{1}{n}\sum_{j=1}^{n} f_{avg}^{j}(i), \tag{12}$$

where i is the generation of GA or ACO; $f_{avg}(i)$ is the average fitness in GA or ACO in the i^{th} generation; $f_{avg}^{j}(i)$ is the fitness of the j^{th} chromosome in GA or ant in ACO; n is the number of chromosomes in GA or ants in ACO. The defined convergence will become small when the speed convergence tends to slow or stop.

HGAC starts by running ACO. By checking the status of the process using Eqs. (10) and 11, the algorithm switches to the other main approach and continues the process until the termination criteria met. During the process, the new operator searches for the better solution alongside the other operators. The penalized objective function is introduced and employed during the optimization process. However, not only the penalized total weight of the structure is obtained, but the total weight of the frame is also obtained to compare the results.

3 Numerical results and discussion

3.1 Ten-storey braced frame

In this case study, multi-objective topology and size optimization of a ten-storey three-span braced frame is considered to evaluate the developed hybrid algorithm, as shown in Fig. 1. Braces may be added to the middle span. Span length and storey height of the frame are set to 5 and 3.5 m, respectively. Uniform distributed gravity loads on the beams are considered to be 50kN/m and a lateral point load of 30 kN is applied at each story. Topology of the braces and cross section of the columns, beams and braces are defined to be design variables.

Two objective functions of the total weight and the maximum roof displacement are converted to a single objective function using Eq. (8). Columns and beams sections may be selected between W4 × 13 and W44 × 335 AISC W-shaped sections. Element groupings are illustrated in Fig. 2, to consider the symmetry for the frame. There are 10 topological and 10 cross sectional design variables for braces, and 40 cross sectional variables for beams and columns. As a whole, a total of 60 design variables are considered in this example.

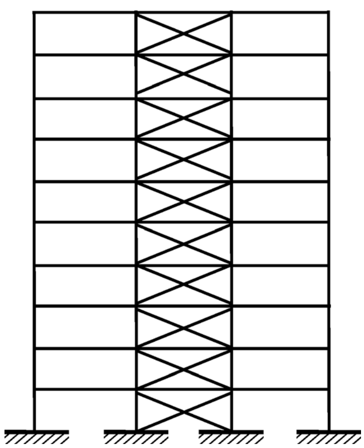

Fig. 1 The topology of the fully braced frame

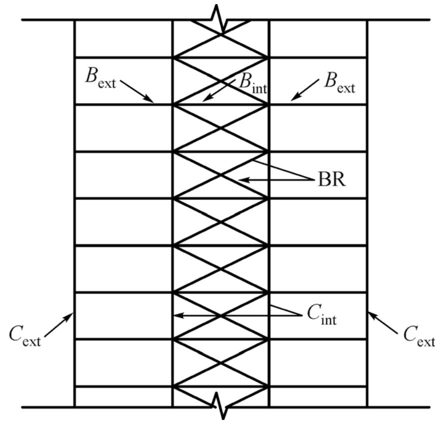

Fig. 2 Grouping scheme for the elements

The termination criterion is applied when the number of iteration is 100 or when the results of 10 consecutive iterations are similar. Since two objective functions are employed, there is not one specific optimal solution for this problem and trade-off between two objectives are obtained using WSM, as illustrated in Fig. 5. As shown in this figure, a large value of α results in for the structural weight, the first objective function, to be important throughout the optimization procedure and decrease the number of braces, but it leads to an increase in the displacement of the best solution, and vice versa. When $\alpha = 0$, the displacement of the frame is very small, however the weight of the structure increases drastically to control the displacement.

The results of the three different scenarios, assuming $\alpha = 1$, 0.6, 0.2, are given in Table 1. The effect of this coefficient on the trade-off between objective functions is clear. Since all constraints are satisfied, both the penalized weight and the net weight of the structure are the same. Figure 3 shows the convergence history for HGAC, GA, and ACO, and Fig. 4 illustrates the effect of sorting elements cross section on the HGAC convergence. Moreover, Fig. 5 displays Pareto front, illustrating the topology of different scenarios. This figure shows that to optimize

Table 1 Optimal results for the scenarios

	scenario 1	scenario 2	scenario 3
penalized weight (kN)	304	471	747.7
net weight (kN)	304	471	747.7
displacement (cm)	3.85	1.63	0.72

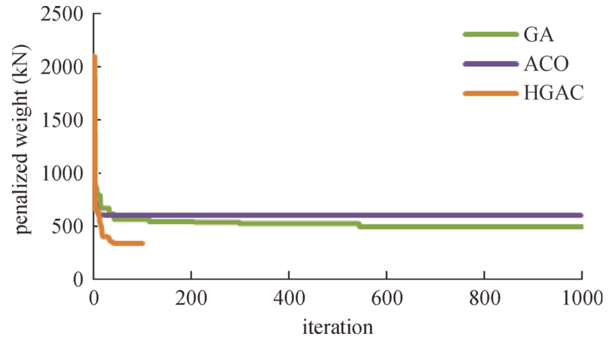

Fig. 3 Convergence history of HGAC, GA and ACO three algorithm for 10-storey braced frame

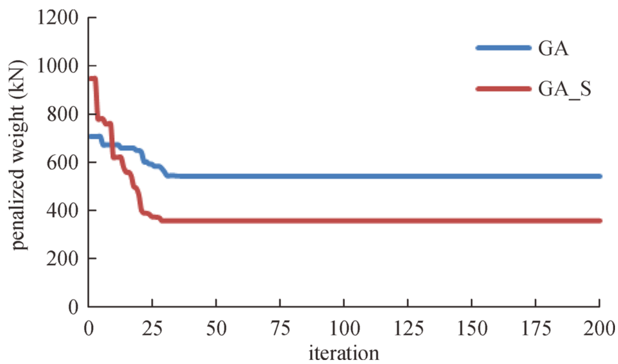

Fig. 4 The effect of new operator in convergence of the GA

the weight of the structure, the frame does not need more braces, while to minimize the displacement more floors (at least 60 percent) need bracing elements. The best final topology of the structure, when the total weight of the frame is minimized by satisfying the displacement constraint, is shown in Fig. 6.

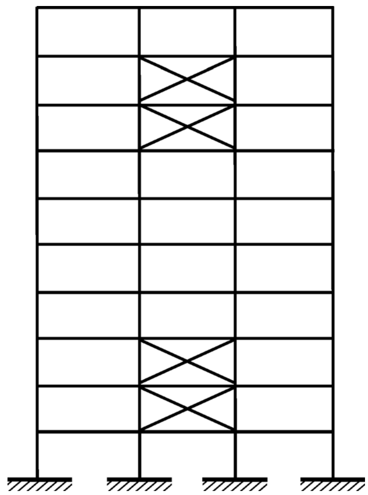

Fig. 6 The best final topology for ten-storey braced frame

3.2 Forty-storey braced frame

In this example, multi-objective topology and sizing optimization of a 40-storey three-span braced frame is considered. The topology and loading conditions are given in Fig. 7. Span length, storey heights, bracing locations, elements grouping and uniform distributed gravity loads on the beams are the same as example 3.1. Topology of the

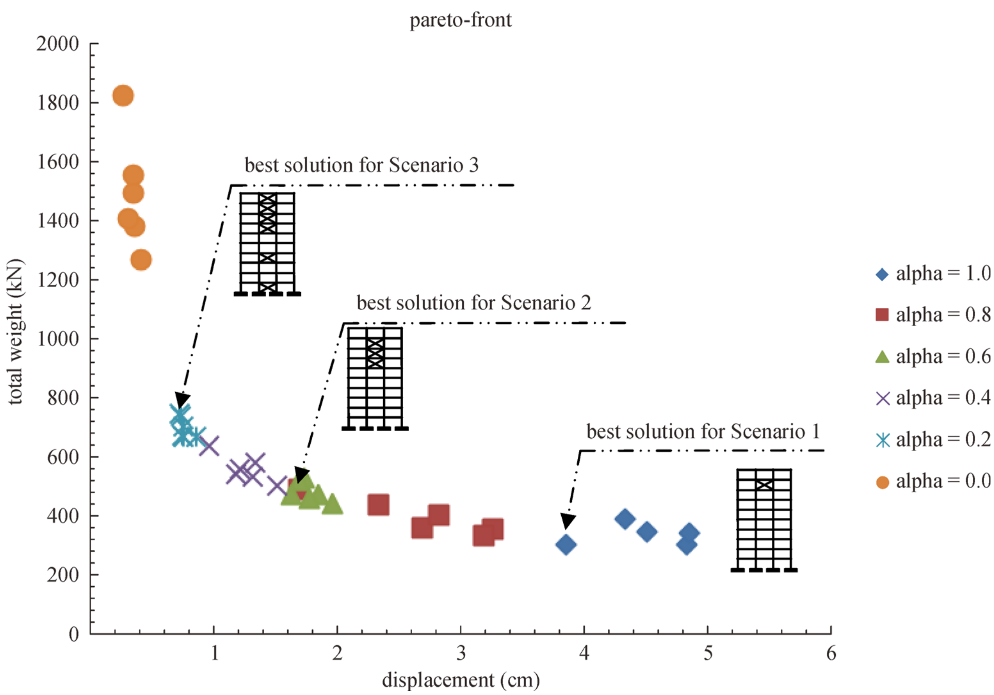

Fig. 5 Trade-off between total weight and roof displacement for 10-storey braced frame

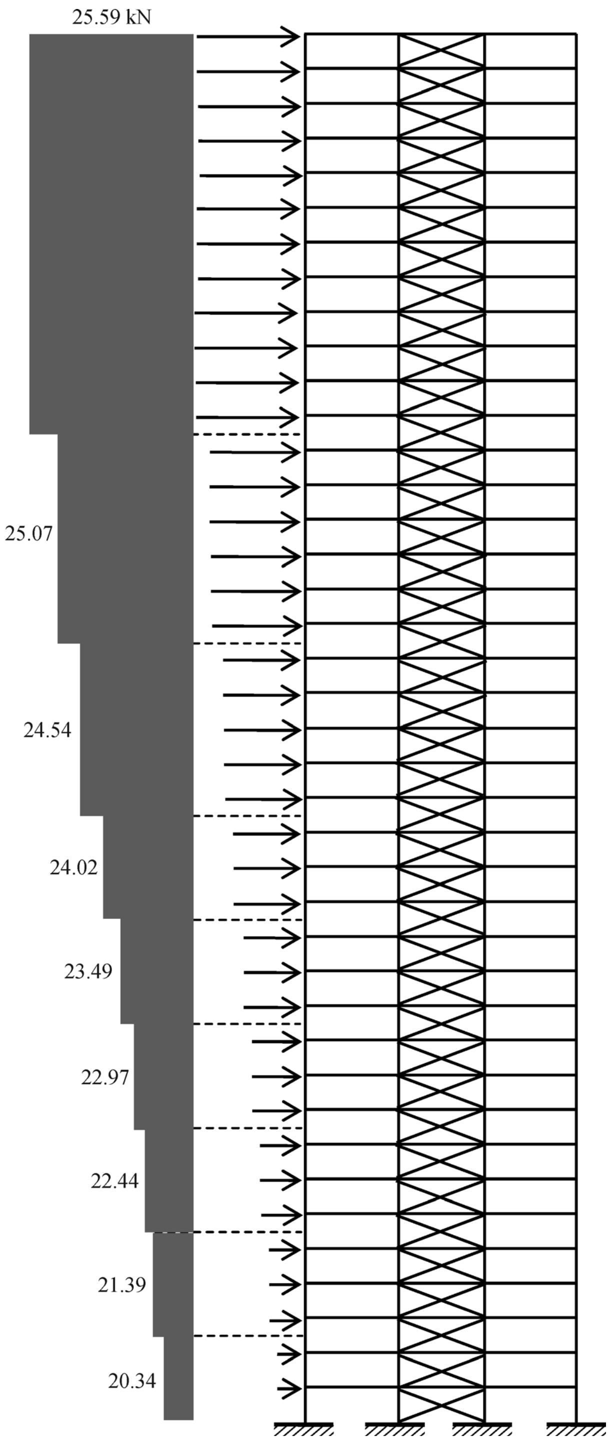

Fig. 7 The topology and loading of the fully braced 40-storey frame.

Table 2 Optimal results obtained for five different runs of HGAC

	run 1	run 2	run 3	run 4	run 5
penalized weight (kN)	3442.4	3361.6	3262.0	3138.4	3121.8
required braces (%)	42	42	45	47	58

braces and cross section of the columns, beams and braces are defined to be the design variables. Two objective functions of the total weight and the maximum roof displacement are converted to a single objective function using Eq. (8). Columns and beams sections may be selected between W4 $\times$ 13 and W44 $\times$ 335, AISC W-shaped sections. There are 40 topological and 40 cross sectional design variables for braces, 160 cross sectional variables for beams and columns, adding up to a total of 240 design variables, to be considered in this example.

A number of 100 chromosomes are set as the initial population. Mutation rate for topology and elements cross sections are set to be 20 and 5 percent, respectively. To explore design space efficiently in order to find better structural forms, a large value of topology mutation rate is assigned.

Figure 8 shows optimal topologies obtained by HGAC for the five different runs of the algorithm. These results are achieved when displacements are set as constraints during the optimization procedure. Since HGAC is a stochastic evolutionary method, different results may be obtained; however, the results have a small variance. The optimal penalized weight and the required bracings (in percentage) for every run are given in Table 2. At least 42 percent of the stories are needed to be strengthened by bracings to control displacements. The convergence history of HGAC for the five different runs and the average of these runs are illustrated in Fig. 9.

It is difficult for designers to make a decision on selecting the best topology based on the obtained forms in Fig. 8, which are not practical. To overcome this problem, another constraint is imposed in the optimization problem to find practical topologies. In this case, the algorithm does not allow a chromosome/ant with a separated bracing layout to be selected as an individual during the optimization procedure. Only ordered bracing patterns remain in the population. The total weight and the maximum displacement are converted to a single objective function using different weights of 0.50 and 0. When α is set to be 0.50, the optimal topologies are obtained assuming the same importance for two objective functions, and both objectives are minimized. In the case of $\alpha = 0$, only the maximum displacement is minimized. The optimal topologies for the second scenario are drawn in Fig. 10 for the five runs. The optimal penalized weight and the number of the required bracings for every run of these two scenarios are given in Table 3. As shown in this table, almost similar results are obtained for a specific scenario.

4 Conclusions

In this paper a novel hybrid genetic ant colony (HGAC) algorithm is developed for optimal design of steel braced frames. The topology and sizing of the braces along with

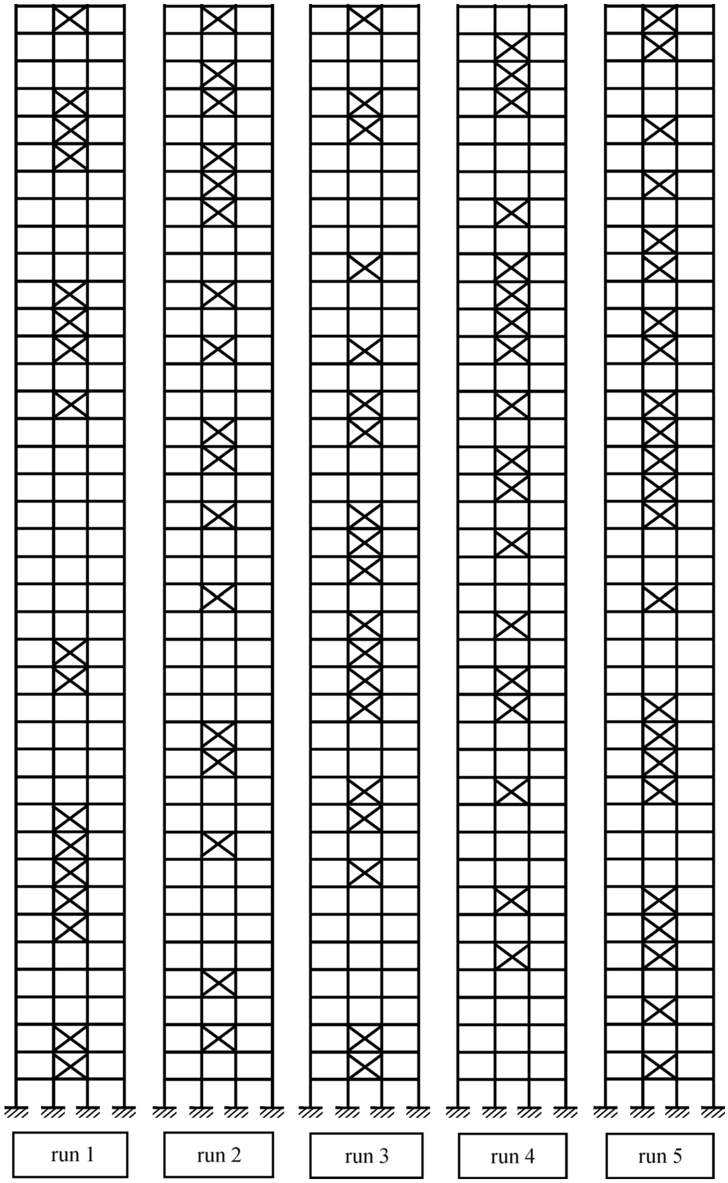

Fig. 8 Final topologies for different runs of HGAC

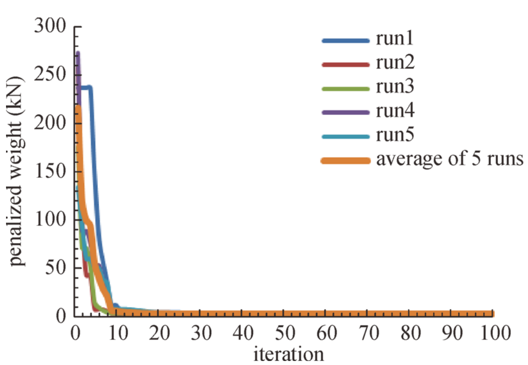

Fig. 9 Convergence history of different runs of HGAC for 40-storey braced frame

sizing of the frame elements are considered as design variables and they are formulated to apply optimization method to the numerical examples. For multi-objective optimization, the total weight and the maximum lateral displacement of the frames are taken as the objectives to be minimized. Formulation of the problem is converted to a single-objective optimization procedure, using weighted sum method (WSM). A trade-off between the objective functions are then obtained and shown. The following results were achieved by applying the algorithm to multi-objective optimization of 10 and 40-storey braced frames: a) the algorithm is capable of finding the optimal topologies and sizing; b) the greater the weight coefficient for displacement objective function, the more braces it

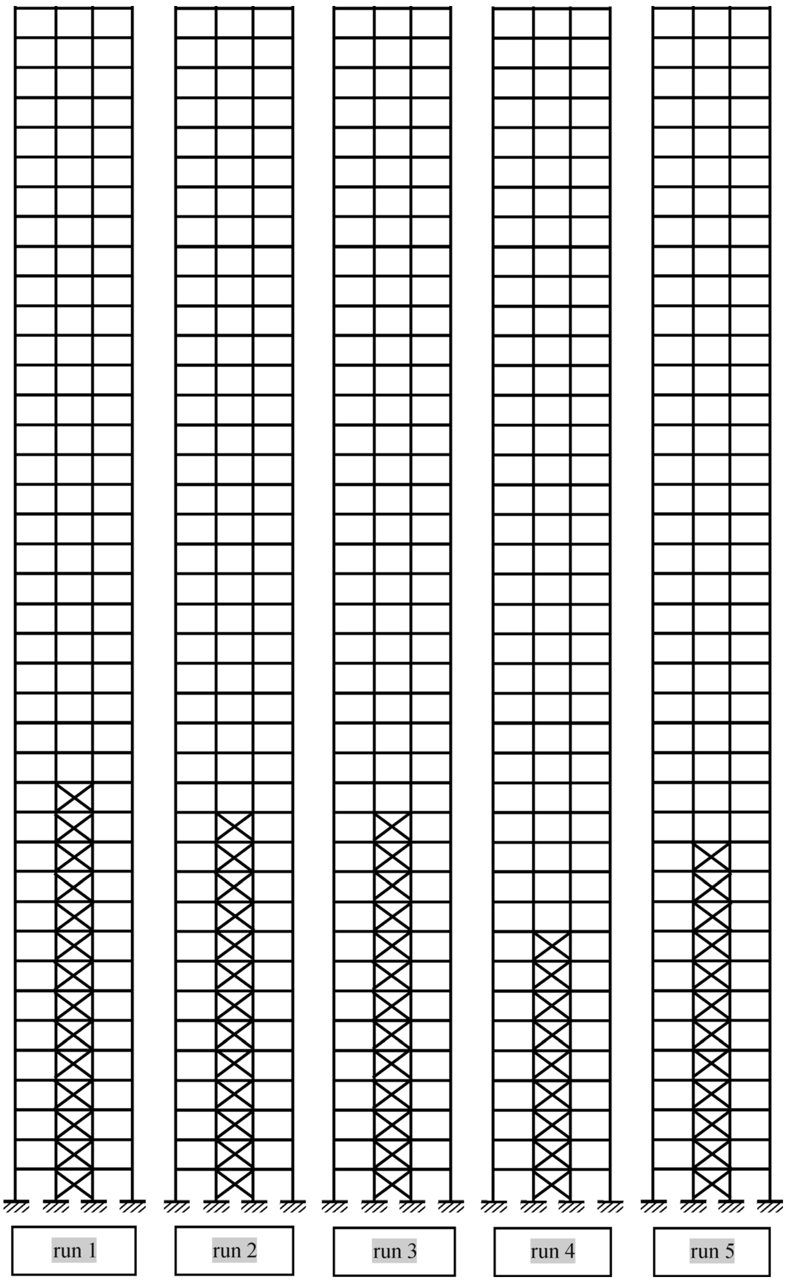

Fig. 10 Final practical topologies: scenario 2, different runs of HGAC

Table 3 Optimal practical results obtained for five different runs of HGAC

	alpha		run 1	run 2	run 3	run 4	run 5
scenario 1	0.50	penalized weight (kN)	3201.4	3223.7	3201.4	3223.7	3264.0
		number of braces	1	0	1	0	3
scenario 2	0.00	penalized weight (kN)	3752.9	3891.3	3743.4	3683.5	3601.7
		number of braces	14	13	13	9	12

needs to control the displacement; c) similarity in the convergence history results for different runs of HGAC; d) proven capability of the algorithm in fast convergence and achieving solutions better than those obtained by GA and ACO.

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