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RESEARCH ARTICLE

Foundations bearing capacity subjected to seepage by the kinematic approach of the limit analysis

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ABSTRACT An estimate of the ultimate load on foundations on soil layers subject to groundwater flow has been presented. The kinematic approach of the limit analysis was employed to find the upper-bound limit of the bearing capacity. Both smooth and rough base strip foundations were considered associated with different collapse patterns. Presence of the groundwater flow leads to a non-symmetric collapse pattern, i.e., a weak side and a strong side in two-sided collapse patterns, depending on the direction of the flow. It was found that the bearing capacity has a decreasing trend with increase in the groundwater flow gradient and hence, a reduction factor has been introduced to the third term in the bearing capacity equation as a function of the flow gradient.

KEYWORDS foundation, bearing capacity, limit analysis, numerical computation, plasticity, seepage

1 Introduction

The renowned bearing capacity equation of Karl Terzaghi has been widely accepted. The bearing capacity is a very important subject in geotechnical engineering with a reasonably long history and many contributions. Prandtl [1] and Reissner [2] derived the closed-form bearing capacity equation for a punch into a semi-infinite space which was later turned into the well-known triple-N bearing capacity equation of Terzaghi [3] with the original form as:

$$q_{ult} = cN_c + qN_q + 0.5\gamma BN_\gamma, \tag{1}$$

where c is cohesion, q is the surcharge pressure, B is the foundation width, γ is the soil unit weight, q_{ult} is the bearing capacity and N coefficients are the bearing capacity factors as functions of the soil friction angle. Unlike the first two terms with closed-form solutions, the third term is the most challenging one.

Development of the theory of plasticity and its consequences [4,5] led to many theoretical attempts to find the third factor under different assumptions and subjected to different conditions which are still under development. Among many, the influence of soil weight on the bearing capacity [6,7], the bearing capacity of soils with variable density in depth [8], bearing capacity factors for strip and circular foundations [9], influence of soil weight, non-associativity and non-symmetric loads by limit analysis [10–12], three-dimensional bearing capacity [13], seismic bearing capacity [14], bearing capacity of ring foundations [15], bearing capacity factor, N_γ, by limit analysis [16], bearing capacity of non-associative materials [17], effect of footing width and roughness on N_γ [18,19], bearing capacity of unsaturated soils [20] and effect of stress level and foundation size on the bearing capacity of both shallow [21–25] and deep foundations [26] can be addressed.

The effect of groundwater flow has been given less consideration in the literature. Very recently, the bearing capacity of foundations subject to groundwater flow has been presented by Kumar and Chakraborty [27] and also by Veiskarami and Kumar [28]. In the current study, the bearing capacity of soils conducting groundwater flow has been studied. The limit analysis has been employed to find the upper-bound estimate of the limit load on the foundation since this method was found to be independent of soil stress-strain relationship and hence, requires only the shear strength parameters to estimate the state of the

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failure [29,30]. Computations are made by numerical techniques and the third bearing capacity term was given the most consideration. A correction factor is introduced to the third bearing capacity term and computed as a function of the hydraulic gradient of the seepage.

2 Limit analysis method for shallow foundations

A discussion on the advantages of the methods based on the bound theorems in plasticity (limit analysis), over other methods, is beyond the scope of this work. However, the methods based on limit analysis do not require a constitutive soil model to be prescribed which is often very difficult in practice. This important advantage makes the limit analysis a very useful and versatile tool in computation of the limit loads on soil and other materials.

Following Drucker and Prager [5] and Drucker et al. [31] the kinematic approach of the limit analysis, leading to an upper-bound estimate of the load has been employed. This method meets the requirements of the upper-bound limit theorem in soil plasticity, i.e., if a compatible collapse pattern is found the rate of external works done by external agencies and internal body forces is not less than the rate of the internal energy dissipation. The major assumption in this approach is associated flow rule or the normality rule defined as Ref. [4]:

$$\mathrm{d}\varepsilon_{ij}^{\mathrm{p}} = \mathrm{d}\lambda \frac{\partial f(\sigma_{ij})}{\partial \sigma_{ij}}, \tag{2}$$

where $\mathrm{d}\varepsilon_{ij}^{\mathrm{p}}$ is the plastic strain increment tensor, σ_{ij} is the stress tensor, $\mathrm{d}\lambda$ is a nonnegative plastic multiplier to be determined by suitable assumptions, and $f(\sigma_{ij})$ is the yield function. The upper-bound theorem can be presented theoretically as follows [10,32]:

$$\int_V \sigma_{ij}\dot{\varepsilon}_{ij}^{\mathrm{p}}\mathrm{d}V \geqslant \int_S T_i^{\mathrm{s}}\dot{v}_i\mathrm{d}s + \int_V X_i\dot{v}_i\mathrm{d}V. \tag{3}$$

In this equation, V is the volume of the body surrounded by the boundary, S, over which, the integration is taken, T_t^s are the surface tractions, $\dot{v}_i$ are components of velocity increments and X_i are components of body force. Note that the Einstein summation convention has been assumed.

An admissible collapse mechanism, i.e., that conforming to the normality rule, can be assumed based on observation of real failures. There are several collapse patterns assumed theoretically based on the movement of rigid blocks (e.g., [1,4,10,29]), rotation of rigid blocks [11,12] and regions containing continuous deformation [5,10]. A complete history and assumptions on different collapse patterns and their applications to various problems can be found in the literature [10,29,32].

As a brief review, the collapse mechanism assumed by Prandtl [1] corresponds to a failure mechanism beneath a rough-base foundation whereas the mechanism assumed by Hill [4] corresponds to a smooth base foundation in which, there is no footing-soil interface energy dissipation. Both mechanisms contain continuous deformation regions which are bounded by a log-spiral curve. A multi-block mechanism can also be considered which is less restrictive in comparison to other mechanisms. A one-sided continuous deformation mechanism can also be applied to the case of rotation failure. Different mechanisms assumed by researchers, which are also adopted in this study, are presented in Fig. 1. It is important to note that the Prandtl mechanism is an upper-bound to both smooth base and rough base foundations, however, the Hill mechanism gives a better estimate (a lower value for the upper-bound) for the smooth base condition and hence, according to Michalowski [10] among others, it was assumed suitable for the bearing capacity of smooth base foundations. As stated by Michalowski [10], when a multi-block failure mechanism is considered, it is possible to seek for the optimum values of the angles α and ω to achieve the best upper-bound limit, i.e., the least ultimate load.

For any of the abovementioned mechanisms, the work done by the external agency and/or the body forces can be found by integrating the infinitesimal works done over each soil element. For example, a rigid block, shown in Fig. 2(a), arbitrarily chosen from a multi-block failure mechanism, experiences a velocity discontinuity at its interfaces with adjacent blocks and the rigid stationery ground underneath. Figure 2(b) shows the same mechanism after the deformation taken place. Figure 2(c) shows the velocity discontinuity in the lowermost part of the rigid block.

According to this figure, the work done by the external agency and/or body force can be computed by the following equation:

$$W_{\mathrm{ext}} = \vec{X} \cdot \vec{v}\, V. \tag{4}$$

In this equation W_{ext} is the work done in the rigid block, ABC, by the body force, $\vec{X}$, under the absolute incremental displacement (with respect to the stationary ground), $\vec{v}$, taken place over the entire volume of the rigid block, V. On the other hand, the internal energy dissipation would take place between the velocity discontinuities, i.e., along AB and BC sides:

$$D_{\mathrm{int}} = cL_{AB}v_{\mathrm{rel}}\cos\phi + cL_{BC}v\cos\phi. \tag{5}$$

In this equation D_{int} is the internal energy dissipation corresponding to the rigid block, ABC, c is the cohesion strength, v_{rel} is the magnitude of the relative velocity between the rigid block under study and the adjacent block and L_{AB} and L_{BC} are the length of the AB and BC sides respectively. All vectors can be found easily from the geometry of the collapse pattern. Similar computations can

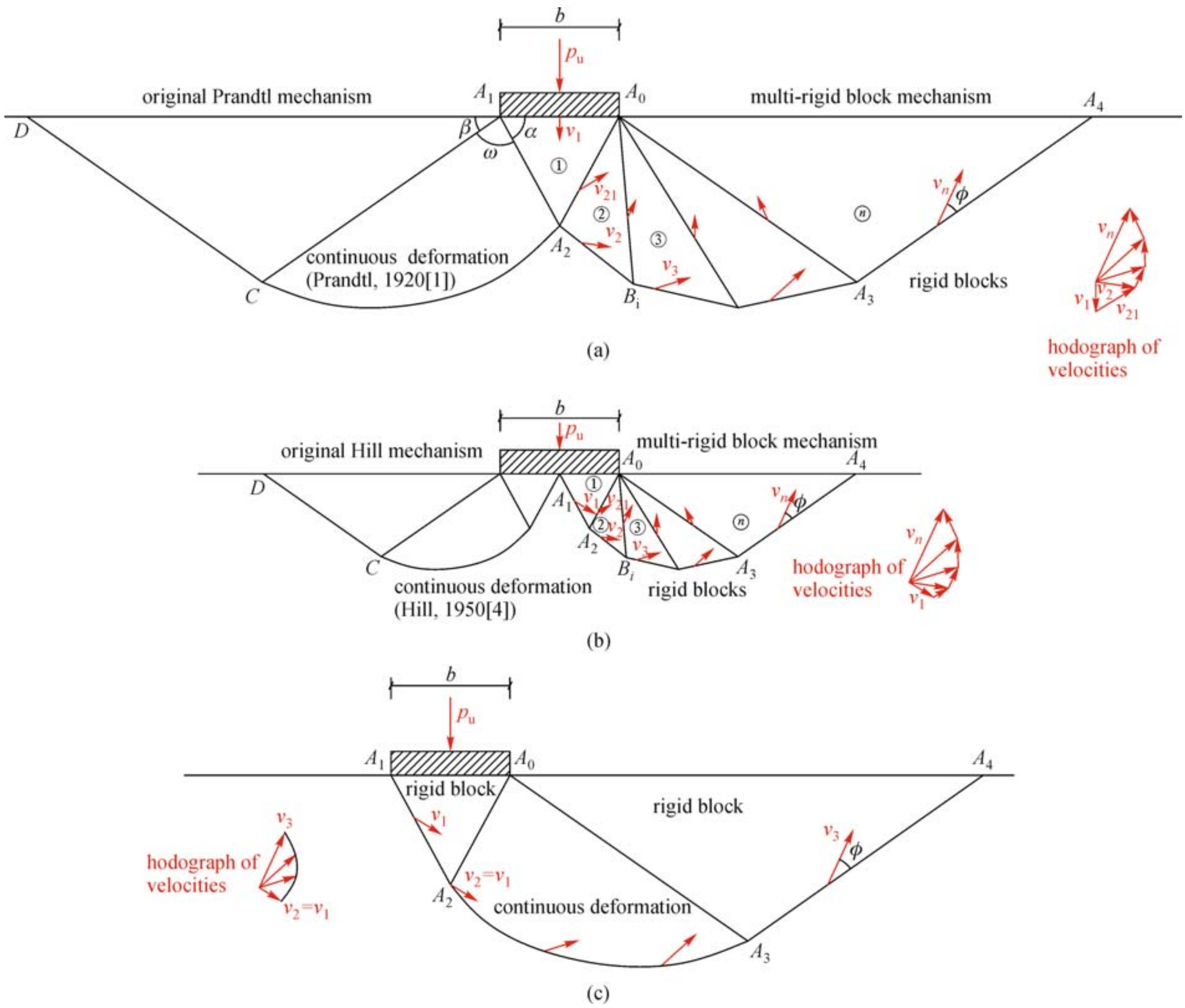

Fig. 1 Failure mechanisms. (a) Original and multi-block Prandtl [1], two-sided mechanism; (b) original and multi-block Hill [4] two-sided mechanism; (c) one-sided collapse pattern with a continuous deformation region [10]

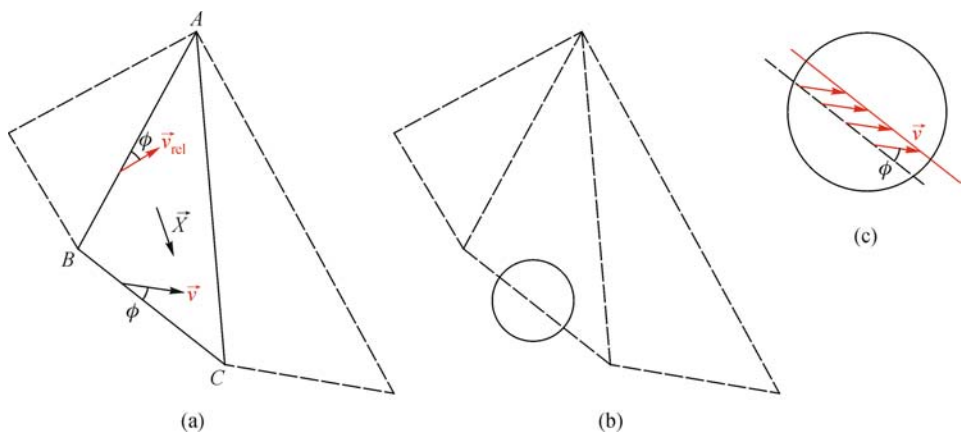

Fig. 2 Incremental work and energy dissipation computation in a multi-block failure mechanism. (a) Position of a rigid block and applied forces before deformation; (b) position of a rigid block after deformation; (c) velocity discontinuity

be performed for any region of different shape and velocity. A complete detail of computation of the external work and internal energy dissipation was given by Michalowski and You [12]. It is noticeable that the change in the bearing capacity has been assumed to be only a consequence of the presence of the seepage flow; the soil shear strength parameters are assumed to remain constant.

3 Influence of the groundwater flow

The presence of the groundwater flow imposes an extra body force in the direction of the groundwater flow. Also, the soil density would be that of a submerged soil. As in most practical situations, the direction of flow is assumed to be nearly horizontal and the whole soil layer is subjected to the groundwater flow. Therefore, the soil is totally submerged and the seepage force is horizontal. In other words, there is a horizontal component for the body force (force per unit volume) with the magnitude $i\gamma_w$ in which, i is the hydraulic gradient and γ_w is the water density. The vertical component of the body force (force per unit volume) is nothing but the submerged soil density, γ_{sub}.

The influence of the groundwater flow force can be considered from two quite different senses depending on the direction of the flow. Depending on the direction of the groundwater flow, it can act as a passive (resistant) or active force. Therefore, two different regions may be formed beneath the foundation, i.e., a strong region (smaller in size) and a weak region. The strong region corresponds to the side in which, the direction of the flow has a stabilizing effect whereas the weak region (larger in size) corresponds to the side in which the flow has a deteriorating effect with respect to the sliding blocks. In such condition, analysis of the ultimate load, at the first look, seems to be complex because the collapse mechanism would be no longer symmetric. However, this latter complexity can be removed by making suitable assumption. It is believed that for an admissible mechanism to be held, the limit load obtained from either side of the failure mechanism must result the same ultimate load. As a result, the non-symmetric collapse pattern must be found such that half of the ultimate load is obtained by equating and minimizing the external load and internal energy dissipation terms in each side. This condition ensures a balanced condition leading to a vertical movement. Therefore, it is possible to take a variable angle, θ, in Prandtl mechanism or the distance, b_R (or alternatively, b_L) in Hill mechanism, and seek for the balanced condition. It is worth noting that such assumptions are not required for one-sided collapse pattern since the pattern is in essence non-symmetric. Figure 3 shows the non-symmetric mechanisms for the

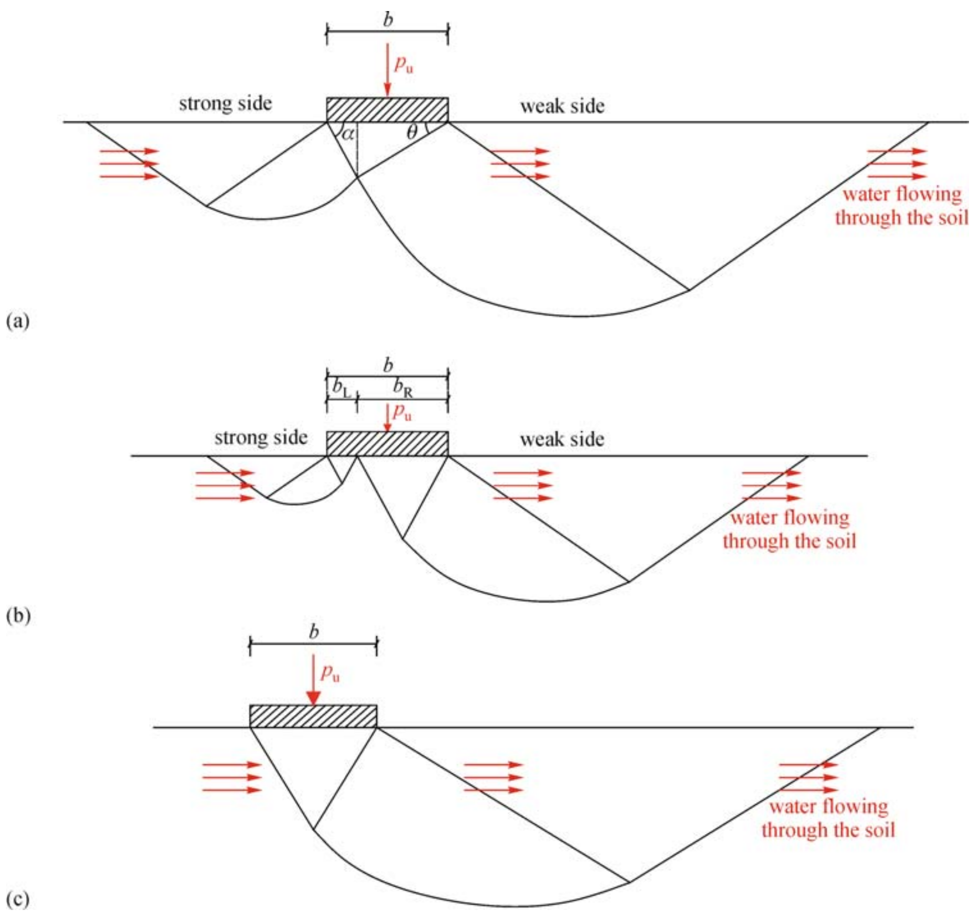

Fig. 3 Influence of the water flow on the formation of the failure mechanisms. (a) Non-symmetric Prandtl failure mechanism; (b) non-symmetric Hill failure mechanism; (c) one-sided failure mechanism

collapse patterns assumed in this paper. Figure 3(a), shows the non-symmetric collapse pattern corresponding to Prandtl's mechanism, used for rough-base foundations. Figure 3(b) shows the non-symmetric Hill's mechanism, used for a smooth-base foundation. Finally, Fig. 3(c) shows the one-sided collapse pattern which is again used for rough-base foundations. It is also noticeable that the so called "balanced condition" may appear to be an unnecessary and restrictive assumption; however, for a vertical movement of the foundation, it is necessary.

4 Analysis and results

As stated earlier, the influence of the seepage on the bearing capacity was investigated by assuming different non-symmetric collapse patterns corresponding to rough and smooth base strip foundations. Attempts were made to compute the third bearing capacity term since it is the most important term for shallow foundations on granular soils. A computer code was developed to compute the ultimate bearing capacity based on the limit analysis method outlined before. Both multi-block failure mechanisms and mechanisms comprising a continuous deformation region were considered. Minimum results were obtained by assuming different values for α and ω to find the minimum ultimate load as the best upper-bound estimate. Results were compared to those existing in the literature. Once the code was verified with the existing data, several cases were analyzed to investigate the influence of groundwater flow on the bearing capacity. The results were then presented in terms of a correction factor (a reduction factor), f_γ, to the third bearing capacity term.

Table 1 presents the results of the analyzed cases of rough base foundations in absence of the groundwater flow (i = 0). In this table, the bearing capacity factor, N_γ, is compared with values calculated by different researchers. Similar results for smooth base foundations are presented in Table 2 in a comparative manner. It is evident that the results reasonably compare with those based on similar assumptions, i.e., Michalowski [10] and Prandtl [1]. Comparisons with other methods indicate that the results of the upper-bound limit analyses are close to the common range suggested by different authors.

In the next step, the correction factor, f_γ, representing the bearing capacity ratio in presence of the seepage at different normalized gradients, i.e., $i\gamma_{\rm w}/\gamma_{\rm sub}$ were computed. This factor shows the "normalized" bearing capacity factor, N_γ, i.e., the ratio of the bearing capacity in presence of the seepage flow to that corresponding to no-flow case. Therefore, the correction factor for a particular mechanism may be lower than the other, although it may be opposite for the bearing capacity factors corresponding to these particular mechanisms. It should be noted that f_γ = 0 corresponds to the case with no seepage. Effect of seepage on the formation of a non-symmetric failure mechanism is

Table 1 Bearing capacity factor, N_γ, for smooth base foundations obtained by different methods

friction angle, ϕ/(°)	present study[a)]	present study[b)]	Michalowski [10]	Bolton and Lau [9]	Kumar [19] [c)]
10	0.55	0.45	0.423	0.29	0.282
20	2.67	2.41	2.332	1.60	1.577
25	5.71	5.18	5.020	3.51	3.457
30	12.3	11.3	10.918	7.74	7.644
35	27.6	25.5	24.749	17.8	17.549
40	68.2	62.1	60.215	44	43.084
45	185.3	169.6	164.308	120	117.146

Notes: a) Continuous mechanism; b) Multi-block mechanism; c) Kumar [19], Table 1, δ/ϕ = 0

Table 2 Bearing capacity factor, N_γ, for rough base foundations obtained by different methods

friction angle, ϕ/(°)	present study[a)]	present study[b)]	Michalowski [10][c)]	Michalowski [10][d)]	Prandtl [1]	Bolton and Lau [9]	Kumar [19][e)]
10	1.32	1.08	0.921	0.706	1.446	1.71	0.430
20	5.95	5.06	5.236	4.468	6.904	5.97	2.822
25	12.49	10.51	11.389	9.765	14.327	11.6	6.458
30	26.79	22.35	24.983	21.394	30.381	23.6	14.683
35	60.59	50.29	57.112	48.681	67.739	51	34.308
40	146.9	123.1	140.479	118.827	163.500	121	85.099
45	395.6	329.7	385.963	322.835	442.750	324	232.648

Notes: a) Continuous one-sided Prantl-type mechanism; b) multi-block Prantl-type mechanism; c) continuous one-sided Prantl-type mechanism; d) Multi-block Prantl-type mechanism; e) Kumar [19], Table 1, $\delta\phi$ = 1

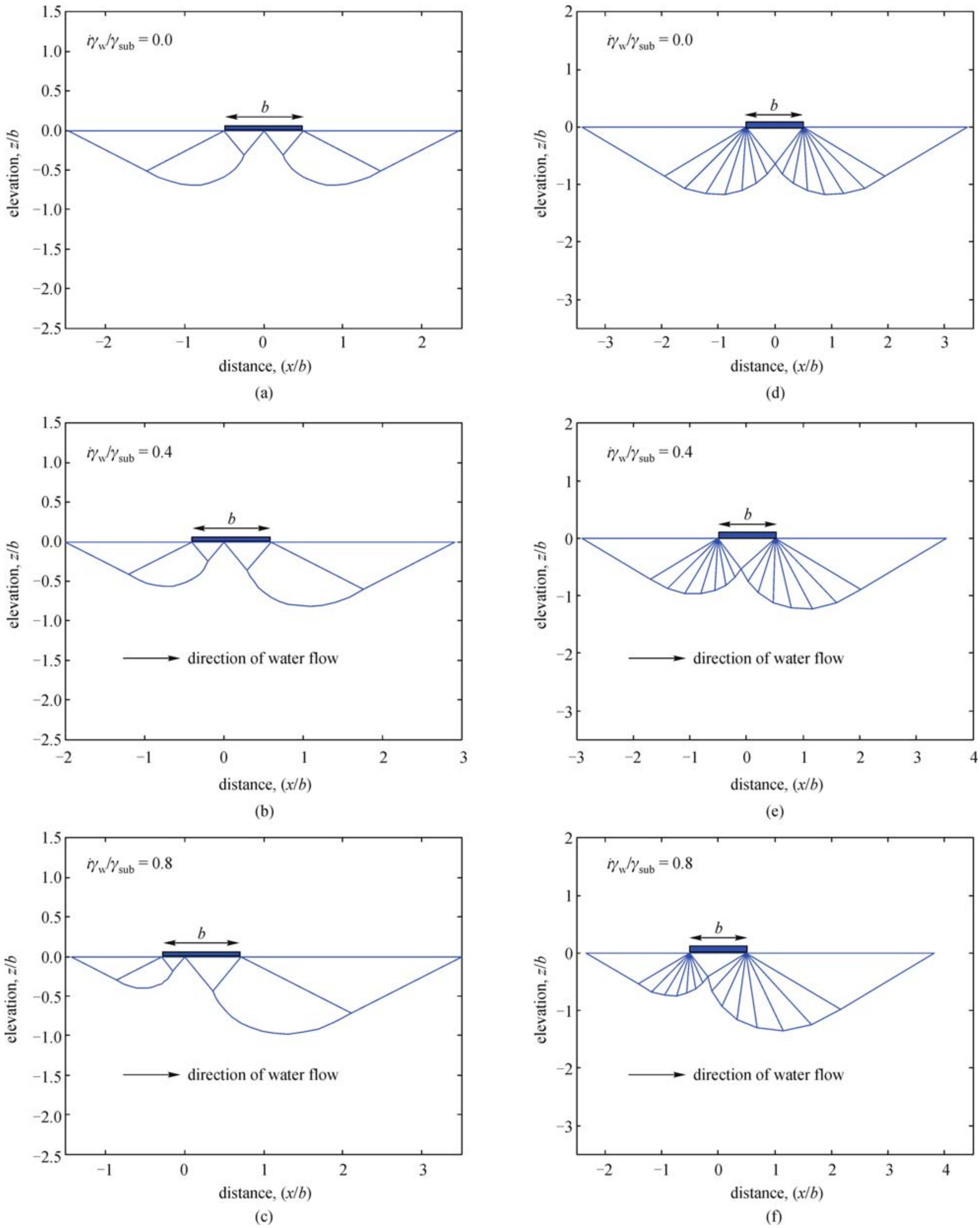

Fig. 4 Failure mechanisms in presence of groundwater flow: (a), (b) and (c) Hill's non-symmetric mechanisms for smooth base foundations and (d), (e) and (f) Prandtl's multi-block mechanisms for rough base foundations (ϕ = 30° in all cases)

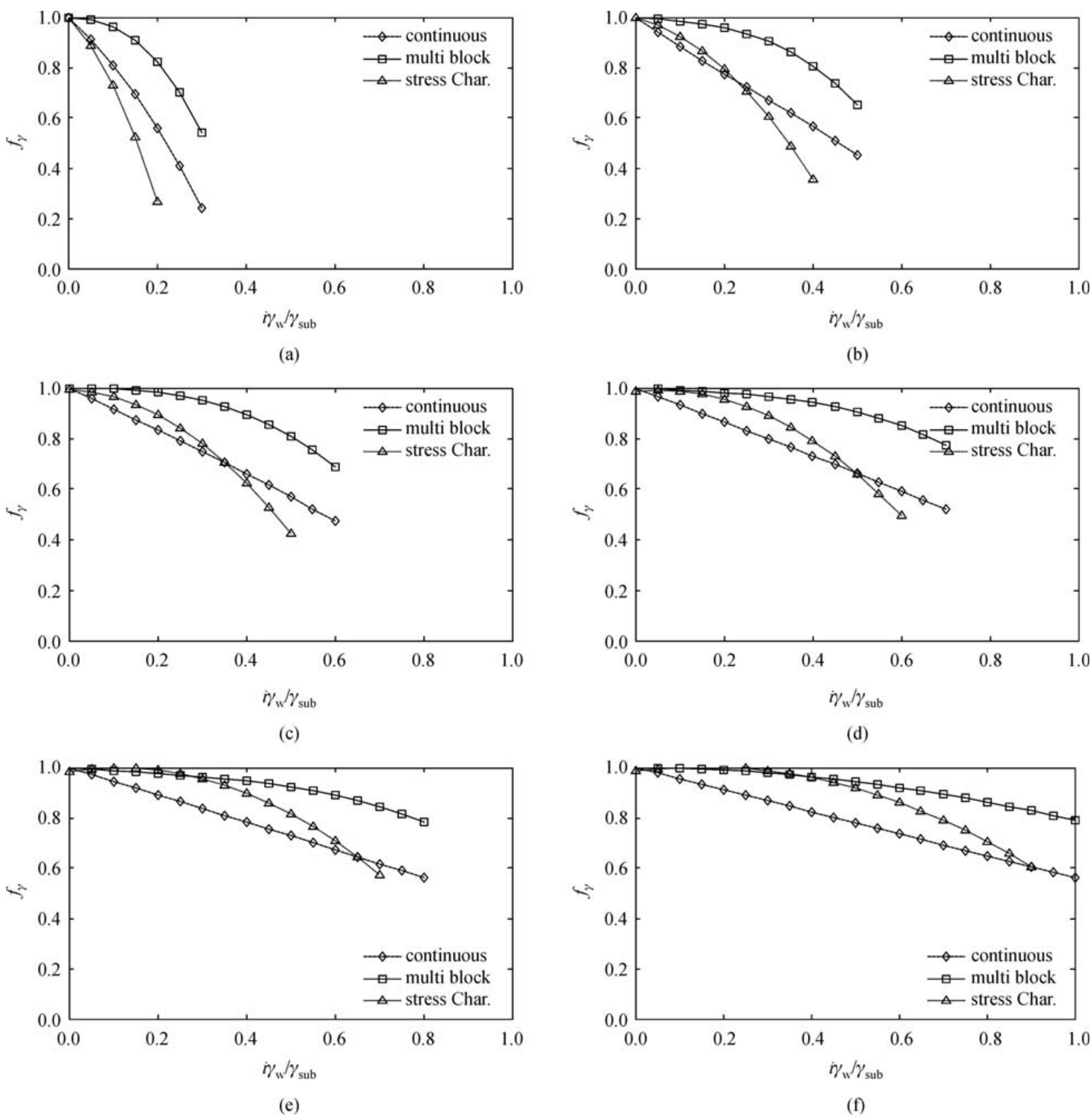

Fig. 5 Variation of the bearing capacity correction factor, f_γ, versus normalized hydraulic gradient, $i\gamma_w/\gamma_{sub}$ for the rough-base strip foundations subjected to the groundwater flow: Continuous: Rough base one-sided mechanism with continuous deformation Multi-Block: Prandtl [1] mechanism with multi-rigid blocks. Stress Char.: Results from Veiskarami and Kumar [28] based on the stress characteristics method. (a) $\phi = 10°$; (b) $\phi = 20°$; (c) $\phi = 25°$; (d) $\phi = 30°$; (e) $\phi = 35°$; (f) $\phi = 40°$

presented in Fig. 4. In this figure, two different cases were selected. The first one is a smooth base foundation over a soil layer with groundwater flow at different rates. Soil friction angle is set to be 30°. A two-sided Hill [4] mechanism with a continuous deformation region is assumed. It is evident that the symmetric mechanism in absence of the seepage is gradually degenerated to a non-symmetric mechanism when the flow rate grows. The results are shown in parts (a), (b) and (c) of this figure. Similar non-symmetric failure patterns for a two-sided Prandtl [1] mechanism with multi-block failure pattern are shown in parts (d), (e) and (f) of the same figure. This latter mechanism corresponds to a rough base. In all cases, the mechanism corresponding to the minimum collapse load was achieved by varying the geometry of sliding blocks to get the best upper-bound estimate, i.e., the least ultimate load.

The correction factor, f_γ, has been computed for a wide

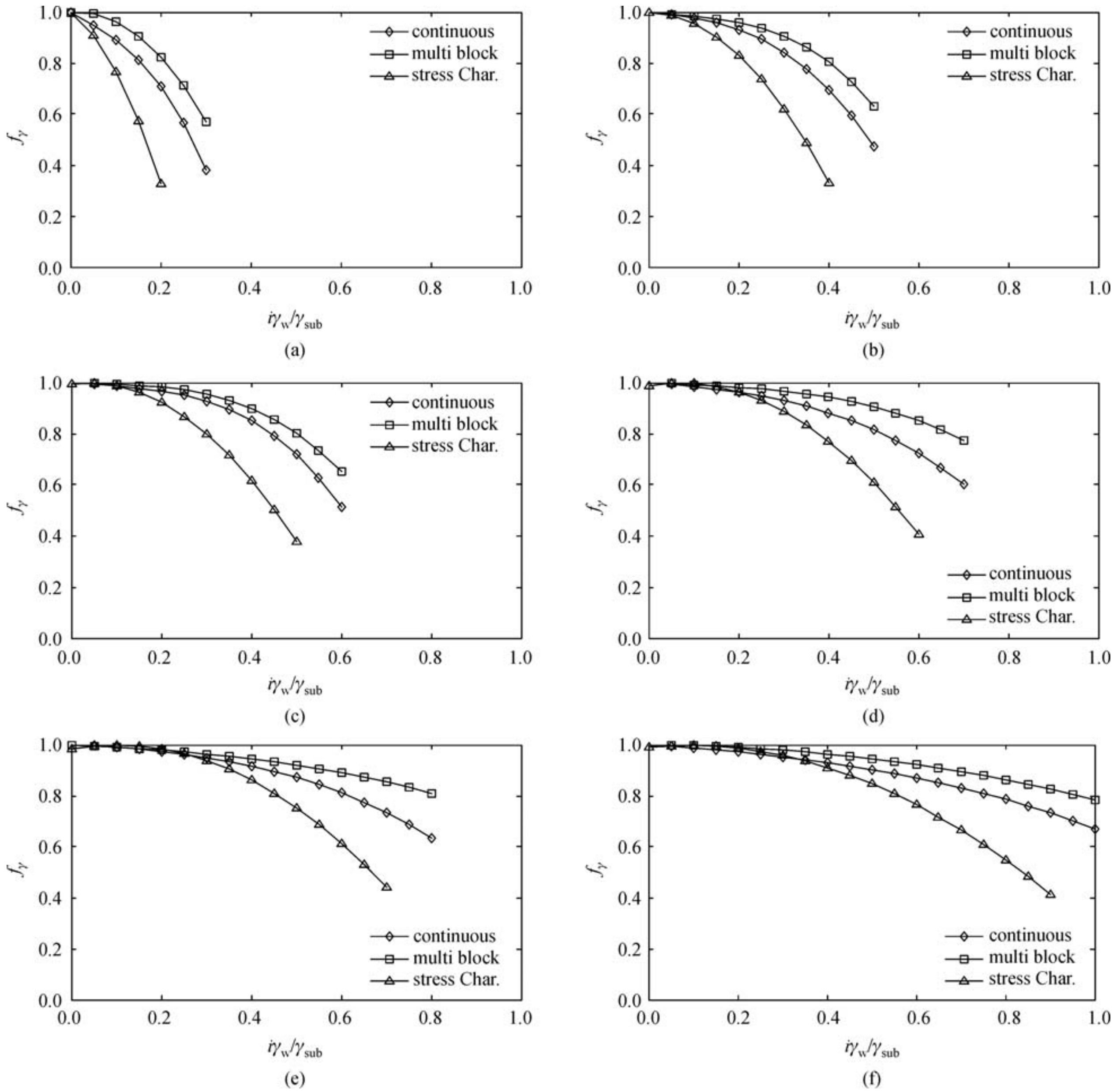

Fig. 6 Variation of the bearing capacity correction factor, f_γ, versus normalized hydraulic gradient, $i\gamma_w/\gamma_{sub}$ for the smooth-base strip foundations subjected to the groundwater flow: Continuous: Smooth base one-sided mechanism with continuous deformation Multi-Block: Hill [4] mechanism with multi-rigid blocks. Stress Char.: Results from Veiskarami and Kumar [28] based on the stress characteristics method. (a) $\phi = 10°$; (b) $\phi = 20°$; (c) $\phi = 25°$; (d) $\phi = 30°$; (e) $\phi = 35°$; (f) $\phi = 40°$

range of $i\gamma_w/\gamma_{sub}$ and different values of soil friction angle. The results are presented in Figs. 5 and 6. In the same figure, results obtained by the method of stress characteristics (based on Veiskarami and Kumar [28]) are also inserted for the sake of comparison. It is worth mentioning that the method of stress characteristics, although cannot be regarded as a complete upper-bound limit to the actual load, is most likely to be close to an upper-bound limit for associative materials [33] and hence, it has been presented here for comparison. It is evident that the correction factor has a decreasing tendency with increase in $i\gamma_w/\gamma_{sub}$. Also, the upper-bound estimate of f_γ, is higher than those obtained in the method of stress characteristics. It is also apparent that by increasing the soil friction angle, the influence of the strength reduction due to the presence of the groundwater flow is mitigated. The reason can be simply related to the available soil shear strength which balances the active body forces in the weak side of the foundation. In both smooth base and rough base foundations, variations of f_γ with groundwater flow gradient, is very closely to be linear.

A summary of all results are shown in Fig. 7 which

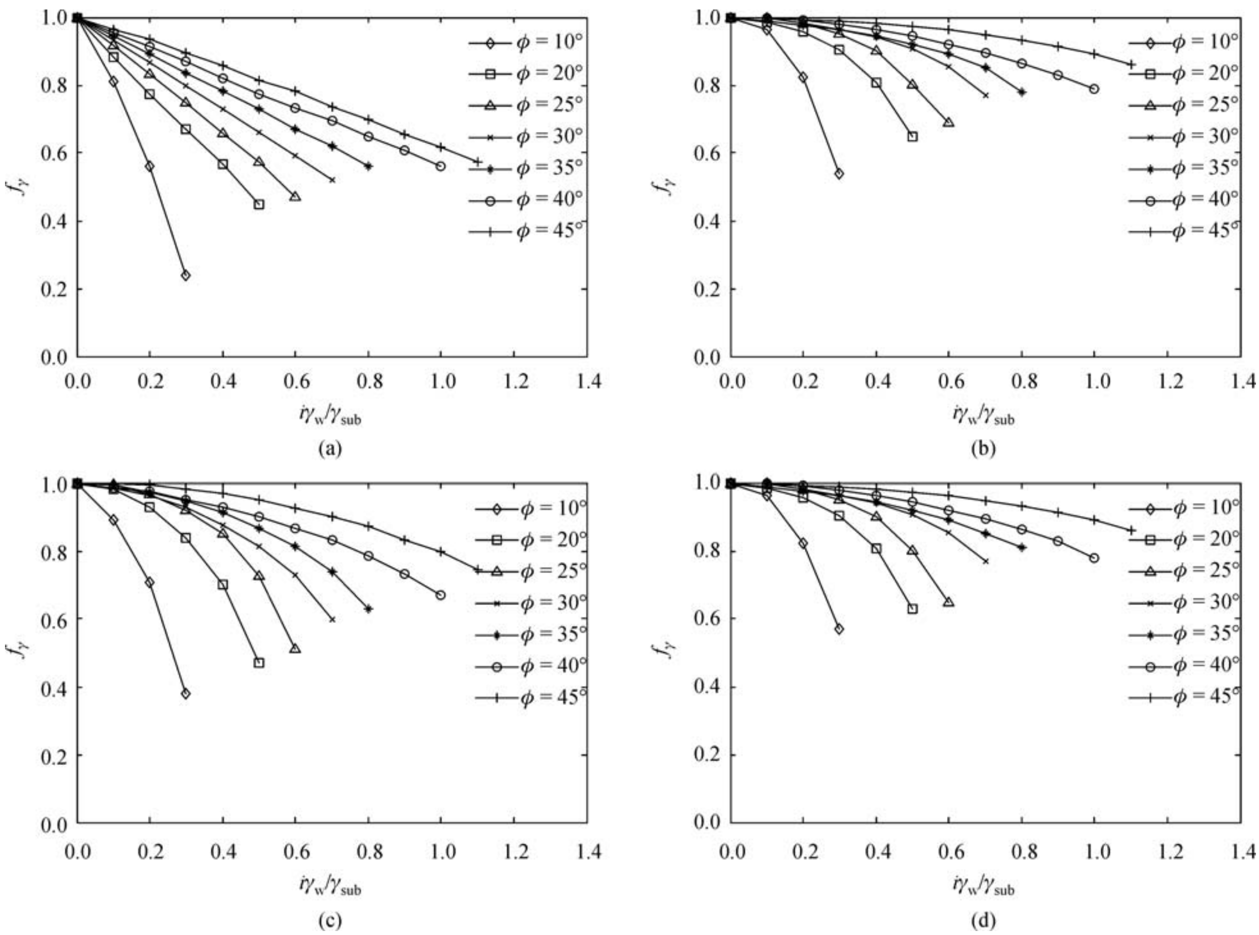

Fig. 7 Variation of the bearing capacity correction factor f_γ, versus normalized hydraulic gradient, $i\gamma_w/\gamma_{sub}$ for (a) Rough base foundations with a one-sided continuous deformation region, (b) rough base foundations with multi-block mechanism, (c) smooth-base foundations with a continuous deformation region and (d) smooth-base foundations with multi-block failure mechanism

reflects the variation of the bearing capacity correction factor, f_γ, for different mechanisms obtained under different assumptions. It is remarkable that in general, since the limit analysis method gives an upper-bound limit of the ultimate load, the results are higher than those obtained by methods based on assumptions involved in the lower-bound limit theorem. In addition, it should be noted that except for very special cases, the groundwater seepage, in particular in a horizontal direction, flows under rather low hydraulic gradients. However, the results have been theoretically extended to cover a wide range of hydraulic gradients which may either frequently or rarely occur in practice.

5 Conclusions

The kinematic approach of the limit analysis was employed to investigate the bearing capacity reduction of strip foundations in the event of the presence of the groundwater flow. Several collapse patterns, including Hill [4] mechanism with a continuous deformation region and with a multi-blocks mechanism for smooth base foundations and Prandtl [1] original and multi-block mechanisms for rough base foundations were considered. The collapse pattern was definitely non-symmetric with a weak and a strong failure blocks formed in either side of the foundation. The ultimate load was found through an iterative procedure to achieve the same ultimate load from each side.

A computer code was developed to find the minimum upper-bound estimate of the bearing capacity in presence of the seepage as the least upper-bound limit load in collapse patterns based on the multi-block failure mechanism by a slight change in geometry of the mechanism. The third bearing capacity factor which has the most significant contribution in shallow foundations on granular soils was focused and the reduction in the bearing capacity was considered by introducing a correction factor, f_γ. Variations in f_γ were presented as a function of the normalized groundwater flow hydraulic gradient, $i\gamma_w/\gamma_{sub}$. It was found that the effect of groundwater flow is more severe for smooth base foundations and the correction factor decreases by increasing the soil friction angle. Results were also compared with those obtained from the stress characteristics method which showed the values based on the limit analysis are fairly higher than those obtained by the method of stress characteristics. This difference is more significant at higher hydraulic gradients.

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