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RESEARCH ARTICLE

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Comparison of optimal capacitor placement methods in radial distribution system with load growth and ZIP load model

Abstract In this paper, a combined power loss sensitivity (PLS) index-based approach is proposed to determine the optimal location of the capacitors in the radial distribution system (RDS) based on the real and reactive combined loss sensitivity index, as capacitor placement not only reduces real power loss with voltage profile improvement but also reduces reactive power loss due to the reactive power compensation in the network. The results have been obtained with the existing methods of power loss index (PLI) and index vector (IV) method for comparison. Besides, the optimal placement has been obtained with the proposed method as well as existing methods and the total kVar support has been obtained. In addition, the results of net cost savings for the 10-, 34-, and 69-bus systems are obtained for comparison. Moreover, the results have been obtained for a large system of 85 buses to validate the results with combined sensitivity based approach. Furthermore, the load growth factor has been considered in the study which is essential for the planning and expansion of the existing systems, whereas the impact of the realistic load model as ZIP load model has been considered for the study of all the systems.

Keywords load growth, load models, reactive power compensation, radial distribution system, power loss index (PLI), power loss sensitivity (PLS), index vector (IV)

1 Introduction

Optimal distribution system planning plays a very important role in the growth of the distribution system network and in the effective use of the distribution system. With the continuing increase in load demand, the future expansion of the network depends on the study of the load flow of the distribution system network and thus is one of the most important research fields in electrical engineering. With the growing effort to reduce system losses and to increase the efficiency of the system and proper voltage profile, research on optimal distribution system planning has been conducted [1].

The methods adopted for the analysis of radial distribution systems were based on the concept of the backward/forward method [2–13]. These methods provided the results of the distribution load flow in one iteration for constant current loads, or in more than one iteration for other types of loads (constant power, mixed, etc.). A direct method for solving the load flow of the radial and meshed distribution networks was presented by Goswami and Basu [2]. The advantage of the direct method is that there is no convergence problem and an accurate solution is guaranteed for any realistic distribution system and composite loads. However, the disadvantage of the direct method is that it is difficult to number the nodes and branches and to assume that no node in the network is the junction of more than three branches. The numerical properties and the convergence rate of the proposed algorithm in Ref. [2] were studied by Chiang [3], who used the iterative solution for three fundamental equations of real power, reactive power, and voltage magnitude at the receiving end.

A load flow method for a radial distribution network (RDS) based on evaluating the total real and reactive power fed through any node was presented by Das et al. [4], who created a unique node, branch, and lateral numbering scheme to enhance the evaluations of the real and reactive loads fed at any node and receiving-end voltages. The advantage of this method is that all data can be stored in vector forms, thus saving an enormous amount of computer memory. The power flow problem of

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distribution systems was formulated in terms of three sets of recursive equations and was analyzed for power flow results for various voltage dependent load models by Haque [5]. The effects of various load models on the convergence pattern of the proposed method were also studied. The effect of voltage dependency of the load on the results of the power flow solution was analyzed by Mok et al. [6]. Ghosh and Das [7] presented a simple method for solving distribution networks assuming an initial flat voltage for all nodes. By numbering the nodes beyond each branch, the loads and charging currents were calculated, and consequently, the branch currents were calculated. The modified nodal voltages were recalculated in each step and thereby the losses in the system. Nanda et al. [8] proposed new findings for distribution system load flow. Aravindhababu et al. [9–13] presented radial distribution system load flow proposing iterative technique including voltage dependent load models.

The voltage profile in the radial distribution system based on the load tapped from the start to the end of the feeders results in a considerable voltage drop. Thus, the capacitors are widely installed in distribution systems for reactive power compensation which helps to achieve power and energy reduction, voltage regulation, and system capacity release. The installation of shunt capacitors in primary distribution systems can also effectively reduce peak power and energy losses by improving the voltage profile and compensating reactive power consumption patterns of loads. The extent of these benefits is based on the location and size of the installed capacitors [14–16]. The objective for capacitor placement can minimize the annual cost of the system, subject to operating constraints under a certain load consumption pattern. Grainger and Lee [17–19] proposed approaches for the calculation of the size of capacitor banks. Bala et al. [20] proposed a sensitivity-based method for the optimal capacitor placement in the distribution system network. Sundhararajan and Pahwa [21] proposed the genetic algorithm (GA) to select capacitors for radial distribution systems. Sensitivity-based approach for optimal capacitor placement in radial distribution system is presented in Refs. [22–24]. However, in the proposed methods for optimal capacitor location, reactive power loss has not been included. With the capacitor in the network, reactive power loss component cannot be neglected and should be included for optimal capacitor placement.

In this paper, a combined power loss index (PLI) based on the real power loss and the reactive power loss has been proposed and the results have been compared with the PLI based on the real power loss sensitivity (PLS) and the index vector (IV) method available in the literature. The optimal capacitor placement has been obtained based on these indices. The results have been obtained for the 10-, 34-, and 69-bus test systems. The results have also been obtained for a large system of 85 buses to validate the results with combined sensitivity based approach for optimal location of capacitors and its comparison with other existing methods. The values of the reactive compensation have been calculated with the three sensitivity methods and are compared for loss reduction, total kVar support, cost savings, and operating cost. The analysis has been conducted by developing codes in MATLAB 7.04 [25].

2 Mathematical model for radial distribution systems

A simple radial distribution system with source at one end and load at the other end with two nodes with voltages and impedance is shown in Fig. 1. The mathematical model is presented taking into account the impact of load growth and realistic static load model. The load flow algorithm used in this paper consists of forward and backward sweep methods. The forward sweep is mainly a voltage drop calculation from the sending end to the receiving end of a feeder or a lateral while the backward sweep is primarily a current summation based on voltage updates from the far end of the feeder to the sending end. Then by using Kirchoffs current law (KCL) and Kirchoffs voltage law (KVL), the voltage drop can be obtained [14].

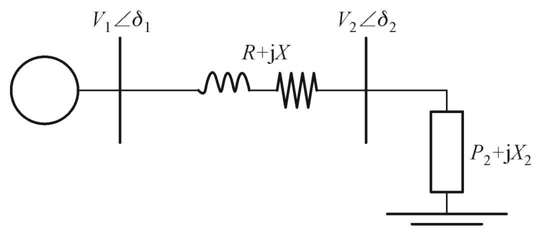

Fig. 1 Equivalent circuit model of RDS

Let the system comprise a large number of buses and branches represented as

n = Number of buses,
n_b = Number of branches,
$I_L(i)$ = Load currents.

Knowing the kVA and voltage $V(i)$ at any bus i, the load currents can be calculated as

$$I_L(i) = \left[\frac{S(i)}{V(i)}\right]^* \text{ for } i = 1,2,...,n, \tag{1}$$

$$I_L\left(\mathrm{se}(k)\right)_{n+1} = I_L\left(\mathrm{se}(k)\right)_n + I_L\left(\mathrm{re}(k)\right) \text{ for } k = 1,2,3,...,n_b, \tag{2}$$

where n is iteration number.

The branch current can be calculated as

$$I_{br} = I_L(\mathrm{re}). \tag{3}$$

The change in voltage drop for all branches is obtained and the voltage at other nodes can be calculated as

$$\Delta V = I \times Z \text{ for all branches,} \tag{4}$$

$$V_2 = V_1 - \Delta V. \tag{5}$$

Knowing branch currents, the real and reactive power losses can be obtained as

$$P_{\text{loss}}(k) = I_{\text{br}(k)}^2 \times R(k) \text{ for } k = 1,2,3,...,n_{\text{b}}, \tag{6}$$

$$Q_{\text{loss}}(k) = I_{\text{br}(k)}^2 \times X(k) \text{ for } k = 1,2,3,...,n_{\text{b}}. \tag{7}$$

The total real as well as reactive power loss TPL and TQL is given in Ref. [23]:

$$\text{TPL} = \sum_{k=1}^{n_{\text{b}}} P_{\text{loss}}(k), \tag{8}$$

$$\text{TQL} = \sum_{k=1}^{n_{\text{b}}} Q_{\text{loss}}(k). \tag{9}$$

2.1 Model of load growth

For future expansion and planning of the distribution systems, it is desirable that a system engineer must know the future estimate of the system solutions for planning and expansion or the efficient operation of the distribution systems. It is essential to know the load growth pattern of the distribution systems for future planning and expansion. In this paper, the load growth has also been considered for the capacitor placement. In the proposed load flow method, the load growth is modeled as

$$\text{Load}_i = \text{Load} \times (1 + r)^m, \tag{10}$$

where r is the annual growth rate and m is the plan period up to which the feeder can take the load.

The load growth is calculated for all the systems to consider the impact of load growth on capacitor placement and accordingly the impact on kVar requirements from capacitors installed in the system.

2.2 Polynomial load model

In this paper, a realistic static load model that represents the power-voltage relationship as a polynomial equation of voltage magnitude has been considered. It is usually referred to as the ZIP model, as it is made up of constant impedance (Z), constant current (I) and constant power (P). The real and reactive power characteristics of the ZIP load model are given in Ref. [22].

$$P = P_0\left[a_{\text{p}}\left(\frac{V}{V_0}\right)^2 + b_{\text{p}}\left(\frac{V}{V_0}\right) + c_{\text{p}}\right], \tag{11}$$

$$Q = Q_0\left[a_{\text{q}}\left(\frac{V}{V_0}\right)^2 + b_{\text{q}}\left(\frac{V}{V_0}\right) + c_{\text{q}}\right], \tag{12}$$

where the sum of the ZIP load coefficients for both P and Q loads is equal to 1.

$$a_{\text{p}} + b_{\text{p}} + c_{\text{p}} = 1,\ a_{\text{q}} + b_{\text{q}} + c_{\text{q}} = 1.$$

In this paper,

$$a_{\text{p}} = a_{\text{q}} = 0.1,\ b_{\text{p}} = b_{\text{q}} = 0.1,\ c_{\text{p}} = c_{\text{q}} = 0.8, \tag{13}$$

P_0 and Q_0 are the real and reactive power consumed at a reference voltage V_0.

Knowing the total power loss in the distribution system, the running cost of the capacitor placed in the system can be calculated as

$$\text{Running cost} = C_{\text{p}} \times \text{TPL}, \tag{14}$$

where C_{p} is \$/(kW·a). In this paper, C_{p} has been taken as US\$168/(kW·a).

The capital cost component called as fixed cost for life span of 15 years can be calculated as

$$\text{Fixed cost} = Q_{\text{c}} \times 13.2\ \$/100\ \text{MVar}\cdot\text{h}. \tag{15}$$

Knowing the fixed as well as operating cost components for capacitor, the total cost recovery for the capacitors can be calculated as

$$\text{Total cost} = \text{Running cost} + \text{Fixed cost}.$$

3 Algorithm for optimal capacitor placement in radial distribution system

A study of a radial distribution system with optimal capacitor placement has been conducted for three IEEE test systems. The optimal location of the capacitors has been obtained utilizing the methods of real power loss index and the IV method reported in the literature. A combined loss sensitivity index has been proposed for an optimal location of the capacitor and the results have been compared with those of the other two methods for the test systems.

3.1 PLI [25]

To determine the power loss index, the following steps have been implemented.

Step 1 Read the given data for balanced RDS.

Step 2 Perform the load flows and calculate the base case total active power loss.

Step 3 By compensating the reactive power injections (QC) at each node (except source node) in all the phases, run the load flows and calculate the active power losses in each case.

Step 4 Calculate the power loss reduction and power loss indices using the following equation:

The PLIs are calculated as

$$\mathrm{PLI}(l)=\frac{X(l)-Y}{Z-Y}\quad \text{for } l=2,3,\ldots,n, \tag{16}$$

where X is the reduction in the loss, Y is the minimum loss reduction, and Z is the maximum loss reduction.

Step 5 Select the candidate node with PLI > tolerance.

Step 6 Stop.

After identifying the candidate nodes, inject the capacitive reactive power which is equal to the reactive power loss obtained in Step 3 until the loss goes on increasing. The power loss indices plots with the ZIP load model taking load growth rate of 0.07 are illustrated in Figs. 2 to 4 for the 10-bus system, the 34-bus system and the 69-bus test system. For the 10-bus system, the PLI at buses 4 to 6 are comparatively higher than that at other buses. For the 34-bus system, the PLI at buses 2 to 11 are quite higher than that at buses 17 to 25. For the 69-bus system, the PLI at buses 7 to 9, and at buses 52 to 60 are higher. Higher PLI buses are candidate nodes for the capacitor placement.

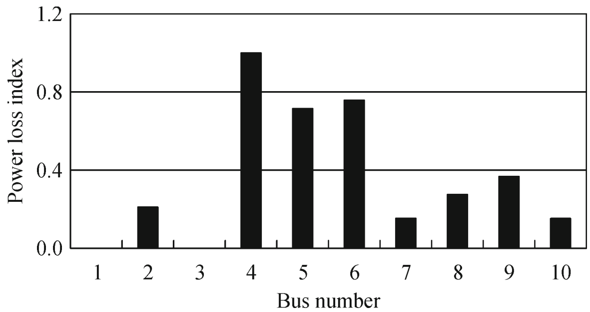

Fig. 2 PLI plot for the 10-bus system

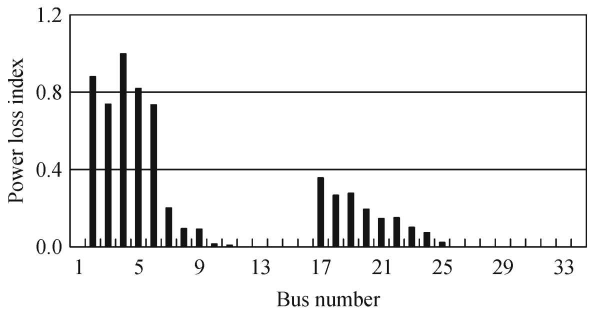

Fig. 3 PLI plot for the 34-bus system

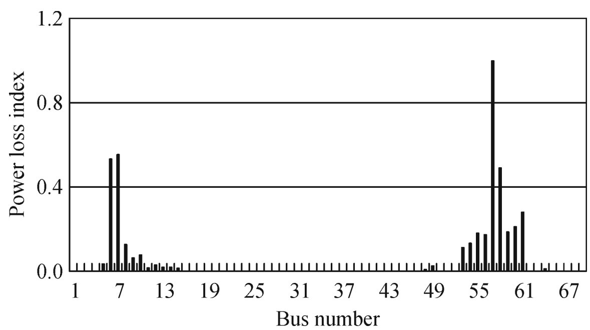

Fig. 4 PLI plot for the 69-bus system

3.2 IV [25]

Index vector is formulated by running the base case load flow on a given radial distribution network, and by calculating the reactive component of the current in the branches and the reactive power load concentration at each node. Based on the elements of the index vector, this method identifies a sequence of nodes to be compensated. The sequence of priority of the nodes is mainly determined by the index vector.

The index vector for bus n is given by

$$\mathrm{Index}(n)=\frac{1}{V_n^2}+\frac{I_{\mathrm{q}}(k)}{I_{\mathrm{p}}(k)}+\frac{Q_{\mathrm{eff}}(n)}{\mathrm{Total}Q}, \tag{17}$$

where, Index(n) is the index for the nth bus, $V(n)$ is the voltage at the nth bus, $I_{\mathrm{q}}(k)$ is the imaginary component of the current in the kth branch, $I_{\mathrm{p}}(k)$ is the real component of the current in the kth branch, $Q_{\mathrm{eff}}(n)$ is the effective load at the nth bus, and Total Q is the total reactive load of the given distribution system.

After calculating the index vectors at all buses, the normalized voltage magnitudes are calculated for all the buses by the following formula

$$\mathrm{Norm}(i)=\frac{V(i)}{0.95}. \tag{18}$$

The norm voltage profile has been determined for all test systems viz. the IEEE 10-bus, 34-bus, and 69-bus systems. Buses, where the index vector is higher and normalized values of voltages at the buses are less than 1.01 are considered as candidate nodes for capacitor placements.

The following steps are followed for optimal placement of capacitors:

1) After formulating the index vector, multiply the index value by the load reactive power at that bus to estimate the size of the capacitor to be placed. Thus, the potential location and size of the capacitor to be placed are obtained directly.

2) Arrange the index vector in descending order so that the highest priority bus will come first and the lowest priority bus will come at the end. Now place the capacitor at the first potential location and run the load flow and estimate the losses.

3) Then assume capacitors at the first two potential locations and perform load flow again and evaluate the corresponding losses. It may be observed that the losses will reduce.

4) Repeat this with estimated capacitors at the first "n" busses till the losses reduce to the minimum and for the first (n + 1) potential locations the loss start increasing.

Then the estimated capacitors at the first n potential locations will give the optimal location and size for the given radial distribution system.

The index vector values obtained for the 10-bus, 34-bus, and 69-bus test systems are depicted in Figs. 5 to 7.

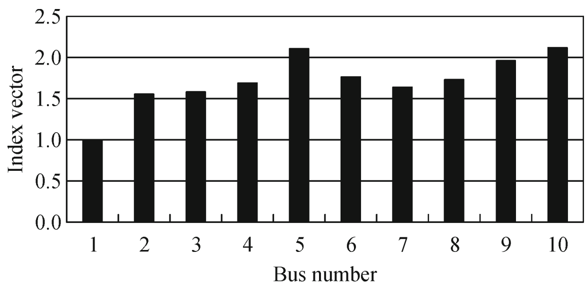

Fig. 5 Index vector for the 10-bus test system

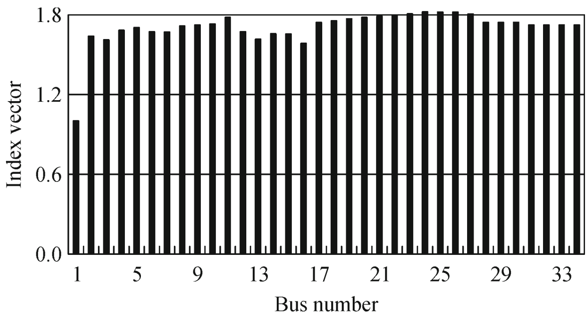

Fig. 6 Index vector for the 34-bus test system

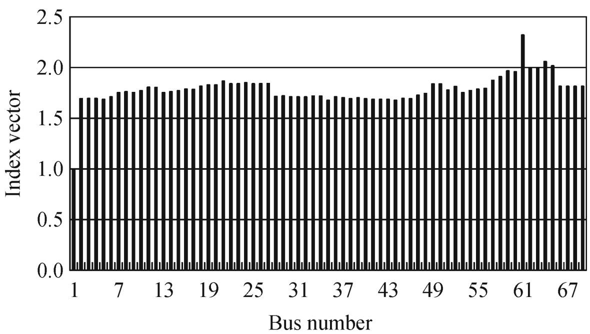

Fig. 7 Index vector for the 69-bus test system

3.3 Proposed combined PLS index

In this proposed method, the loss sensitivity factors are calculated for determining the candidate nodes for placement of capacitors. Estimation of these sensitive nodes helps to reduce the search space. The real power loss and reactive PLS can be calculated as

$$\frac{\partial P_{\text{loss}}}{\partial Q_2} = \frac{2Q_{2R(j)}}{V_2^2}, \tag{19}$$

$$\frac{\partial Q_{\text{loss}}}{\partial Q_2} = \frac{2Q_{2X(j)}}{V_2^2}. \tag{20}$$

After obtaining, the loss sensitivity, the combined loss sensitivity with respect to the reactive power can be obtained as

$$\frac{\partial S_{\text{loss}}}{\partial Q_2} = \frac{\partial P_{\text{loss}}}{\partial Q_2} + j\frac{\partial Q_{\text{loss}}}{\partial Q_2}.$$

In a similar way, the loss sensitivity corresponding to the real power can be obtained and the combined loss sensitivity with respect to the real power can be represented as

$$\frac{\partial S_{\text{loss}}}{\partial P_2} = \frac{\partial P_{\text{loss}}}{\partial P_2} + j\frac{\partial Q_{\text{loss}}}{\partial P_2}.$$

Based on the loss sensitivities, the loss sensitivity matrix can be formulated as

$$\text{Loss sensitivity matrix} = \begin{pmatrix} \dfrac{\partial P_{\text{loss}}}{\partial P_2} & \dfrac{\partial P_{\text{loss}}}{\partial Q_2} \\[2ex] \dfrac{\partial Q_{\text{loss}}}{\partial P_2} & \dfrac{\partial Q_{\text{loss}}}{\partial Q_2} \end{pmatrix}. \tag{21}$$

The loss sensitivity obtained for the 10-bus, 34-bus, and 69-bus test systems are demonstrated in Figs. 8–10.

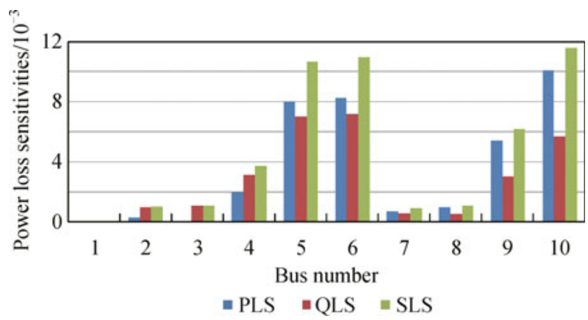

Fig. 8 Combined PLS for the 10-bus system

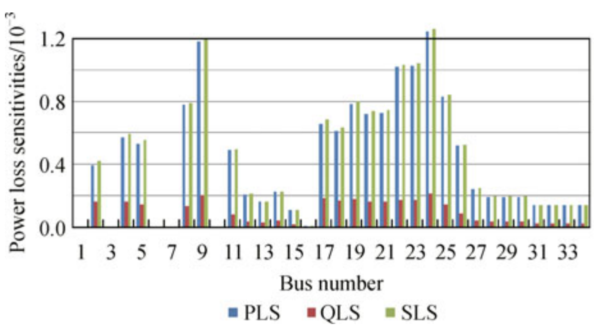

Fig. 9 Combined PLS for the 34-bus system

The buses with higher loss sensitivities can be observed in the potential candidate buses for capacitor placement in

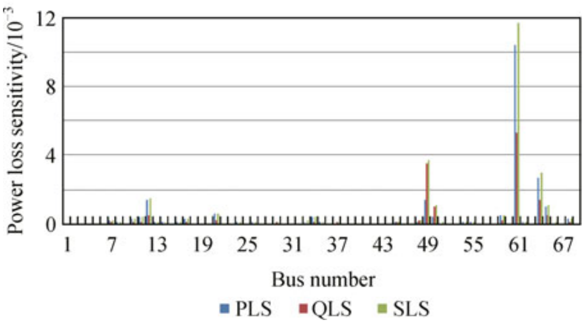

Fig. 10 Combined PLS for the 69-bus system

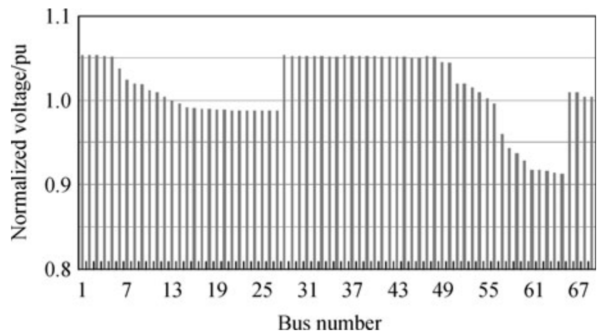

Fig. 13 Norm voltage profile for the 69-bus system

Figs. 8–10. Loss sensitivity factors are calculated from load flow analysis and values are arranged in descending order for all the buses. Normalized voltage magnitudes are calculated for all the buses using Eq. (18). The buses with higher loss sensitivity factors and normalized voltages at the buses are less than 1.01 and are considered the potential buses for capacitor placement. The norm of voltage profile for all test bus systems is displayed in Figs. 11–13.

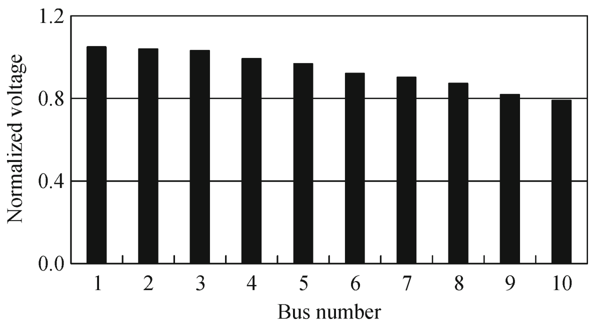

Fig. 11 Norm voltage profile for the 10-bus system

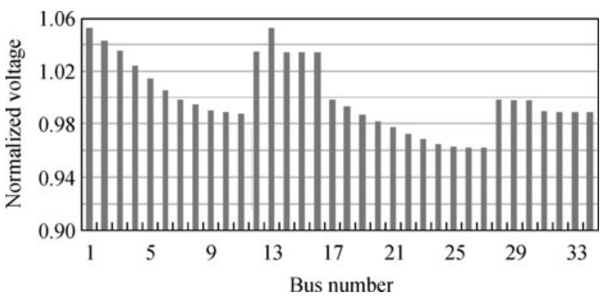

Fig. 12 Norm voltage profile for the 34-bus system

4 Results with the ZIP load model and discussion

The results for the three test systems have been obtained for voltage profile, loss profile, and cost of loss component without and with capacitors. The results are obtained with the ZIP load model for total power loss, running cost, fixed cost, total cost for capacitor installation, and reduction in total power loss, and the savings are also calculated for the three test systems.

4.1 Results for the IEEE 10-bus test system

The results obtained for the IEEE 10-bus test system and the results without compensation and with compensation considering three different methods are given in Table 1. The reduction in the total power loss, the real power loss and the reactive power loss with the PLI method is lower compared to those of the other methods. The running cost and the total cost obtained with the IV method is observed to be minimum. The total savings obtained with the PLS-based method are higher than that of the other methods. The reactive compensation obtained at the corresponding buses with the three methods is tabulated in Table 2. The total kVar obtained with the PLI method is minimum compared to that of the other two methods. The respective positions obtained for capacitor placement are presented in Table 2.

The voltage profile, real power loss and reactive power loss without and with compensation are sketched in Figs. 14–16. With compensation, the loss reduction with all methods and the voltage profile obtained is better. For the 10-bus system without capacitor placement, the TPL is 1801 kW, the total reactive power loss (TQL) is 2333.8 kVar, the minimum bus voltage is 0.75 pu and the total reactive power load is 5871.1 kVar. With the PLI approach, the capacitors are placed at buses 4 and 6. The Total Q_c injected is 3870.52 kVar, and the remaining 2000.58 kVar is supplied by the system itself. After capacitor allocation, the minimum bus voltage is increased to 0.7990, the TPL is reduced to 1530.9 kW, the TQL is reduced to 1946.9 kVar, which give a saving of $ 40887.6780. By the PLS approach, the capacitors are placed at buses 4, 5, 6, 7, 8, 9 and 10. The Total Q_c injected is 4483.4 kVar, and the remaining 1387.7 kVar is supplied by the system itself. After capacitor allocation, the minimum bus voltage is increased to 0.8317, the TPL is reduced to 1483.7.1 kW, the TQL is reduced to 1901.8 kVar, which give a saving of $ 48106.4451. By the IV approach, the capacitors are placed at buses 4, 5, 6,

Table 1 Results for the 10-bus system

	Without compensation	With compensation		
		PLI	PLS	IV
Min voltage/pu	0.7500	0.7990	0.8317	0.8318
TPL/kW	1801.0	1530.9	1483.7	1467.35
TQL/kVar	2333.8	1946.9	1901.8	1888.42
Running cost/$	302563.9423	257191.2	249261.6	246514.8
Fixed cost/$	0	4489.1219	5199.9549	6149.3868
Total cost/$	302563.9423	261680.322	254461.5549	252664.1868
Reduction in PL/kW	0	270.1	317.3	333.65
Reduction in QL/kVar	0	386.9	432	445.38
Net savings/$	0	40887.6780	48106.4451	49903.8131

Table 2 Capacitors placement and total kVar with the three methods

Bus	Without compensation Q load	Q injected with compensation		
		PLI	PLS	IV
1	0	0	0	0
2	645.2	0	0	0
3	476.9	0	0	0
4	625.5	1935.3	621.9	479
5	2580.7	0	2502.6	2907
6	841.5	1935.3	774.2	960
7	154.3	0	139.4	174
8	84.2	0	73.5	101
9	182.3	0	150.3	250.4
10	280.5	0	221.4	430.6
Total Q/kVar	5871.1	3870.52	4483.4	5302

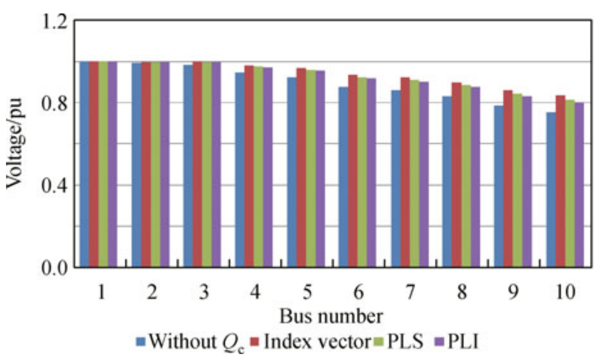

Fig. 14 Voltage profile for the 10-bus system with the ZIP load without and with compensation

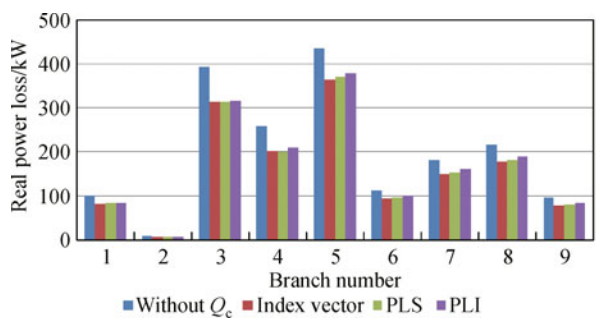

Fig. 15 Real power loss profile for the 10-bus system with the ZIP load without and with compensation

7, 8, 9 and 10. The Total Q_c injected is 5302 kVar, and the remaining 569.1 kVar is supplied by the system itself. After capacitor allocation, the minimum bus voltage is increased to 0.7964, the TPL reduced to 1467.35 kW, the TQL is reduced to 1888.42 kVar, which give a saving of $ 49903.8131. Of these three methods, the PLS and IV methods give better results in terms of loss reduction and overall cost savings.

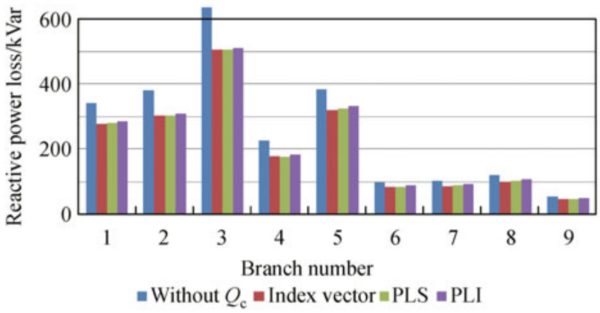

Fig. 16 Reactive power loss profile for the 10-bus system with the ZIP load without and with compensation

4.2 Results for the 34-bus test system

For the 34-bus system, the results obtained with the ZIP load model for the total power loss, running cost, fixed cost, total cost, and reduction in the total power loss, and the savings are given in Table 3 where the results without compensation and with compensation, and the results obtained with three different methods are also presented. The reduction in the total power loss, the real power loss and the reactive power loss with the IV method is lower compared to those of the other methods. The running cost obtained with the IV method is observed minimum. However, the total cost is obtained minimum with the PLI based method. The total savings obtained with the PLS based method are higher than that of the other methods. The reactive compensation obtained at the corresponding buses with the three methods is listed in Table 4. The respective positions obtained for capacitor placement are given in Table 4.

The voltage profile, the real power loss and the reactive power loss without and with compensation are shown in Figs. 17–19. With compensation, the loss reduction with all methods and the voltage profile obtained is better. For the 34-bus system without capacitor placement, the TPL is 504.67 kW, the TQL is 148.67 kVar, the minimum bus voltage is 0.9137, and the total reactive power load is 4030.2 kVar. By the PLI approach, the capacitors are placed at buses 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 17, 18, 19, 20, 21, 22, 23, 24 and 25. The Total Q_c injected is 2017.5 kVar, and the remaining 2012.7 kVar is supplied by the system itself. After capacitor allocation, the minimum bus voltage is increased to 0.922, the TPL is reduced to 382.0424 kW, the TQL is reduced to 112.0125 kVar, which give a saving of $ 18263.0710. By the PLS approach, the capacitors are placed at buses 8, 9, 11, 17, 18, 19, 20, 21, 22, 23, 24, 25 and 26. The Total Q_c injected is 2545.8 kVar, and the remaining 1484.4 kVar is supplied by the system itself. After capacitor allocation, the minimum bus voltage is increased to 0.9276, the TPL is reduced to 365.8427 kW, the TQL is reduced to 107.4560 kVar, which give a saving of $ 20371.8857. By the IV approach, the capacitors are placed at buses 12, 21, 22, 23, 24, 25, 26, 27 and 28. The Total Q_c injected is 2219.3 kVar, and the remaining 1810.9 kVar is supplied by the system itself. After capacitor allocation, the minimum bus voltage is increased to 0.9285, the TPL is reduced to 384.5327 kW, the TQL is reduced to 111.7754 kVar, which give a saving of $ 17610.6482. Of these three methods, the PLS gives best results compared to the other methods.

4.3 Results for the 69-bus test system

For the 69-bus system, the results obtained with the ZIP load model for the total power loss, running cost, fixed cost, total cost, and reduction in total power loss, and the savings are given in Table 5 where the results without compensation and with compensation, and the results obtained with the three different methods are also given. The reduction in the total power loss, the real power loss and the reactive power loss with the IV method are lower compared to those of the other methods. The running cost obtained with the IV method is observed minimum. However, the total cost is obtained minimum with the PLI based method. The total savings obtained with the PLS based method are higher than that of the other methods.

Table 3 Results for the 34-bus system

	Without compensation	With compensation		
		PLI	PLS	IV
Min. voltage/pu	0.9137	0.9222	0.9276	0.9285
TPL/kW	504.6794	382.0424	365.8427	384.5327
TQL/kVar	148.3767	112.0125	107.4560	111.7754
Running cost/$	84786.1430	64183.1232	61461.5736	64601.4936
Fixed cost/$	0	2339.9449	2952.6799	2573.9974
Total cost/$	84786.1430	66523.0681	64414.2535	67175.491
Reduction in TPLk	0	122.637	138.8367	120.1467
Reduction in TQL/kVar	0	36.3642	40.9207	36.6013
Net savings/$	0	18263.0710	20371.8857	17610.6482

Table 4 Capacitors placement and total kVar with the three methods

Bus	Without compensation Q load	With Q_c		
		PLI	PLS	IV
1	0	0	0	0
2	199.8636	106.1830	0	0
3	0	106.1830	0	0
4	199.8636	106.1830	0	0
5	199.8636	106.1830	0	0
6	0	106.1830	0	0
7	0	106.1830	0	0
8	199.8636	106.1830	198.7700	0
9	199.8636	106.1830	197.8300	0
10	0	106.1830	0	0
11	199.8636	106.1830	197.3300	0
12	117.8143	0	0	255.2800
13	63.1148	0	0	0
14	63.1148	0	0	0
15	63.1148	0	0	0
16	10.5191	0	0	0
17	199.8636	106.1830	199.5500	0
18	199.8636	106.1830	198.4200	0
19	199.8636	106.1830	197.1800	0
20	199.8636	106.1830	196.1800	0
21	199.8636	106.1830	195.3200	254.8500
22	199.8636	106.1830	194.3200	256.1700
23	199.8636	106.1830	193.5100	257.7200
24	199.8636	106.1830	192.7500	259.0100
25	199.8636	106.1830	192.3800	260.2600
26	199.8636	0	192.2500	260.8600
27	119.2169	0	0	261.0900
28	67.3225	0	0	154.1000
29	67.3225	0	0	0
30	67.3225	0	0	0
31	48.3880	0	0	0
32	48.3880	0	0	0
33	48.3880	0	0	0
34	48.3880	0	0	0
Total Q/kVar	4030.2	2017.5	2545.8	2219.3

The reactive compensation obtained at the corresponding buses with the three methods is given in Table 6. The total kVar obtained with the PLS method is minimum compared to that of the other two methods. The respective positions obtained for capacitor placement are given in Table 6.

The voltage profile, the real power loss and the reactive power loss without and with compensation are shown in Figs. 20–22. With compensation, the loss reduction with all methods and voltage profile obtained is better. For the 69-bus system without capacitor placement, the TPL is 479.3812 kW, the TQL is 216.6932 kVar, the minimum bus voltage is 0.8671 and the total reactive power load is 3777.9 kVar. By the PLI approach, the capacitors are placed at buses 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 48, 52, 53, 54, 55, 56, 57, 58, 59 and 60. The Total Q_c injected is 2708 kVar, and the remaining 1069.9 kVar is supplied by the

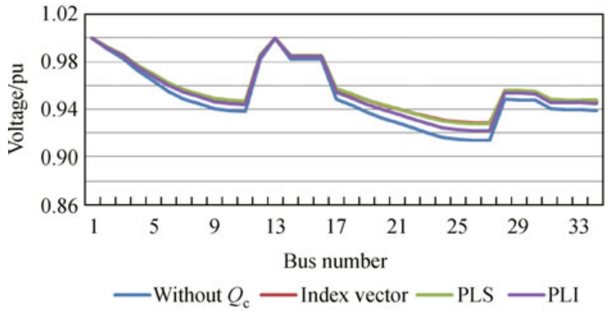

Fig. 17 Voltage profile for the 34-bus system with the ZIP load without and with compensation

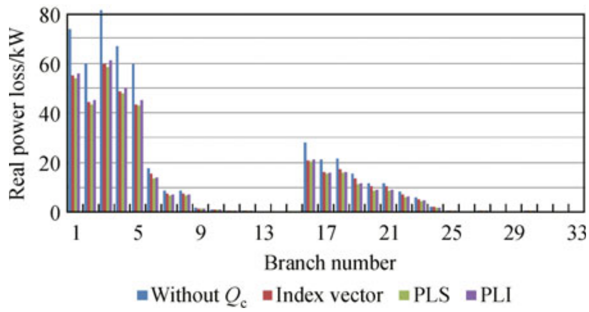

Fig. 18 Real power loss profile for the 34-bus system with the ZIP load without and with compensation

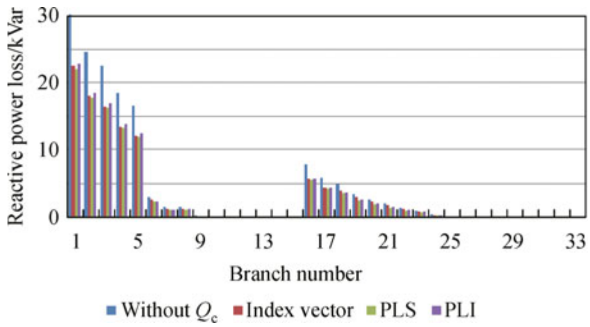

Fig. 19 Reactive power loss profile for the 34-bus system with the ZIP load without and with compensation

system itself. After capacitor allocation, the minimum bus voltage is increased to 0.888, the TPL is reduced to 350.4831 kW, the TQL is reduced to 158.8266 kVar, which give a saving of $ 18514.0774. By the PLS approach, the capacitors are placed at buses 1, 12, 16, 17, 21, 24, 54, 59, 61, 64, 65 and 68. The Total Q_c injected is 2077.7 kVar, and the remaining 1700.2 kVar is supplied by the system itself. After capacitor allocation, the minimum bus voltage is increased to 0.9073, the TPL is reduced to 304.4692 kW, the TQL is reduced to 141.0962 kVar, which give a saving of $ 26975.4497. By the IV approach, the capacitors are placed at buses 11, 18, 20, 21, 22, 24, 26, 27, 49, 50, 52, 62, 64, 65, 66, 67, 68 and 69. The Total Q_c injected is 3115.4 kVar, and the remaining 662.5 kVar is supplied by the system itself. After capacitor allocation, the minimum bus voltage is increased to 0.8956, the TPL reduced to 323.1405 kW, the TQL is reduced to 152.6056 kVar, which give a saving of $ 22635.1219. Of these three methods, the PLS gives the best results. With the PLS based method, the net kVar requirements for voltage and loss reduction are less, and, therefore, the capital investment is lower compared to that of the other methods and overall cost is lower for capacitor installation. The system data for the 10 and 34-bus systems are given in Ref. [3], and for the 69-bus system and 85-bus system are given in Refs. [4,7].

4.4 Results for the 85-bus test system

For the 85-bus system, the results obtained with the ZIP load model for the total power loss, running cost, fixed cost, total cost, and the reduction in total power loss, and the savings are given in Table 6, where the results without compensation and with compensation, and the results obtained with the three different methods are also given. The reduction in the total power loss, the real power loss and the reactive power loss with the IV method are lower compared to those of the other methods. The running cost obtained with the PLS method is observed minimum. However, the total cost is obtained minimum with the PLS based method. The total savings obtained with the PLS based method are higher than that of the other methods.

Table 5 Results for the 69-bus system

	Without compensation	With compensation		
		PLI	PLS	IV
Min. voltage	0.8671	0.8880	0.9073	0.8956
TPL/kW	479.3812	350.4831	304.4692	323.1405
TQL/kVar	216.6932	158.8266	141.0962	152.6056
Reduction in TPL/kW	0	128.8981	174.912	156.2407
Reduction in TQL/kVar	0	57.8666	75.597	64.0876
Running cost/$	80536.0416	58881.1608	51150.8256	54287.604
Fixed cost/$	0	3140.8033	2409.7663	3613.3156
Total cost/$	80536.0416	62021.9641	53560.5919	57900.9196
Net savings/$	0	18514.0774	26975.4497	22635.12191

Table 6 Capacitors placement and total kVar with the three methods

	Without compensation Q load	With compensation		
		PLI	PLS	IV
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	135.4000	0	0
5	0	135.4000	0	0
6	0.0031	135.4000	0	0
7	0.0421	135.4000	0	0
8	0.0757	135.4000	0	0
9	0.0309	135.4000	0	0
10	0.0266	0	0	0
11	0.1459	135.4000	0.1472	262.8700
12	0.1459	135.4000	0.1465	0
13	0.0070	135.4000	0	0
14	0.0070	135.4000	0	0
15	0	0	0	0
16	0.0421	0	0.0417	0
17	0.0491	0	0.0486	0
18	0.0491	0	0	88.6800
19	0	0	0	0
20	0.0008	0	0	1.5300
21	0.1136	0	0.1122	211.3220
22	0.0049	0	0	8.9900
23	0	0	0	0
24	0.0281	0	0.0277	51.6300
25	0	0	0	0
26	0.0140	0	0	25.7700
27	0.0140	0	0	25.7700
28	0.0261	0	0	0
29	0.0261	0	0	0
30	0	0	0	0
31	0	0	0	0
32	0	0	0	0
33	0.0140	0	0	0
34	0.0196	0	0	0
35	0.0056	0	0	0
36	0.0260	0	0	0
37	0.0260	0	0	0
38	0	0	0	0
39	0.0238	0	0	0
40	0.0238	0	0	0
41	0.0014	0	0	0
42	0	0	0	0
43	0.0060	0	0	0

(*Continued*)

	Without compensation Q load	With compensation		
		PLI	PLS	IV
44	0	0	0	0
45	0.0369	0	0	0
46	0.0369	0	0	0
47	0	0	0	0
48	0.0791	135.4000	0	0
49	0.3850	0	0	707.6400
50	0.3850	0	0	708.3900
51	0.0397	0	0	0
52	0.0038	135.4000	0	6.8000
53	0.0049	135.4000	0	0
54	0.0266	135.4000	0.0269	0
55	0.0241	135.4000	0	0
56	0	135.4000	0	0
57	0	135.4000	0	0
58	0	135.4000	0	0
59	0.1010	135.4000	0.0946	198.0800
60	0	135.4000	0	0
61	1.2455	0	1.1427	0
62	0.0323	0	0	64.4000
63	0	0	0	0
64	0.2272	0	0.2076	467.5200
65	0.0589	0	0.0538	118.6400
66	0.0182	0	0	32.9300
67	0.0182	0	0	32.9300
68	0.0281	0	0.0282	50.7700
69	0.0281	0	0	50.7700
Total Q/MVar	3.7779	2.708	2.0777	3.1154

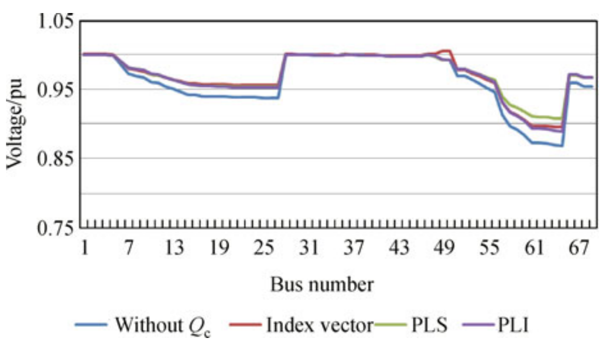

Fig. 20 Voltage profile for the 69-bus system with the ZIP load without and with compensation

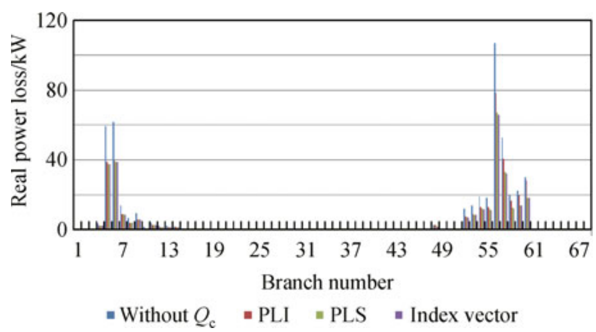

Fig. 21 Real power loss profile for the 69-bus system with the ZIP load without and with compensation

The reactive compensation obtained at the corresponding buses with the three methods is given in Table 7. The total kVar obtained with the PLS method is minimum compared to that of the other two methods. The respective positions

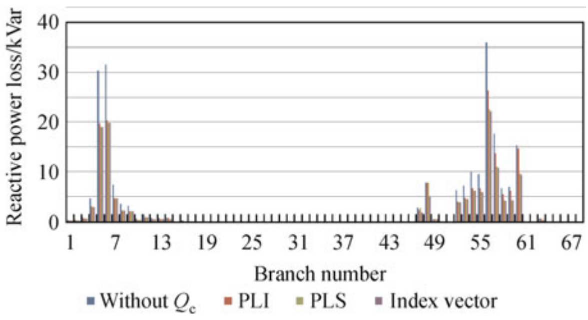

Fig. 22 Reactive power loss profile for the 69-bus system with the ZIP load without and with compensation

obtained for capacitor placement are given in Table 8.

The voltage profile, the real power loss and the reactive power loss without and with compensation are shown in Figs. 23–25. With compensation, the loss reduction with all methods and voltage profile obtained is better. For 85-bus system without capacitor placement the TPL is 563.86 kW, the TQL is 353.23 kVar, the minimum bus voltage is 0.82582 and the total reactive power load is 3401.4 kVar. By the PLI approach, the Total Q_c injected is 2513.6 kVar, and the remaining 887.8 kVar is supplied by the system itself. After capacitor allocation, the minimum bus voltage is increased to 0.86419, the TPL is reduced to 327.48 kW, the TQL is reduced to 201.43 kVar, which give a saving of $ 36797.3014. By the PLS approach, the Total Q_c injected

Table 7 Results for the 85-bus system

	Without compensation	With compensation		
		PLI	PLS	IV
Min. voltage	0.82582	0.86419	0.89135	0.90052
TPL/kW	563.86	327.48	226.08	305.68
TQL/kVar	353.23	201.43	141.5	185.67
Reduction in TPL/kW	0	236.38	337.78	258.18
Reduction in TQL/kVar	0	151.8	211.73	167.56
Running cost/$	94728.5393	55015.9043	37980.9503	51354.9623
Fixed cost/$	0	2915.3336	2650.6002	2759.5634
Total cost/$	94728.5393	57931.2379	40631.5506	54114.5257
Net savings/$	0	36797.3014	54096.9887	40614.0136

Table 8 Capacitors placement and total kVar with the three methods

	Without compensation Q load	With compensation		
		PLI	PLS	IV
1	0	0	0	0
2	0	157.1	0	0
3	0	157.1	0	0
4	79.587	157.1	0	0
5	0	157.1	0	0
6	49.78	157.1	49.793	104.04
7	0	157.1	0	0
8	48.886	0	45.693	0
9	0	0	0	0
10	0	0	0	0
11	77.353	0	71.405	0
12	0	0	0	0
13	0	0	0	0
14	48.664	0	44.669	0
15	48.661	0	44.654	0
16	50.427	0	0	0
17	159.65	0	0	0
18	79.372	0	0	0

(*Continued*)

	Without compensation Q load	With compensation		
		PLI	PLS	IV
19	79.316	0	0	0
20	49.957	0	0	0
21	49.943	0	0	0
22	49.929	0	0	0
23	79.312	0	0	0
24	49.601	157.1	48.974	0
25	48.777	157.1	45.19	0
26	77.291	157.1	71.118	0
27	0	157.1	0	0
28	77.038	157.1	69.95	0
29	0	157.1	0	0
30	48.358	0	43.252	0
31	48.318	0	43.07	0
32	0	0	0	0
33	19.157	157.1	0	0
34	0	0	0	0
35	0	0	0	0
36	48.142	0	0	115.47
37	77.283	0	71.081	0
38	77.096	0	70.216	0
39	76.879	0	69.215	0
40	48.282	0	42.902	114.49
41	0	0	0	0
42	48.263	0	42.813	114.65
43	48.261	0	42.805	114.66
44	48.159	0	42.332	115.3
45	48.14	0	42.244	115.46
46	48.13	0	42.195	115.56
47	19.099	0	0	45.696
48	0	0	0	0
49	0	0	0	0
50	49.454	0	43.203	119.13
51	76.326	0	66.647	184.56
52	0	0	0	0
53	48.042	0	41.788	116.17
54	76.247	0	66.28	185.12
55	76.262	0	66.35	185.01
56	19.086	0	16.684	45.784
57	77.482	157.1	71.998	0
58	0	0	0	0
59	77.302	157.1	71.169	0
60	0	0	0	0
61	0	0	0	0

(*Continued*)

	Without compensation Q load	With compensation		
		PLI	PLS	IV
62	77.164	0	70.53	0
63	19.293	157.1	0	0
64	0	0	0	0
65	0	0	0	0
66	77.046	0	69.985	0
67	0	0	0	0
68	0	0	0	0
69	76.842	0	69.044	181.13
70	0	0	0	0
71	48.396	0	43.431	0
72	76.99	0	69.728	0
73	0	0	0	0
74	76.858	0	69.117	181.07
75	48.416	0	43.52	0
76	76.815	0	68.916	181.34
77	19.262	0	0	44.655
78	77.424	0	71.729	0
79	48.502	0	43.918	0
80	77.237	0	70.867	0
81	0	0	0	0
82	77.22	0	0	0
83	48.636	0	44.541	0
84	19.299	0	17.669	0
85	48.663	0	44.662	0
Total Q/kVar	3401.4	2513.6	2285.3	2379.3

is 2285.3 kVar, and the remaining 1116.1 kVar is supplied by the system itself. After capacitor allocation, the minimum bus voltage is increased to 0.89135, the TPL is reduced to 226.08 kW, the TQL is reduced to 141.5 kVar, which give a saving of $ 54096.9887. By the IV approach, the Total Q_c injected is 2379.3 kVar, and the remaining 1022.1 kVar is supplied by the system itself. After capacitor allocation, the minimum bus voltage is increased to 0.90052, the TPL is reduced to 305.68 kW, the TQL is reduced to 185.67 kVar, which give a saving of $ 40614.0136. Of these three methods, the PLS gives the best results. With PLS based method, the net kVar requirements for voltage and loss reduction are less, and, therefore, the capital investment is lower compared to that of the other methods and the overall cost is lower for capacitor installation. The system data for the 10- and 34-bus systems are given in Ref. [3], for the 69-bus system are given in Ref. [7], and for the 85-bus system are given in Ref. [4].

5 Conclusions

In this paper, an optimal location and size for capacitors have been obtained using existing methods of power loss index, IV method, and proposed PLS-based method. The sizes of capacitors and kVar supplied to the system, voltage profile, and power losses are compared for all the methods. It is interesting to find that the locations obtained are not the same and the sizes obtained are also different in all the methods. After placing the capacitors, the reactive power supplied by the system decreases. For the 34-, 69-, and 85-bus test system, the PLS approach gives the best results in terms of loss and overall cost savings. For the10-bus test system, the results obtained with the IV and the PLS are comparable and better than those obtained with the PLI-based method. This is due to the fact that for bigger systems, the reactive power consumptions are higher and cannot be ignored for optimal placement of capacitors. Thus, both of the indices obtained from the real power

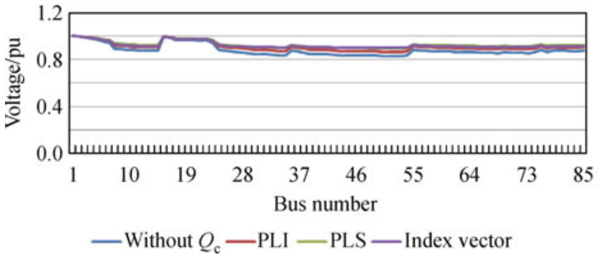

Fig. 23 Voltage profile for the 85-bus system with the ZIP load without and with compensation

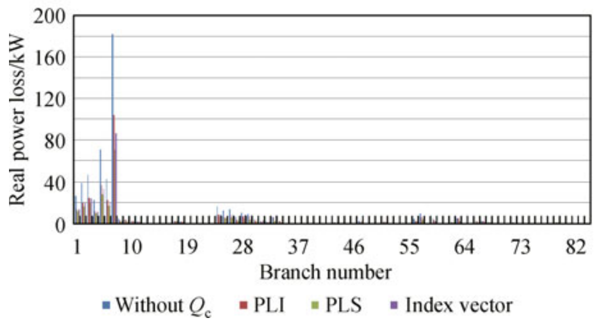

Fig. 24 Real power loss profile for the 85-bus system with the ZIP load without and with compensation

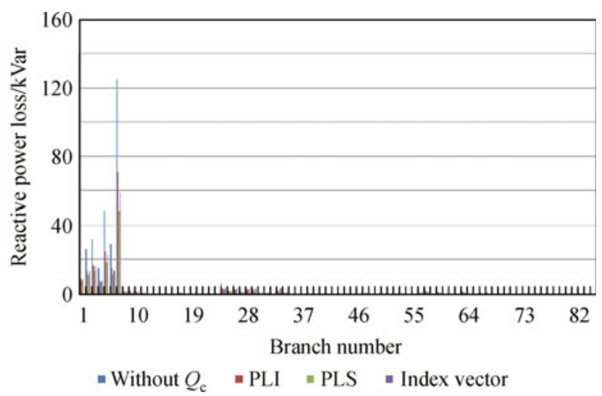

Fig. 25 Reactive power loss profile for the 85-bus system with the ZIP load without and with compensation

loss and the reactive power loss should be considered for the optimal location of the capacitors, especially in the system with realistic loads for overall losses reduction and net cost savings, and lower capital cost investment for capacitor.

Acknowledgements This work was supported by the project SR/S3/EECE/0035/2012 of the Department of Science and Technology, GOI, New Delhi. Thanks also goes to DST, GOI, New Delhi for sponsoring the project.

Notations

n	Number of buses
n_b	Number of branches
se	Sending end nodes
re	Receiving end nodes
$I_L(i)$	Load currents at each bus
Z	Impedance for all branches
V_n	Voltage at receiving end node
V_s	Voltage at sending end node
ΔV	Voltage drops in each branch
TPL	Total real power loss
TQL	Total reactive power loss
$P_{loss}(k)$	Real power loss for branch k
$Q_{loss}(k)$	Reactive power loss for branch k
r	Annual rate growth
m	Plan period up to which feeder can take the load
PLI	Power loss index
IV	Index vector
PLS	Combined power loss sensitivity
Norm (i)	Normalized voltage at any bus i

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