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# Toward a nonlinear control of an AC-DC-PWM converter dedicated to induction heating

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**Abstract** In this paper, a nonlinear control strategy applied to an AC-DC-pulse width modulation (PWM) converter is developed and simulated. First a nonlinear system modeling is derived with state variables of the input current and the output voltage by using power balance of the input and output. The system is linearized and decoupled, and then a state feedback law is obtained. For robust control of parameter perturbation, integrators are added to the exact feedback control law. The simulation is provided to verify the validity of the control algorithm.

**Keywords** DC-DC converter, AC-DC-pulse width modulation (PWM) converter, induction heating, nonlinear control

## 1 Introduction

In the past few years, remarkable progress has been made in development of high power density DC/DC converters using resonant link schemes which utilize high speed devices such as fast recovery transistors and gate turn-off thyristors (GTOs). These new converters not only have high power density but also possess very low switching losses since switching of the devices are made at zero-voltage instants and thus enable the whole system to operate at very high frequencies compared to the conventional DC link transistorized converters. Although these resonant link converters are intended to operate at high power density, almost all the systems presented in the

past require self-commutated transistors and have some difficulty performing conversion at very high power levels because of the relatively low voltage and current margins that self-commutated devices such as transistors typically have.

These new converters with high frequencies and high power densities are necessary in induction heating application which leads to the increase of the switching frequency. However, increasing the switching frequency leads to significant switching losses, which will deteriorate the overall system efficiency [1].

In recent years, three-phase voltage-source pulse width modulation (PWM) converters have been increasingly used for applications such as uninterruptible power supply (UPS) systems, electric traction and induction heating. The attractive features of these converters are constant DC bus voltage, low harmonic distortion of the utility currents, bidirectional power flow and controllable power factor [2–4].

In Ref. [5], the design and performance of voltage and current PI controllers have been analyzed, which are composed of an inner current control loop and outer voltage control loop in a cascade structure. In Refs. [6,7], the PWM converter has been modelled in a nonlinear system. In Ref. [8], the nonlinear systems have been analyzed and the controllers have been designed using small signal analysis which is valid only around operating points, on which linear control is based.

In this paper, a nonlinear control technique for a PWM three-phase voltage-source AC-DC converter associated with a power circuit of the passively clamped two switch quasi resonant DC link converter (QRDCL) [9] is investigated. First, it is shown that it is feasible to apply nonlinear multiple input-multiple output (MIMO) feedback linearization technique to such a system that is operated in high frequency regimes. Next, the effect of parameter perturbation on the control performance is investigated. Finally, an integral control is introduced to the exact feedback control law in order to eliminate the steady state error [10,11].

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## 2 Modelling of the proposed converter

The power circuit of the PWM three-phase voltage-source AC-DC converter associated with the power circuit of the QRDCL converter feeding induction heating is introduced in Fig. 1. This circuit could be modelled and an equivalent circuit is derived Fig. 2 [4]. It is assumed that a resistive load RL is connected to the output terminal. A voltage equation is derived from Fig. 2 as

$$e_s = Ri_s + L \frac{di_s}{dt} + v_F. \quad (1)$$

Considering the terms of the three-phases, if Eq. (1) is transformed into a synchronous reference frame, then

$$L \frac{di_{sd}}{dt} - \omega Li_{sq} + Ri_{sd} = e_{sd} - v_{Fd}, \quad (2)$$

$$L \frac{di_{sq}}{dt} + \omega Li_{sd} + Ri_{sq} = e_{sq} - v_{Fq}, \quad (3)$$

where  $e_{sd}$  and  $e_{sq}$  are the  $d$ - $q$  axis source voltage,  $i_{sd}$  and  $i_{sq}$  the  $d$ - $q$  axis source current, and  $v_{sd}$  and  $v_{sq}$  are the  $d$ - $q$  axis converter input voltage.  $R$  and  $L$  represent the line resistance and the input inductance, respectively.  $\omega$  is the angular frequency of the source voltage.

Supposing a sinusoidal power supply, the main resulting current will be also sinusoidal with a shift angle compared to the voltage.

So Eqs. (4) and (5) are obtained.

$$V_s(t) = V_s \sqrt{2} \sin \omega t, \quad (4)$$

$$i_{\text{load}}(t) = i_{\text{load}} \sqrt{2} \sin(\omega t - \varphi), \quad (5)$$

where  $\varphi$  is the phase between the voltage and the load current.

For fast voltage control, the input power should supply instantaneously the sum of load power and charging rate of the capacitor energy. The average rate of change of energy associated with AC link and DC link is given by

$$P = \frac{3}{2}(e_{sd}i_{sd} + e_{sq}i_{sq}) = V_C i_{dC}, \quad (6)$$

where the input resistance loss and switching device loss are neglected. With regard to the output, Eq. (7) can be obtained.

$$i_{dC} = C \frac{dV_C}{dt} + \frac{V_C}{R_{ch}}. \quad (7)$$

From Eqs. (6) and (7), Eq. (8) can be obtained.

$$\frac{3}{2}(e_{sd}i_{sd} + e_{sq}i_{sq}) = CV_C \frac{dV_C}{dt} + \frac{V_C^2}{R_{ch}}. \quad (8)$$

Equation (8) leads to nonlinear system with regard to  $V_C$ . The combination of Eqs. (2), (3) and (8) describes a nonlinear model as

$$\begin{bmatrix} \dot{i}_{sd} \\ \dot{i}_{sq} \\ \dot{V}_C \end{bmatrix} = \begin{bmatrix} -\frac{R}{L}i_{sd} + \omega i_{sq} \\ -\frac{R}{L}i_{sq} - \omega i_{sd} \\ \frac{3}{2CV_C}(e_{sd}i_{sd} + e_{sq}i_{sq}) - \frac{V_C}{R_{ch}C} \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_{sd} - v_{sd} \\ e_{sq} - v_{sq} \end{bmatrix}. \quad (9)$$

The system is of the third order which has a two control inputs.

## 3 Feedback linearization

A feedback linearizing control of nonlinear system multiple input multiple output corresponding to the model in Ref. [9] is designed. Before applying nonlinear control to the associated PWM AC-DC-QRDCL converter, a feedback linearization theory is first described. For a suitable choice of a nonlinear state feedback control law, a nonlinear control system may be transformed into a linear input-output [5].

An MIMO system is considered as

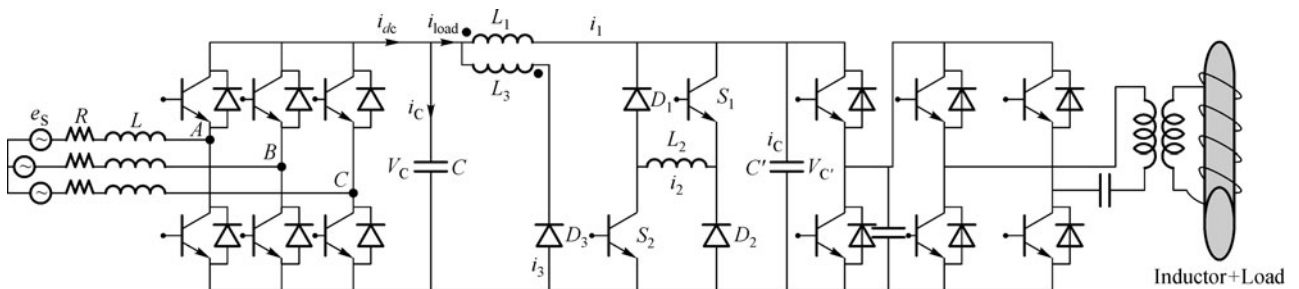


Fig. 1 Power circuit of the PWM converter associated with the power circuit of the QRDCL converter

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}(t), \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}),\end{aligned}\quad (10)$$

where  $\mathbf{x}$  is the state vector,  $\mathbf{u}$  is the control input vector,  $\mathbf{y}$  is the output vector,  $\mathbf{f}$  and  $\mathbf{g}$  are smooth vector fields, and  $\mathbf{h}$  is the smooth scalar function. The approach to obtaining the input-output linearization of the MIMO systems is to differentiate each output  $y_j$  of the system until the inputs appear [6].

## 4 Nonlinear model

The nonlinear system model can be determined as follows. Let's consider  $x_1, x_2$  and  $x_3$  to be the three states variables,

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad (11)$$

where  $x_1 = i_{sd}$ ,  $x_2 = i_{sq}$  and

$$x_3 = V_C.$$

And

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad (12)$$

where  $u_1 = e_{sd} - v_{sd}$  and  $u_2 = e_{sq} - v_{sq}$  are inputs.

Equation (9) is transformed to

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}. \quad (13)$$

This expression represents a bilinear system, where

$$\mathbf{f} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} g_1 & 0 \\ 0 & g_2 \\ 0 & 0 \end{bmatrix}. \quad (14)$$

Therefore,

$$\begin{cases} f_1(\mathbf{x}) = -\frac{R}{L}i_{sd} + \omega i_{sq}, \\ f_2(\mathbf{x}) = -\frac{R}{L}i_{sq} - \omega i_{sd}, \\ f_3(\mathbf{x}) = \frac{3}{2CV_C}(e_{sd}i_{sd} + e_{sq}i_{sq}) - \frac{V_C}{R_{ch}C}, \end{cases} \quad (15)$$

and

$$g_1 = g_2 = \frac{1}{L}. \quad (16)$$

## 5 Principle of nonlinear control

The first stage of any nonlinear control involves defining

which output variables of the system should be controlled. In this case, since there are two controls variables  $e_{sd} - v_{sd}$  and  $e_{sq} - v_{sq}$ , there will thus be the possibility of regulating two outputs  $y_1$  and  $y_2$  independently (decoupling phenomenon).

It is then the problem of writing a matrix differential equation coupled to an  $n$ th derivative of the output  $y_1$  and an  $m$ th derivative of the output  $y_2$  associated to the controls  $u_1$  and  $u_2$  respectively.

In the majority of the cases, based on the sufficient number of derivations of the output variables based on the theory of Lie [7], Eq. (17) can be obtained.

$$\begin{bmatrix} \frac{d^n y_1}{dt^n} \\ \frac{d^m y_2}{dt^m} \end{bmatrix} = \mathbf{B}_0 + \mathbf{A}_0 \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad (17)$$

where  $\mathbf{B}_0$  is a vector of size 2, and  $\mathbf{A}_0$  is the matrix of decoupling (square matrix of order 2 which can be reversed to whatever the equilibrium point of operation of the system). It should be noticed that the components of  $\mathbf{A}_0$  and  $\mathbf{B}_0$  can depend on the state variable  $x$ . According to Eq. (17), the control mechanism can be adopted as

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \mathbf{A}_0^{-1} \left( -\mathbf{B}_0 + \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \right). \quad (18)$$

At this level, it is arranged to decouple the system because each external input  $r_i \in \{1, 2\}$  acts only on one output variable of  $y_i$ . Therefore, the dynamic behaviour of the outputs can be obtained according to

$$\frac{d^n y_1}{dt^n} = r_1 \quad (19)$$

and

$$\frac{d^m y_2}{dt^m} = r_2. \quad (20)$$

In a static regime, it is forced that  $y_1 = y_{1ref}$  and  $y_2 = y_{2ref}$  and a dynamics behaviour on the error should be controlled by an equation of

$$\frac{d^n e_1}{dt^n} = K_{n-1} \frac{d^{n-1} e_1}{dt^{n-1}} + K_{n-2} \frac{d^{n-2} e_1}{dt^{n-2}} + \dots + K_0 e_1, \quad (21)$$

with  $e_1 = y_{1ref} - y_1$  and

$$\begin{aligned} \frac{d^m e_2}{dt^m} &= K'_{m-1} \frac{d^{m-1} e_2}{dt^{m-1}} + K'_{m-2} \frac{d^{m-2} e_2}{dt^{m-2}} + \dots \\ &+ K'_0 e_2, \end{aligned} \quad (22)$$

and with  $e_2 = y_{2ref} - y_2$ ,  $e_1$  and  $e_2$  represent the errors compared to the commands on the output  $y_1$  and  $y_2$  respectively. It is suitable to choose a law of control of form

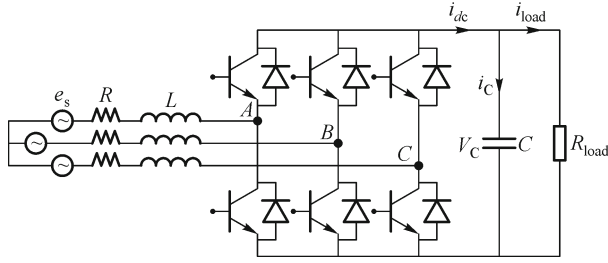


Fig. 2 Power circuit of the PWM converter

$$\frac{d^n y_1}{dt^n} = r_1 = K_{n-1} \frac{d^{n-1} y_1}{dt^{n-1}} + K_{n-2} \frac{d^{n-2} y_1}{dt^{n-2}} + \dots + K_0 (y_{1\text{ref}} - y_1), \quad (23)$$

$$\frac{d^m y_2}{dt^m} = r_2 = K'_{m-1} \frac{d^{m-1} y_2}{dt^{m-1}} + K'_{m-2} \frac{d^{m-2} y_2}{dt^{m-2}} + \dots + K'_0 (y_{2\text{ref}} - y_2). \quad (24)$$

The behaviour of the system is thus linearized. The coefficients  $K_0, K_1, \dots, K_{n-1}, K'_0, K'_1, \dots, K'_{m-1}$ , are selected according to the static and dynamic desired behaviour. An independent differential equation is obtained, thus, regulating the outputs. The static regime can be regulated while placing the new controls.

## 6 Control of voltage by using nonlinear control

Since there are two control inputs for input-output decoupling, thus, one input is used to control the line current and the other is used to control the capacitor voltage. Therefore, the outputs can be chosen as

$$y_1 = \alpha i_{sd} + \beta i_{sq}, \\ y_2 = V_C.$$

Differentiating the output variable  $y_1$  until a control input appears as in the form of Eq. (17).

$$\frac{dy_1}{dt} = \alpha f_1 + \beta f_2 + [\alpha g_1 \quad \beta g_2] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \quad (25)$$

The functions  $g_1$  and  $g_2$  are always different from zero with a proper choice of non-zero coefficients  $\alpha$  or  $\beta$ . The differentiation of the second output  $y_2$  leads to

$$\frac{dy_2}{dt} = f_3. \quad (26)$$

It is seen that the derivative of the second output utilizes neither  $u_1$  nor  $u_2$ . It should, therefore, be differentiated. With the end, the following differential equation can, then, be written:

$$\begin{bmatrix} \frac{dy_1}{dt} \\ \frac{d^2 y_2}{dt^2} \end{bmatrix} = \mathbf{B}_0 + \mathbf{A}_0 \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (27)$$

with

$$\mathbf{B}_0 = \begin{bmatrix} \alpha f_1 + \beta f_2 \\ \frac{3}{2CV_C} (e_{sd} f_1 + e_{sq} f_2) \\ - \left[ \frac{3}{2CV_C^2} (e_{sd} i_{sd} + e_{sq} i_{sq}) + \frac{1}{R_{ch} C} \right] f_3 \end{bmatrix}, \quad (28)$$

$$\mathbf{A}_0 = \begin{bmatrix} \alpha g_1 & \beta g_2 \\ \frac{3e_{sd} g_1}{2CV_C} & \frac{3e_{sq} g_2}{2CV_C} \end{bmatrix}. \quad (29)$$

If the determinant of the matrix of decoupling  $\mathbf{A}_0$  is non-zero, the law of control can, then, be applied:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \mathbf{A}_0^{-1} \left( -\mathbf{B}_0 + \begin{bmatrix} K_{11} (y_{1\text{ref}} - y_1) \\ -K_{21} \frac{dy_2}{dt} + K_{22} (y_{2\text{ref}} - y_2) \end{bmatrix} \right), \quad (30)$$

where

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \mathbf{A}_0^{-1} \left( -\mathbf{B}_0 + \begin{bmatrix} K_{11} (y_{1\text{ref}} - y_1) \\ -K_{21} f_3 + K_{22} (y_{2\text{ref}} - y_2) \end{bmatrix} \right). \quad (31)$$

This makes it possible to impose the static regime  $y_1 = y_{1\text{ref}}$  and  $y_2 = y_{2\text{ref}}$  and a dynamic regime on the error which is governed by the equation of

$$\frac{de_1}{dt} + K_{11} e_1 = 0$$

with

$$e_1 = y_{1\text{ref}} - y_1, \quad (32)$$

and

$$\frac{d^2 e_2}{dt^2} + K_{21} \frac{de_2}{dt} + K_{22} e_2 = 0$$

with

$$e_2 = y_{2\text{ref}} - y_2. \quad (33)$$

It is now easy to place the poles of the closed loop system, by adjusting the coefficients  $K_{11}$ , and  $K_{22}$ . If it is considered that the control is selected, then

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \dot{y}_{1\text{ref}} - K_{11} e_1 \\ \dot{y}_{2\text{ref}} - K_{21} \dot{e}_2 - K_{22} e_2 \end{bmatrix}. \quad (34)$$

After some mathematical manipulation, the control law can be obtained. The proposed nonlinear control block diagram of the proposed converter is illustrated in Fig. 3.

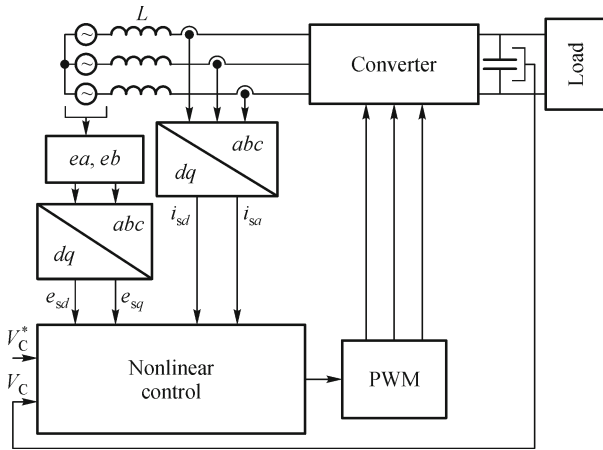


Fig. 3 Block diagram of the adaptive scheme used

## 7 Simulation results

The performances of the closed loops system have been evaluated at the nominal power operation using MATLAB simulation tool. Only linear load with uncertainty was tested.

The simulation of the steady state operation was performed at nominal power  $P_n = 10$  kW.

Figures 4 and 5 indicate that the voltage  $V_C$  and the current  $i_{sd}$  are controlled, in the case of a step change of the load, with the desired values. Figure 6 depicts the line currents which are sinusoidal and reach the expected nominal value. Figure 7 demonstrates that the source power factor is controlled as unity as usual. For the control of this factor, the constants  $\alpha$  and  $\beta$  in Eq. (25) can be also adjusted. It is observed that the supply current is close to sinusoidal and remains in phase with the supply voltage. Therefore, unity power factor is maintained at the output of supply system.

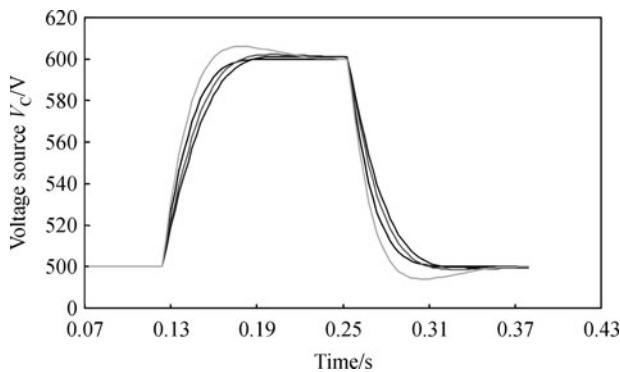


Fig. 4 Transient responses of capacitor voltage  $V_C$  for step change of load

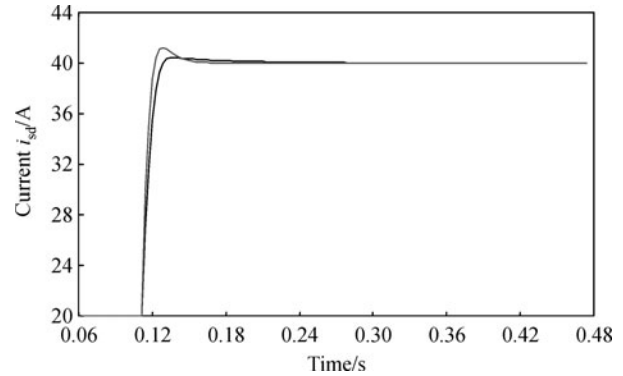


Fig. 5 Transient responses of current  $i_{sd}$  for step change of load

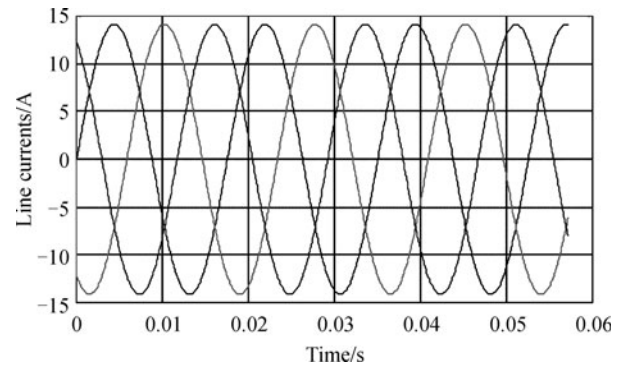


Fig. 6 Line currents

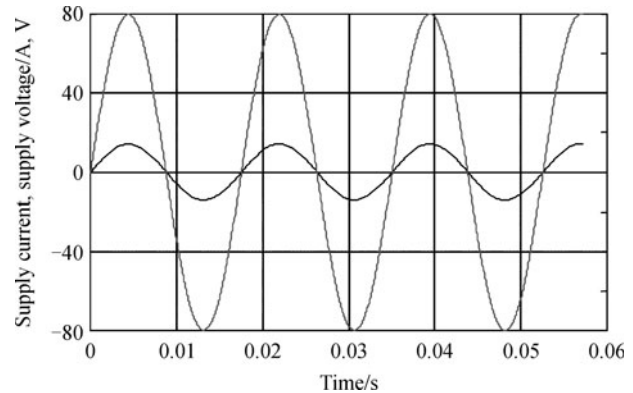


Fig. 7 Supply current and supply voltage (Unity power factor)

## 8 Conclusions

In this paper, a nonlinear MIMO state space mathematical model of the PWM three-phase voltage-source AC-DC converter associated with a power circuit of the passively clamped two switch quasi resonant DC link converter by feedback linearization is developed. Moreover, a nonlinear control is designed in order to diminish the influence of the unknown load uncertainties and disturbances and to reduce the number of sensors in the system. The proposed control

scheme gives satisfactory simulation results with nominal load.

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